

$$\begin{array}{c}
\frac{p \supset q}{p} \\
\hline
/ \therefore q \\
p \supset q \\
\sim q \\
\hline
/ \therefore \sim p \\
p \supset q \\
q \supset r \\
\hline
/ \therefore p \subset r \\
\frac{p \cdot q}{p \cdot q} \\
\hline
/ \therefore p \\
\frac{p \cdot q}{p \cdot q} \\
\hline
/ \therefore q \\
p \\
q \\
\hline
/ \therefore p \cdot q \\
p \supset q \\
r \supset s \\
p \vee r \\
\hline
/ \therefore q \vee s \\
p \vee q \\
\sim p \\
\hline
/ \therefore q \\
p \vee q \\
\sim q \\
\hline
/ \therefore p \\
\frac{p}{p} \\
\hline
/ \therefore p \vee q \\
\frac{q}{q} \\
\hline
/ \therefore p \vee q
\end{array}$$

Double Negation

$$p :: \sim \sim p$$

Duplication

$$p :: (p \vee p)$$

$$p :: (p \cdot p)$$

Commutation

$$(p \vee q) :: (q \vee p)$$

$$(p \cdot q) :: (q \cdot p)$$

Association

$$((p \vee q) \vee r) :: (p \vee (q \vee r))$$

$$((p \cdot q) \cdot r) :: (p \cdot (q \cdot r))$$

Contraposition

$$(p \supset q) :: (\sim q \supset \sim p)$$

DeMorgan's

$$\sim (p \vee q) :: (\sim p \cdot \sim q)$$

$$\sim (p \cdot q) :: (\sim p \vee \sim q)$$

Biconditional Exchange

$$(p \equiv q) :: ((p \supset q) \cdot (q \supset p))$$

Conditional Exchange

$$(p \supset q) :: (\sim p \vee q)$$

Distribution

$$(p \cdot (q \vee r)) :: ((p \cdot q) \vee (p \cdot r))$$

$$(p \vee (q \cdot r)) :: ((p \vee q) \cdot (p \vee r))$$

Exportation

$$((p \cdot q) \supset r) :: (p \supset (q \supset r))$$

Double Negation

$$p :: \sim \sim p$$

Duplication

$$p :: (p \vee p)$$

$$p :: (p \cdot p)$$

Commutation

$$(p \vee q) :: (q \vee p)$$

$$(p \cdot q) :: (q \cdot p)$$

Association

$$((p \vee q) \vee r) :: (p \vee (q \vee r))$$

$$((p \cdot q) \cdot r) :: (p \cdot (q \cdot r))$$

Contraposition

$$(p \supset q) :: (\sim q \supset \sim p)$$

DeMorgan's

$$\sim (p \vee q) :: (\sim p \cdot \sim q)$$

$$\sim (p \cdot q) :: (\sim p \vee \sim q)$$

Biconditional Exchange

$$(p \equiv q) :: ((p \supset q) \cdot (q \supset p))$$

Conditional Exchange

$$(p \supset q) :: (\sim p \vee q)$$

Distribution

$$(p \cdot (q \vee r)) :: ((p \cdot q) \vee (p \cdot r))$$

$$(p \vee (q \cdot r)) :: ((p \vee q) \cdot (p \vee r))$$

Exportation

$$((p \cdot q) \supset r) :: (p \supset (q \supset r))$$