Lab 7

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#Rcpp

We will get some experience with speeding up R code using C++ via the `Rcpp` package.

First, clear the workspace and load the `Rcpp` package.

```
pacman::p_load(Rcpp)
```

Create a variable `n` to be 10 and a vaiable `Nvec` to be 100 initially. Create a random vector via `rnorm` `Nvec` times and load it into a `Nvec` x `n` dimensional matrix.

```
n = 10
Nvec = 100
X = matrix(data = rnorm(Nvec*n), nrow = Nvec)
head(X)
```

```
##            [,1]       [,2]        [,3]       [,4]       [,5]       [,6]
## [1,]  0.4260257  1.2973338  1.03448725 -0.8507652 -0.7347756 -1.3315476
## [2,] -2.2130863  0.4661540  0.95936180 -0.2144699  0.1785635  0.3738078
## [3,]  0.4440189  0.4538967  0.05999678 -0.5592860 -0.2128973 -1.1990671
## [4,]  1.0721152 -0.5254999  2.27247787 -2.4164931 -0.4996500 -0.7334628
## [5,] -0.1415513  0.9640003 -1.36869581  0.2966006  1.3242731 -1.4212075
## [6,]  0.6900924 -0.9908338 -0.41251624  1.1219566 -0.5570731  1.2885302
##            [,7]       [,8]        [,9]      [,10]
## [1,] -0.1545601 -1.2464010 -0.48478071 -1.7792610
## [2,] -0.0589727 -0.9611672 -0.83796395  0.7440142
## [3,] -0.4402798 -1.7604354  1.65256816  1.9876773
## [4,] -1.1360924  0.6150288  0.06968694 -0.1718873
## [5,]  1.1805591  0.5307111 -1.58765972  0.5176311
## [6,]  1.5708393 -1.7860063 -0.76632504  0.1162629
```

Write a function `all_angles` that measures the angle between each of the pairs of vectors. You should measure the vector on a scale of 0 to 180 degrees with negative angles coerced to be positive.

```
angle = function(u,v){
  (acos(sum(u*v)/sqrt(sum(u^2)*sum(v^2))))*(180/pi)
}
all_angles = function(X){
  A = matrix(NA, nrow = nrow(X), ncol = nrow(X))
  for(i in 1:(nrow(X)-1)){
    for(j in (i+1):nrow(X)){
      A[i,j] = angle(X[i,],X[j,])
    }
  }
  A
}
```

```
all_angles(X)
```

```
##       [,1]     [,2]     [,3]     [,4]      [,5]      [,6]     [,7]      [,8]
##  [1,]   NA 87.03322 85.06439 65.96173  92.87329  97.97792 78.60834  80.45545
##  [2,]   NA       NA 85.85670 95.07828  88.65895  89.35291 79.37790 114.77863
##  [3,]   NA       NA       NA 81.88730  97.39674  93.96639 97.76103  96.84914
##  [4,]   NA       NA       NA       NA 114.35762 117.82550 63.01259 107.93154
##  [5,]   NA       NA       NA       NA        NA  92.78557 78.06001  88.18199
##  [6,]   NA       NA       NA       NA        NA        NA 89.33137  70.38268
##  [7,]   NA       NA       NA       NA        NA        NA       NA 106.71094
##  [8,]   NA       NA       NA       NA        NA        NA       NA        NA
##  [9,]   NA       NA       NA       NA        NA        NA       NA        NA
## [10,]   NA       NA       NA       NA        NA        NA       NA        NA
## [11,]   NA       NA       NA       NA        NA        NA       NA        NA
## [12,]   NA       NA       NA       NA        NA        NA       NA        NA
## [13,]   NA       NA       NA       NA        NA        NA       NA        NA
## [14,]   NA       NA       NA       NA        NA        NA       NA        NA
## [15,]   NA       NA       NA       NA        NA        NA       NA        NA
## [16,]   NA       NA       NA       NA        NA        NA       NA        NA
## [17,]   NA       NA       NA       NA        NA        NA       NA        NA
## [18,]   NA       NA       NA       NA        NA        NA       NA        NA
## [19,]   NA       NA       NA       NA        NA        NA       NA        NA
## [20,]   NA       NA       NA       NA        NA        NA       NA        NA
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##  [99,]   NA       NA       NA       NA        NA        NA       NA        NA
## [100,]   NA       NA       NA       NA        NA        NA       NA        NA
##              [,9]     [,10]     [,11]     [,12]     [,13]     [,14]     [,15]
##   [1,] 132.06125  70.84126  80.04000  43.28448  98.63177  81.69807  69.97621
##   [2,] 100.23764 108.23255  63.13898  99.63234  96.88888  70.76450 107.57016
##   [3,]  87.93427 120.86547 112.27872  83.79255  49.56119  94.25867  83.98631
##   [4,]  93.75242  86.48565  94.30545  69.66669 115.37372  91.14354  80.78943
##   [5,]  95.76812  64.00527  85.09790 107.02883  84.10345  93.01699  70.40911
##   [6,]  87.76705  98.32406 100.84880 108.43590  78.93962  75.60750 119.72984
##   [7,]  86.89192  84.91214 109.64309 103.70157 120.68271  88.50980  85.06256
##   [8,]  74.90009  63.77872  90.71757  86.62391  87.24726  79.76454  87.95130
##   [9,]        NA  90.30943 108.07550 130.44677  92.51948 105.26899  92.83186
##  [10,]        NA        NA  68.12499  85.47604 110.99941 103.59615  61.61401
##  [11,]        NA        NA        NA  74.44729  97.64708  71.39824  88.76731
##  [12,]        NA        NA        NA        NA  79.33158  79.38302  64.51129
##  [13,]        NA        NA        NA        NA        NA  94.31175  74.54071
##  [14,]        NA        NA        NA        NA        NA        NA 118.65271
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##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,16]     [,17]     [,18]     [,19]     [,20]     [,21]     [,22]
##   [1,]  75.10570 107.14889  79.62090  97.04677  62.39233  91.20019  64.09498
##   [2,]  78.88351  82.80395  97.14230  93.98541  99.62136  80.90830  66.54523
##   [3,] 106.08354  53.08866 110.28772  86.62378  89.87988  81.98477  73.08674
##   [4,]  85.64167 123.47508  84.14713  62.64603  90.36197  96.59773  72.35778
##   [5,] 101.64207  75.44388  91.29055  61.99213 106.25372 103.85532 103.83335
##   [6,]  95.28175  86.63592 101.59379 116.37181  70.38779 111.03544 129.76488
##   [7,]  62.83915 119.74813  88.35835  62.39157  86.67044 134.92574  96.12849
##   [8,]  92.60211  77.67553  63.78997 113.73263  79.57347  79.53127 105.55392
##   [9,]  92.64293  71.69903  82.49047  85.66184 119.20962  81.75325 113.21526
##  [10,]  98.12268 100.30556  66.29981  78.93687 102.66087  82.26664 105.84587
##  [11,] 102.05595  90.10760  89.55687  93.99496 114.47216  52.99653  80.47258
##  [12,]  79.05041 100.79736  99.10129 104.85599  74.12550  74.36067  64.22387
##  [13,] 109.18486  38.89036 134.05162 105.18947  98.98341  73.44279  98.47943
##  [14,]  88.18466  99.63873  90.36378  91.30950  82.00435  97.95065  83.97733
##  [15,]  80.73166  77.96326 103.96753  82.56940 109.64313  75.84394  88.75812
##  [16,]        NA 110.02267  84.46648 111.30001  70.27487 103.47611  73.89537
##  [17,]        NA        NA 108.13005 101.91837 113.25699  58.62948  91.87315
##  [18,]        NA        NA        NA  87.51445  70.90123  90.25949  72.16693
##  [19,]        NA        NA        NA        NA 113.44613 110.06170  91.59723
##  [20,]        NA        NA        NA        NA        NA 118.20213  75.68663
##  [21,]        NA        NA        NA        NA        NA        NA  72.81121
##  [22,]        NA        NA        NA        NA        NA        NA        NA
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##  [69,]        NA        NA        NA        NA        NA        NA        NA
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##  [71,]        NA        NA        NA        NA        NA        NA        NA
##  [72,]        NA        NA        NA        NA        NA        NA        NA
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##  [76,]        NA        NA        NA        NA        NA        NA        NA
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##  [86,]        NA        NA        NA        NA        NA        NA        NA
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##            [,23]     [,24]     [,25]     [,26]     [,27]     [,28]     [,29]
##   [1,]  75.22200  61.17734  60.85289  94.28625 101.18644 117.89758  70.28904
##   [2,]  63.94025  91.96119  55.09228  42.82717  60.59513  77.13405  42.49355
##   [3,]  81.00270  93.37918  68.70192  84.42751  87.28788  76.85452  91.43348
##   [4,]  78.52592  95.13486  84.58126  97.05563 138.36633  89.43544  89.84066
##   [5,]  77.31480 111.40959  90.96619  93.66843  67.05618  83.57134 113.54499
##   [6,]  98.29728  78.35187  82.79074  90.40360  72.05258  87.52377  90.33776
##   [7,]  60.51010 100.42198  86.59324  73.99285 102.25476  62.99014  98.81170
##   [8,] 106.72334  64.59062 115.34546  93.34610 106.31035 117.40177 106.34970
##   [9,] 109.98487 104.28999 134.17783  76.16023 109.31939  72.59347 116.80703
##  [10,] 108.19640  89.97839 106.80139 110.67393 109.00597 121.67911 106.28778
##  [11,]  99.02525  81.07593  71.02607  90.63854  79.24175 126.86486  54.59748
##  [12,]  94.33294  48.09333  63.29454 113.28583  97.48872 134.28319  66.39385
##  [13,] 109.40368  76.71418  69.65710 106.97014  60.37471 100.22874  90.47725
##  [14,]  61.37900  73.98630  72.58632  75.91376  86.50546  96.12504  72.98802
##  [15,] 109.79649  75.23203  87.42173 112.27416  93.94002 115.85377  98.49136
##  [16,]  77.10801  58.50504  90.63321  63.46231  87.27746  85.92369  70.54088
##  [17,] 105.44506  89.40939  85.65577  81.62782  63.95647  91.80093  91.91834
##  [18,]  74.54634 100.69817 125.62931  71.66357 119.97312  85.49812 101.87416
##  [19,]  66.39542 138.53616  96.32077  98.16958 108.31452  66.21294 120.89262
##  [20,]  68.71307  73.22380  84.61827  86.99648  91.17494  84.07565  85.48328
##  [21,] 121.39373  75.57632  88.19314  89.15121  88.56927 124.48030  65.92778
##  [22,]  58.84165  85.64316  74.15079  63.93625  91.87025  86.87804  60.66255
##  [23,]        NA 106.15912  75.68254  57.06377  86.58093  55.48195  85.85176
##  [24,]        NA        NA  74.00398  92.55630  85.56241 132.32386  60.97916
##  [25,]        NA        NA        NA  88.55494  60.71353 100.30514  47.63945
##  [26,]        NA        NA        NA        NA  79.40645  60.77147  67.79732
##  [27,]        NA        NA        NA        NA        NA  84.58250  66.84317
##  [28,]        NA        NA        NA        NA        NA        NA 107.37001
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##   [2,]  97.91830 102.98109  74.78874 103.93558  92.20330 107.53259  91.77137
##   [3,]  77.12073  86.48877  66.67524  68.28304  84.62997  97.50626  82.70013
##   [4,]  78.07482  77.14452  84.46950 106.82985  86.65321  81.70701  57.37613
##   [5,] 104.92290  83.81699  65.19326  80.74012  94.24172  78.03642  80.03026
##   [6,] 124.44251  76.31881  97.80766  82.11093  72.28258 104.92162 104.65621
##   [7,] 119.14276  70.57360  62.17008  90.55140 104.96860  69.65381  71.95699
##   [8,]  85.43609 118.01875 109.08751  80.27048  67.78291 105.59117 100.22764
##   [9,] 104.99747  93.56520 122.75111  68.29038  99.65724  90.93974  73.49699
##  [10,]  82.00041 106.09053  96.70292  93.52787  62.77273  93.77656  71.01380
##  [11,]  81.95442 110.20862 104.01023 112.36628  72.40831 104.07895  87.01995
##  [12,]  63.31080 102.81917  74.59493  90.78038  85.24000  76.25107 110.49699
##  [13,]  91.70719  85.97411  78.62726  47.67719  90.75016  81.88845 108.73028
##  [14,] 110.46296  83.55749  91.29675 112.69567  96.36387  89.23525  98.34684
##  [15,]  78.01765 103.81817  67.06765  53.18944  94.86101  59.91218  90.81160
##  [16,]  96.14274 109.54425  72.40665  80.67749 122.82658  67.09197 125.24851
##  [17,]  85.16222 108.30506  86.95116  52.87711  85.10137 101.43739  96.37659
##  [18,]  62.58735 116.79390 104.74926 124.15836  75.76970 118.92152  78.78244
##  [19,]  98.50556  56.24822  78.78858 103.91414  98.78831  74.27444  37.53286
##  [20,]  76.55931  91.61726  74.05226 106.39112  83.35165 101.19065 116.47841
##  [21,]  57.94033 130.92322 112.86779  83.78520  73.81230 109.73667  93.28530
##  [22,]  45.89281 117.24217  69.62735 112.13041  96.77448 101.01720  97.03726
##  [23,]  93.01546  80.65722  55.80846 118.25520 105.70118  90.80591  83.58127
##  [24,]  88.30907 117.55460  86.45373  70.63545  90.24775  79.74587 136.10928
##  [25,]  93.12045  83.96024  54.04379  90.67392  79.95270  91.51334  98.75351
##  [26,]  93.33565 115.07238  81.70433  98.40222 102.79930 109.15561  95.78299
##  [27,] 104.72068  91.23600  69.63200  75.96095 101.15975  88.23247 122.06504
##  [28,] 105.48925  65.11707  78.21984  96.01589 113.91814  90.27371  75.79668
##  [29,]  81.02026 114.00856  83.36784 102.08060  85.80127 103.60928 114.20859
##  [30,]        NA 123.62228  89.73060 105.16010  69.88753 114.99222  89.71448
##  [31,]        NA        NA  86.18253  94.33531 102.41920  66.36904  69.82168
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##  [33,]        NA        NA        NA        NA 103.22030  64.46513 107.20204
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##            [,37]     [,38]     [,39]     [,40]     [,41]     [,42]     [,43]
##   [1,]  80.97844  85.34459  78.01481  77.68322  77.31242 109.42695  97.95383
##   [2,] 129.72505  47.83797 103.21176  84.11380  96.02816  99.15746  98.02187
##   [3,]  92.69796  84.15030  95.83154 118.22699 111.57804  89.82944 122.96881
##   [4,]  88.25175  94.97571  75.51934 104.79698  86.24818  95.89259  94.38034
##   [5,] 110.28711  87.46071  83.66481 100.09921  94.04379 102.48218  54.07608
##   [6,]  67.21597 108.65192  82.73873  75.99896  88.45163  71.50285 104.13218
##   [7,] 102.22473 105.47086  76.95605  99.88840  78.12445 103.03408  78.86086
##   [8,]  75.75632 100.96352  92.71517  94.91512 114.59719 100.13935 103.61184
##   [9,]  98.34471 105.15924 117.59630 125.70369 124.10602  96.95730  96.30346
##  [10,]  83.80443  99.50588  95.24517  93.07757  93.48617 117.37364  58.24933
##  [11,] 113.42381  58.99414  95.23011  82.21233 102.11824  89.13769  83.93051
##  [12,]  81.59058  92.29772  75.15788  81.16347  80.90786  79.02427 110.43911
##  [13,]  88.29367 100.32538  97.65970 108.16965 109.04766  65.16846 117.84614
##  [14,] 113.44108  71.70712  50.38073  87.89490 105.44295  63.27434 112.73275
##  [15,]  95.55062 112.36836 106.02729 114.18973  95.52143 103.41652  85.26870
##  [16,]  98.58264  99.29485  97.20474  75.76279  70.03799  99.00951 100.54278
##  [17,] 105.01841  81.14346 121.43561 117.38738 128.91953  93.45606 109.37405
##  [18,]  82.07022  68.91014  91.63361  73.42584  85.05225 131.62743  68.98235
##  [19,] 109.47154  86.48228  67.08684 116.72817  94.75282  94.57704  58.86911
##  [20,]  53.10565  91.55430  68.92895  46.41631  51.18946  93.74688  95.56238
##  [21,] 101.99625  67.97315 127.52564 100.50700 119.76591  97.88443 105.71753
##  [22,] 106.44006  45.43609  93.66352  78.22818  82.37653 111.32987  97.60912
##  [23,] 112.95354  59.85110  60.24899  82.57384  79.43233 100.31655  84.48791
##  [24,]  84.14079 104.98592  93.04775  81.51878  92.82424  77.62972 130.18903
##  [25,] 100.45501  75.89931  80.86215  82.97348  83.85229  76.83794 109.18200
##  [26,] 123.00237  55.73061 108.74272  88.98632 100.89432 115.11848 101.63880
##  [27,] 104.07679  76.84713  99.40146  72.34648  80.79449  77.57475  88.54928
##  [28,]  98.47538  82.18212  86.82610  92.58849  79.60138  96.96749  77.60653
##  [29,] 105.18259  61.31745 102.98310  67.51332  83.98494  86.99091 112.61519
##  [30,]  74.73162  64.36628 106.40263  76.13131  81.93999 119.86416  86.20018
##  [31,]  81.44039 112.31028  50.79941  96.69021  76.61681  52.95436  79.21841
##  [32,] 100.87969  87.39819  80.12321  90.49475  74.07518 102.61099  89.11237
##  [33,]  91.20332 127.53068 115.04469 121.36269 108.85393  83.68005 111.38244
##  [34,]  63.14477  82.26628  98.42946  82.98173  96.97162 110.00301  88.48451
##  [35,] 103.08183 124.52254  71.08867 106.43006  82.47359  62.05963  89.17751
##  [36,]  96.67477  84.03304  85.36068 119.14462 107.01011 106.26796  68.90768
##  [37,]        NA 117.70788  85.99217  62.97536  59.63222  88.30471  85.65001
##  [38,]        NA        NA  94.93168  76.57665  96.24041 108.90345  84.87710
##  [39,]        NA        NA        NA  82.31614  76.81341  59.25417  86.44644
##  [40,]        NA        NA        NA        NA  37.13396  91.51560  73.00187
##  [41,]        NA        NA        NA        NA        NA  91.00158  62.38668
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##  [71,]        NA        NA        NA        NA        NA        NA        NA
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##  [73,]        NA        NA        NA        NA        NA        NA        NA
##  [74,]        NA        NA        NA        NA        NA        NA        NA
##  [75,]        NA        NA        NA        NA        NA        NA        NA
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##  [78,]        NA        NA        NA        NA        NA        NA        NA
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## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,44]     [,45]     [,46]     [,47]     [,48]     [,49]     [,50]
##   [1,]  85.29509  82.48971  90.12522  33.72969  94.32991 109.71545  77.11860
##   [2,]  84.80848 102.04074  83.58924  96.00329 101.94008 110.66055 111.36373
##   [3,]  88.64597  70.45000  81.32613  89.52357  96.04603  63.15953  91.89560
##   [4,] 107.19013  92.05161 100.04417  65.81687  74.82910 103.50257  93.68700
##   [5,]  75.91875  55.49000  57.73075 113.01221  91.11427  73.77606  89.27965
##   [6,]  62.40954 100.29673  75.84275  99.59023 126.99918  93.89020  98.47227
##   [7,] 100.41692  92.69902  70.69607  82.91110  82.13280 124.17037 117.76264
##   [8,]  63.54099  86.11750  78.79730  68.70561 125.81041  77.23829  43.56924
##   [9,]  82.30329 107.97207  71.69943 108.70431  98.03356  80.12956  88.92016
##  [10,]  60.99244  89.03611  67.36532  77.73471  96.13578  91.63345  70.19774
##  [11,]  66.48182  94.44248  88.95873  90.58195 107.97464  92.01429  71.63752
##  [12,]  96.98388  80.91991 106.63533  45.59100  86.77145  97.24596  62.66560
##  [13,]  73.64134  71.90134  73.83025 103.28512 103.91262  57.61693  80.58866
##  [14,]  91.68937  68.39156  99.93002  85.72183 124.81970  88.02585  64.44143
##  [15,]  81.46362  83.24243  62.46060  72.04225  76.48244  91.67034  78.32637
##  [16,] 116.15027 114.52792  96.25660  56.58783  74.87930 145.34900  99.28180
##  [17,]  64.14882  79.43859  63.11604 107.94489 109.86514  55.14346  79.29964
##  [18,] 100.06644  92.84493 109.84844  70.91832  83.51134  93.12147  76.40957
##  [19,] 101.15424  57.53013  81.50369 113.36878  73.70589  72.02407  98.13092
##  [20,] 111.33104  89.35271 120.65824  59.34057  81.91390 106.32319  94.84990
##  [21,]  69.49797 103.59626  89.56858  84.64891 100.75676  75.68800  64.80525
##  [22,] 120.88419  84.26289 120.98380  63.37880  68.59264  97.08276  86.92190
##  [23,] 119.55929  62.66039 103.30157  86.04292  82.65441  96.53602 100.79438
##  [24,]  81.28876 104.47732  89.43939  45.74844 110.66694 113.23869  63.40907
##  [25,]  79.18734  83.14119  82.63008  82.25033 102.30776 100.66407 102.52852
##  [26,]  99.18928 104.13512  89.36409  86.39338  97.14063 109.83979 103.59735
##  [27,]  81.80692  87.30743  80.20166 113.58637  93.98321  91.16154 108.19228
##  [28,] 118.58534  86.15289  95.37861 117.95185  69.12274  89.79499 131.30600
##  [29,]  87.06409 115.11017 101.16926  71.70785  98.61003 120.40060  96.78963
##  [30,] 100.87939  89.86586 117.65426  63.57490  67.68374  80.14897  76.85762
##  [31,] 101.75746  70.65877  91.47552 122.41446  81.82473  78.60676 111.76686
##  [32,]  99.23870  66.61328  75.23879  75.40564  78.27276 101.65730 106.82781
##  [33,]  73.32079  93.65321  51.17333  91.51580  97.00244  85.65726  88.69034
##  [34,]  45.72706  95.36498  78.89463  79.13717 116.25411  79.32029  82.48341
##  [35,] 113.26272  76.92820  84.08288  91.76868  70.44207  98.99986  88.26757
##  [36,]  78.61533  78.69717  75.40430 107.63297  90.52197  71.37365  97.14100
##  [37,]  88.06868 102.57014 106.33422  77.04581  80.65003  89.75291  95.79955
##  [38,]  96.29021  81.44198 109.47283  94.30652  91.12395  81.98635  88.14238
##  [39,] 108.87862  50.46952 108.54513  89.84142  93.06974  81.34317  77.66280
##  [40,] 105.24198 105.81722 127.26167  79.47426  73.80809 113.75699 104.47238
##  [41,] 120.14998 100.89902 119.88377  81.66617  48.17776 119.19834 122.34655
##  [42,]  96.28793  79.78266 101.25602 109.24815 100.49897  79.01819  82.60773
##  [43,]  96.22964  83.24038  91.81816 111.35317  59.15069  89.19738 107.68689
##  [44,]        NA  96.27642  43.36824  95.45458 138.15810  79.40051  78.39437
##  [45,]        NA        NA  88.16428 100.17296  93.98539  48.45817  66.49652
##  [46,]        NA        NA        NA  99.20253 118.33337  87.92744  92.26631
##  [47,]        NA        NA        NA        NA  91.24119 118.57916  70.26644
##  [48,]        NA        NA        NA        NA        NA 100.77507 116.70579
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## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,51]     [,52]     [,53]     [,54]     [,55]     [,56]     [,57]
##   [1,]  95.82785 101.24231  88.07074  78.35417  75.57602  94.90172  79.31213
##   [2,]  86.09716  66.93117  79.21725 107.12831  79.97593  76.26014  72.54321
##   [3,]  51.50567  93.97755  93.14401  79.38773  95.53333  62.24727  84.49032
##   [4,]  91.62162  88.05090  96.54114  87.53148  91.74333 107.40934 112.26163
##   [5,]  70.98083  81.04248  71.63959 106.11794  78.12078  82.27548  95.86101
##   [6,]  93.43182  89.99621  89.06898  93.03139  87.07091  84.81662  49.64741
##   [7,]  70.77988  73.75650  83.72977 105.74191  48.83328 129.71204 111.07626
##   [8,] 111.28840 153.61346 120.70689  40.67952 104.49184  94.15165  66.30349
##   [9,]  83.68766 110.04693 129.94598  62.95775 101.26239 105.76483 100.31269
##  [10,] 110.10308 107.05922 109.67952  70.96279  97.69279 100.98070  87.37420
##  [11,] 119.41621  85.50861  83.93103  94.57111 114.47423  65.95201  72.31048
##  [12,]  95.76318  98.60111  75.91933  84.56269  88.42408  86.32579  95.17317
##  [13,]  54.63601  88.35442  78.40755  89.24374  96.59260  53.91160  82.13462
##  [14,] 106.27123 100.18296  63.55199 101.60616  97.41183  78.51400  76.08823
##  [15,]  62.59028  93.32436  94.56798  77.53317  73.79645 101.13990 118.24310
##  [16,]  84.35704  90.91988  88.12126  88.51748  36.19001 140.66954 111.13922
##  [17,]  62.13319 101.59095 101.92373  71.11439 103.20999  55.14086  74.97946
##  [18,] 130.03392 124.32375 119.76672  61.50242  99.22546 107.53698  82.47940
##  [19,]  76.44157  73.89479  76.29984 111.78161  95.53103  89.15246 119.00662
##  [20,] 103.21405  98.56937  83.05691  88.85930  70.28432 102.44285  74.64154
##  [21,] 102.90599 107.14047 114.41372  60.77229 123.85178  63.93010  77.59910
##  [22,]  94.41780  93.53890  86.26692  85.57528  85.77910  84.28859  95.93376
##  [23,]  83.81503  82.16225  64.36508 111.39247  69.66178  92.77480  94.88122
##  [24,]  95.11813 111.36972  91.18483  70.33174  75.55992 101.08866  81.05598
##  [25,]  72.52537  60.44669  58.38186 115.18747  79.09894  62.99648  72.17200
##  [26,]  87.88163  94.86653 100.83758  83.10538  73.06117  98.79436  80.99324
##  [27,]  70.29447  59.28239  58.36460 119.94276  72.25506  66.20871  75.45991
##  [28,]  68.47427  69.40720  83.63524 110.94015  72.59822  99.14597 103.08517
##  [29,] 100.55218  74.84487  80.36789  97.78841  83.94980  78.73338  70.34370
##  [30,] 107.18547 108.09774 110.15907  63.48665 107.58862  78.17157  85.57452
##  [31,]  73.71267  56.62944  56.89893 133.59826  89.58270  86.57013 105.72276
##  [32,]  49.58957  71.37583  63.22165 107.95790  46.73812  91.56624  99.02829
##  [33,]  43.03111  94.67731  98.69119  75.73519  70.72907  96.35620 102.21815
##  [34,] 112.79651 104.28729 116.95091  65.81633 120.36728  65.24856  39.34918
##  [35,]  61.19757  75.02591  59.19898 113.59558  60.31461 115.82807 145.60511
##  [36,]  90.67644  86.19968 105.22744  88.89137 115.73556  82.13580  93.61746
##  [37,] 104.95006  96.46237 103.91737  78.71593  95.43887  95.67194  74.79575
##  [38,] 108.30265  88.49210  85.58971  93.74179 107.32553  60.47735  70.97641
##  [39,]  97.77784  85.92602  48.63371 118.69617  90.41643  88.26427  97.46689
##  [40,] 122.28158  76.70748  75.81074 106.41122  79.57397  93.70786  71.35894
##  [41,]  97.91901  59.59112  68.12071 119.55135  61.43663 109.46786  98.85312
##  [42,]  85.65183  74.16312  53.13765 119.46467  98.73637  76.66218  96.80277
##  [43,] 101.24497  69.72234  83.12341 110.88467  87.34058  97.70325 102.78980
##  [44,]  91.95913 100.21766 110.54649  70.64169 109.53243  67.31729  49.76031
##  [45,]  75.48075  96.23333  59.56203 102.78713  98.10657  65.65445  97.91547
##  [46,]  58.21428  90.06976 101.68421  80.85179  79.57087  89.10299  83.61033
##  [47,] 101.93738 118.92446 105.48402  59.23864  73.52664 114.06726  87.85232
##  [48,]  82.15159  65.58105  80.38083 108.52331  70.80832 110.43407 132.15109
##  [49,]  84.56309 105.23488  89.52454  82.77927 133.15370  46.10082  83.43977
##  [50,] 110.57480 141.34128  97.73484  58.42651 117.21609  80.15967  85.67019
##  [51,]        NA  70.14933  76.32069 102.34338  60.31331  89.02663 111.78412
##  [52,]        NA        NA  52.99872 150.80728  71.82571  83.42811 101.46465
##  [53,]        NA        NA        NA 150.56951  73.25526  77.40807 103.64388
##  [54,]        NA        NA        NA        NA 106.28968  97.29974  75.51589
##  [55,]        NA        NA        NA        NA        NA 129.08860 111.40088
##  [56,]        NA        NA        NA        NA        NA        NA  58.24384
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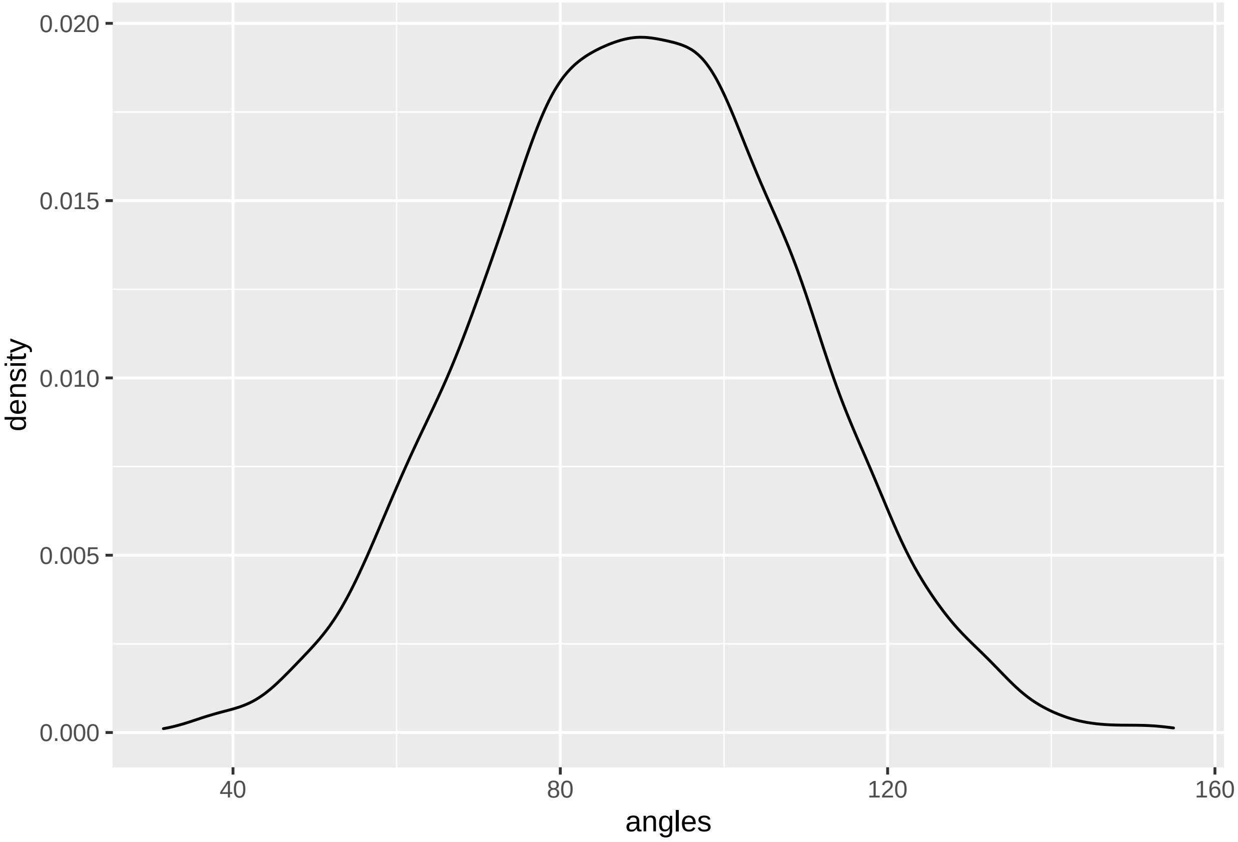
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,58]     [,59]     [,60]     [,61]     [,62]     [,63]     [,64]
##   [1,] 116.11767 108.87620  97.77046  84.31468  91.46507 109.98340  76.27271
##   [2,]  79.73684  52.39516  94.62069  74.40949 115.63178 120.62278  89.14223
##   [3,]  98.80057  98.20472  70.07310  36.12420  85.32042  72.08508  83.96839
##   [4,]  91.80619  81.28940 105.14459  61.82293  73.03462  82.05799  96.77774
##   [5,]  96.65468 107.60191  90.24130  95.37871 101.11044  98.15393  57.83089
##   [6,]  75.47532  94.62291  68.62312 107.23219 104.19755  85.72861  86.32533
##   [7,]  82.60616  70.12882  90.70184  87.80233 117.75671 100.83799  88.98840
##   [8,]  96.52486 113.92602  77.04840 114.30492  87.85182 102.93817  94.32725
##   [9,]  56.82509  75.31654  76.99810  91.01760  85.69863  78.61167 113.18382
##  [10,]  85.27534 117.94408 102.94750 108.93454  69.47256  97.51207  75.17129
##  [11,]  79.12123  91.08818 127.32270  90.56607  70.84990 105.43331  70.65428
##  [12,] 111.16666 117.80465 120.80183  84.13546  71.68946  87.10158  69.80761
##  [13,]  85.70961 123.60079  89.11931  68.96210  75.33045  56.81051  59.15070
##  [14,] 102.28854  76.68530 105.01776  89.70949 108.10855 112.96712  80.74154
##  [15,]  84.32523 128.31529 114.83648  83.76703  69.05876  73.98917  58.50366
##  [16,]  92.29048  69.55292  97.53285 115.85504 127.72301 116.13060 104.62815
##  [17,]  80.42347 106.20510  76.68118  69.76737  83.41707  79.29371  76.17605
##  [18,] 117.49388  74.05079  65.62484 111.87842 102.99870 131.67747 131.84380
##  [19,]  94.35192  84.84632  97.18516  64.58513  82.59013  81.39326  78.92709
##  [20,] 131.71163  89.16074  60.59489 109.59886 117.82776 109.25542 111.05141
##  [21,]  76.75606  98.98231 106.30241  78.32033  56.99678  87.98551  88.58961
##  [22,] 126.01143  70.36479  89.33959  72.27724 102.46538 120.52154 107.48423
##  [23,] 126.44792  59.46553  76.57487  78.39763 134.78872 126.83018  98.45616
##  [24,]  88.47658 108.01782 110.39020 107.01684  91.89615  96.40617  78.45608
##  [25,]  90.86792  95.56089 106.01819  61.36705  92.18277  89.17623  55.94314
##  [26,]  91.04744  38.81510  70.62504  87.99192 136.90167 136.12363 118.70145
##  [27,]  86.59568  90.94055  89.52120  96.18281 110.90247  94.55461  67.59242
##  [28,]  95.41960  50.10242  55.18094  79.92244 124.72078  95.04980 116.53181
##  [29,]  83.66874  72.05497 109.62690  84.98649  97.24870 110.26264  87.66879
##  [30,] 121.85289  98.97602  79.74972  81.09819  72.42714  99.94449 109.17459
##  [31,]  84.51987  90.02672  93.22351  76.81231  84.86356  55.40452  75.37245
##  [32,] 112.66264  98.37493  86.48876  72.46628 115.73862 100.09136  65.65205
##  [33,]  65.46900 117.42420  91.84628  86.00162  84.89181  61.09840  67.70391
##  [34,]  88.69056 111.40881  74.39525  80.63261  64.82502  90.12686  86.51504
##  [35,]  84.25029 100.85951 124.85290  96.22631  91.44497  68.91029  64.93743
##  [36,]  79.71007  84.58617  86.62427  57.77465  67.31280  79.29001  90.20100
##  [37,] 100.05913 113.79149  65.60568 106.50664  74.49146  69.47294 103.82994
##  [38,] 111.69375  61.76747  83.46853  75.44120 102.67335 128.76981 103.57184
##  [39,] 118.49788  94.69618  99.92595  90.35732  96.99346  89.66663  75.89892
##  [40,] 113.57534  81.03647  79.44042 125.19583 108.03546 112.75406 107.79807
##  [41,] 111.01623  85.33328  83.61512 114.09095 104.83466  94.28979 100.88116
##  [42,]  79.27299  97.43444 115.32602  89.03349  78.66134  57.48561  68.82000
##  [43,]  98.20434  89.80196  86.74579 109.02416  89.71431  96.90650  91.20680
##  [44,]  59.15200 117.19378  91.87847  85.37447  68.48212  82.10321  62.40959
##  [45,] 125.84087 111.35145  90.18883  72.01483  91.07010  88.58199  62.96458
##  [46,]  55.82303 109.84834  93.79238  79.30074  86.50665  80.09739  55.67293
##  [47,] 109.92414  99.42532  95.06470  97.09493  94.46780 111.18007  97.47557
##  [48,] 107.50120  80.33535  88.66549  94.32146  91.83083  83.96346 107.84378
##  [49,] 102.27555 113.39415  78.34802  67.82151  65.11050  68.28742  78.80862
##  [50,] 104.67063 116.74258 107.37747  96.58251  72.10615  96.38761  78.81703
##  [51,]  80.48620  99.69421  86.93855  61.44254  98.67789  66.43006  65.17599
##  [52,]  75.56116  77.07208 102.84008  79.19194  93.25215  74.03247  73.61478
##  [53,] 106.54228  91.45322 112.26391  88.08614 103.89496  86.67240  58.98117
##  [54,]  89.35887 102.67075  73.55396  92.49097  77.83563  98.09367 109.78036
##  [55,]  93.91053  81.68150  88.63478 105.88050 132.75069 104.13061  86.07696
##  [56,]  95.53331 104.72151  87.27936  59.07447  68.99488  78.69705  68.67892
##  [57,]  89.37234  94.71469  66.40956  90.53816  91.14978 104.38836  91.19008
##  [58,]        NA  85.31269 108.83963  86.78038  71.62373  65.05901  79.52418
##  [59,]        NA        NA  80.26395  89.50131 123.05994 118.35311 130.20549
##  [60,]        NA        NA        NA  91.07998 113.30169 100.63533 124.11842
##  [61,]        NA        NA        NA        NA  71.63275  70.50394  76.26333
##  [62,]        NA        NA        NA        NA        NA  47.26693  72.75330
##  [63,]        NA        NA        NA        NA        NA        NA  69.51403
##  [64,]        NA        NA        NA        NA        NA        NA        NA
##  [65,]        NA        NA        NA        NA        NA        NA        NA
##  [66,]        NA        NA        NA        NA        NA        NA        NA
##  [67,]        NA        NA        NA        NA        NA        NA        NA
##  [68,]        NA        NA        NA        NA        NA        NA        NA
##  [69,]        NA        NA        NA        NA        NA        NA        NA
##  [70,]        NA        NA        NA        NA        NA        NA        NA
##  [71,]        NA        NA        NA        NA        NA        NA        NA
##  [72,]        NA        NA        NA        NA        NA        NA        NA
##  [73,]        NA        NA        NA        NA        NA        NA        NA
##  [74,]        NA        NA        NA        NA        NA        NA        NA
##  [75,]        NA        NA        NA        NA        NA        NA        NA
##  [76,]        NA        NA        NA        NA        NA        NA        NA
##  [77,]        NA        NA        NA        NA        NA        NA        NA
##  [78,]        NA        NA        NA        NA        NA        NA        NA
##  [79,]        NA        NA        NA        NA        NA        NA        NA
##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
##  [84,]        NA        NA        NA        NA        NA        NA        NA
##  [85,]        NA        NA        NA        NA        NA        NA        NA
##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,65]     [,66]     [,67]     [,68]     [,69]     [,70]     [,71]
##   [1,]  70.35039  64.39936  66.75852  98.13638  72.63811  78.10585  84.48459
##   [2,]  89.52185  72.18255  61.89316  98.23555  88.28858 111.36608 120.33964
##   [3,]  94.78564  92.03731 105.67577 114.73520 104.07252  85.23726 119.86841
##   [4,] 106.80632  91.59444  71.21231  98.54106  96.95732 116.80624  93.91924
##   [5,]  82.54355 119.46634 101.74140 104.45820  58.91692  77.76358  83.88739
##    [6,] 108.11086  81.33319  97.86843 102.38772 100.53194  93.39696  93.90922
##    [7,] 130.49751  94.52643  75.78861 118.92399  61.20533 125.97136  92.25396
##    [8,]  76.25806  65.31375  79.70959  87.62840  99.23572  81.22669  47.23162
##    [9,] 114.28289  96.44220  96.21395 101.46100 108.68020 116.85152  78.96432
##   [10,]  70.54798  99.59259  81.96011  95.02541  77.75212  81.80059  57.08683
##   [11,]  48.94872  80.69769  66.93622  79.55543 106.39543  91.39221  95.64878
##   [12,]  51.26257  66.32142  85.74064  84.19002  92.45147  69.12201  94.04163
##   [13,]  69.70176  96.27079 134.60248 109.52713 106.91050  63.50906 121.22699
##   [14,]  88.82388  63.01063  56.59650  82.17887 107.51087 113.95106  89.98875
##   [15,]  62.20334  98.62760 112.07149 115.94530  72.22918  71.27312  92.58042
##   [16,]  94.43443  52.81938  74.09437  89.50633  58.24796  98.63111  83.11026
##   [17,]  71.07313  92.69960 114.10143 108.90699 104.20822  73.36509 107.30481
##   [18,]  94.36742  77.38547  50.34814  58.03815  75.04218  89.86353  31.49219
##   [19,] 111.31579 135.16068  89.43030 100.06646  82.73710 110.13191  91.51747
##   [20,]  99.27016  66.61982  79.45375  69.13999  67.91692  70.04436  72.12205
##   [21,]  44.46500  76.16582  87.15853  82.70697 119.29560  77.81488  95.66693
##   [22,]  73.24840  67.05928  61.40110  68.59924  77.81353  81.89342  92.10374
##   [23,] 111.02094  83.81458  58.55461  84.42678  64.59568 105.55448  89.30712
##   [24,]  61.01589  37.49550  84.06022  94.56424  95.34336  81.01781  89.30162
##   [25,]  75.90147  80.24712  88.69083 111.21452  92.10087  87.04613 139.09048
##   [26,] 105.50206  60.64802  53.86522  90.15851  79.36577 115.26303  91.11650
##   [27,]  73.53754  91.41179 109.79898  93.62568  75.98414  68.42784 119.67441
##   [28,] 142.97012 109.71077  92.08366  91.03126  72.95468 108.74511  97.38978
##   [29,]  66.68530  52.53432  66.93140  85.37234  97.11960  92.79982 117.66590
##   [30,]  61.07742  82.11624  81.19290  62.98685  86.08199  56.31523  75.14234
##   [31,] 125.25725 129.94090 113.33759  99.42764  96.48839 104.91867 109.66223
##   [32,]  89.79554  90.64198  94.52021 115.75897  49.30617  78.40947 109.73027
##   [33,]  84.50409  91.96774 132.37752 129.78384  88.83414  80.33526 106.50684
##   [34,]  79.38381  89.58270  80.83959  96.82286 106.51537  78.58266  83.81685
##   [35,]  91.40537 100.99282 113.44807 101.98481  75.64933  94.98604  98.52172
##   [36,] 112.92463 125.71556  81.48031 105.29775 102.79190 116.31183  89.88261
##   [37,]  96.58171  95.48432 109.64376  78.08499  89.82355  61.48476  74.34668
##   [38,]  75.13282  79.12903  54.71405  65.38793  91.04420  89.78766  92.64378
##   [39,] 102.09266  96.58635  81.55675  79.36324  89.46386  97.63840  83.51304
##   [40,]  82.27675  76.53801  79.54205  49.60497  68.74698  64.16585  77.83618
##   [41,]  97.36162  96.94512  96.16562  66.79695  52.66987  67.59690  86.26555
##   [42,]  90.54958  94.47183 111.92802  86.83237 119.60879  95.85895 113.51770
##   [43,]  93.45326 130.77267  93.84904  73.58058  56.86124  77.26853  69.53584
##   [44,]  74.47234  90.84078  95.02480 121.66082 110.58644  89.44869  96.51357
##   [45,]  83.31820 109.50799  95.84558  90.79768  84.38786  78.58176  86.26682
##   [46,]  91.60167 100.76097 104.08768 151.29484  87.75288 101.22918 103.31762
##   [47,]  74.45215  44.54208  63.94058  89.49861  78.55300  83.78960  72.05460
##   [48,]  95.70323 112.38880 104.36488  67.99181  57.44518  72.44007  87.27667
##   [49,]  80.68385 118.06243 111.03291  85.37601 114.56853  73.82420  87.78729
##   [50,]  55.51326  69.33475  77.43316  78.54790 110.96429  82.31875  61.84006
##   [51,] 101.63129 107.33673 125.36377 135.03167  76.29383  89.59935 126.02439
##   [52,] 101.33045 119.21211 109.20129  97.71046  77.37522  92.23926 133.45686
##   [53,]  85.49141 101.54987 100.00743  84.39091  77.20210  83.90384 113.69203
##   [54,]  78.52271  63.94525  78.00960  93.52658 103.24622  85.90790  60.48938
##   [55,] 103.52755  78.93870  93.80120 107.48005  39.07882  90.31876  96.19078
##   [56,]  66.91902 102.71731 100.24784  88.32923 118.49775  72.26857 115.20823
##   [57,]  81.79415  72.05486  76.25137  90.45460 107.93836  81.77586  91.21288
##   [58,]  98.63728  97.15397 103.58745 121.34214 114.82738 118.19663 113.87241
##   [59,] 121.71139  77.52302  56.62321  75.00186  88.39999 128.36155  93.88529
##  [60,] 117.92701  92.74702  92.07895  85.50777  78.18985  78.82618  75.23792
##  [61,]  95.93343 103.89420  93.79435 117.41283 112.25483 103.44483 129.57371
##  [62,]  67.71669 111.70984 110.32693  93.20904 126.19578  80.18622  96.92539
##  [63,]  94.98435 124.31889 144.29038 108.98388 115.61170  83.84302 115.93227
##  [64,]  64.91568 104.97379 112.30111 122.59785  91.31108  80.69021 116.29833
##  [65,]        NA  75.95574  92.52498  77.69143  93.30756  53.25078  88.56146
##  [66,]        NA        NA  54.53699  83.02969  94.53127  96.89260  80.98587
##  [67,]        NA        NA        NA  74.86305  89.94647 118.63440  69.31452
##  [68,]        NA        NA        NA        NA  88.30924  71.46637  60.55145
##  [69,]        NA        NA        NA        NA        NA  71.07459  75.06556
##  [70,]        NA        NA        NA        NA        NA        NA  81.97771
##  [71,]        NA        NA        NA        NA        NA        NA        NA
##  [72,]        NA        NA        NA        NA        NA        NA        NA
##  [73,]        NA        NA        NA        NA        NA        NA        NA
##  [74,]        NA        NA        NA        NA        NA        NA        NA
##  [75,]        NA        NA        NA        NA        NA        NA        NA
##  [76,]        NA        NA        NA        NA        NA        NA        NA
##  [77,]        NA        NA        NA        NA        NA        NA        NA
##  [78,]        NA        NA        NA        NA        NA        NA        NA
##  [79,]        NA        NA        NA        NA        NA        NA        NA
##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
##  [84,]        NA        NA        NA        NA        NA        NA        NA
##  [85,]        NA        NA        NA        NA        NA        NA        NA
##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,72]     [,73]     [,74]     [,75]     [,76]     [,77]     [,78]
##   [1,]  97.66500  80.06757  89.52190 110.57493  91.62748  93.20537  80.57943
##   [2,]  89.36213 116.64393 123.14914  64.13135 115.99826  73.11351  87.37705
##   [3,]  97.56631 103.35764  96.18122 103.66130  74.42263  68.31850 100.45677
##   [4,]  80.12679  90.10306  71.64871  97.65608  93.90610 104.56535  96.17076
##   [5,] 112.51826  99.67374 103.85339  70.69990 115.88677  62.09425 101.92390
##   [6,]  86.57597  88.56037 109.77694  68.88678  83.47260 114.90845 102.27673
##   [7,]  65.82800  80.97038  95.13688  46.88471 129.80993 102.38585 100.01899
##   [8,] 115.64332  50.86425  99.59426 107.71313  55.07163 115.83052  81.16845
##   [9,]  92.63373  67.97466  93.62121  77.36542  84.59172 101.16964  92.41368
##  [10,] 124.53600  72.52620  84.90183 100.47616 102.39647  93.71944  90.45886
##  [11,] 124.53894 114.90942 111.67041  96.55465  96.28089  76.26053  74.22472
##  [12,]  95.16004  88.40098  86.96652 124.85764  74.33922  83.64266  57.13745
##  [13,] 107.81673 106.37939 108.22139 101.76402  74.14074  55.13900  78.80927
##  [14,]  92.34952 104.38708 125.48390  73.67973  70.02140 108.66532  82.74096
##  [15,] 108.61177  76.23426  89.45733 104.16901 107.81715  56.60846  62.68654
##  [16,]  50.85157  55.03821  93.77204  73.89168 107.08599  99.11335  52.83079
##  [17,] 121.68569  97.55943 114.06750  98.59598  79.28844  52.91598  84.83038
##  [18,]  90.14613  58.76582  65.00786 106.68710  76.40294 120.84546 105.53899
##  [19,]  95.80317 111.88484  81.16682  74.50213 108.44972  81.06953 121.12352
##  [20,]  58.54358  70.16655  68.01451 102.66696  72.90486 119.12052  97.94671
##  [21,] 126.34858  94.52415  98.42652 124.61270  74.44369  67.28211  65.57995
##  [22,]  81.34495  93.25330  79.25579 113.91688  84.13160  76.33272  80.82614
##  [23,]  69.85671 100.52252  94.31416  70.03136  97.65825  93.99733 108.74215
##  [24,]  89.84913  65.87273 110.88306 102.09893  74.80182  97.30127  36.45319
##  [25,]  94.91092 127.67795 115.70312  84.47442 105.33261  65.65213  84.19142
##  [26,]  76.08737  82.40109 110.30709  66.95512  99.95071  92.69693  88.62334
##  [27,]  89.45907 115.37164 115.08449  71.76015 108.96358  53.73978  81.18761
##  [28,]  55.49205  96.62976  79.33051  59.09696 104.77136  92.69325 124.24606
##  [29,]  86.48053 106.38017 111.09366  91.17147  98.86930  79.49777  63.31119
##  [30,] 100.23282  81.76469  55.24376 152.92101  70.61378  80.54081  89.84586
##  [31,]  72.22534 119.10525  83.18348  64.67481  97.52936  96.34558 118.55557
##  [32,]  81.52602  96.85935  98.12285  79.64059 116.54650  62.60727  90.27464
##  [33,]  96.50591  73.71522 107.89855  84.56740  96.04605  65.78919  65.22299
##  [34,] 126.60043  94.28695  86.33631 115.72597  80.14381 100.19612 114.65639
##  [35,]  70.76031  86.02115  94.30986  70.89500 104.65024  77.75219  61.38955
##  [36,] 110.40565 107.32879  80.69174  85.90297 100.27642  93.24240 131.99825
##  [37,]  78.66440  72.24437  49.97908 117.87657  71.70731 115.87364 106.29132
##  [38,] 101.34329 114.41709  95.96792  98.39845  86.68583  76.55960 100.23750
##  [39,]  78.81574 104.88217  88.77932  80.99698  76.06997 110.94115 103.29490
##  [40,]  65.38400  88.43376  67.85314  98.83102  90.73347 104.81290  92.61541
##  [41,]  48.96336  88.01168  52.39014  91.22654 108.32979  97.16072  97.33346
##  [42,]  79.85599 114.46493 104.34574  78.79590  73.15929  93.22969  77.17975
##  [43,]  90.71100  95.59522  61.66477  84.23771 119.08583  83.59396 116.24774
##  [44,] 141.16516  95.68035 120.74896  89.18196  93.96667  84.04257  93.44349
##  [45,] 107.62762 108.82949  95.70789  94.08838  75.06737  75.36830 105.26827
##  [46,] 120.75431  88.61500 125.86595  65.61786 117.08753  71.14025  89.13154
##  [47,]  84.81571  54.05449  84.16867 115.30595  79.03713 107.42197  63.87787
##  [48,]  55.97819  86.68166  40.00455  99.94083 107.55383  76.83618  92.95022
##  [49,] 123.38470 109.52909  88.08789 111.00863  57.06912  75.35852 111.94513
##  [50,] 123.54856  75.09664 103.05643 117.85572  46.93949  98.02050  66.82297
##  [51,]  84.22391  98.22376 104.50544  70.77465 111.97645  53.15242  87.18677
##  [52,]  69.74919 131.73260  87.89886  63.10805 133.88926  65.05336 100.52810
##  [53,]  74.84281 126.21818 100.98537  70.24114 101.26337  72.48756  86.38571
##  [54,] 112.54782  46.94887  87.41115 121.88792  61.41547 105.02535  79.67590
##  [55,]  53.19261  70.60798  94.59988  61.09362 125.87005  83.99726  73.26155
##  [56,] 126.07569 137.56818 104.07223 107.30524  72.53067  62.96949 106.17466
##  [57,] 113.80424  98.64217 106.73316  98.44238  75.03921 101.65153 106.79456
##  [58,]  99.21919  95.34263 113.42619  64.30649 112.83139  85.85740  79.18248
##  [59,]  57.28518  94.22524  91.53729  62.07449 101.70692 105.19560  98.24968
##  [60,]  79.22559  74.86038  71.28720  94.68885  77.68790 104.57384 128.82543
##  [61,] 105.61302 123.94186  98.43402  95.72389  89.85710  65.75815 106.43065
##  [62,] 123.19911 103.53036  77.24986 123.50517  76.90570  80.61088  89.06331
##  [63,]  93.93218 105.15491  79.94596  96.94650  85.63927  78.05792  93.45855
##  [64,] 120.08346 116.50631 121.18523  81.24772 105.75276  57.15136  77.46863
##  [65,] 124.06325  94.25497  99.15666 124.86786  79.92892  60.64902  55.12593
##  [66,]  82.44053  63.56042 110.61619  96.72710  71.46509 109.75154  54.15504
##  [67,]  90.13551  81.93548  99.82000  86.33139  86.09416 117.39118  89.93516
##  [68,]  74.86703  86.76589  58.61147 114.07951  64.25989 106.09790  90.42261
##  [69,]  68.28534  71.86159  74.89496  78.80108 123.41913  79.53353  91.11555
##  [70,]  99.01481  86.40762  66.86368 129.51511  78.94509  66.98009  85.68315
##  [71,]  92.26305  45.78059  67.64332 105.98851  69.48971 124.14521  93.44617
##  [72,]        NA  77.60715  68.21039  71.78346  98.53891 109.24585  88.08351
##  [73,]        NA        NA  76.73879  99.71084  79.24633 117.83886  71.32708
##  [74,]        NA        NA        NA 120.97114  82.97922 101.92529 110.49952
##  [75,]        NA        NA        NA        NA 126.18846  93.39268  94.90682
##  [76,]        NA        NA        NA        NA        NA 112.90568  88.04557
##  [77,]        NA        NA        NA        NA        NA        NA  78.59519
##  [78,]        NA        NA        NA        NA        NA        NA        NA
##  [79,]        NA        NA        NA        NA        NA        NA        NA
##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
##  [84,]        NA        NA        NA        NA        NA        NA        NA
##  [85,]        NA        NA        NA        NA        NA        NA        NA
##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,79]     [,80]     [,81]     [,82]     [,83]     [,84]     [,85]
##   [1,]  96.09750  75.03341  92.77528  90.76325  64.93650  72.89020  98.07725
##   [2,] 103.53993 108.39807 102.65893  58.75029 106.21194  95.68132 103.62964
##   [3,]  79.12229  44.95206  88.70662  97.12197  64.27556  96.48337  78.02317
##   [4,]  97.93492  88.25386 110.83485  89.29357  81.15923  71.43550  98.49276
##   [5,] 100.63046  83.13891  68.01755  50.82110  77.83179 130.13946  36.11335
##   [6,]  68.56474  99.85847  63.62419 112.50167 100.56595  88.88921 111.09556
##   [7,]  88.20993 102.82992  83.11875  71.53521  74.60221 111.11327  82.34430
##   [8,] 101.53378  69.91393  59.65852  98.38130  94.74287  87.59873  93.15282
##   [9,]  99.04807  86.29444  81.64734  86.14355 113.97628 100.99289  88.25058
##  [10,] 120.61284  90.08014  86.15020  71.02771 102.92853  79.89039  80.23439
##  [11,] 115.49392 102.95145  99.02980  60.91226 126.69500  73.46694 102.94053
##  [12,]  75.51542  66.59341  90.00413 103.98061  68.46386  71.24825 100.00785
##  [13,]  56.74533  45.52078  67.23377 103.51261  76.17861 102.48149  76.38322
##  [14,]  88.02153  93.60029  63.17211  75.75262  90.66466 105.43941  95.49081
##  [15,]  83.77957  53.73575  80.88569  79.48841  71.66882  98.15919  66.56895
##  [16,]  74.57074  99.38815  86.06208  94.13510  74.64398 100.68757 104.70004
##  [17,]  89.03395  51.59820  75.23805  82.08799  92.45797 106.32697  72.51145
##  [18,] 138.68094 106.98535 108.51290  84.67858  94.03499  77.00207  91.82607
##  [19,] 107.52708  91.11695  93.96685  55.78331  79.98240 111.31886  48.69727
##  [20,]  77.17710  97.90967  94.55119 127.50862  58.57287  76.80613 105.66874
##  [21,] 109.74890  73.60281 105.04855  83.52797 119.58482  66.66843 103.54923
##  [22,] 108.35509  85.54783 121.91395  81.68101  71.22545  81.92516  91.59529
##  [23,] 100.81727  99.17367  92.28109  67.47450  60.19138 115.40828  72.13998
##  [24,]  65.25942  71.67958  67.32645 109.55193  84.70807  81.17098 117.88249
##  [25,]  72.89714  82.29765  90.83203  84.65981  79.63209  85.87159 100.04411
##  [26,] 109.89896 103.20619  96.98065  67.92426  95.79345 105.42864  98.42856
##  [27,]  70.02313  94.80557  81.03054  83.17005  88.90814 109.64607  83.53125
##  [28,]  89.82331 107.49512 100.60532  84.95198  76.93269 114.78960  73.98162
##  [29,]  88.33742 103.02419 107.57994  87.45504 105.45103  69.50258 128.97391
##  [30,] 115.80089  78.12087 129.88353 100.99198  81.32450  56.24610  93.65992
##  [31,]  63.16738  99.86480  81.04801  96.87585  79.38504 103.50933  75.75891
##  [32,]  76.61394  70.86757  83.20046  77.54125  40.14351 114.17923  65.25823
##  [33,]  57.06122  49.76726  58.76460  98.11745  78.12080 111.88837  77.28394
##  [34,] 116.00033  86.65049 102.43541  97.15467 108.25697  48.81088 108.46374
##  [35,]  51.92320  78.54061  63.75790  86.48186  65.50784 123.58270  66.72406
##  [36,] 124.09783  92.65866 105.22139  65.02346 102.70410  84.76058  74.78827
##  [37,]  75.83078  93.51235 101.03938 148.74750  82.66156  52.97645 109.55809
##  [38,] 129.01543 102.95422 117.77749  62.48751 100.19878  85.36689  90.08206
##  [39,]  76.17697  94.02846  70.66330  91.49057  65.15323 103.54634  76.24249
##  [40,]  88.04163 128.95795 113.21371 113.68829  90.07737  65.13986 114.98541
##  [41,]  75.07129 122.42534 115.34705 114.80153  71.29815  74.93817  99.17028
##  [42,]  44.04482  89.08510  65.75658 107.73922  91.43874  98.45715  95.78909
##  [43,] 114.52999 120.09492 109.57479  71.52672  93.64320  92.31415  65.07910
##  [44,] 100.99971  77.03108  71.81592  79.49022 119.40285  80.03027  97.75485
##  [45,]  90.39576  62.10057  66.34681  71.56077  54.88686 122.42315  37.58606
##  [46,]  90.11411  67.63701  59.32272  64.11722  96.53374 112.10961  73.42426
##  [47,]  91.77440  75.06927  92.07048 104.50150  70.53511  69.49157 111.40403
##  [48,]  86.44074 104.06460 126.48680  99.09088  68.87486  86.89385  76.96380
##  [49,]  97.74460  60.92989  79.53540  86.50748  86.57577  99.92501  57.01181
##  [50,]  99.95179  59.78596  62.76372  85.16018  90.00729  92.04834  82.38331
##  [51,]  60.13697  57.24967  72.18754  83.32460  58.92971 126.10255  60.38691
##  [52,]  71.18150 116.44997 108.41361  84.14512  91.16576  93.45510  88.68184
##  [53,]  60.92993  96.97307  76.98049  82.15538  66.86807 115.23598  71.65539
##  [54,] 111.74331  62.90847  87.10787  99.51725  97.39016  72.69642 101.36686
##  [55,]  64.77955  93.94800  79.37593  89.62926  58.04092 115.45892  84.17862
##  [56,]  96.45090  74.97026  93.46908  83.58377  98.90819  83.50644  82.62992
##  [57,] 103.81446  94.77311  90.23904  98.37381 111.28476  65.67171 116.50388
##  [58,]  82.07105  97.74072  83.27287  83.36597 133.31725  86.32473 109.33169
##  [59,] 105.05969 129.08937 112.98694  76.98700 105.62113  94.29226 106.91086
##  [60,]  99.94891  91.09828  99.23713 108.84829  77.94659  88.32197  87.45504
##  [61,]  93.55854  62.42178 100.68643  77.22383  81.41395  89.86798  79.73130
##  [62,]  93.86638  73.77796 100.65040  98.67541 110.18801  58.47059  94.42897
##  [63,]  56.71897  71.04072  84.57834 114.02825  90.35913  83.57689  85.60718
##  [64,]  70.61683  64.76063  58.64703  71.20666  81.15823 108.26749  67.83173
##  [65,]  95.68411  70.71109  89.16772  83.52406  94.69411  78.64280  88.84135
##  [66,]  88.52167  86.33217  81.84022  99.17289  91.22481  80.56007 125.57581
##  [67,] 128.51085 110.20234 100.79665  67.59443 101.94692  80.93120 108.84964
##  [68,] 103.72184 117.84378 114.98816 104.03209  95.94049  72.66839  98.81770
##  [69,]  91.29416  99.33863  93.96355  80.63593  56.63880 109.82344  66.83950
##  [70,]  81.86756  74.57723  97.19770 112.51127  69.77453  78.90173  78.21718
##  [71,] 117.20972  97.40604  85.91074  92.78297  88.90209  86.82201  83.77808
##  [72,]  62.69103 116.41087 101.94786 115.53181  70.49007  94.47952 103.49695
##  [73,]  92.18740  82.30660  79.38996 107.55926  80.61159  88.61885  97.54717
##  [74,]  96.97412 103.19394 129.89029 119.32309  77.32862  62.56368  89.62152
##  [75,]  78.15989 112.68033  67.73039  65.34207  96.64758 127.54413  83.77703
##  [76,]  88.30603  68.93562  80.21527 117.90726  86.27328  77.74571  97.67178
##  [77,]  85.47670  67.54588  94.24109  66.91287  81.10520 107.26632  61.16641
##  [78,]  69.21291  75.41200  73.11616  91.45669  90.85434  95.09420 102.82457
##  [79,]        NA  77.44122  64.77243 123.16877  67.39238 102.89335  94.41154
##  [80,]        NA        NA  63.41338  91.75129  61.57654 104.45286  66.29923
##  [81,]        NA        NA        NA  83.52151  77.87467 130.29732  71.37240
##  [82,]        NA        NA        NA        NA 100.02140 119.84989  61.06985
##  [83,]        NA        NA        NA        NA        NA 113.52520  59.58670
##  [84,]        NA        NA        NA        NA        NA        NA 136.53688
##  [85,]        NA        NA        NA        NA        NA        NA        NA
##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,86]     [,87]     [,88]     [,89]     [,90]     [,91]     [,92]
##   [1,] 101.50582 108.57299  93.79898  89.38009  83.11784  85.96762  93.00783
##   [2,] 110.52784  75.17713  91.50349  97.88544 105.59292  73.57327 100.63752
##   [3,]  66.78435  85.93164  90.55587 128.05903 100.53608  75.39358  99.51457
##   [4,] 117.92182  73.67218  89.12971 112.03185  70.25618 106.16318  77.97495
##   [5,]  87.61280  90.97495  67.79237  77.75571  90.01490 102.34006 104.68502
##   [6,]  50.55605 113.88326  69.39285  76.01030  95.86211  81.41725 116.70433
##   [7,]  99.27973  80.09452  78.51434  87.09754  74.91475 119.54520  93.02658
##   [8,]  78.20170 119.17424  94.23341  86.04663  67.69109  64.32095 104.20112
##   [9,]  82.57223  87.70573  93.35367  94.88755  83.70742  80.43073 100.06202
##  [10,] 105.22012 118.03096  73.67471  61.89368  82.65139  90.87238 109.86105
##  [11,] 125.55122  86.05512  92.02587  87.60082  93.04870  62.51176  92.49491
##  [12,]  98.70502  92.02711 118.82402  99.45265  81.95916  76.28505  62.75611
##  [13,]  45.91946  95.72524  99.68431 103.60597 110.56627  62.07073  92.41457
##  [14,] 107.49793  59.32646  96.01948 123.52272  49.22744  75.81985  73.35077
##  [15,]  86.33753 103.22608 108.12815  79.71944  97.38564  82.99113  84.95991
##  [16,]  96.71944  88.56394 130.90577  75.69213  84.40590  93.70174  64.97605
##  [17,]  62.58818  99.18591  95.98095 101.83095 112.52627  51.78139 110.02087
##  [18,] 118.25800 100.20684  82.32670  81.90051  70.39835 103.57599  97.27076
##  [19,] 112.63966  58.64156  61.44150 109.13267  70.62727 124.12276  88.32336
##  [20,]  76.77284 107.42252  89.56030  80.31456  86.05380 109.36077  85.00537
##  [21,] 102.31481  97.73849 112.08845  95.96775 106.87690  41.55124  93.57632
##  [22,] 121.76571  73.85182 113.19789 110.12965  90.62773  88.37023  70.93038
##  [23,] 111.64329  59.95551  82.01442 113.62531  68.59830 114.57313  81.13491
##  [24,]  80.14577 106.32459 132.75624  86.16537  84.24483  54.00250  74.35362
##  [25,]  86.90946  86.90919  88.43385 101.08365 108.23923  75.45974  94.64839
##  [26,] 106.02879  78.11552 101.15667  99.95641  89.85511  80.02563  97.04118
##   [27,]  69.18964  92.51253  92.46976  74.82775 124.54515  79.41153  95.18512
##   [28,]  85.08529  68.28913  72.81191  98.59679  93.02581 125.15258  93.14912
##   [29,] 106.60822  88.23847 112.60147  90.13308 109.18671  61.71208  84.77695
##   [30,] 105.31446 104.96165  99.33392  90.39511 102.83789  86.65538  88.48320
##   [31,]  77.24554  65.67927  61.71120 101.32036  80.66309 125.31525  81.95904
##   [32,]  82.09035  89.67305  89.44330  92.86607  98.30664 102.94124  90.94480
##   [33,]  46.31778 105.95387 109.94834  84.54539 105.83245  68.63172  94.46008
##   [34,]  85.25617 131.71463  55.02517  80.37002 103.76644  74.64277 139.35022
##   [35,]  86.42244  64.26382 115.65185  93.09725  72.83109 105.12582  47.14815
##   [36,] 110.08898  80.21377  48.43310 105.90316  80.40676 105.89744 113.78548
##   [37,]  59.90962 129.91158  74.14317  64.30257 103.05348 106.38862 101.04429
##   [38,] 125.37007  73.17939  86.95434 106.90232  94.62578  81.04000  94.11257
##   [39,]  97.75565  62.58737  77.09092 111.80474  48.31580 116.19036  66.22138
##   [40,]  93.00044 106.35037  87.09959  56.68098 105.68152 107.77887  84.98464
##   [41,]  88.37731  99.63259  84.16362  55.43371 106.63114 134.55067  77.18348
##   [42,]  74.63786  63.73111 100.00423 107.71068  78.23677  85.27790  60.02631
##   [43,] 106.57449  94.21964  58.61698  56.77349  97.70430 133.12042  98.19600
##   [44,]  73.66644 124.66222  67.93629  78.38581 104.23773  53.33622 139.50206
##   [45,]  90.42169  67.87431  77.03217 120.72389  63.69872 100.80688  80.73432
##   [46,]  70.85475 108.08116  76.08636  82.58924  98.65983  70.75962 126.06296
##   [47,]  99.93566 109.02096 116.64213  88.72767  76.73576  79.13027  80.78624
##   [48,] 100.31248  80.22468  97.95105  74.82527 104.25252 134.31654  64.13569
##   [49,]  77.69631  81.87744  72.79707 116.94350  83.77034  82.80744  96.99516
##   [50,] 100.20377  87.93610 109.84812 110.74797  53.22725  59.89592  75.80824
##   [51,]  60.03281  82.85963  94.85481 100.93261 105.49680  91.73594  91.66059
##   [52,]  88.95108  75.46265  75.27007  77.21759 120.50444 114.49273  87.09523
##   [53,]  91.51521  58.08054  90.58755  99.00072  84.10852 108.24900  60.78749
##   [54,]  85.99750 119.86570 103.43741  92.29466  83.18776  59.37561 107.09329
##   [55,]  79.36198  93.20470 106.12692  69.80161  96.20407 108.86299  79.50077
##   [56,]  82.85950  87.03920  71.81025 110.98273 107.01243  67.14723 106.73398
##   [57,]  76.43489 122.03172  66.10488  83.76593 105.66239  62.90411 133.45954
##   [58,]  81.31373  95.12869  89.31980  78.84271 106.43933  68.74882 106.42211
##   [59,] 117.67747  60.26169  94.22889 100.20503  85.27121 101.31328  82.05434
##   [60,]  64.14724 112.64510  65.86716  84.46936 102.79835 103.49906 120.85499
##   [61,]  92.49808  71.24834  79.80656 134.04789  95.91782  80.51848  99.42003
##   [62,]  90.93144  98.45031  83.76472  93.59930  97.20942  76.17667  93.75356
##   [63,]  58.68561  90.96704  83.78476  94.02148 103.56067  91.66052  87.49495
##   [64,]  78.87176  89.40793  84.73006  92.65112  92.46442  76.34278  92.59419
##   [65,]  98.54872 102.24234 113.80191  83.01881 100.64603  56.90193  81.44577
##   [66,]  97.69636  96.70569 128.16410  96.44500  77.63552  56.59939  79.87180
##   [67,] 138.28172  78.53153  92.85394 103.37083  61.57647  84.27149  89.66424
##   [68,] 112.40289  80.02715  99.34850  84.37396  82.08569 104.55340  63.43267
##   [69,]  91.29671 100.43637  89.15557  59.12720  97.90061 124.41548  87.72378
##   [70,]  67.46048 120.51736  92.87720  65.43068 117.77500  91.37286  90.90442
##   [71,] 103.42735 104.25589  89.11223  75.11737  60.39762 102.82437  87.78477
##   [72,]  86.18924  74.84444 104.76852  83.26528  88.98601 124.97296  60.99505
##   [73,]  82.66285 115.96329 112.39096  70.97718  75.68172  89.37999  87.11673
##   [74,]  93.12597  99.95533  80.99552  73.25159  97.96567 131.40516  81.31314
##   [75,]  88.68494  69.50852  79.87186  87.57637  82.91659 101.77675  93.48576
##   [76,]  82.01768  88.29181 101.91406 118.00529  66.21363  71.68459  77.47861
##   [77,]  85.22947  84.71036  97.79074  90.90552 122.96851  78.50124  90.73242
##   [78,]  94.29110  86.95049 154.94198  86.94785  86.08477  58.01592  56.92998
##   [79,]  49.33452  86.14538 109.43914  89.58647  96.34324  91.39415  66.33420
##   [80,]  67.44510  95.92724 105.78258 116.41377  84.34919  63.53828  90.36278
##  [81,]  65.80180  87.28802  97.63424 100.08310  62.17270  73.03981  86.32369
##  [82,] 125.50523  66.49970  82.29725 101.64447  77.85755  85.16537  97.04610
##  [83,]  74.94066  84.71825  97.97280 102.05078  79.55368 113.53409  71.43238
##  [84,]  97.08425 117.94779  83.35514  75.16752 109.12461  83.21630 102.54551
##  [85,]  87.85692  73.38553  79.12445 100.96285  78.32434 109.42306  85.65442
##  [86,]        NA 118.99646  83.93254  77.81007 113.22307  84.06241 107.47974
##  [87,]        NA        NA 102.88827 132.92169  60.56060 100.74789  49.62701
##  [88,]        NA        NA        NA  81.46765  97.25275 114.12368 133.63633
##  [89,]        NA        NA        NA        NA 121.10050 103.24271 106.82181
##  [90,]        NA        NA        NA        NA        NA  95.88544  64.55779
##  [91,]        NA        NA        NA        NA        NA        NA 100.49837
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,93]     [,94]     [,95]     [,96]     [,97]     [,98]     [,99]
##   [1,]  80.81589 108.19858  90.68020  92.34368 101.36703  99.39962  81.11930
##   [2,] 108.19349  67.98799  84.80279  84.64880 122.58567  71.16729  95.83898
##   [3,]  94.42032  94.32817 126.62882 123.76844  90.16156 105.73226  94.62354
##   [4,] 100.47673  97.50000  71.78670  95.64006  85.50002  88.82022  84.92394
##   [5,]  78.59224 110.21257  84.57518  69.59563  87.48781  58.74412  56.21207
##   [6,]  58.55308  66.34793  97.92320 101.81934  70.80689  92.38108 107.27840
##   [7,]  74.45344  84.95673  48.18394  81.90634  58.74596  74.77771  89.36499
##   [8,]  74.24380 106.69183  90.11874 105.97358  77.37648 116.81192 100.24540
##   [9,]  87.56800  94.86777  79.03122 112.62247  60.45635  98.90802 119.56113
##  [10,]  69.55350 137.61789  76.29108  86.50264  87.61762  73.49051  76.29325
##  [11,] 116.87552  93.60794  92.76677  68.84477 147.12026  70.89213  78.84546
##  [12,] 106.31272  96.60398 108.02345  77.43545 113.54298 117.76934  85.05780
##  [13,]  92.92000  90.60126 152.40348  99.83233  95.96059 110.84465  97.97529
##  [14,] 121.45190  50.21934  75.23488  63.90934 109.44490  93.80529  75.88569
##  [15,]  84.49645 133.44582 103.82634  85.69088  85.68172 102.06346  92.24655
##  [16,]  88.82394  74.75418  64.44794  75.22286  76.55697 117.20915 117.41166
##  [17,]  92.97728 103.13890 132.76940 112.85077  99.15556 101.49311 102.67056
##  [18,]  81.81078 107.28535  55.36906 101.00871  86.17845  85.19356  78.66125
##  [19,]  99.77266 101.83654  67.54986  76.71822  83.48582  50.78265  48.09708
##  [20,]  67.22403  76.68861  85.74482  97.02527  77.68167 107.07373  87.56109
##  [21,] 114.22658 109.66503 119.09545 103.14161 129.73795 102.84972 105.45045
##  [22,] 118.11597  90.79953  88.96791  90.09423 122.78535  98.04945  80.32147
##  [23,] 101.88930  69.83232  61.36624  76.19987  94.07543  74.53145  60.74525
##  [24,]  92.96429  82.06740 103.10713  84.78319  96.26689 140.06150 120.53530
##  [25,]  98.16924  76.06756 115.05646  83.34618 120.43353  83.58233  85.40421
##  [26,]  99.68206  71.14614  67.87083  98.42005  95.38175  90.56044 106.86461
##  [27,]  89.52487  71.74628 114.43102  73.56938 109.51414  80.64608  90.33558
##  [28,]  84.29269  70.21387  70.06675  97.26128  65.95544  71.40429  85.09579
##  [29,] 110.65554  69.08501 100.37992  83.01505 133.93178  95.84621 107.61045
##  [30,]  94.75171 121.91970 106.40949 110.69143 113.61500  99.52215  81.87814
##  [31,]  88.06114  67.90778  85.60604  75.05606  70.33981  67.62490  68.32214
##  [32,]  79.76097  95.62629  95.12337  83.50190  87.97984  86.60872  75.52179
##  [33,]  75.31382 102.50094 118.09554 102.09726  65.32479 119.46939 122.31590
##  [34,]  63.73630 118.26474 108.16043 125.69703 100.47257  76.86360  86.88455
##  [35,] 103.08549  80.68828  81.14715  50.17494  72.55174 102.91507  87.27208
##  [36,]  88.71669 112.42390  75.56052 105.33027  84.95590  51.48908  67.74049
##  [37,]  49.22047 100.96108 105.02858 114.65030  66.83615 100.80885  96.23101
##  [38,] 118.61083  84.41378  87.24465  88.29273 139.14926  69.71241  69.57266
##  [39,] 101.31781  66.17448  73.12616  58.87079  87.21992  82.30680  49.74379
##  [40,]  76.96150  74.68592  85.15245  78.42988 102.77797  83.71458  83.48589
##  [41,]  69.40548  82.28411  80.41786  75.06783  83.37021  79.48832  79.13328
##  [42,] 113.01456  48.76257 104.66581  63.05436  93.82033 100.99239  89.13406
##  [43,]  73.65923 112.03234  67.49856  72.55611  86.75284  41.44096  54.29229
##  [44,]  69.24723 111.69186 111.64423 110.00057  95.55595  80.83127 101.24703
##  [45,] 103.70458  94.91019  95.08695  72.91189  95.71123  80.84504  38.01594
##  [46,]  68.09625 112.26062  97.45046 100.75190  74.13964  81.43372 102.23930
##  [47,]  84.22685 102.65260  84.27000  98.74055  89.77241 125.35946 104.35246
##  [48,]  90.30746  98.61477  81.02615  79.36040  83.61025  82.80627  78.23381
##  [49,] 101.16505 103.40830 117.11273 100.36832  97.32730  84.66989  63.45274
##  [50,] 111.69834 101.67778  94.90733  82.24208 103.92347 116.62778  81.40256
##  [51,]  83.75531  92.95503 109.44907  95.19244  70.98763  95.88323  97.66204
##  [52,]  90.73998  72.33593  93.99048  69.45192  98.77776  55.34936  80.23032
##  [53,] 109.84346  58.65315  91.63517  40.08796 105.94380  76.15157  57.78556
##  [54,]  78.96781 121.29455  96.11361 130.88366  80.42707 122.96993 116.55726
##  [55,]  71.34633  80.85824  73.46652  75.37054  66.00072  98.84609 102.51982
##  [56,] 105.85525  90.45651 133.26114  98.12787 129.68703  74.89246  69.58538
##  [57,]  71.81344  87.72729 109.58277 116.95139 106.54744  84.09916  96.31294
##  [58,]  86.26519  87.25343  93.85291  96.46438  81.95423  84.81766 126.60333
##  [59,] 108.63155  55.67989  54.62124  86.93242  93.54072  78.07220  99.41402
##  [60,]  55.41068  94.34151  92.76684 132.62671  65.10457  89.16828  94.00174
##  [61,] 107.97615  95.88910 112.51445 110.62274 104.00487  82.16546  82.39468
##  [62,]  96.88924 118.14805 117.86389 100.18247 102.87782  88.36855  87.17462
##  [63,]  85.29756  95.64672 122.67094  98.54366  74.75273  94.93023  97.08829
##  [64,]  93.12938  98.69909 113.15497  66.84164 102.79184  80.98440  73.35030
##  [65,] 107.31960 110.51889 118.80586  76.21996 133.47049 102.04810  84.03899
##  [66,] 100.17394  72.46031  84.88725  93.76535 101.31475 130.67739 120.29568
##  [67,] 106.81127  80.67562  51.42031  86.08406 108.41126  84.70836  84.75488
##  [68,] 108.09160  77.89311  78.76485  74.07011 109.64419  90.36880  72.71195
##  [69,]  65.61300 102.79539  69.47359  75.24308  74.86790  79.73675  76.20303
##  [70,]  73.33050 114.03713 126.46545  92.97982 100.32086  98.51014  78.01356
##  [71,]  76.92291 108.63882  59.07545  89.77938  73.19698  96.92000  79.79422
##  [72,]  86.66878  54.80601  67.73991  80.00278  66.34579 100.90176 102.40399
##  [73,]  66.18394 106.29086  70.93358 104.75599  54.17207 126.58088 117.68714
##  [74,]  74.62489 108.63454  85.13631 101.49419  76.21294  86.14934  77.77247
##  [75,]  87.31139  59.12348  60.93404  68.88099  71.79212  69.29536  94.87298
##  [76,] 104.72100  83.10065 105.83016 104.44176  94.28472 123.89003  89.84464
##  [77,] 102.85345 107.03926 122.90753  81.81001 115.29974  79.68567  81.09480
##  [78,] 112.29312  83.80260  96.30595  67.92271 102.60273 131.11354 118.03562
##  [79,]  86.17766  62.03750 111.51109  77.50869  71.72323 117.56422 107.67741
##  [80,]  93.17874 113.73890 124.10765 107.25495  85.84258 122.04906  93.73321
##  [81,]  87.41694  79.59170  90.83454  74.67050  70.48541 109.19660  93.19888
##  [82,] 111.05901  99.37481  68.05539  68.89478 109.63464  57.53092  67.35624
##  [83,]  82.59001  93.32045  93.88079  85.22281  72.73276 106.64288  71.74105
##  [84,]  80.31532 102.58827 103.31468 115.26590 105.33680  91.62505 102.03598
##  [85,]  93.08632 109.69024  88.12255  74.45864  81.40734  74.16520  51.32521
##  [86,]  54.34959  88.83706 126.56561 113.24305  59.56044 109.62637 111.24796
##  [87,] 144.93278  56.74232  70.41304  56.40225 103.33903  82.62057  70.09575
##  [88,]  60.42026 101.94206  86.33515 104.53123  80.95148  39.25819  61.59203
##  [89,]  47.96179 107.07540  86.25957  87.85720  77.66151  78.82965 100.94670
##  [90,] 111.37394  77.99450  53.94062  69.31040  79.86058 100.62079  72.29755
##  [91,] 106.05533  91.42128 119.19929 103.85025 115.06746 116.86701 120.07142
##  [92,] 130.40918  64.97738  78.60723  49.08281  96.93126 113.57997  83.67284
##  [93,]        NA 112.70374  92.54037 118.15422  51.06446  84.35078 100.69106
##  [94,]        NA        NA  79.58701  67.89987  93.99843  96.36365  97.91725
##  [95,]        NA        NA        NA  69.14573  72.98757  74.72424  80.63592
##  [96,]        NA        NA        NA        NA 105.88700  78.80817  64.04181
##  [97,]        NA        NA        NA        NA        NA 101.55396 106.36846
##  [98,]        NA        NA        NA        NA        NA        NA  53.51095
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##           [,100]
##   [1,]  78.94680
##   [2,] 121.49223
##   [3,]  99.14476
##   [4,]  77.18366
##   [5,] 103.55200
##   [6,]  84.74581
##   [7,]  99.20628
##   [8,]  78.91843
##   [9,] 108.82518
##  [10,]  90.05572
##  [11,]  87.95720
##  [12,]  53.50522
##  [13,]  83.36781
##  [14,]  61.72910
##  [15,]  89.17519
##  [16,]  93.29111
##  [17,] 113.03480
##  [18,] 102.41578
##  [19,]  91.50780
##  [20,]  77.49573
##  [21,]  99.54707
##  [22,] 100.96011
##  [23,]  97.20603
##  [24,]  65.37145
##  [25,]  86.82141
##  [26,] 129.58312
##  [27,] 105.76693
##  [28,] 117.76002
##  [29,]  96.09249
##  [30,]  98.11562
##  [31,]  67.44666
##  [32,]  99.65166
##  [33,]  95.16615
##  [34,]  96.63758
##  [35,]  62.56479
##  [36,] 101.86202
##  [37,]  74.68409
##  [38,] 113.62732
##  [39,]  43.16430
##  [40,]  86.80242
##  [41,]  85.21358
##  [42,]  43.85247
##  [43,] 104.43833
##  [44,]  99.58928
##  [45,]  70.60165
##  [46,] 109.35328
##  [47,]  77.25732
##  [48,]  97.32128
##  [49,]  82.48705
##  [50,]  57.99295
##  [51,] 103.64781
##  [52,]  99.21090
##  [53,]  65.09936
##  [54,]  98.64014
##  [55,] 100.13197
##  [56,]  91.16289
##  [57,] 100.40732
##  [58,] 101.57837
##  [59,] 113.92233
##  [60,] 118.87987
##  [61,]  98.33413
##  [62,]  71.74484
##  [63,]  70.77360
##  [64,]  71.76289
##  [65,]  80.15241
##  [66,]  84.37791
##  [67,]  94.85221
##  [68,]  77.99862
##  [69,] 108.53100
##  [70,]  87.59236
##  [71,]  83.21352
##  [72,]  86.62395
##  [73,]  91.26676
##  [74,]  88.37869
##  [75,] 101.16250
##  [76,]  60.36960
##  [77,] 110.18712
##  [78,]  73.93069
##  [79,]  61.75438
##  [80,]  80.66203
##  [81,]  66.10584
##  [82,] 110.95267
##  [83,]  77.69737
##  [84,]  86.58204
##  [85,]  93.63708
##  [86,]  89.04993
##  [87,]  75.69121
##  [88,] 102.15256
##  [89,] 105.39525
##  [90,]  54.80335
##  [91,]  91.85249
##  [92,]  54.05458
##  [93,] 106.21592
##  [94,]  73.61121
##  [95,]  91.87943
##  [96,]  63.13026
##  [97,]  90.05208
##  [98,] 110.24180
##  [99,]  76.76564
## [100,]        NA
```

Plot the density of these angles.

```
pacman::p_load(ggplot2)
ggplot(data.frame(angles=c(all_angles(X)))) + aes(x = angles) + geom_density()
```

```
## Warning: Removed 5050 rows containing non-finite values (stat_density).
```



Write an Rcpp function `all_angles_cpp` that does the same thing. Use an IDE if you want, but write it below in-line.

```
cppFunction(
'
  NumericMatrix all_angles_cpp(NumericMatrix X) {
    int n = X.nrow();
    int p = X.ncol();
    NumericMatrix A(n, n);
    std::fill(A.begin(), A.end(), NA_REAL);
    for (int i_1 = 0; i_1 < (n - 1); i_1++){
      for (int i_2 = i_1 + 1; i_2 < n; i_2++){
        double sum_sqd_u = 0;
        double sum_sqd_v = 0;
        double sum_u_v = 0;
        for (int j = 0; j < p; j++){

          //sqd_diff += pow(X(i_1, j) - X(i_2, j), 2); //by default the cmath library in std is loaded

          sum_sqd_u += pow(X(i_1, j), 2);
          sum_sqd_v += pow(X(i_2, j), 2);
          sum_u_v += X(i_1, j) * X(i_2, j);


        }
        A(i_1, i_2) = acos(sum_u_v / sqrt(sum_sqd_u * sum_sqd_v)) * (180 / M_PI); //by default the cmat
      }
    }
    return A;
  }
'
)
head(all_angles_cpp(X))
```

```
##      [,1]     [,2]     [,3]     [,4]      [,5]      [,6]     [,7]      [,8]
## [1,]   NA 87.03322 85.06439 65.96173  92.87329  97.97792 78.60834  80.45545
## [2,]   NA       NA 85.85670 95.07828  88.65895  89.35291 79.37790 114.77863
## [3,]   NA       NA       NA 81.88730  97.39674  93.96639 97.76103  96.84914
## [4,]   NA       NA       NA       NA 114.35762 117.82550 63.01259 107.93154
## [5,]   NA       NA       NA       NA        NA  92.78557 78.06001  88.18199
## [6,]   NA       NA       NA       NA        NA        NA 89.33137  70.38268
##           [,9]     [,10]     [,11]     [,12]     [,13]    [,14]     [,15]
## [1,] 132.06125  70.84126  80.04000  43.28448  98.63177 81.69807  69.97621
## [2,] 100.23764 108.23255  63.13898  99.63234  96.88888 70.76450 107.57016
## [3,]  87.93427 120.86547 112.27872  83.79255  49.56119 94.25867  83.98631
## [4,]  93.75242  86.48565  94.30545  69.66669 115.37372 91.14354  80.78943
## [5,]  95.76812  64.00527  85.09790 107.02883  84.10345 93.01699  70.40911
## [6,]  87.76705  98.32406 100.84880 108.43590  78.93962 75.60750 119.72984
##          [,16]     [,17]     [,18]     [,19]     [,20]     [,21]     [,22]
## [1,]  75.10570 107.14889  79.62090  97.04677  62.39233  91.20019  64.09498
## [2,]  78.88351  82.80395  97.14230  93.98541  99.62136  80.90830  66.54523
## [3,] 106.08354  53.08866 110.28772  86.62378  89.87988  81.98477  73.08674
## [4,]  85.64167 123.47508  84.14713  62.64603  90.36197  96.59773  72.35778
## [5,] 101.64207  75.44388  91.29055  61.99213 106.25372 103.85532 103.83335
## [6,]  95.28175  86.63592 101.59379 116.37181  70.38779 111.03544 129.76488
##         [,23]     [,24]    [,25]    [,26]     [,27]     [,28]     [,29]
## [1,] 75.22200  61.17734 60.85289 94.28625 101.18644 117.89758  70.28904
## [2,] 63.94025  91.96119 55.09228 42.82717  60.59513  77.13405  42.49355
## [3,] 81.00270  93.37918 68.70192 84.42751  87.28788  76.85452  91.43348
## [4,] 78.52592  95.13486 84.58126 97.05563 138.36633  89.43544  89.84066
## [5,] 77.31480 111.40959 90.96619 93.66843  67.05618  83.57134 113.54499
## [6,] 98.29728  78.35187 82.79074 90.40360  72.05258  87.52377  90.33776
##          [,30]     [,31]    [,32]     [,33]    [,34]     [,35]     [,36]
## [1,]  66.27180 108.76104 57.01916  99.57992 66.57676  95.44355  94.97961
## [2,]  97.91830 102.98109 74.78874 103.93558 92.20330 107.53259  91.77137
## [3,]  77.12073  86.48877 66.67524  68.28304 84.62997  97.50626  82.70013
## [4,]  78.07482  77.14452 84.46950 106.82985 86.65321  81.70701  57.37613
## [5,] 104.92290  83.81699 65.19326  80.74012 94.24172  78.03642  80.03026
## [6,] 124.44251  76.31881 97.80766  82.11093 72.28258 104.92162 104.65621
##          [,37]     [,38]     [,39]     [,40]     [,41]     [,42]     [,43]
## [1,]  80.97844  85.34459  78.01481  77.68322  77.31242 109.42695  97.95383
## [2,] 129.72505  47.83797 103.21176  84.11380  96.02816  99.15746  98.02187
## [3,]  92.69796  84.15030  95.83154 118.22699 111.57804  89.82944 122.96881
## [4,]  88.25175  94.97571  75.51934 104.79698  86.24818  95.89259  94.38034
## [5,] 110.28711  87.46071  83.66481 100.09921  94.04379 102.48218  54.07608
## [6,]  67.21597 108.65192  82.73873  75.99896  88.45163  71.50285 104.13218
##          [,44]     [,45]     [,46]     [,47]     [,48]     [,49]     [,50]
## [1,]  85.29509  82.48971  90.12522  33.72969  94.32991 109.71545  77.11860
## [2,]  84.80848 102.04074  83.58924  96.00329 101.94008 110.66055 111.36373
## [3,]  88.64597  70.45000  81.32613  89.52357  96.04603  63.15953  91.89560
## [4,] 107.19013  92.05161 100.04417  65.81687  74.82910 103.50257  93.68700
## [5,]  75.91875  55.49000  57.73075 113.01221  91.11427  73.77606  89.27965
## [6,]  62.40954 100.29673  75.84275  99.59023 126.99918  93.89020  98.47227
##         [,51]     [,52]    [,53]     [,54]    [,55]     [,56]     [,57]
## [1,] 95.82785 101.24231 88.07074  78.35417 75.57602  94.90172  79.31213
## [2,] 86.09716  66.93117 79.21725 107.12831 79.97593  76.26014  72.54321
## [3,] 51.50567  93.97755 93.14401  79.38773 95.53333  62.24727  84.49032
## [4,] 91.62162  88.05090 96.54114  87.53148 91.74333 107.40934 112.26163
## [5,] 70.98083  81.04248 71.63959 106.11794 78.12078  82.27548  95.86101
## [6,] 93.43182  89.99621 89.06898  93.03139 87.07091  84.81662  49.64741
##          [,58]     [,59]     [,60]     [,61]     [,62]     [,63]    [,64]
## [1,] 116.11767 108.87620  97.77046  84.31468  91.46507 109.98340 76.27271
## [2,]  79.73684  52.39516  94.62069  74.40949 115.63178 120.62278 89.14223
## [3,]  98.80057  98.20472  70.07310  36.12420  85.32042  72.08508 83.96839
## [4,]  91.80619  81.28940 105.14459  61.82293  73.03462  82.05799 96.77774
## [5,]  96.65468 107.60191  90.24130  95.37871 101.11044  98.15393 57.83089
## [6,]  75.47532  94.62291  68.62312 107.23219 104.19755  85.72861 86.32533
##          [,65]     [,66]     [,67]     [,68]     [,69]     [,70]     [,71]
## [1,]  70.35039  64.39936  66.75852  98.13638  72.63811  78.10585  84.48459
## [2,]  89.52185  72.18255  61.89316  98.23555  88.28858 111.36608 120.33964
## [3,]  94.78564  92.03731 105.67577 114.73520 104.07252  85.23726 119.86841
## [4,] 106.80632  91.59444  71.21231  98.54106  96.95732 116.80624  93.91924
## [5,]  82.54355 119.46634 101.74140 104.45820  58.91692  77.76358  83.88739
## [6,] 108.11086  81.33319  97.86843 102.38772 100.53194  93.39696  93.90922
##          [,72]     [,73]     [,74]     [,75]     [,76]     [,77]     [,78]
## [1,]  97.66500  80.06757  89.52190 110.57493  91.62748  93.20537  80.57943
## [2,]  89.36213 116.64393 123.14914  64.13135 115.99826  73.11351  87.37705
## [3,]  97.56631 103.35764  96.18122 103.66130  74.42263  68.31850 100.45677
## [4,]  80.12679  90.10306  71.64871  97.65608  93.90610 104.56535  96.17076
## [5,] 112.51826  99.67374 103.85339  70.69990 115.88677  62.09425 101.92390
## [6,]  86.57597  88.56037 109.77694  68.88678  83.47260 114.90845 102.27673
##          [,79]     [,80]     [,81]     [,82]     [,83]     [,84]     [,85]
## [1,]  96.09750  75.03341  92.77528  90.76325  64.93650  72.89020  98.07725
## [2,] 103.53993 108.39807 102.65893  58.75029 106.21194  95.68132 103.62964
## [3,]  79.12229  44.95206  88.70662  97.12197  64.27556  96.48337  78.02317
## [4,]  97.93492  88.25386 110.83485  89.29357  81.15923  71.43550  98.49276
## [5,] 100.63046  83.13891  68.01755  50.82110  77.83179 130.13946  36.11335
## [6,]  68.56474  99.85847  63.62419 112.50167 100.56595  88.88921 111.09556
##          [,86]     [,87]    [,88]     [,89]     [,90]     [,91]     [,92]
## [1,] 101.50582 108.57299 93.79898  89.38009  83.11784  85.96762  93.00783
## [2,] 110.52784  75.17713 91.50349  97.88544 105.59292  73.57327 100.63752
## [3,]  66.78435  85.93164 90.55587 128.05903 100.53608  75.39358  99.51457
## [4,] 117.92182  73.67218 89.12971 112.03185  70.25618 106.16318  77.97495
## [5,]  87.61280  90.97495 67.79237  77.75571  90.01490 102.34006 104.68502
## [6,]  50.55605 113.88326 69.39285  76.01030  95.86211  81.41725 116.70433
##          [,93]     [,94]     [,95]     [,96]     [,97]     [,98]     [,99]
## [1,]  80.81589 108.19858  90.68020  92.34368 101.36703  99.39962  81.11930
## [2,] 108.19349  67.98799  84.80279  84.64880 122.58567  71.16729  95.83898
## [3,]  94.42032  94.32817 126.62882 123.76844  90.16156 105.73226  94.62354
## [4,] 100.47673  97.50000  71.78670  95.64006  85.50002  88.82022  84.92394
## [5,]  78.59224 110.21257  84.57518  69.59563  87.48781  58.74412  56.21207
## [6,]  58.55308  66.34793  97.92320 101.81934  70.80689  92.38108 107.27840
##         [,100]
## [1,]  78.94680
## [2,] 121.49223
## [3,]  99.14476
## [4,]  77.18366
## [5,] 103.55200
## [6,]  84.74581
```

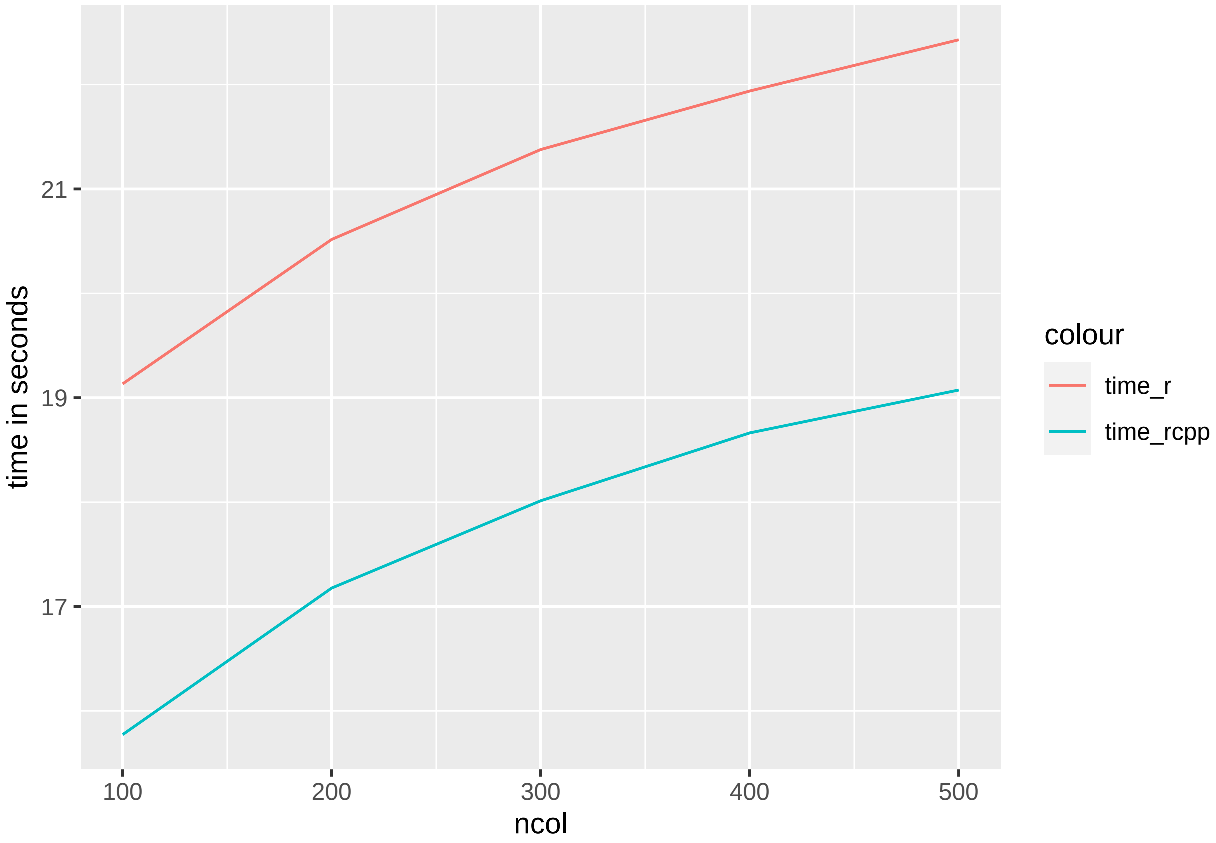
Test the time difference between these functions for `n = 1000` and `Nvec = 100, 500, 1000, 5000` using the package `microbenchmark`. Store the results in a matrix with rows representing `Nvec` and two columns for base R and Rcpp.

```
pacman::p_load(microbenchmark)

n = 1000
Nvec = c(100, 200,300, 400,500)
time_r = c()
time_cpp = c()
x_list = c()
for (i in 1:length(Nvec)){
  X = c()
  for (j in 1:n){
    x = rnorm(Nvec[i])
    X = cbind(X, x)
  }
  x_list = c(x_list,X)
  time_r = c(time_r, mean(microbenchmark(angles_r = all_angles(X), times = 3, unit = "s")$time))
  time_cpp = c(time_cpp, mean(microbenchmark(angles_cpp = all_angles_cpp(X), times = 3, unit = "s")$tim
}
```

Plot the divergence of performance (in log seconds) over n using a line geometry. Use two different colors for the R and CPP functions. Make sure there's a color legend on your plot. We wil see later how to create "long" matrices that make such plots easier.

```
pacman::p_load(ggplot2)
ggplot()+
  geom_line(aes(x = Nvec, y = log(time_r), col = "time_r"))+
  geom_line(aes(x = Nvec, y = log(time_cpp), col = "time_rcpp"))+
  xlab("ncol")+
  ylab("time in seconds")
```



Let `Nvec = 10000` and vary `n` to be 10, 100, 1000. Plot the density of angles for all three values of `n` on one plot using color to signify `n`. Make sure you have a color legend. This is not easy.

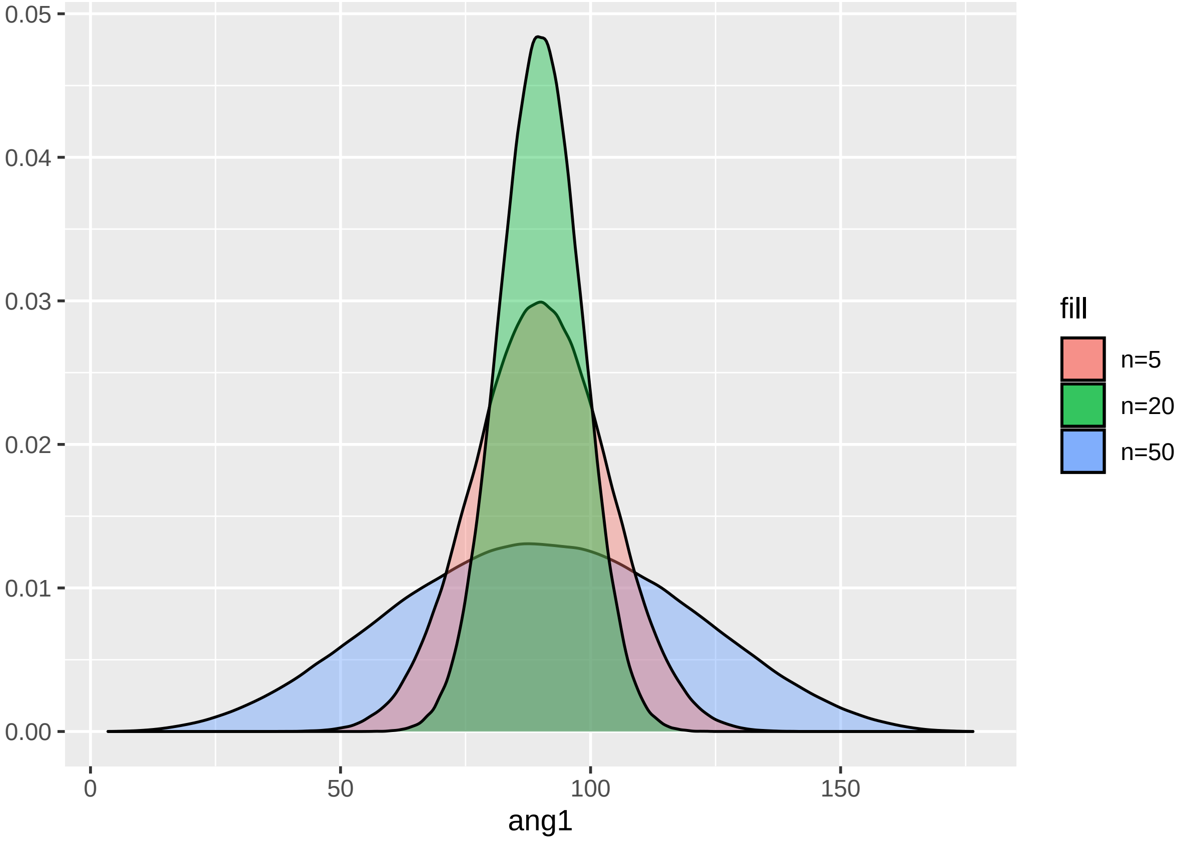
```
Nvec = 1000
X <- c()
for (i in 1:5){
  x <- rnorm(Nvec)
  X <- cbind(X, x)
}
ang1 <- all_angles(X)
X <- c()
for (i in 1:20){
  x <- rnorm(Nvec)
  X <- cbind(X, x)
}
ang2 <- all_angles(X)
X <- c()
for (i in 1:50){
  x <- rnorm(Nvec)
  X <- cbind(X, x)
}
ang3 <- all_angles(X)

ggplot() +
  geom_density(aes(x = ang1, fill = "red"), alpha = .4) +
  geom_density(aes(x = ang2, fill = "blue"), alpha = .4) +
  geom_density(aes(x = ang3, fill = "green"), alpha = .4) +
  scale_fill_discrete(labels = c("n=5", "n=20", "n=50")) +
  ylab("Density of Angles") +
  ylab("")
```

```
## Warning: Removed 500500 rows containing non-finite values (stat_density).

## Warning: Removed 500500 rows containing non-finite values (stat_density).

## Warning: Removed 500500 rows containing non-finite values (stat_density).
```



Write an R function `nth_fibonnaci` that finds the nth Fibonnaci number via recursion but allows you to specify the starting number. For instance, if the sequency started at 1, you get the familiar 1, 1, 2, 3, 5, etc. But if it started at 0.01, you would get 0.01, 0.01, 0.02, 0.03, 0.05, etc.

```
nth_fibonacci <- function(n, start){
  if (n == 1 | n == 2) return(start)
  else return(nth_fibonacci(n-1, start) + nth_fibonacci(n-2, start))
}
nth_fibonacci(10, 1)
```

```
## [1] 55
```

Write an Rcpp function `nth_fibonnaci_cpp` that does the same thing. Use an IDE if ou want, but write it below in-line.

```
cppFunction(
  'double nth_fibonacci_cpp(int n, double start){
    if (n == 1 || n == 2) return start;
    else return (nth_fibonacci_cpp(n-1, start) + nth_fibonacci_cpp(n-2, start));
  }'
)
nth_fibonacci_cpp(10, 1)
```

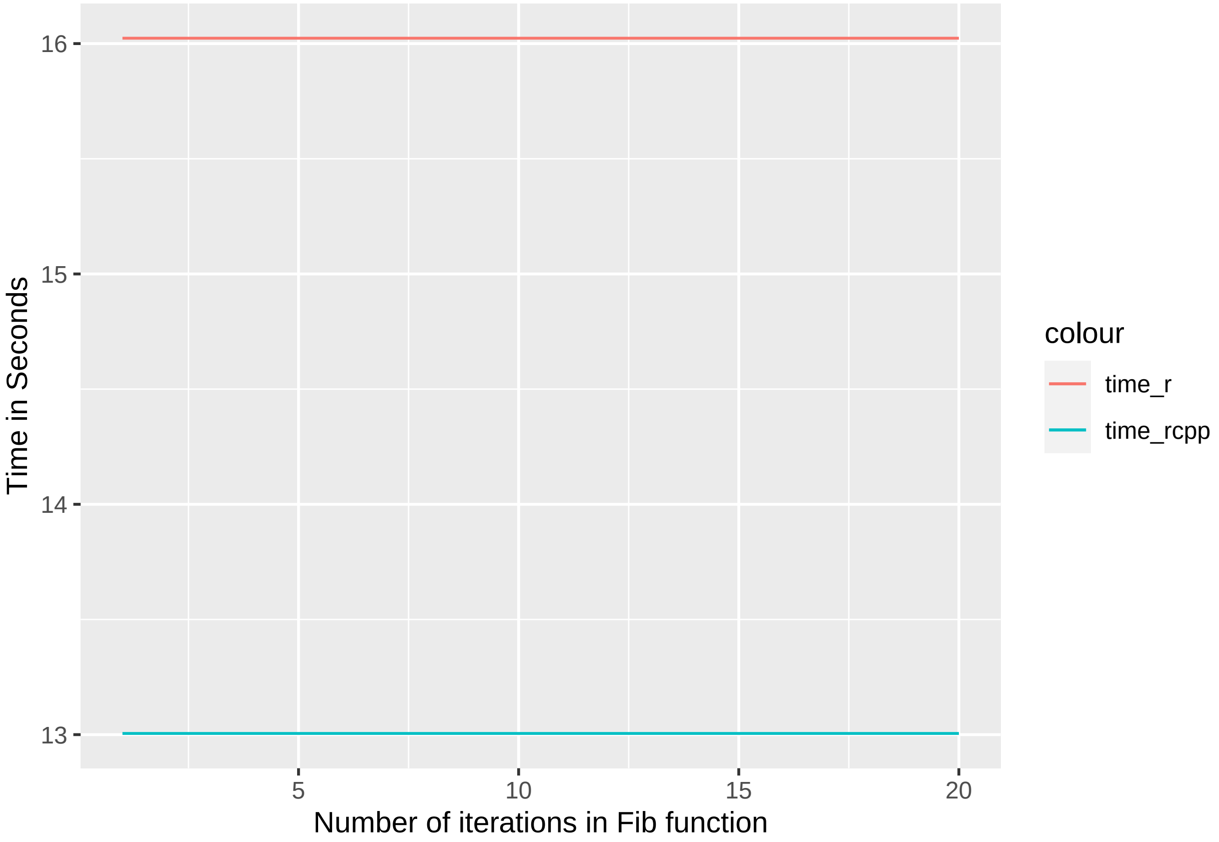
```
## [1] 55
```

Time the difference in these functions for n = 100, 200, ..., 1500 while starting the sequence at the smallest possible floating point value in R. Store the results in a matrix.

```
n=20
time_r = c()
time_rcpp = c()
for(i in n){
  time_r = c(time_r, mean(microbenchmark(fib_r = nth_fibonacci(i, .Machine$double.xmin), times = 3, uni
  time_rcpp = c(time_rcpp, mean(microbenchmark(fib_rcpp = nth_fibonacci_cpp(i, .Machine$double.xmin), t
}
```

Plot the divergence of performance (in log seconds) over n using a line geometry. Use two different colors for the R and CPP functions. Make sure there's a color legend on your plot.

```
ggplot() +
  geom_line(aes(x = 1:n, y = log(time_r), col = "time_r")) +
  geom_line(aes(x = 1:n, y = log(time_rcpp), col = "time_rcpp")) +
  xlab("Number of iterations in Fib function") +
  ylab("Time in Seconds")
```



Data Wrangling / Munging / Carpentry

Throughout this assignment you can use either the `tidyverse` package suite or `data.table` to answer but not base R. You can mix `data.table` with `magrittr` piping if you wish but don't go back and forth between `tbl_df`'s and `data.table` objects.

```
pacman::p_load(dplyr, magrittr, data.table)
```

Load the `storms` dataset from the `dplyr` package and investigate it using `str` and `summary` and `head`. Which two columns should be converted to type factor? Do so below.

```
data(storms)
str(storms)
```

```
## tibble[,13] [10,010 x 13] (S3: tbl_df/tbl/data.frame)
##  $ name       : chr [1:10010] "Amy" "Amy" "Amy" "Amy" ...
##  $ year       : num [1:10010] 1975 1975 1975 1975 1975 ...
##  $ month      : num [1:10010] 6 6 6 6 6 6 6 6 6 6 ...
##  $ day        : int [1:10010] 27 27 27 27 28 28 28 28 29 29 ...
##  $ hour       : num [1:10010] 0 6 12 18 0 6 12 18 0 6 ...
##  $ lat        : num [1:10010] 27.5 28.5 29.5 30.5 31.5 32.4 33.3 34 34.4 34 ...
##  $ long       : num [1:10010] -79 -79 -79 -79 -78.8 -78.7 -78 -77 -75.8 -74.8 ...
##  $ status     : chr [1:10010] "tropical depression" "tropical depression" "tropical depression" "trop
##  $ category   : Ord.factor w/ 7 levels "-1"<"0"<"1"<"2"<..: 1 1 1 1 1 1 1 1 2 2 ...
##  $ wind       : int [1:10010] 25 25 25 25 25 25 25 30 35 40 ...
##  $ pressure   : int [1:10010] 1013 1013 1013 1013 1012 1012 1011 1006 1004 1002 ...
##  $ ts_diameter: num [1:10010] NA NA NA NA NA NA NA NA NA NA ...
##  $ hu_diameter: num [1:10010] NA NA NA NA NA NA NA NA NA NA ...
```

Reorder the columns so name is first, status is second, category is third and the rest are the same.

```
storms%>%
  select(name, status, category,everything())
```

```
## # A tibble: 10,010 x 13
##    name  status      category   year month   day  hour   lat  long  wind pressure
##    <chr> <chr>       <ord>    <dbl> <dbl> <int> <dbl> <dbl> <dbl> <int>    <int>
##  1 Amy   tropical d~ -1        1975     6    27     0  27.5 -79      25     1013
##  2 Amy   tropical d~ -1        1975     6    27     6  28.5 -79      25     1013
##  3 Amy   tropical d~ -1        1975     6    27    12  29.5 -79      25     1013
##  4 Amy   tropical d~ -1        1975     6    27    18  30.5 -79      25     1013
##  5 Amy   tropical d~ -1        1975     6    28     0  31.5 -78.8    25     1012
##  6 Amy   tropical d~ -1        1975     6    28     6  32.4 -78.7    25     1012
##  7 Amy   tropical d~ -1        1975     6    28    12  33.3 -78      25     1011
##  8 Amy   tropical d~ -1        1975     6    28    18  34   -77      30     1006
##  9 Amy   tropical s~ 0         1975     6    29     0  34.4 -75.8    35     1004
## 10 Amy   tropical s~ 0         1975     6    29     6  34   -74.8    40     1002
## # ... with 10,000 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Find a subset of the data of storms only in the 1970's.

```
storms%>%
  filter(year>=1970 & year<=1979)
```

```
## # A tibble: 546 x 13
##    name   year month   day  hour   lat  long  status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Amy    1975     6    27     0  27.5 -79   tropical d~ -1          25     1013
##  2 Amy    1975     6    27     6  28.5 -79   tropical d~ -1          25     1013
##  3 Amy    1975     6    27    12  29.5 -79   tropical d~ -1          25     1013
##  4 Amy    1975     6    27    18  30.5 -79   tropical d~ -1          25     1013
##  5 Amy    1975     6    28     0  31.5 -78.8 tropical d~ -1          25     1012
##  6 Amy    1975     6    28     6  32.4 -78.7 tropical d~ -1          25     1012
##  7 Amy    1975     6    28    12  33.3 -78   tropical d~ -1          25     1011
##  8 Amy    1975     6    28    18  34   -77   tropical d~ -1          30     1006
##  9 Amy    1975     6    29     0  34.4 -75.8 tropical s~ 0           35     1004
## 10 Amy    1975     6    29     6  34   -74.8 tropical s~ 0           40     1002
## # ... with 536 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Find a subset of the data of storm observations only with category 4 and above and wind speed 100MPH and above.

```
storms%>%
  filter(category >= 4 & wind >=100)
```

```
## # A tibble: 416 x 13
##    name   year month   day  hour   lat  long status    category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>     <ord>    <int>    <int>
##  1 Anita  1977     9     2     0  24.6 -96.2 hurricane 5          140      931
##  2 Anita  1977     9     2     6  24.2 -97.1 hurricane 5          150      926
##  3 Anita  1977     9     2    12  23.7 -98   hurricane 4          120      940
##  4 David  1979     8    28     0  12.2 -52.9 hurricane 4          115      947
##  5 David  1979     8    28     6  12.5 -54.4 hurricane 4          125      941
##  6 David  1979     8    28    12  12.8 -55.7 hurricane 4          130      938
##  7 David  1979     8    28    18  13.2 -56.9 hurricane 4          125      941
##  8 David  1979     8    29     0  13.7 -58   hurricane 4          120      944
##  9 David  1979     8    29     6  14.2 -59.2 hurricane 4          120      942
## 10 David  1979     8    29    12  14.8 -60.3 hurricane 4          125      938
## # ... with 406 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Create a new feature `wind_speed_per_unit_pressure`.

```
storms %<>%
  mutate(wind_speed_per_unit_pressure = wind/pressure)
```

Create a new feature: `average_diameter` which averages the two diameter metrics. If one is missing, then use the value of the one that is present. If both are missing, leave missing.

```
storms %>%
  rowwise() %>%
  arrange(desc(year)) %>%
  mutate(average_diameter = mean(c(ts_diameter, hu_diameter), na.rm = TRUE))
```

```
## # A tibble: 10,010 x 15
## # Rowwise:
##    name   year month   day  hour   lat  long status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Ana    2015     5     9     6  32.2 -77.5 tropical s~ 0           50      998
##  2 Ana    2015     5     9    12  32.5 -77.8 tropical s~ 0           50     1001
##  3 Ana    2015     5     9    18  32.7 -78   tropical s~ 0           45     1001
##  4 Ana    2015     5    10     0  33.1 -78.3 tropical s~ 0           45     1001
##  5 Ana    2015     5    10     6  33.5 -78.6 tropical s~ 0           40     1002
##  6 Ana    2015     5    10    10  33.8 -78.8 tropical s~ 0           40     1002
##  7 Ana    2015     5    10    12  33.9 -78.8 tropical s~ 0           35     1002
##  8 Ana    2015     5    10    18  34.3 -78.7 tropical d~ -1          30     1006
##  9 Ana    2015     5    11     0  34.7 -78.5 tropical d~ -1          30     1009
## 10 Ana    2015     5    11     6  35.5 -78   tropical d~ -1          30     1010
## # ... with 10,000 more rows, and 4 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>,
## #   average_diameter <dbl>
```

For each storm, summarize the maximum wind speed. "Summarize" means create a new dataframe with only the summary metrics you care about.

```
storms %>%
  group_by(name) %>%
  summarise(max_wind_speed = max(wind, na.rm=TRUE))
```

```
## # A tibble: 198 x 2
##    name     max_wind_speed
##    <chr>             <int>
##  1 AL011993             30
##  2 AL012000             25
##  3 AL021992             30
##  4 AL021994             30
##  5 AL021999             30
##  6 AL022000             30
##  7 AL022001             25
##  8 AL022003             30
##  9 AL022006             45
## 10 AL031987             40
## # ... with 188 more rows
```

Order your dataset by maximum wind speed storm but within the rows of storm show the observations in time order from early to late.

```
storms %>%
  group_by(name) %>%
  mutate(max_wind_by_storm = max(wind, na.rm = TRUE)) %>%
  select(name, max_wind_by_storm, everything()) %>%
  arrange(desc(max_wind_by_storm), year, month, day, hour)
```

```
## # A tibble: 10,010 x 15
## # Groups:   name [198]
##    name   max_wind_by_sto~  year month   day  hour   lat  long status   category
##    <chr>             <int> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>    <ord>
##  1 Gilbe~              160  1988     9     8    18  12   -54   tropica~ -1
##  2 Gilbe~              160  1988     9     9     0  12.7 -55.6 tropica~ -1
##  3 Gilbe~              160  1988     9     9     6  13.3 -57.1 tropica~ -1
##  4 Gilbe~              160  1988     9     9    12  14   -58.6 tropica~ -1
##  5 Gilbe~              160  1988     9     9    18  14.5 -60.1 tropica~ 0
##  6 Gilbe~              160  1988     9    10     0  14.8 -61.5 tropica~ 0
##  7 Gilbe~              160  1988     9    10     6  15   -62.8 tropica~ 0
##  8 Gilbe~              160  1988     9    10    12  15.3 -64.1 tropica~ 0
##  9 Gilbe~              160  1988     9    10    18  15.7 -65.4 tropica~ 0
## 10 Gilbe~              160  1988     9    11     0  15.9 -66.8 hurrica~ 1
## # ... with 10,000 more rows, and 5 more variables: wind <int>, pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>
```

Find the strongest storm by wind speed per year.

```
storms %>%
  group_by(year) %>%
  arrange(year, desc(wind))%>%
  slice(1)%>%
  select(name,year)
```

```
## # A tibble: 41 x 2
## # Groups:   year [41]
##    name      year
##    <chr>    <dbl>
##  1 Caroline  1975
##  2 Belle     1976
##  3 Anita     1977
##  4 Cora      1978
##  5 David     1979
##  6 Ivan      1980
##  7 Harvey    1981
##  8 Debby     1982
##  9 Alicia    1983
## 10 Diana     1984
## # ... with 31 more rows
```

For each named storm, find its maximum category, wind speed, pressure and diameters. Do not allow the max to be NA (unless all the measurements for that storm were NA).

```
storms %>%
  group_by(name)%>%
  mutate(max_wind = max(wind,na.rm = TRUE))%>%
  mutate(max_pressure = max(pressure,na.rm = TRUE))%>%
  mutate(max_ts_diameter = max(ts_diameter, na.rm = TRUE))%>%
  mutate(max_hu_diameter = max(hu_diameter, na.rm = TRUE))%>%
  slice(1)%>%
  select(name, max_wind, max_pressure, max_ts_diameter, max_hu_diameter)
```

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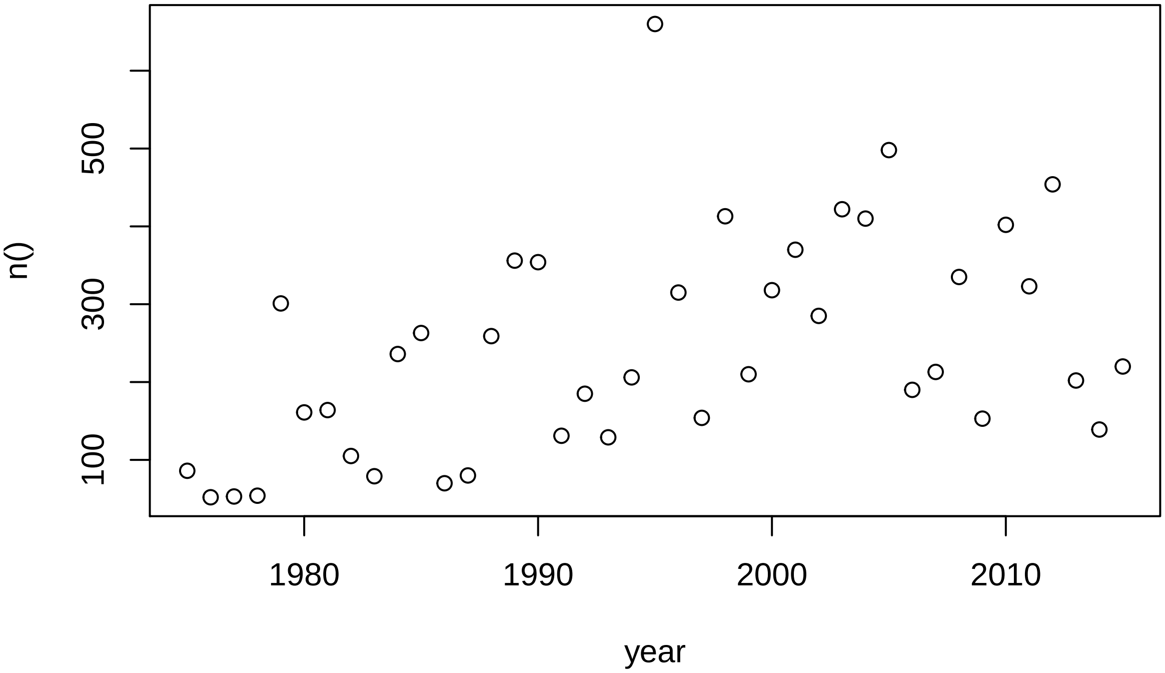
```

```
## # A tibble: 198 x 5
## # Groups:   name [198]
##    name     max_wind max_pressure max_ts_diameter max_hu_diameter
##    <chr>       <int>        <int>           <dbl>           <dbl>
##  1 AL011993       30         1003          -Inf            -Inf
##  2 AL012000       25         1010          -Inf            -Inf
##  3 AL021992       30         1009          -Inf            -Inf
##  4 AL021994       30         1017          -Inf            -Inf
##  5 AL021999       30         1006          -Inf            -Inf
##  6 AL022000       30         1010          -Inf            -Inf
##  7 AL022001       25         1012          -Inf            -Inf
##  8 AL022003       30         1010          -Inf            -Inf
##  9 AL022006       45         1008            69.0             0
## 10 AL031987       40         1015          -Inf            -Inf
## # ... with 188 more rows
```

For each year in the dataset, tally the number of storms. “Tally” is a fancy word for “count the number of”. Plot the number of storms by year. Any pattern?

```
storms%>%
  group_by(year)%>%
  summarise(n())%>%
  plot()
```



For each year in the dataset, tally the storms by category.

```
storms%>%
  group_by(year, category)%>%
  summarise(n())
```

```
## `summarise()` has grouped output by 'year'. You can override using the `.groups` argument.
```

```
## # A tibble: 233 x 3
## # Groups:   year [41]
##     year category `n()`
##    <dbl> <ord>    <int>
##  1  1975 -1          30
##   2  1975 0           33
##   3  1975 1           12
##   4  1975 2            9
##   5  1975 3            2
##   6  1976 -1          10
##   7  1976 0           20
##   8  1976 1           10
##   9  1976 2            9
##  10  1976 3            3
## # ... with 223 more rows
```

For each year in the dataset, find the maximum wind speed per status level.

```
storms%>%
  group_by(year,status)%>%
  summarise(max_wind = max(wind))
```

```
## `summarise()` has grouped output by 'year'. You can override using the `.groups` argument.
```

```
## # A tibble: 123 x 3
## # Groups:   year [41]
##     year status              max_wind
##    <dbl> <chr>                  <int>
##   1  1975 hurricane               100
##   2  1975 tropical depression      30
##   3  1975 tropical storm           60
##   4  1976 hurricane               105
##   5  1976 tropical depression      30
##   6  1976 tropical storm           60
##   7  1977 hurricane               150
##   8  1977 tropical depression      30
##   9  1977 tropical storm           60
##  10  1978 hurricane                80
## # ... with 113 more rows
```

For each storm, summarize its average location in latitude / longitude coordinates.

```
storms%>%
  group_by(name)%>%
  summarise(avg_latitude = mean(lat), avg_longitude = mean(long))
```

```
## # A tibble: 198 x 3
##    name     avg_latitude avg_longitude
##    <chr>           <dbl>         <dbl>
##   1 AL011993        24.7          -78.0
##   2 AL012000        20.8          -93.1
##   3 AL021992        26.7          -84.5
##   4 AL021994        33.6          -79.7
##   5 AL021999        20.4          -96.4
##   6 AL022000         9.9          -28.5
##   7 AL022001        11.9          -45.3
##   8 AL022003         9.62         -43.4
##   9 AL022006        41.3          -63.5
##  10 AL031987        30.8          -88.7
## # ... with 188 more rows
```

For each storm, summarize its duration in number of hours (to the nearest 6hr increment).

```
storms %>%
  group_by(name) %>%
  mutate(duration = (n()-1)*6) %>%
  select(name, duration) %>%
  distinct
```

```
## # A tibble: 198 x 2
## # Groups:   name [198]
##    name     duration
##    <chr>       <dbl>
##  1 Amy           174
##  2 Caroline      192
##  3 Doris         132
##  4 Belle         102
##  5 Gloria        744
##  6 Anita         114
##  7 Clara         138
##  8 Evelyn         48
##  9 Amelia         30
## 10 Bess           72
## # ... with 188 more rows
```

For storm in a category, create a variable `storm_number` that enumerates the storms 1, 2, . . . (in date order).

```
storms %>%
  group_by(category, name) %>%
  slice(1) %>%
  group_by(category) %>%
  mutate(storm_number = dense_rank(paste(year, as.numeric(month), day))) %>%
  select(category, storm_number, year, month, day, name) %>%
  distinct %>%
  arrange(category, storm_number)
```

```
## # A tibble: 687 x 6
## # Groups:   category [7]
##    category storm_number  year month   day name
##    <ord>           <int> <dbl> <dbl> <int> <chr>
##  1 -1                  1  1975     6    27 Amy
##  2 -1                  2  1975     8    24 Caroline
##  3 -1                  3  1976     8     6 Belle
##  4 -1                  4  1976     9    26 Gloria
##  5 -1                  5  1977    10    13 Evelyn
##  6 -1                  6  1977     8    29 Anita
##  7 -1                  7  1977     9     5 Clara
##  8 -1                  8  1978    10     7 Juliet
##  9 -1                  9  1978     7    30 Amelia
## 10 -1                 10  1978     8     5 Bess
## # ... with 677 more rows
```

```
#since some storms span multiple days, and go into different categories as time processes, I counted th
```

Convert year, month, day, hour into the variable `timestamp` using the `lubridate` package. Although the new package `clock` just came out, `lubridate` still seems to be standard. Next year I'll probably switch the class to be using `clock`.

```
pacman :: p_load(lubridate)
storms %<>%
  mutate(timestamp = make_datetime(year, month, day, hour)) %>%
  select(timestamp, everything())
```

Using the `lubridate` package, create new variables `day_of_week` which is a factor with levels "Sunday", "Monday", ... "Saturday" and `week_of_year` which is integer 1, 2, . . . , 52.

```
storms%>%
  mutate(day_of_week = weekdays(as.Date(paste(year, month, day,hour, sep = "-"))))%>%
  mutate(week_of_year = week(as.Date(paste(year, month, day, hour,sep = "-"))))
```

```
## # A tibble: 10,010 x 17
##    timestamp           name   year month   day  hour   lat  long status category
##    <dttm>              <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>  <ord>
##  1 1975-06-27 00:00:00 Amy    1975     6    27     0  27.5 -79   tropi~ -1
##  2 1975-06-27 06:00:00 Amy    1975     6    27     6  28.5 -79   tropi~ -1
##  3 1975-06-27 12:00:00 Amy    1975     6    27    12  29.5 -79   tropi~ -1
##  4 1975-06-27 18:00:00 Amy    1975     6    27    18  30.5 -79   tropi~ -1
##  5 1975-06-28 00:00:00 Amy    1975     6    28     0  31.5 -78.8 tropi~ -1
##  6 1975-06-28 06:00:00 Amy    1975     6    28     6  32.4 -78.7 tropi~ -1
##  7 1975-06-28 12:00:00 Amy    1975     6    28    12  33.3 -78   tropi~ -1
##  8 1975-06-28 18:00:00 Amy    1975     6    28    18  34   -77   tropi~ -1
##  9 1975-06-29 00:00:00 Amy    1975     6    29     0  34.4 -75.8 tropi~ 0
## 10 1975-06-29 06:00:00 Amy    1975     6    29     6  34   -74.8 tropi~ 0
## # ... with 10,000 more rows, and 7 more variables: wind <int>, pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>,
## #   day_of_week <chr>, week_of_year <dbl>
```

For each storm, summarize the day in which is started in the following format "Friday, June 27, 1975".

```
storms %>%
  group_by(name) %>%
  summarize(start_date = min(timestamp)) %>%
  mutate(start_date = paste(weekdays(start_date),
                            paste(months(start_date), day(start_date), sep = " "),
                            year(start_date), sep = ", "))
```

```
## # A tibble: 198 x 2
##    name     start_date
##    <chr>    <chr>
##  1 AL011993 Monday, May 31, 1993
##  2 AL012000 Wednesday, June 7, 2000
##  3 AL021992 Thursday, June 25, 1992
##  4 AL021994 Wednesday, July 20, 1994
##  5 AL021999 Friday, July 2, 1999
##  6 AL022000 Friday, June 23, 2000
##  7 AL022001 Wednesday, July 11, 2001
##  8 AL022003 Wednesday, June 11, 2003
##  9 AL022006 Monday, July 17, 2006
## 10 AL031987 Sunday, August 9, 1987
## # ... with 188 more rows
```

Create a new factor variable `decile_windspeed` by binning wind speed into 10 bins.

```
bins = 0:10
storms%>%
  mutate(decile_windspeed = factor(ntile(wind,10)))
```

```
## # A tibble: 10,010 x 16
##    timestamp           name   year month   day  hour   lat  long status category
##    <dttm>              <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>  <ord>
##  1 1975-06-27 00:00:00 Amy    1975     6    27     0  27.5 -79   tropi~ -1
##  2 1975-06-27 06:00:00 Amy    1975     6    27     6  28.5 -79   tropi~ -1
##  3 1975-06-27 12:00:00 Amy    1975     6    27    12  29.5 -79   tropi~ -1
##  4 1975-06-27 18:00:00 Amy    1975     6    27    18  30.5 -79   tropi~ -1
##  5 1975-06-28 00:00:00 Amy    1975     6    28     0  31.5 -78.8 tropi~ -1
##  6 1975-06-28 06:00:00 Amy    1975     6    28     6  32.4 -78.7 tropi~ -1
##  7 1975-06-28 12:00:00 Amy    1975     6    28    12  33.3 -78   tropi~ -1
##  8 1975-06-28 18:00:00 Amy    1975     6    28    18  34   -77   tropi~ -1
##  9 1975-06-29 00:00:00 Amy    1975     6    29     0  34.4 -75.8 tropi~ 0
## 10 1975-06-29 06:00:00 Amy    1975     6    29     6  34   -74.8 tropi~ 0
## # ... with 10,000 more rows, and 6 more variables: wind <int>, pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>,
## #   decile_windspeed <fct>
```

Create a new data frame `serious_storms` which are category 3 and above hurricanes.

```
serious_storms = storms%>%
  filter(category >=3)
serious_storms
```

```
## # A tibble: 779 x 15
##    timestamp           name   year month   day  hour   lat  long status category
##    <dttm>              <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>  <ord>
##  1 1975-08-31 00:00:00 Caro~  1975     8    31     0  24   -97   hurri~ 3
##  2 1975-08-31 06:00:00 Caro~  1975     8    31     6  24.1 -97.5 hurri~ 3
##  3 1976-08-08 18:00:00 Belle  1976     8     8    18  29.5 -75.3 hurri~ 3
##  4 1976-08-09 00:00:00 Belle  1976     8     9     0  30.9 -75.3 hurri~ 3
##  5 1976-08-09 06:00:00 Belle  1976     8     9     6  32.5 -75.2 hurri~ 3
##  6 1977-09-01 18:00:00 Anita  1977     9     1    18  25.2 -95.5 hurri~ 3
##  7 1977-09-02 00:00:00 Anita  1977     9     2     0  24.6 -96.2 hurri~ 5
##  8 1977-09-02 06:00:00 Anita  1977     9     2     6  24.2 -97.1 hurri~ 5
##  9 1977-09-02 12:00:00 Anita  1977     9     2    12  23.7 -98   hurri~ 4
## 10 1979-08-28 00:00:00 David  1979     8    28     0  12.2 -52.9 hurri~ 4
## # ... with 769 more rows, and 5 more variables: wind <int>, pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>
```

In `serious_storms`, merge the variables lat and long together into `lat_long` with values `lat / long` as a string.

```
#serious_storms %<>%
 # unite(lat_long, lat, long, sep = " / ")
```

Let's return now to the original storms data frame. For each category, find the average wind speed, pressure and diameters (do not count the NA's in your averaging).

```
storms %>%
  group_by(category) %>%
  summarize(avg_wind_speed = mean(wind),
            avg_pressure = mean(pressure),
            avg_ts_diam = mean(ts_diameter, na.rm = TRUE),
            avg_hu_diam = mean(hu_diameter, na.rm = TRUE))
```

```
## # A tibble: 7 x 5
##   category avg_wind_speed avg_pressure avg_ts_diam avg_hu_diam
##   <ord>             <dbl>        <dbl>       <dbl>       <dbl>
## 1 -1                 27.3        1008.          0          0
## 2 0                  45.8         999.        160.         0
## 3 1                  70.9         982.        278.        57.3
## 4 2                  89.4         967.        282.        78.8
## 5 3                 105.          954.        307.        91.4
## 6 4                 122.          940.        315.       102.
## 7 5                 145.          916.        317.       120.
```

For each named storm, find its maximum category, wind speed, pressure and diameters (do not allow the max to be NA) and the number of readings (i.e. observations).

```
storms %>%
  filter(!is.na(ts_diameter), !is.na(hu_diameter)) %>%
  group_by(name) %>%
  mutate(max_category = max(category),
            max_wind = max(wind),
            max_pressure = max(pressure),
            max_ts_diam = max(ts_diameter),
            max_hu_diam = max(hu_diameter))
```

```
## # A tibble: 3,482 x 20
## # Groups:   name [114]
##    timestamp           name   year month   day  hour   lat  long status category
##    <dttm>              <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>  <ord>
##  1 2004-07-31 18:00:00 Alex   2004     7    31    18  30.3 -78.3 tropi~ -1
##  2 2004-08-01 00:00:00 Alex   2004     8     1     0  31   -78.8 tropi~ -1
##  3 2004-08-01 06:00:00 Alex   2004     8     1     6  31.5 -79   tropi~ -1
##  4 2004-08-01 12:00:00 Alex   2004     8     1    12  31.6 -79.1 tropi~ -1
##  5 2004-08-01 18:00:00 Alex   2004     8     1    18  31.6 -79.2 tropi~ 0
##  6 2004-08-02 00:00:00 Alex   2004     8     2     0  31.5 -79.3 tropi~ 0
##  7 2004-08-02 06:00:00 Alex   2004     8     2     6  31.4 -79.4 tropi~ 0
##  8 2004-08-02 12:00:00 Alex   2004     8     2    12  31.3 -79   tropi~ 0
##  9 2004-08-02 18:00:00 Alex   2004     8     2    18  31.8 -78.7 tropi~ 0
## 10 2004-08-03 00:00:00 Alex   2004     8     3     0  32.4 -78.2 tropi~ 0
## # ... with 3,472 more rows, and 10 more variables: wind <int>, pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>,
## #   max_category <ord>, max_wind <int>, max_pressure <int>, max_ts_diam <dbl>,
## #   max_hu_diam <dbl>
```

Calculate the distance from each storm observation to Miami in a new variable `distance_to_miami`. This is very challenging. You will need a function that computes distances from two sets of latitude / longitude coordinates.

```
MIAMI_LAT_LONG_COORDS = c(25.7617, -80.1918)

find_distance = function(lat_1, long_1, lat_2, long_2){
  lat_1 = lat_1*180/pi
  long_1 = long_1*180/pi
  lat_2 = lat_2*180/pi
  long_2 = long_2*180/pi
  a = sin(lat_2 - lat_1 / 2)^2 + (cos(lat_2) * cos(lat_1)) * sin(long_2 - long_1 / 2)^2
  c = 2 * atan2(sqrt(a), sqrt(1-a))
  distance = 6373.0 * c #multiply by radius of earth in km
  distance
}

storms %>%
  mutate(distance_to_miami = find_distance(lat,long, MIAMI_LAT_LONG_COORDS[1], MIAMI_LAT_LONG_COORDS[2])
```

```
## Warning in sqrt(a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced
```

```
## # A tibble: 10,010 x 16
##    timestamp           name   year month   day  hour   lat  long status category
##    <dttm>              <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>  <ord>
##  1 1975-06-27 00:00:00 Amy    1975     6    27     0  27.5 -79   tropi~ -1
##  2 1975-06-27 06:00:00 Amy    1975     6    27     6  28.5 -79   tropi~ -1
##  3 1975-06-27 12:00:00 Amy    1975     6    27    12  29.5 -79   tropi~ -1
##  4 1975-06-27 18:00:00 Amy    1975     6    27    18  30.5 -79   tropi~ -1
##  5 1975-06-28 00:00:00 Amy    1975     6    28     0  31.5 -78.8 tropi~ -1
##  6 1975-06-28 06:00:00 Amy    1975     6    28     6  32.4 -78.7 tropi~ -1
##  7 1975-06-28 12:00:00 Amy    1975     6    28    12  33.3 -78   tropi~ -1
##  8 1975-06-28 18:00:00 Amy    1975     6    28    18  34   -77   tropi~ -1
##  9 1975-06-29 00:00:00 Amy    1975     6    29     0  34.4 -75.8 tropi~ 0
## 10 1975-06-29 06:00:00 Amy    1975     6    29     6  34   -74.8 tropi~ 0
## # ... with 10,000 more rows, and 6 more variables: wind <int>, pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>,
## #   distance_to_miami <dbl>
```

For each storm observation, use the function from the previous question to calculate the distance it moved since the previous observation.

```
storms %<>%

  mutate(dist_from_prev = ifelse(name != lag(name), 0, find_distance(lat, long, lag(lat), lag(long)))) %
  mutate(dist_from_prev = ifelse(is.na(dist_from_prev), 0, dist_from_prev))
```

```
## Warning in sqrt(a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced
```

```
head(storms)
```

```
## # A tibble: 6 x 16
##   timestamp           name   year month   day  hour   lat  long status   category
##   <dttm>              <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>    <ord>
## 1 1975-06-27 00:00:00 Amy    1975     6    27     0  27.5 -79   tropic~ -1
## 2 1975-06-27 06:00:00 Amy    1975     6    27     6  28.5 -79   tropic~ -1
## 3 1975-06-27 12:00:00 Amy    1975     6    27    12  29.5 -79   tropic~ -1
## 4 1975-06-27 18:00:00 Amy    1975     6    27    18  30.5 -79   tropic~ -1
## 5 1975-06-28 00:00:00 Amy    1975     6    28     0  31.5 -78.8 tropic~ -1
## 6 1975-06-28 06:00:00 Amy    1975     6    28     6  32.4 -78.7 tropic~ -1
## # ... with 6 more variables: wind <int>, pressure <int>, ts_diameter <dbl>,
## #   hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>, dist_from_prev <dbl>
```

For each storm, find the total distance it moved over its observations and its total displacement. “Distance” is a scalar quantity that refers to “how much ground an object has covered” during its motion. “Displacement” is a vector quantity that refers to “how far out of place an object is”; it is the object’s overall change in position.

```
storms%>%
  group_by(name)%>%
  mutate(distance = sum(dist_from_prev), displacement = paste(round(last(lat)-first(lat),2),round(last(
```

```
## # A tibble: 10,010 x 18
## # Groups:   name [198]
##    timestamp           name   year month   day  hour   lat  long status category
##    <dttm>              <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>  <ord>
##  1 1975-06-27 00:00:00 Amy    1975     6    27     0  27.5 -79   tropi~ -1
##  2 1975-06-27 06:00:00 Amy    1975     6    27     6  28.5 -79   tropi~ -1
##  3 1975-06-27 12:00:00 Amy    1975     6    27    12  29.5 -79   tropi~ -1
##  4 1975-06-27 18:00:00 Amy    1975     6    27    18  30.5 -79   tropi~ -1
##  5 1975-06-28 00:00:00 Amy    1975     6    28     0  31.5 -78.8 tropi~ -1
##  6 1975-06-28 06:00:00 Amy    1975     6    28     6  32.4 -78.7 tropi~ -1
##  7 1975-06-28 12:00:00 Amy    1975     6    28    12  33.3 -78   tropi~ -1
##  8 1975-06-28 18:00:00 Amy    1975     6    28    18  34   -77   tropi~ -1
##  9 1975-06-29 00:00:00 Amy    1975     6    29     0  34.4 -75.8 tropi~ 0
## 10 1975-06-29 06:00:00 Amy    1975     6    29     6  34   -74.8 tropi~ 0
## # ... with 10,000 more rows, and 8 more variables: wind <int>, pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>,
## #   dist_from_prev <dbl>, distance <dbl>, displacement <chr>
```

For each storm observation, calculate the average speed the storm moved in location.

```
storms %<>%
  mutate(speed = dist_from_prev/6)
```

For each storm, calculate its average ground speed (how fast its eye is moving which is different from windspeed around the eye).

```
storms%>%
  group_by(name)%>%
  mutate(avg_ground_speed = mean(speed,na.rm=TRUE))
```

```
## # A tibble: 10,010 x 18
## # Groups:   name [198]
##    timestamp           name   year month   day  hour   lat  long status category
##    <dttm>              <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>  <ord>
##  1 1975-06-27 00:00:00 Amy    1975     6    27     0  27.5 -79   tropi~ -1
##  2 1975-06-27 06:00:00 Amy    1975     6    27     6  28.5 -79   tropi~ -1
##  3 1975-06-27 12:00:00 Amy    1975     6    27    12  29.5 -79   tropi~ -1
##  4 1975-06-27 18:00:00 Amy    1975     6    27    18  30.5 -79   tropi~ -1
##  5 1975-06-28 00:00:00 Amy    1975     6    28     0  31.5 -78.8 tropi~ -1
##  6 1975-06-28 06:00:00 Amy    1975     6    28     6  32.4 -78.7 tropi~ -1
##  7 1975-06-28 12:00:00 Amy    1975     6    28    12  33.3 -78   tropi~ -1
##  8 1975-06-28 18:00:00 Amy    1975     6    28    18  34   -77   tropi~ -1
##  9 1975-06-29 00:00:00 Amy    1975     6    29     0  34.4 -75.8 tropi~ 0
## 10 1975-06-29 06:00:00 Amy    1975     6    29     6  34   -74.8 tropi~ 0
## # ... with 10,000 more rows, and 8 more variables: wind <int>, pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>,
## #   dist_from_prev <dbl>, speed <dbl>, avg_ground_speed <dbl>
```

Is there a relationship between average ground speed and maximum category attained? Use a dataframe summary (not a regression).

```
speed_and_category <- storms %>%
  group_by(name) %>%
  summarize(avg_ground_speed = mean(speed), maximum_category = as.numeric(max(category)))

cor(speed_and_category$avg_ground_speed, speed_and_category$maximum_category)
```

```
## [1] 0.2554108
```

Now we want to transition to building real design matrices for prediction. This is more in tune with what happens in the real world. Large data dump and you convert it into X and y how you see fit.

Suppose we wish to predict the following: given the first three readings of a storm, can you predict its maximum wind speed? Identify the `y` and identify which features you need $x_1, ... x_p$ and build that matrix with `dplyr` functions. This is not easy, but it is what it's all about. Feel free to "featurize" as creatively as you would like. You aren't going to overfit if you only build a few features relative to the total 198 storms.

Fit your model. Validate it.

Assess your level of success at this endeavor.

#TO-DO # The Forward Stepwise Procedure for Probability Estimation Models

Set a seed and load the `adult` dataset and remove missingness and randomize the order.

```
set.seed(1)
pacman::p_load_gh("coatless/ucidata")
```

```
## * checking for file '/tmp/RtmpfIaeE4/remotes3b736a57438/coatless-ucidata-edcdc13/DESCRIPTION' ... OK
## * preparing 'ucidata':
## * checking DESCRIPTION meta-information ... OK
## * checking for LF line-endings in source and make files and shell scripts
## * checking for empty or unneeded directories
## * building 'ucidata_0.0.3.tar.gz'
```

```
data(adult)
adult = na.omit(adult)
adult = adult[sample(1 : nrow(adult)), ]
```

Copy from the previous lab all cleanups you did to this dataset.

```
adult$fnlwgt = NULL

adult$marital_status = as.character(adult$marital_status)
adult$marital_status = ifelse(adult$marital_status == "Married-AF-spouse" | adult$marital_status == "Ma
adult$marital_status = as.factor(adult$marital_status)

adult$education = as.character(adult$education)
adult$education = ifelse(adult$education == "1st-4th" | adult$education == "Preschool", "<=4th", adult$e
adult$education = as.factor(adult$education)
adult$education = NULL

tab = sort(table(adult$native_country))
adult$native_country = as.character(adult$native_country)
adult$native_country= ifelse(adult$native_country %in% names(tab[tab<50]), "Other", adult$native_country
adult$native_country= as.factor(adult$native_country)

adult$worktype = paste(adult$occupation, adult$workclass, sep = ":")
tab_worktype = sort(table(adult$worktype))
```

```
adult$occupation = NULL
adult$workclass = NULL

adult$worktype = as.character(adult$worktype)
adult$worktype = ifelse(adult$worktype %in% names(tab_worktype[tab_worktype<100]), "Other", adult$workty
adult$worktype = as.factor(adult$worktype)

adult$status = paste(as.character(adult$relationship), as.character(adult$marital_status), sep = ":")
adult$status = as.character(adult$status)
tab_status = sort(table(adult$status))
adult$relationship = NULL
adult$marital_status = NULL
adult$status = as.factor(adult$status)
```

We will be doing model selection. We will split the dataset into 3 distinct subsets. Set the size of our splits here. For simplicitiy, all three splits will be identically sized. We are making it small so the stepwise algorithm can compute quickly. If you have a faster machine, feel free to increase this.

```
Nsplitsize = 1000
```

Now create the following variables: `Xtrain`, `ytrain`, `Xselect`, `yselect`, `Xtest`, `ytest` with `Nsplitsize` observations. Binarize the y values.

```
Xtrain = adult[1 : Nsplitsize, ]
Xtrain$income = NULL
ytrain = ifelse(adult[1 : Nsplitsize, "income"] == ">50K", 1, 0)
Xselect = adult[(Nsplitsize + 1) : (2 * Nsplitsize), ]
Xselect$income = NULL
yselect = ifelse(adult[(Nsplitsize + 1) : (2 * Nsplitsize), "income"] ==">50K", 1, 0)
Xtest = adult[(2 * Nsplitsize + 1) : (3 * Nsplitsize), ]
Xtest$income = NULL
ytest = ifelse(adult[(2 * Nsplitsize + 1) : (3 * Nsplitsize), "income"] == ">50K", 1, 0)
```

Fit a vanilla logistic regression on the training set.

```
logistic_mod = glm(ytrain ~ ., Xtrain, family = binomial(link = logit))
```

```
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

and report the log scoring rule, the Brier scoring rule.

```
p_hat_train = predict(logistic_mod, Xtrain, type = 'response')
```

```
## Warning in predict.lm(object, newdata, se.fit, scale = 1, type = if (type == :
## prediction from a rank-deficient fit may be misleading
```

```
#in sample log scoring rule
mean(ytrain * log(p_hat_train) + (1 - ytrain) * log(1 - p_hat_train))
```

```
## [1] -0.2671121
```

```
#in sample Brier scoring rule
mean(-(ytrain - p_hat_train)^2)
```

```
## [1] -0.08715781
```

We will be doing model selection using a basis of linear features consisting of all first-order interactions of the 14 raw features (this will include square terms as squares are interactions with oneself).

Create a model matrix from the training data containing all these features. Make sure it has an intercept column too (the one vector is usually an important feature). Cast it as a data frame so we can use it more easily for modeling later on. We're going to need those model matrices (as data frames) for both the select and test sets. So make them here too (copy-paste). Make sure their dimensions are sensible.

```
Xmm_train = data.frame(model.matrix( ~ . , Xtrain))
Xmm_select = data.frame(model.matrix( ~ . , Xselect))
Xmm_test = data.frame(model.matrix( ~ . , Xtest))

dim(Xmm_train)
```

```
## [1] 1000   93
```

```
dim(Xmm_select)
```

```
## [1] 1000   93
```

```
dim(Xmm_test)
```

```
## [1] 1000   93
```

Write code that will fit a model stepwise. You can refer to the chunk in the practice lecture. Use the negative Brier score to do the selection. The negative of the Brier score is always positive and lower means better making this metric kind of like s_e so the picture will be the same as the canonical U-shape for oos performance.

Run the code and hit "stop" when you begin to the see the Brier score degrade appreciably oos. Be patient as it will wobble.

```
pacman::p_load(Matrix)
p_plus_one = ncol(Xmm_train)
predictor_by_iteration = c() #keep a growing list of predictors by iteration
in_sample_brier_by_iteration = c() #keep a growing list of briers by iteration
oos_brier_by_iteration = c() #keep a growing list of briers by iteration
i = 1

repeat {

  #get all predictors left to try
  all_briers = array(NA, p_plus_one) #record all possibilities
  for (j_try in 1 : p_plus_one){
    if (j_try %in% predictor_by_iteration){
      next
    }
    Xmm_sub = Xmm_train[, c(predictor_by_iteration, j_try), drop = FALSE]
    logistic_mod = suppressWarnings(glm(ytrain ~ ., Xmm_sub, family = "binomial"))
    phat_train = suppressWarnings(predict(logistic_mod, Xmm_sub, type = 'response'))
    all_briers[j_try] = -mean(-(ytrain - phat_train)^2)
  }
  j_star = which.max(all_briers)
  predictor_by_iteration = c(predictor_by_iteration, j_star)
  in_sample_brier_by_iteration = c(in_sample_brier_by_iteration, all_briers[j_star])

  #now let's look at oos
  Xmm_sub = Xmm_train[, predictor_by_iteration, drop = FALSE]

    logistic_mod = suppressWarnings(glm(ytrain ~ ., Xmm_sub, family = "binomial"))
    phat_train = suppressWarnings(predict(logistic_mod, Xmm_sub, type = 'response'))
    all_briers[j_try] = -mean(-(ytrain - phat_train)^2)
    phat_select = suppressWarnings(predict(logistic_mod, Xmm_select[, predictor_by_iteration, drop = FA

    oos_brier = -mean(-(yselect - phat_select)^2)
    oos_brier_by_iteration = c(oos_brier_by_iteration, oos_brier)

  cat("i =", i, "in-sample_brier =", all_briers[j_star], "oos_brier =", oos_brier, "\n   predictor adde

  i = i + 1

  if (i > Nsplitsize || i > p_plus_one){
    break
  }
}
```

```
## i = 1 in-sample_brier = 0.181356 oos_brier = 0.185548
##    predictor added: X.Intercept.
## i = 2 in-sample_brier = 0.181356 oos_brier = 0.185548
##    predictor added: native_countryPoland
## i = 3 in-sample_brier = 0.181356 oos_brier = 0.185548
##    predictor added: statusNot.in.family.Married
## i = 4 in-sample_brier = 0.181356 oos_brier = 0.185548
##    predictor added: statusOther.relative.Separated
## i = 5 in-sample_brier = 0.181356 oos_brier = 0.185548
##    predictor added: statusOther.relative.Widowed
## i = 6 in-sample_brier = 0.181356 oos_brier = 0.185548
##    predictor added: statusOwn.child.Widowed
## i = 7 in-sample_brier = 0.1813554 oos_brier = 0.1855417
##    predictor added: worktypeTransport.moving.Self.emp.not.inc
## i = 8 in-sample_brier = 0.1813548 oos_brier = 0.1855661
##    predictor added: statusUnmarried.Married.spouse.absent
## i = 9 in-sample_brier = 0.1813542 oos_brier = 0.1855927
##    predictor added: worktypeSales.Self.emp.not.inc
## i = 10 in-sample_brier = 0.181353 oos_brier = 0.1856649
##    predictor added: statusUnmarried.Widowed
## i = 11 in-sample_brier = 0.1813499 oos_brier = 0.1856563
##    predictor added: worktypeCraft.repair.Private
## i = 12 in-sample_brier = 0.1813447 oos_brier = 0.1856134
##    predictor added: native_countryIndia
## i = 13 in-sample_brier = 0.1813373 oos_brier = 0.1856355
##    predictor added: native_countryPuerto.Rico
## i = 14 in-sample_brier = 0.1813246 oos_brier = 0.1859607
##    predictor added: worktypeFarming.fishing.Private
## i = 15 in-sample_brier = 0.1813123 oos_brier = 0.1857883
##    predictor added: worktypeFarming.fishing.Self.emp.not.inc
## i = 16 in-sample_brier = 0.1812982 oos_brier = 0.1856838
##    predictor added: statusNot.in.family.Separated
## i = 17 in-sample_brier = 0.1812717 oos_brier = 0.1857482
##    predictor added: native_countryGuatemala
## i = 18 in-sample_brier = 0.1812449 oos_brier = 0.1861397
##    predictor added: worktypeCraft.repair.Local.gov
## i = 19 in-sample_brier = 0.181218 oos_brier = 0.1857469
##    predictor added: worktypeProf.specialty.Federal.gov
## i = 20 in-sample_brier = 0.1811902 oos_brier = 0.1856173
##    predictor added: raceOther
## i = 21 in-sample_brier = 0.1811586 oos_brier = 0.1855962
##    predictor added: worktypeExec.managerial.State.gov
## i = 22 in-sample_brier = 0.1811215 oos_brier = 0.1859505
##    predictor added: worktypeAdm.clerical.Local.gov
## i = 23 in-sample_brier = 0.1810644 oos_brier = 0.185881
##    predictor added: native_countryVietnam
## i = 24 in-sample_brier = 0.1810073 oos_brier = 0.186112
##    predictor added: statusOwn.child.Married
## i = 25 in-sample_brier = 0.1809499 oos_brier = 0.1860419
##    predictor added: native_countryDominican.Republic
## i = 26 in-sample_brier = 0.1809499 oos_brier = 0.1860419
##    predictor added: statusOwn.child.Married.spouse.absent
## i = 27 in-sample_brier = 0.1808553 oos_brier = 0.1860526
##    predictor added: native_countryOther
## i = 28 in-sample_brier = 0.1807887 oos_brier = 0.1862179
##    predictor added: native_countryUnited.States
## i = 29 in-sample_brier = 0.180699 oos_brier = 0.1868485
##    predictor added: worktypeTech.support.Private
## i = 30 in-sample_brier = 0.1805934 oos_brier = 0.1864382
##    predictor added: worktypeOther.service.Local.gov
## i = 31 in-sample_brier = 0.1804642 oos_brier = 0.1848996
##    predictor added: worktypeExec.managerial.Self.emp.inc
## i = 32 in-sample_brier = 0.1803137 oos_brier = 0.1846994
##    predictor added: native_countryJapan
## i = 33 in-sample_brier = 0.1801419 oos_brier = 0.1849772
##    predictor added: worktypeProtective.serv.State.gov
## i = 34 in-sample_brier = 0.1799592 oos_brier = 0.1847671
##    predictor added: statusOther.relative.Divorced
## i = 35 in-sample_brier = 0.179768 oos_brier = 0.1846089
##    predictor added: worktypeProtective.serv.Private
## i = 36 in-sample_brier = 0.1795723 oos_brier = 0.1842935
##    predictor added: worktypeProf.specialty.Local.gov
## i = 37 in-sample_brier = 0.179356 oos_brier = 0.1841564
##    predictor added: native_countryChina
## i = 38 in-sample_brier = 0.1791469 oos_brier = 0.1840683
##    predictor added: native_countryColumbia
## i = 39 in-sample_brier = 0.1789191 oos_brier = 0.1840311
##    predictor added: worktypeOther.service.State.gov
## i = 40 in-sample_brier = 0.1786884 oos_brier = 0.1838212
##    predictor added: statusOwn.child.Divorced
## i = 41 in-sample_brier = 0.1784501 oos_brier = 0.1841729
##    predictor added: native_countryEngland
## i = 42 in-sample_brier = 0.1782019 oos_brier = 0.1839516
##    predictor added: worktypeCraft.repair.Self.emp.not.inc
## i = 43 in-sample_brier = 0.1779541 oos_brier = 0.1839752
##    predictor added: worktypeSales.Self.emp.inc
## i = 44 in-sample_brier = 0.1777008 oos_brier = 0.1845176
##    predictor added: worktypeAdm.clerical.State.gov
## i = 45 in-sample_brier = 0.1774312 oos_brier = 0.1850813
##    predictor added: statusOther.relative.Married.spouse.absent
## i = 46 in-sample_brier = 0.1772429 oos_brier = 0.1851188
##    predictor added: native_countryEl.Salvador
## i = 47 in-sample_brier = 0.1770012 oos_brier = 0.1848479
##    predictor added: worktypeTransport.moving.Local.gov
## i = 48 in-sample_brier = 0.1766289 oos_brier = 0.1852858
##    predictor added: native_countryItaly
## i = 49 in-sample_brier = 0.1762576 oos_brier = 0.1850986
##    predictor added: worktypeTransport.moving.Private
## i = 50 in-sample_brier = 0.1759073 oos_brier = 0.185645
##    predictor added: statusOther.relative.Married
## i = 51 in-sample_brier = 0.1755777 oos_brier = 0.1855656
##    predictor added: worktypePriv.house.serv.Private
## i = 52 in-sample_brier = 0.1752024 oos_brier = 0.1858937
##    predictor added: worktypeOther
## i = 53 in-sample_brier = 0.1748781 oos_brier = 0.1858285
##    predictor added: native_countryGermany
## i = 54 in-sample_brier = 0.1744952 oos_brier = 0.1864225
##    predictor added: native_countryCuba
## i = 55 in-sample_brier = 0.1741871 oos_brier = 0.186287
##    predictor added: statusOwn.child.Separated
## i = 56 in-sample_brier = 0.1737656 oos_brier = 0.1862193
##    predictor added: native_countrySouth
## i = 57 in-sample_brier = 0.1733164 oos_brier = 0.1853527
##    predictor added: worktypeOther.service.Self.emp.not.inc
## i = 58 in-sample_brier = 0.1728051 oos_brier = 0.1853208
##    predictor added: worktypeProf.specialty.Self.emp.inc
## i = 59 in-sample_brier = 0.1722497 oos_brier = 0.1846987
##    predictor added: worktypeSales.Private
## i = 60 in-sample_brier = 0.1717164 oos_brier = 0.1863781
##    predictor added: worktypeProtective.serv.Local.gov
## i = 61 in-sample_brier = 0.1711044 oos_brier = 0.1860013
##    predictor added: statusNot.in.family.Widowed
## i = 62 in-sample_brier = 0.1705002 oos_brier = 0.1857051
##    predictor added: worktypeExec.managerial.Self.emp.not.inc
## i = 63 in-sample_brier = 0.1698833 oos_brier = 0.1865027
##    predictor added: native_countryJamaica
## i = 64 in-sample_brier = 0.1693691 oos_brier = 0.1866908
##    predictor added: raceWhite
## i = 65 in-sample_brier = 0.1686613 oos_brier = 0.1859704
##    predictor added: statusUnmarried.Separated
## i = 66 in-sample_brier = 0.1678313 oos_brier = 0.1864843
##    predictor added: raceBlack
## i = 67 in-sample_brier = 0.1671104 oos_brier = 0.1841216
##    predictor added: worktypeMachine.op.inspct.Private
## i = 68 in-sample_brier = 0.1664096 oos_brier = 0.1846154
##    predictor added: raceAsian.Pac.Islander
## i = 69 in-sample_brier = 0.165671 oos_brier = 0.1834925
##    predictor added: worktypeProf.specialty.Self.emp.not.inc
## i = 70 in-sample_brier = 0.164799 oos_brier = 0.1839977
##    predictor added: native_countryPhilippines
## i = 71 in-sample_brier = 0.1639532 oos_brier = 0.1829634
##    predictor added: statusOther.relative.Never.married
## i = 72 in-sample_brier = 0.1630177 oos_brier = 0.1798843
##    predictor added: worktypeProf.specialty.Private
## i = 73 in-sample_brier = 0.161836 oos_brier = 0.178388
##    predictor added: worktypeHandlers.cleaners.Private
## i = 74 in-sample_brier = 0.1604635 oos_brier = 0.1780931
##    predictor added: worktypeExec.managerial.Local.gov
## i = 75 in-sample_brier = 0.1590754 oos_brier = 0.1803847
##    predictor added: native_countryMexico
## i = 76 in-sample_brier = 0.1576239 oos_brier = 0.18131
##    predictor added: statusNot.in.family.Married.spouse.absent
## i = 77 in-sample_brier = 0.1561724 oos_brier = 0.1814974
##    predictor added: worktypeExec.managerial.Federal.gov
## i = 78 in-sample_brier = 0.154877 oos_brier = 0.1792748
##    predictor added: worktypeAdm.clerical.Private
## i = 79 in-sample_brier = 0.1530984 oos_brier = 0.1792153
##    predictor added: worktypeProf.specialty.State.gov
## i = 80 in-sample_brier = 0.1512046 oos_brier = 0.1803241
##    predictor added: statusUnmarried.Divorced
## i = 81 in-sample_brier = 0.1486265 oos_brier = 0.1798221
##    predictor added: statusUnmarried.Never.married
## i = 82 in-sample_brier = 0.1455114 oos_brier = 0.1793399
##    predictor added: statusWife.Married
## i = 83 in-sample_brier = 0.141789 oos_brier = 0.179233
##    predictor added: statusNot.in.family.Divorced
## i = 84 in-sample_brier = 0.1375809 oos_brier = 0.1772499
##    predictor added: capital_loss
## i = 85 in-sample_brier = 0.1330105 oos_brier = 0.1663411
##    predictor added: hours_per_week
## i = 86 in-sample_brier = 0.1290151 oos_brier = 0.1591097
##    predictor added: worktypeExec.managerial.Private
## i = 87 in-sample_brier = 0.1283621 oos_brier = 0.1569123
##    predictor added: worktypeOther.service.Private
## i = 88 in-sample_brier = 0.1242607 oos_brier = 0.1476126
##    predictor added: education_num
## i = 89 in-sample_brier = 0.1209538 oos_brier = 0.1422338
##    predictor added: statusOwn.child.Never.married
## i = 90 in-sample_brier = 0.1133092 oos_brier = 0.1362918
##    predictor added: sexMale
## i = 91 in-sample_brier = 0.1027663 oos_brier = 0.1329848
##    predictor added: statusNot.in.family.Never.married
## i = 92 in-sample_brier = 0.09516563 oos_brier = 0.1313902
##    predictor added: age
## i = 93 in-sample_brier = 0.08715781 oos_brier = 0.1264595
##    predictor added: capital_gain
```