Explaining the Negative Returns to VIX Futures and ETNs: An Equilibrium Approach

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Abstract

We study the returns to investing in VIX futures and VIX Exchange Traded Notes (ETNs). We document a substantial negative return premium for both ETNs and the futures. For example, the a constant maturity portfolio of one-month VIX futures loses about 30% per year over our sample period (2006-2013). We propose an equilibrium model to explain these negative returns. In this model, increases in volatility endogenously lead to decreasing stock prices. Our model explains the negative expected returns to VIX futures and ETNs as well as several other stylized facts about the returns to VIX futures and VIX futures ETNs.

JEL classification: G12, G13, C22, C58.

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1 Introduction

In 2004 the CBOE Futures Exchange introduced cash settled futures contracts on the CBOE VIX volatility index. While initially sparsely traded, the VIX futures market has become very liquid in recent years. In addition to the futures market itself, since 2009, more than a dozen VIX futures Exchange Traded Notes (ETNs) have been introduced, allowing retail investors to trade VIX futures through regular brokerage accounts. The ETNs follow simple, pre-specified, dynamic trading programs, and in most cases offer constant maturity exposure to n-month futures positions.

The interest in VIX futures and ETNs trading is due at least in part to the perceived positive diversification benefits of the contracts. The CBOE notes through various marketing materials that the VIX correlates negatively with the S&P 500 returns and therefore provides diversification benefits. The CBOE's own estimates of the VIX-return correlation range from -75% to -86%. Additionally, since the VIX is significantly more volatile than the S&P 500 itself, the VIX, and thus VIX futures, have substantial negative market betas.

The first objective of our paper is to provide descriptive statistics on the average returns to VIX futures positions and the associated ETNs. Szado (2009), Alexander and Korovilas (2012) and Whaley (2013) report negative annualized VIX futures returns. We collect futures data from January 2006 to April 2013 and confirm these findings. For example, if an investor were to invest in VIX futures in January 2006 and roll the position at end-of-day futures prices reported by the CBOE, she would have lost more than 97% of the initial investment by the end of March 2013. This corresponds to an annualized return of about negative 30%. This number is staggering considering that during the first part of the sample period the investor would have more than doubled the initial investment through the peak of the 2008 financial crisis. Not surprisingly, the VIX ETNs perform as badly, if not worse, than the underlying futures. In fact, since the first two VIX ETNs were introduced on January 30, 2009, the VXX and VXZ, which offer exposure to short and medium term futures respectively, have lost an average of 34 and 14 basis points per day (simple returns).

Our second and major objective is to understand what causes this large negative return premium in VIX futures and ETNs. we are interested in testing whether the negative returns are consistent with some notion of equilibrium. Since VIX futures have negative market betas, one might expect that they earn negative returns in a CAPM equilibrium. However, we show that standard linear factor models including the CAPM and the Fama-French three factor models cannot explain the returns. We use the equilibrium model of Eraker and Wang (2011) to derive equilibrium VIX futures prices. This model is based on a dynamic present-value framework where investors worry about downside jump-risk differently from what is the case in the traditional, static CAPM. We show that the model produces a very sizable volatility risk premium. Importantly, the model generates an upward sloping equilibrium futures curve (contango) in steady state. This means that, ceteris paribus, investors who purchase VIX futures pay more than the value of the spot VIX at expiration of the futures contract, on average. The equilibrium model produces a negative premium in all states of the world, whether or not the VIX is above or below its steady state value. Even if the futures curve is in backwardation (downward sloping), the futures may imply a negative risk premium because the physical speed of mean reversion will be faster than the Q measure speed of mean reversion implicit in the futures prices. These pricing implications are entirely equilibrium outcomes. If the representative agent in the model is risk-neutral, none of these pricing implications hold. In particular, there is no volatility risk premium, the steady state futures curve is essentially flat, and the expected return on VIX futures is zero. We elaborate further below.

Our paper is connected to the extant literature in several ways. Our theoretical model is related to long-run risk models (Bansal and Yaron (2004)) that deliver large volatility risk premia such as those of Eraker and Shaliastovich (2008), and Drechsler and Yaron (2011). Other theoretical justifications for large volatility risk premia include the heterogeneous beliefs model of Buraschi, Trojani, and Vedolin (2011). Bollerslev, Tauchen, and Zhou (2009), Andersen and Todorov (2013) among others show that the volatility risk premium can predict stock market returns. Eraker (2011) shows that a large volatility risk premium is consistent with large negative equity options returns such as found empirically in Bondarenko (2003), Bakshi and Kapadia (2003) and Eraker (2013), among others. Broadie, Chernov, and Johannes (2007) conclude that jump risk premium, not volatility risk premium, is the primary driver of risk premia in option returns. Recently, Andersen and Todorov (2013) propose a model with a self-exciting jump process but find that this "tail factor" has no incremental power in predicting equity return above the level of volatility itself. This empirical finding lends support to the specification of models in which jump-risks are not disentangled from the diffusive variance, as in our model.

In our model, jump and volatility risk premia obtain endogenously and both are increasing in the level of risk-aversion exerted by the agents. Simplified, if agents are risk averse, they care about the volatility of future cash flows. Their aversion toward high volatility is essentially similar across diffusive and jump driven increments to volatility. Yet, the equilibrium price process we use has characteristics that are similar to existing reduced-form, no-arbitrage models. Our most general model has a two factor volatility specification where jumps in the volatility endogenously lead to negative jumps in the equilibrium price. This is similar, but not identical to, the volatility co-jump models in Duffie, Pan, and Singleton (2000), Eraker, Johannes, and Polson (2003), Bandi and Reno (2012) and Andersen and Todorov (2013) where the correlations between volatility and prices are assumed.

While our paper is the first to attempt a structural explanation for the negative VIX futures return premium, several papers fit statistical models to futures prices and judge the resulting empirical model fit using RMSE or other distance metrics based on the difference between the model and market prices. For example, Zhang and Zhu (2006) analyze the model fit based on Heston (1993), while Lin (2007) and Zhu and Lian (2012) analyze models in the more general class of Duffie, Pan, and Singleton (2000). These papers generally conclude that models with more complicated volatility dynamics (i.e, jumps) are preferable. Egloff, Leippold, and Wu (2010) find that the two factor model of Bates (2000) outperforms Heston's model. Some studies from related markets include Song (2012) who studies returns on VIX options. He finds that both diffusive volatilty-of-volatility and volatility jumps are important in capturing VIX option returns. Carr and Wu (2006) study a sample of returns to variance-swap contracts. Their sample, collected from 1990-2005, contains strikingly large positive (negative) returns to sellers (buyers) of variance swaps.

In our empirical examination we first confirm the large negative returns to VIX futures reported elsewhere. We verify that the negative returns to futures translate into correspondingly negative returns to VIX ETNs that invest in long positions. The returns are particularly bad for short maturity futures and VIX ETNs. Yet, we show that our equilibrium model is generating returns that are almost identical to those we observe in our sample. Our empirical implementation can be summarized as follows: we first estimate our return equilibrium model. This is done using Bayesian MCMC sampler, extending the method in Eraker, Johannes, and Polson (2003) to a structural setting. The advantage of the structural model is that we estimate the risk aversion of the representative agent from returns data alone. We therefore recover the pricing kernel without the use of additional data from derivatives markets. This contrasts empirical studies of reduced form, no-arbitrage models that simultaneously use derivatives and returns data to back out market risk prices, as in Pan (2002), Chernov and Ghysels (2000), Eraker (2004) , Andersen and Todorov (2013), and Jackwerth and Vilkov (2014).

We show that the equilibrium model can explain the negative returns to VIX futures and VIX ETNs almost exactly. In particular, the model that includes volatility jumps fits all moments of maturities that are less than four months. For four and five month futures, the model underestimates the variability (standard deviation) of the futures returns. We argue that the model's inability to account for the return standard deviation of longer maturity contracts is consistent with similar model failures in the affine term structure literature in capturing low frequency movements. We demonstrate that a generalized model that allows for a second volatility factor (i.e, a "central tendency" factor) can be calibrated such that all the moments of the VIX futures data can be matched.

The rest of the paper is organized as follows: in the next section we present basic descriptive evidence on the returns to VIX futures contracts and VIX ETNs. Section 3 presents the equilibrium framework and structural parameter estimates. Section 4 presents our empirical evaluation of the model and compares it to data on VIX futures and VIX ETN options. Section 5 concludes.

2 VIX Futures and ETN Returns

In the following section we provide descriptive evidence of the statistical behavior of VIX futures and ETNs. We start with the futures. Before presenting the evidence, it is worthwhile noting that there are some subtle issues involved in measuring the average returns of VIX futures. In particular, the futures price, like the VIX itself, is extremely volatile. The return distribution also displays right skewness - as the VIX occasionally jumps, a long futures position provides a large positive return. High frequency estimates of average arithmetic returns are upward biased estimates of long horizon buy and hold returns (see Blume (1974)). We therefore report, in addition to the arithmetic returns, the annualized mean log returns as well as the geometric returns. These are defined as follows: let V_t represent a time t value of a portfolio that rolls a futures position at daily closing prices. We compute $V_t = V_{t-1}(1 + r_p(t))$ where

$$1 + r_p(t) = w_{t-1}\frac{F_t(T_1)}{F_{t-1}(T_1)} + (1 - w_{t-1})\frac{F_t(T_2)}{F_{t-1}(T_2)} \tag{1}$$

is the return to a portfolio of futures with maturities T_1 and T_2, and w_{t-1} is the weight in the front month futures. We report the average daily arithmetic return,

$$R^1 := \frac{1}{T}\sum_t r_p(t),$$

the average daily log-return,

$$R^2 := \frac{1}{T}\sum_t \ln(1 + r_p(t))$$

as well as the annualized geometric return

$$R^3 := \left(\frac{V_T}{V_0}\right)^{\frac{252}{T}} - 1.$$

While the geometric return is known to be biased for the expected annual return (Blume (1974)), it represents a monotonic transform of the total return over the sample period and we include it for this reason.

[Table 1 about here.]

Table 1 presents measures of return and higher order return moments. Though introduced in 2004, low liquidity in the first two years leads us to start the sample in January 2006. As can be seen from the table, VIX futures averaged negative returns over the sample period. Largest were the losses for the short-term maturity futures, with one month contracts losing an average of 12 basis points per day which roughly annualizes to $252 \times -0.12 = -30\%$. In terms of log-returns the one-month futures averaged negative 20 basis points per day, or -50.4% annually. The geometric average return was -39.4%. The returns tend to increase with maturity, and the five-month contract loses a comparably small amount, with "only" -7.33% percent per annum geometric average loss. Both the average loss for the short maturity contracts, as well as the comparably smaller loss for the long maturities are interesting features of the data. These features of the data are seen in Figure 1 which plots the value and log-value of a dollar invested in rolled positions in VIX futures on January 3rd, 2006.

[Figure 1 about here.]

[Table 2 about here.]

In order to get a first pass at whether or not the returns are consistent with an equilibrium story, we compute α's using several factor model specifications. As the futures prices are first order dependent on the underlying VIX, we use log changes in VIX ($\triangle VIX_t := \ln(VIX_t/VIX_{t-1})$) in addition to standard Fama-French risk factors. The results are reported in Table 2. As can be seen, the various specifications give large negative α's. The short maturity one to two month α's are statistically significantly negative while the longer maturities are not. We also report tests of the null hypothesis $\alpha_i = 0 \; \forall \; i = 1, ..., 5$ which are rejected for all model specifications. We conclude accordingly that neither of these asset pricing models can explain the average returns to VIX futures.

[Table 3 about here.]

To understand where the negative returns come from, we present the average values of the VIX spot and the various maturity futures over the sample period in Table 3. This table shows that on average, the futures curve is in contango and prices monotonically increase with maturity of the contract. This, mechanically, is the reason why long positions in VIX futures lose money on average. Consider, for example, an investor who buys a one-month contract and holds it until it expires. Her average return would be $20.57/21.48 - 1 = -4.24\%$ per month, or -40.52% per year when compounded. This is close to the annually average compounding geometric return in Table 1. Similarly, the annualized, average one-month holding period returns for two to seven month contracts are reported in the row labelled "Implied Return" in Table 3. While not identical, the numbers are on the same order of magnitude as the actual returns reported in Table 1. This shows that in order to understand why the returns to VIX futures are so low, we must understand why the futures curve is on average severely upward sloping in the data. An equilibrium explanation for the negative returns, therefore, will need to generate a sharply upward sloping steady-state futures curve.

2.1 Returns to ETNs

[Table 4 about here.]

The first Exchange Traded Notes (ETNs) linked to VIX futures were introduced in January 2009. The VXX and VXZ offer long exposures to the one and five month futures, respectively. The large negative returns earned on the VIX ETNs are typically thought to be a result of the unfortunate timing of their inception, either during the height of the financial crisis as with the VXX or VXZ, or in its aftermath. It is not true, however, that the decimation of these securities' value is solely a consequence of the directional move in the VIX over the sample period. The simple evidence of this is the fact that the securities also lose value during periods for which there is no change in the underlying VIX index. For example, during the period March 1, 2010 to June 21, 2012, the VIX went from 19.26 to 20.08, a marginal positive change, but the VXX lost 82.74% of its value over this sample period while the VXZ lost 29.21%. Clearly, the directional move in the VIX was not the reason why the VXX lost almost 83% of its value over this period!

[Figure 2 about here.]

The performance of the VIX ETNs is closely tied, if not identical, to the performance of the synthetic VIX futures portfolios that we have analyzed above. To see this, Figure 2 shows the log value of our synthetic one-month portfolio alongside VXX's log net asset value. As can be seen, the two are highly correlated and essentially identical with the exception that the VXX depreciates at a slightly higher rate over the sample period. Specifically, the annualized geometric return to the VXX was -65.33% vs. -64.78% for the synthetic portfolio. The difference is only about 0.55% per year, of which 0.89% comprises its management fee. The remaining -34 basis points are presumably due to differences in execution between the VXX and our synthetic portfolio.

[Table 5 about here.]

It is also interesting to study the relative performance of the three ETNs, VXX, TVIX and XIV which span the one-month space with single long, double long, and single short positions, respectively. Since the TVIX and XIV can be replicated by trading in the VXX, we compute the terminal values of the replicating portfolios and compare them to the respective terminal values of the XIV and TVIX. These results are reported in Table 5. We use two performance measures, $G1$ and $G2$. $G1$ denotes the total return difference (annualized) between actual trade price of TVIX and XIV and the value of synthetic securities. $G2$ denotes the corresponding differential based on the net asset value. As can be seen, the TVIX does about 161 basis points better than the synthetic security of VXX per annum based on the net asset value. Based on the actual trade price, it gains 336 basis points on the synthetic counterpart. The XIV loses 180 and 190 basis points.

[Figure 3 about here.]

Table 5 reveals that the market value of the TVIX tends to deviate from its net asset value. Most VIX ETNs tend to track their net asset values (NAV) relatively closely. The TVIX is an exception to this, and this stock has at times traded significantly above its net asset value. In particular, on March 21, 2012, the TVIX was traded 89% above its net asset value after the issuer, Credit Suisse, had announced it would stop share issuance. On March 22, Credit Suisse announced that it would issue more shares, resulting in a collapse in the spread between the price and the net asset value. The stock lost almost 60% of its value over the next two days. The stock has traded about 8% above fair value on average since then.

The sharp increase in the price of the TVIX from July 2011 seen in Figure 3 was caused by the second European sovereign debt crisis during which the VIX increased sharply from about 20 to 47 while the S&P 500 dropped about 28%. Anecdotal evidence suggests that there is a high retail investors demand for long ETNs as they provide negative market betas and therefore act as a hedge against financial crisis. VIX futures, in particular the double long TVIX and UVXY ETNs, provide hedges against financial crisis regimes. This mechanism is essential in the equilibrium model, to be discussed next.

3 An Equilibrium Model of VIX Futures

In the following section we outline an equilibrium framework to understand and analyze the returns to VIX futures, and implicitly, ETNs. Our model aims to explain how VIX futures earn high negative expected returns. We take a first pass at this by estimating the (negative) beta of the futures returns and see if the CAPM can explain the returns. With a beta of -2 and a market risk premium of 5% to 7.5%, we end up at a return premium of -10% to -15% if we assume a zero risk free rate, which is about right at least for the latter part of the sample period. The negative beta thus gives us the right sign, but the CAPM still explains only half or less of the magnitude of the negative returns to one month VIX futures.

Of course, it is more compelling to develop a model that does not just present a negative VIX futures beta as an exogenous quantity, but rather one where the negative return volatility relation is endogenous. We develop such a model. Our model also provides an upward sloping term structure of VIX futures. The slope of this futures curve, as well as the negative correlation between the VIX and VIX futures and the stock market return, is endogenous and increasing (in absolute value) in the amount of risk aversion of the representative agent in our model.

If agents are risk-neutral, there is no volatility risk premium, and thus no negative returns to VIX futures or ETNs in our model. Conversely, when agents are risk-averse, exogenous macro volatility carries a positive risk premium. Positive (negative) shocks to this volatility factor leads to increases (decreases) in the expected rate of return of the stock market, which again leads to negative (positive) stock price reactions. This volatility feedback channel generates a negative market beta for VIX futures and an overall negative variance risk premium. Mechanically, we will show that the VIX futures curve is upward sloping and the degree of convexity is determined by risk aversion.

3.1 Model

The aggregate economy consists of bonds in zero supply and one asset in unit net supply which we call the stock market.[1] By investing in the stock market, investors will receive a random lump sum payment, $\tilde{x}_T$ at time T. For now we assume a finite horizon, $T < \infty$. We later work with the infinite horizon limit. The physical investment that produces $\tilde{x}_T$ is irreversible - while investors can sell shares of claims to $\tilde{x}_T$, they cannot, in aggregate, disinvest.

Investors learn about $\tilde{x}_T$ over time. In a simple model with time-homogeneous moments we can think of x_t as representing a stochastic process, observed by investors, in which the terminal value is $x_T = \tilde{x}_T$. We typically assume x_t is part of a larger system of stochastic processes, $X_t = (x_t, Y_t)$. We assume that X_t is exponential affine and Markov such that $E\{. \mid \mathcal{F}_t\} = E(. \mid X_t)$. We can think of x_T as the investors' retirement savings. Investors are unable to control x_T. We can therefore think of the exogenous nature of x_T as representing an irreversibility of investment. This contrasts the standard consumption based CAPM model where investors choose optimal consumption and investment at each point in time.

At date $T-1$ the price of the terminal T claim is

$$P_{T-1} = \frac{E_{T-1}\{u'(\tilde{x}_T)\tilde{x}_T\}}{E_{T-1}\{u'(\tilde{x}_T)e^{r_{T-1}}\}} \tag{2}$$

[1]See Eraker and Wang (2011) for a discussion of individual stocks.

In the following we assume a constant risk free rate, r, as the risk free rate itself typically has no particular importance in valuing short term equity derivatives as VIX futures essentially are. Backward induction now gives

$$
\begin{aligned}
P_{T-2} = \frac{E_{T-2}\left\{u'(\tilde{x}_T)P_{T-1}\right\}}{E_{T-2}\left\{u'(\tilde{x}_T)e^r\right\}} &= E_{T-2}\left\{u'(\tilde{x}_T)\frac{E_{T-1}\left\{u'(\tilde{x}_T)\tilde{x}_T\right\}}{E_{T-1}\left\{u'(\tilde{x}_T)\right\}e^r}\right\} \times \frac{1}{E_{T-2}\left\{u'(\tilde{x}_T)\right\}e^r} \\
&= E_{T-2}E_{T-1}\left\{\frac{u'(\tilde{x}_T)}{E_{T-2}\left\{u'(\tilde{x}_T)\right\}}\frac{E_{T-1}\left\{u'(\tilde{x}_T)\tilde{x}_T\right\}}{E_{T-1}\left\{u'(\tilde{x}_T)\right\}e^{2r}}\right\} = \frac{E_{T-2}\left\{u'(\tilde{x}_T)\tilde{x}_T\right\}}{E_{T-2}\left\{u'(\tilde{x}_T)\right\}e^{2r}}. \quad (3)
\end{aligned}
$$

and we can deduce that

$$
P_t = \frac{E_t\left\{u'(\tilde{x}_T)\tilde{x}_T\right\}}{E_t\left\{u'(\tilde{x}_T)\right\}}e^{-(T-t)r}. \quad (4)
$$

Subject to regularity conditions this equation applies generally. It is analytically tractable in the case of exponential affine processes for x_T in the class of Duffie, Pan, and Singleton (2000), and power utility, $u'(x) = \beta x^{-\gamma}$. Specifically, we assume that

$$
\begin{aligned}
\frac{dx_t}{x_t} &= \mu dt + \sigma_t dB_t^x && (5) \\
d\sigma_t^2 &= \kappa(\theta - \sigma_t^2)dt + \sigma_v\sigma_t dB_t^v + \xi_t dN_t && (6) \\
\xi_t &\sim \mathrm{Exp}(\mu_\xi) && (7) \\
Corr(dB_t^x, dB_t^v) &= 0. && (8)
\end{aligned}
$$

where N_t is a Poisson process with arrival intensity λ and where $\tilde{x}_T = x_T$. σ_t is the volatility of x_t and is driven by a diffusion, B_t^v, and a compound Poisson process, $\xi_t dN_t$, where the counting process, N_t, has Poisson arrivals with intensity, $l(\sigma_t^2) = l_0 + l_1\sigma_t^2$ where again l_0, l_1 are parameters such that $l_0 > 0, l_1 = 0$ implies constant arrivals and $l_0 = 0, l_1 > 0$ implies that arrivals are proportional to σ_t^2. We shall assume that the parameters are chosen so that σ_t^2 is stationary and positive.

It now follows rather straightforwardly that the expectations in equation (4) can be written as exponential affine functions[2]. Specifically,

$$d\ln P_t = rdt + \lambda_0 dt + \lambda_\sigma d\sigma_t^2 + \sigma_t dB_t^x \tag{9}$$

where λ_0 and λ_σ can be computed from the model parameters. We discuss these parameters below.

3.2 Properties of Equilibrium Stock Returns

[Figure 4 about here.]

The parameter λ_σ is an important endogenous parameter, as it determines the impact of economic uncertainty on asset prices. λ_σ is negative and increasing (in absolute value) as a function of risk aversion, γ, and also the inverse of the persistence parameter κ. We show this in Figure 4 (right panel). As seen, λ_σ is uniformly decreasing in γ. By taking unconditional expectations on both sides of Equation(4) we see that $Ed\ln P/dt - r = \lambda_0$ is the unconditional equity premium. The left plot in Figure (4) shows that the equity premium is increasing in γ and reaches realistically large values for values of γ that are plausibly small, suggesting a resolution the equity premium puzzle. The equity premium is larger when persistence in volatility is high (κ close to zero). This is analogous to long-run-risk models. In fact, the model can be seen as a limiting case of long-run-risk when intertemporal elasticity of substitution is high.

[Figure 5 about here.]

It is clear from equation (9) that λ_σ determines the endogenous impact of volatility shocks on stock prices. Since λ_σ increases (in absolute value) in γ, the correlation between volatility shocks and stock prices is negative (since λ_σ is negative) and increasing in absolute value as γ increases. We illustrate this through scatter plots of daily volatility changes and daily returns simulated

[2]Generally, let $\phi(u, X_t) = E_t\{\tilde{x}_T^u\} = e^{\alpha(u)+\beta(u)X_t}$ be the conditional moment generating function of x_t. Then $P_t = \phi(1-\gamma, X_t)/\phi(-\gamma, X_t)e^{-r(T-t)}$ is the equilibrium price.

from the model using $\gamma = 3$ and $\gamma = 8$ in Figure 5. The correlations are -0.26 and -0.63 respectively.

3.3 Estimation

We estimate the model using structural estimation. The estimation approach is similar to that of Eraker, Johannes, and Polson (2003) (hereby EJP) in that we draw the parameters of the model from the posterior distribution $\Theta_j \sim p(\Theta \mid \mathcal{Y}_T)$ using MCMC sampling. The latent conditional variances, σ_t^2, the jump times, dN_t, and jump sizes, ξ_t, are drawn from the respective conditional posterior distributions in a manner similar to that of EJP. The main difference is that our structural model leads to non-standard posterior distributions. This contrasts EJP and similar papers that draw from conjugate densities such as Normal and Gamma. Since our structural model gives rise to complicated non-linear relationships between the "deep" parameters and the affine model coefficients such as λ_σ, it is not possible to draw any of the parameters directly from conditional distributions. We therefore draw the entire parameter vector directly from the posterior using a Metropolis Hastings draw.

[Table 6 about here.]

Table 6 reports structural estimates of the parameters in the model. All parameter estimates should be interpreted to be based on a unit of time being one day. This makes the time-series parameters directly comparable to estimates based on daily returns. Depending on whether the model includes jumps or not, κ is estimated to be 0.014 and 0.0091, respectively. These estimates imply daily autocorrelatons of $\exp(-0.014) = 0.9861$ and $\exp(-0.0091) = 0.991$. These numbers are broadly consistent with estimates reported elsewhere (see Singleton (2006) for a review).

The most interesting parameter in Table 6 is the risk-aversion parameter, γ. We estimate this to be more than 10 posterior standard deviations away from zero for the SVVJ jump model. This contrasts conflicting evidence from the conditional CAPM literature where the risk aversion parameters are typically found to be statistically insignificant.

3.4 Risk-Neutral Dynamics

Under the equivalent measure Q, the variance process follows

$$d\sigma_t^2 = \kappa^Q(\theta^Q - \sigma_t^2)dt + \sigma_v \sigma_t dB_t^v + \xi_t^Q dN_t^Q \tag{10}$$

where

$$\begin{aligned}
\kappa^Q &= \kappa + \lambda_\sigma \sigma_v && (11)\\
\theta^Q &= \frac{\theta\kappa}{\kappa^Q} && (12)\\
\lambda^Q &= \lambda \frac{\mu_\xi}{\mu_\xi + \lambda_\sigma} && (13)\\
\mu_\xi^Q &= \mu_\xi + \lambda_\sigma && (14)\\
\lambda_\sigma &= \frac{-\kappa + \sqrt{\kappa^2 - \sigma_v^2(\gamma^2 + \gamma)}}{\sigma_v^2} && (15)
\end{aligned}$$

are functions of model parameters, notably risk aversion, γ.

The stock price follows

$$d\ln P_t = r_{f,t}dt + \lambda_\sigma d\sigma_t^2 + \sigma_t dB_t^Q \tag{16}$$

under the risk-neutral measure where $d\sigma_t^2$ is given by Equation (10).

Of essential interest is the τ period ahead conditional variance of the log-return, which we can show to be a linear function of the spot variance σ_t^2 under both measures $i = \{P, Q\}$, as

$$\text{Var}_t^i(\ln P_{t+\tau}) = a_i(\tau) + b_i(\tau)\sigma_t^2. \tag{17}$$

Notice that VIX is defined as

$$VIX_t = \text{Std}_t^Q(\ln P_{t+21}) = \sqrt{a_Q(21) + b_Q(21)\sigma_t^2} \tag{18}$$

where we have defined a unit of time to be one business day.

[Figure 6 about here.]

Figure 6 illustrates some of the key pricing implications through a comparison of the P and Q measures for the stock price. As can be seen, the difference between the P and Q densities increases as γ is increasing. The plot also shows that the two different values of γ give widely different Black-Scholes implied volatilities for the underlying stock which suggests that the model can generate a substantial volatility risk premium. This is of course key in explaining the negative returns to VIX futures.

3.5 A Digression on Variance Futures

The non-linearity introduced by the square root in (18) necessitates numerical computation of futures prices. While we can do this through a single one-dimensional numerical integration, for purposes of illustration, we discuss the shape of a hypothetical futures contract written on VIX squared. These can be computed directly, and since the squared VIX is a linear function of σ_t^2, it inherits the properties of the risk premia embedded into the differences between the objective P and risk-neutral Q probability measures. These hypothetical variance futures are priced

$$F_t^{\mathrm{VIX}^2}(\tau) = E_t^Q\left\{VIX_{t+\tau}^2\right\} = E_t^Q\left\{a_Q(21) + b_Q(21)\sigma_{t+\tau}^2\right\} \tag{19}$$

$$= a_Q(21) + b_Q(21)E_t^Q\left\{\sigma_{t+\tau}^2\right\} \tag{20}$$

In *steady-state*, the futures curve is

$$F_t^{\mathrm{VIX}^2}(\tau) = a_Q(21) + b_Q(21)E^Q\left\{\sigma_{t+\tau}^2 \mid \sigma_t^2 = E(\sigma_t^2)\right\} \tag{21}$$

The following basic facts about VIX-squared futures are easy to verify:

Proposition 1. *If $\gamma > 0$, then the volatility risk premium is negative and*

1. *The futures curve is upward sloping (contango) in steady state.*
2. *Long positions in VIX squared futures earn negative expected returns irrespective of the state of the economy.*

3. *VIX square futures have negative market betas. The size of the beta is a function of risk aversion.*

It is worth commenting on some of these features. Some believe that if the futures curve is upward sloping, they will capture a negative roll by purchasing the futures. This is true only in relation to the physical drift rate of the underlying VIX or σ_t^2 processes. For example, if the underlying σ_t^2 process is in steady state, ($\sigma_t^2 = E(\sigma_t^2)$), the negative roll is indicative of the actual expected returns because the expected terminal value of the VIX squared is its present value $E_t(VIX_{t+\tau}^2) = VIX_t^2$. A positive slope of the futures curve reflects only risk premium in steady state. The larger the risk aversion in our model, the steeper the slope, and the higher (lower) the risk premium earned by short (long) positions. It is important to understand that the expected return to a long position in VIX futures is determined not by the shape of the futures curve per se, but rather by the shape of the curve relative to the expected value of the VIX at expiration of the futures. Accordingly, in principle, if the futures curve were upward sloping but the objective measure drift was greater than that implied by the futures curve, the expected return to a long VIX futures position would have been positive. This, however, is never true in our model economy and investors earn a strictly negative premium. [3]

3.6 VIX Futures

[Figure 7 about here.]

How do the actual VIX futures prices differ from the hypothetical VIX squared futures prices we analyzed in the preceding section? To answer this question, Figure 7 shows the various term structures under different assumptions about initial volatility states, σ_t. We plot the term structure for the pure diffusive (SV) model as well as for the jump model (SVVJ). We compare two things: the actual, model-implied futures curve labeled SV_Q and $SVVJ_Q$ (signifying expectation under Q), and also the corresponding objective measure expectation, labeled

[3] The variance risk premium is a function of the difference in drift rate for σ_t^2 under the two measures. It is easy to see that the Q minus P drift $-\lambda_\sigma \sigma_v \sigma_t^2$ is a positive number regardless the level of σ_t^2. $\kappa^Q < \kappa$ and $\theta^Q > \theta$ suggest that the physical mean reversion rate is faster than the risk-neutral and the physical mean reversion level is lower, which gives a negative risk premium for VIX long futures.

SV_P and $SVVJ_P$, respectively. The difference between the Q and the P expectations is due to volatility risk premium. The right hand figures show the expected holding period return, $E_t^P(VIX_{t+\tau})/E_t^Q(VIX_{t+\tau}) - 1$, to a long futures position. The expected returns are negative and more sharply so when volatility is high.

Examining the plots to the left in Figure 7, we see the obvious relations between spot volatility and the shape of the futures curve: when spot volatility is high (low) the curve is in backwardation (contango). When in steady state the futures curve is first convex and then concave. The convexity created at the short end of the curve is due to a Jensen's inequality term created by the concavity of the square root function.[4] The total expected holding period returns are negative for all maturities and for all initial values of spot volatility. The jump model (SVVJ) provides a larger short-term risk premium (in absolute value) as can be seen by the steeper slope of the expected return graphs. We expect, therefore, that the jump model will generate a higher average return to short term VIX futures.

4 Model Performance

[Table 7 about here.]

To obtain estimates of the model's expected return, standard deviation, skewness, and kurtosis, we simulate data from the underlying equilibrium stock price process using the estimated parameters in Table 6. We simulate $10,000$ data sets of length $T = 1816$, the length of our futures time-series and use these to compute theoretical futures prices. We then compute the returns to rolled futures positions similarly to our procedure for the real data. We compute sample moments from the simulated data and compare these to the real data.

Table 7 reports the results. We include the first three estimators of the average returns from the data which we previously reported in Table 1 for convenience. Note that the higher order moments are of logarithmic daily returns. Although slightly higher, the returns produced by the

[4]The Jensen's inequality term is $E_t(VIX_{t+\tau}) - \sqrt{E_t(VIX^2_{t+\tau})} < 0$ since the square root function is concave.

SVVJ model are quite close to what we see in the data for the one-month maturity contracts. For the longer maturities both models overstate the size of the negative return premium. Overall however, both models do surprisingly well in fitting average returns. The models also do seemingly well in fitting the higher order moments, with the exception of the SVVJ model's too high kurtosis. In general, the simulated model returns have moments in the ballpark of that seen in the real data. We further test the null hypothesis of zero difference between the model and the data below.

[Figure 8 about here.]

[Figure 9 about here.]

To carry out these significance tests, we use a model-based bootstrap. We wish to avoid the use of test-statistics based on asymptotic normality because the higher order moments are very non-normally distributed in small samples. The evidence is presented in Figures 9 and 8. The figures plot the kernel-smoothed densities of moments implied by the model and the corresponding data moment represented by the vertical bars. The non-parametric density estimates should be interpreted as a model-based bootstrap of the sampling distribution for each respective moment.

In examining Figures 9 and 8 we see that the average returns for the SV model are insignificantly different between the real data and the model generated data. For the standard deviations and higher order moments, the results are very different. The SV model basically fails to fit any moment higher than the first. The standard deviations are all significantly different between the model and the data. The skewness and kurtosis are so far off that the vertical bars are absent from the plots[5].

On the other hand, for the SVVJ model, the moments are surprisingly well matched. With the exception of the standard deviation for the four and five month maturity contracts, the data and the model are insignificantly different. From Table 7 we see that the SVVJ model generates sample kurtosis coefficients that range from 20 to 27 which compares to coefficients that range

[5]This happens when the real data moments are outside of the empirical support of the sampling distributions under the model.

from 6.02 to 7.28 in the data (see Table 1). This may seem like a large difference, however, Figure 9 reveals that the sampling distribution under the model has most of its mass below what we find in the data. In fact, the medians of the small sample distributions for the kurtosis coefficients under the SVVJ model range from 4.4 to 5. Thus, while it may seem from Table 1 and 7 that the SVVJ model generates too high kurtosis, the differences are in fact almost non-existent.

4.1 A Two-Factor Volatility Model

As we can see in the previous section, the SVVJ model captures all but one aspect of the observed futures returns - it significantly underestimates the variability of longer term contracts. Since the SVVJ model is a single factor affine volatility model, it cannot generate long-run memory like behavior in conditional volatility, as empirically documented by Bollerslev and Mikkelsen (1996) among others. Bates (2000) and Chernov, Gallant, Ghysels, and Tauchen (2003) propose two factor nested conditional variance specification in a no-arbitrage, reduced form model. We propose propose a similar model,

$$\begin{aligned}
\frac{dx_t}{x_t} &= \mu dt + \sigma_t dB_t^x && (22)\\
d\sigma_t^2 &= \kappa(\theta_t - \sigma_t^2)dt + \sigma_v \sigma_t dB_t^v + \xi_t dN_t && (23)\\
d\theta_t &= \kappa_\theta(\theta - \theta_t)dt + \sigma_\theta \sqrt{\theta_t} dB_t^\theta + \xi_t^\theta dN_t^\theta && (24)\\
\xi_t &\sim \mathrm{Exp}(\mu_\xi) && (25)\\
Corr(dB_t^x, dB_t^v) &= 0 && (26)\\
Corr(dB_t^x, dB_t^\theta) &= 0 && (27)\\
Corr(dB_t^v, dB_t^\theta) &= 0. && (28)
\end{aligned}$$

where N_t and $N_t\theta$ are independent Poisson processes with arrival intensities λ and λ_θ respectively. In this specification the conditional variance, σ_t^2, mean-reverts to a stochastic mean, θ_t, which again follows a square root process. If we assume that the persistence in θ_t is stronger than for σ_t^2 (i.e., κ_θ is "small"), then θ_t will generate low frequency movements in volatilities while σ_t accounts for higher frequency movements.

The equilibrium stock price can now be seen to be given by

$$d \ln P_t = r_{f,t} dt + \lambda_0 dt + \lambda_\sigma d\sigma_t^2 + \lambda_\theta d\theta_t + \sigma_t dB_t^x \tag{29}$$

where $\lambda_j, j = \{0, \sigma, \theta\}$ are equilibrium coefficients which again are nonlinear functions of the parameters. The squared VIX index is again a linear function of the state-variables

$$VIX_t^2 = \mathrm{Var}_t^Q(\ln P_{t+21}) = a + b\sigma_t^2 + c\theta_t \tag{30}$$

where we can solve for constants a, b and c (see Appendix). Since VIX_t^2 is a function of two processes with different autocorrelation functions, the autocorrelation for the VIX_t^2 itself is a mixture, and thus displays long memory-like behavior.

[Figure 10 about here.]

It's difficult to take our two-factor volatility model and estimate it using return data alone, as we did with the models in the previous section. The purpose of our exercise here is to demonstrate that the model is capable of matching the moments of the futures returns data. In Figure 10 we show the sampling distributions for the VIX futures data under the model, using calibrated parameters. The parameters are $\kappa = 0.0055$, $\sigma_v = 0.0012$, $\theta \times 10000 = 2.166$, $\gamma = 1.53$, $\lambda_v = 0.0045$, $\mu_v \times 10000 = 1.27$, $\kappa_\theta = 0.008$, $\sigma_\theta \times 10000 = 0.435$, $\mu_\theta \times 10000 = 5.08$, $\lambda_\theta = 0.0003$. . As can be seen, the resulting moments are all well inside the tails of the sampling distributions suggesting that our two factor volatility specification provides a plausible description of the true data generating process. Note that the standard deviation of the longer maturity contracts is matched almost exactly.

4.2 Options on VIX ETNs

[Figure 11 about here.]

Since September 2012 it has been possible to trade equity options on selected VIX ETNs. Since our model endogenizes the entire return distribution to VIX ETNs, we can use it to price

ETNs options. We can use potential discrepancies between model and market prices to infer potential ways in which the dynamic specification of the model can be improved to capture distributional and dynamic features implicit in option prices.

In order to compute prices of VXX options we simulate the VIX futures prices under the Q measure. From these prices we compute the Q measure return distributions conditional upon the initial value of the VIX. This produces a sequence of simulated daily returns. From these we can compute the returns on any maturity option. We choose one month. From the simulated one month returns we can compute theoretical options prices which we again convert into implied Black-Scholes volatilities[6].

Figure 11 presents Optionmetrics implied volatility data for the VXX along with the implied volatility from our two factor model. Over the limited period of available sample data, we pick two days corresponding to (relatively) high volatility (12/28/2012) where the VIX closed at 20.8 on fears of the US congress failing to reach a deal on its debt ceiling. We also pick 3/14/2013 when the VIX closed at 11.3 to represent a low volatility regime. The latter is shown to the left in Figure 11.

As can be seen, the model matches the observed data really well when the VIX is low. When the VIX is high, there is large difference in the market and model implied volatilities. This suggests that the market implied risk-neutral density of VXX returns in much more heavily skewed when the VIX is high. This is hard to accommodate with our model, and perhaps any model. As we have seen, the returns in the SVVJ model are actually quite heavily right skewed. This skewness is however greatly reduced when aggregating to monthly returns as the returns look more and more normally distributed when aggregating to longer horizons. Our two-factor models also fails to generate sufficiently right skewed risk-neutral VXX returns to capture observed option smile when the VIX is high. It is possible that this could be captured bynon-linearities in the specification of the diffusion equation for the underlying[7].

---

[6]OptionMetrics provide implied volatility data based on the Cox-Ross-Rubenstein binomial model for pricing American options with log-normally distributed returns. Our implied volatilities are computed from Call options for which early exercise is not optimal which are in theory comparable to the American Calls.

[7]Mijatovic and Schneider (2013) and Eraker and Wang (2013) develop reduced form models of the variance risk premium that captures non-affine variance dynamics.

[Figure 12 about here.]

Figure 12 shows option implied skewness for VXX options estimated from daily data from Sept. 4, 2012 until Aug. 30, 2013. Skewness is defined as in Bakshi, Kapadia, and Madan (2003)[8]. The figure shows that the skewness is positive for all maturities. We fit a non-parametric regression to represent the average relationship between skew and maturity and as can be seen, the result suggests that skew increases up to maturities of about one month, and then decreases. The theoretical risk-neutral skewness is shows for the SVVJ and the two-factor models alongside the data and the kernel estimate. As can be seen, the models produce positive average skewness, as is consistent with the data but fails to capture the term-structure of option implied skewness. The two factor model comes closest to matching the overall option implied skew, but produces a convex term structure whereas the data appears overall to exhibit concavity. Volatility jumps generate a convex, downward sloping term structure, as the near term options will exhibit greater skewness. By contrast, the data seem to suggest almost short term skew and high skewness for 20-40 day maturities. This points to persistence beyond what is captured by our various model specifications. Chacko and Viceira (2003) show that volatility persistence estimated from weekly or monthly data suggests much longer volatility half-lives than do estimates based on daily data. The fact that the skewness shown in Figure 12 is increasing from almost zero (short maturity) to about 1.2 (30 days maturity), suggests a similar underlying dynamic - there are positive shocks affecting volatility at one-month aggregation level but have a negligible impact on short run VIX.

5 Concluding Remarks

In this paper we show that the average returns earned on VIX futures and ETNs are abysmal. Investors who hold long positions in VIX futures will gradually see their wealth vanish as they realize negative average rates of returns over the long run. We show that this is a statistically significant feature of the data.

[8] We use code from DeMiguel, Plyakha, Uppal, and Vilkov (2013)

We argue that the negative returns are consistent with equilibrium. Though the size of the negative return premium is not consistent with a traditional CAPM, which delivers a risk premium of -15% per year, we show that the equilibrium model of Eraker and Wang (2011) is capable of explaining the -36% per year average return to a one-month (rolled) futures position. The SVVJ model, which includes volatility jumps, is our preferred one-factor model. This model generates a higher short-term volatility risk premium than does the simpler SV model.

It is important to understand the mechanisms that cause these models to assign high volatility risk premia and negative market betas to the VIX futures. The stock market pays a single terminal cash flow such that its current market value is the present value of the cash flow, essentially discounted at an expected rate of return which is proportional to the current volatility, σ_t. The sensitivity of the expected rate of return with respect to changes in volatility is an increasing function of risk aversion, γ, in the model. Thus, in equilibrium, positive (negative) shocks to σ_t give rise to negative (positive) stock price shocks. The absolute magnitude of this negative return/ volatility correlation is an endogenous quantity that increases (in absolute value) with γ. If we then think about volatility as an asset class, relative to the CAPM, volatility, as measured by the VIX, is a negative beta asset. Since VIX futures prices are positively dependent on current spot VIX, they too are negative beta assets.

The overall size of the volatility risk premium in our model depends on risk-aversion, volatility persistence, and the specification of volatility jumps in the model. Investors dislike volatility jumps and large jumps lead to discontinuities in stock prices reminiscent of financial crisis. Overall, this leads to a higher volatility risk premium in the short term. Expected returns, accordingly, are a steeper function of maturity under the SVVJ model than the SV model. The SV model comparably generates a higher volatility risk premium by assigning a slightly lower speed of volatility mean reversion. This leads to a somewhat higher negative return premium for long term futures contracts. It also leads to more volatility in futures returns at longer maturities (i.e. five months), in a manner that is consistent with the real market data.

Appendix

A Equilibrium Prices

In our most general model the process x that generates the terminal (log) payoff $x_T = \tilde{x}_T$ to the aggregate stock market is given by

$$
\begin{aligned}
\frac{dx_t}{x_t} &= \mu dt + \sigma_t dB_t^x && (31)\\
d\sigma_t^2 &= \kappa(\theta_t - \sigma_t^2)dt + \sigma_v \sigma_t dB_t^v + \xi_t dN_t && (32)\\
d\theta_t &= \kappa_\theta(\theta - \theta_t)dt + \sigma_\theta \sqrt{\theta_t} dB_t^\theta && (33)\\
\xi_t &\sim \mathrm{Exp}(\mu_\xi) && (34)\\
N_t &\sim \mathrm{Poisson}(\lambda t) && (35)\\
Corr(dB_t^x, dB_t^v) &= 0 && (36)\\
Corr(dB_t^x, dB_t^\theta) &= 0 && (37)\\
Corr(dB_t^v, dB_t^\theta) &= 0 && (38)
\end{aligned}
$$

The state variable X_t in this economy defined as $(\ln x_t, \sigma_t^2, \theta_t)$ and follows an affine process. Therefore, $\mathbb{E}_t\, e^{u' X_t} = e^{\alpha(u,t,T)+\beta'(u,t,T)X_t}$, where α and β solve a system of ordinary differential equations (see Duffie Pan, and Singleton (2000)), given by

$$
\frac{\partial \alpha}{\partial t} = -\begin{pmatrix} \mu & 0 & \kappa_\theta \theta \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} - \begin{pmatrix} 0 & \lambda & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{\mu_\xi}{\mu_\xi - \beta_2} - 1 \\ 0 \end{pmatrix} \tag{39}
$$

$$
\begin{pmatrix} \frac{\partial \beta_1}{\partial t} \\ \frac{\partial \beta_2}{\partial t} \\ \frac{\partial \beta_3}{\partial t} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{2} & \kappa & 0 \\ 0 & -\kappa & \kappa_\theta \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 0 \\ \begin{pmatrix} \beta_1 & \beta_2 & \beta_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} \\ \begin{pmatrix} \beta_1 & \beta_2 & \beta_3 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_\theta^2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} \end{pmatrix} \tag{40}
$$

$$
\begin{aligned}
\alpha(u, T, T) &= 0 && (41)\\
\beta(u, T, T) &= u. && (42)
\end{aligned}
$$

The analytical solutions for β_1 and β_2 can be obtained as

$$\begin{aligned} \beta_1(u,t,T) &= u(1), && (43)\\ \beta_2(u,t,T) &= a_2 + (a_2 - a_1)\frac{C(t)}{1-C(t)}, && (44) \end{aligned}$$

while,

$$\begin{aligned} a_2 &= \frac{\kappa - \sqrt{\kappa^2 - \sigma_v^2(u(1)^2 - u(1))}}{\sigma_v^2}, && (45)\\ a_1 &= \frac{\kappa + \sqrt{\kappa^2 - \sigma_v^2(u(1)^2 - u(1))}}{\sigma_v^2}, && (46)\\ c_1 &= \frac{u(2) - a_2}{u(2) - a_1}, && (47)\\ c_2 &= \frac{(a_2 - a_1)\sigma_v^2}{2}, && (48)\\ C(t) &= c_1 e^{c_2(T-t)}. && (49) \end{aligned}$$

We solve β_3 and α numerically. However, when $T \to \infty$, the analytical solution for β_3 is

$$\beta_3 = \frac{\kappa_\theta - \sqrt{\kappa_\theta^2 - 2\kappa a_2 \sigma_\theta^2}}{\sigma_\theta^2}. \tag{50}$$

In the case of $T \to \infty$, solution for β_1 is the same and $\beta_2 = a_2$.

A.1 Equilibrium Stock Returns

Let P_t denote the price of the risky asset at t and a be the number of shares the representative agent holds, who has power utility and enjoys consumption at the final date. The equilibrium asset price can be derived by optimal portfolio problem:

$$\max_a \mathbb{E}_t\, u\left(a\tilde{x}_T - (a-1)\,P_t e^{r_f(T-t)}\right) \tag{51}$$

From the first order condition and the fixed supply $a^* = 1$ of the risky asset we we find that its price is

$$\begin{aligned} P_t &= \frac{\mathbb{E}_t\, u'(\tilde{x}_T)\tilde{x}_T e^{-r_f T}}{\mathbb{E}_t\, u'(\tilde{x}_T) e^{-r_f t}} = \frac{\mathbb{E}_t\, \tilde{x}_T^{1-\gamma} e^{-r_f T}}{\mathbb{E}_t\, \tilde{x}_T^{-\gamma} e^{-r_f t}} && (52)\\ &= \exp\{\alpha(u_{1-\gamma},t,T) - \alpha(u_{-\gamma},t,T) + (\beta'(u_{1-\gamma},t,T) - \beta'(u_{-\gamma},t,T))X_t - r_f(T-t)\}\\ u_{1-\gamma} &= \begin{pmatrix} 1-\gamma & 0 & 0 \end{pmatrix}', u_{-\gamma} = \begin{pmatrix} -\gamma & 0 & 0 \end{pmatrix}' && (53) \end{aligned}$$

If we take the limit $T \to \infty$, then β does not depend on t. Define

$$\lambda_\sigma = \beta_2(u_{1-\gamma}) - \beta_2(u_{-\gamma}), \tag{54}$$

$$\lambda_\theta = \beta_3(u_{1-\gamma}) - \beta_3(u_{-\gamma}). \tag{55}$$

The solution for α is

$$\left(-\mu\beta_1 - \kappa_\theta\theta\beta_3 - \lambda\left(\frac{\mu_\xi}{\mu_\xi - \beta_2} - 1\right)\right)(t-T). \tag{56}$$

In order for the stock price to be finite and positive we need

$$\alpha(u_{1-\gamma}, t, T) - \alpha(u_{-\gamma}, t, T) - r_f(T-t) = 0. \tag{57}$$

This gives μ as

$$\mu = r_f - \kappa_\theta\theta\lambda_\theta - \lambda\left(\frac{\mu_\xi}{\mu_\xi - \beta_2(u_{1-\gamma})} - \frac{\mu_\xi}{\mu_\xi - \beta_2(u_{-\gamma})}\right) \tag{58}$$

Therefore, the equilibrium stock price is

$$\ln P_t = \ln x_t + \lambda_\sigma\sigma_t^2 + \lambda_\theta\theta_t \equiv \lambda_X' X_t \tag{59}$$

$$d\ln P_t = d\ln x_t + \lambda_\sigma d\sigma_t^2 + \lambda_\theta d\theta_t \tag{60}$$

$$= (\mu - \frac{1}{2}\sigma_t^2)dt + \sigma_t dB_t^x + \lambda_\sigma d\sigma_t^2 + \lambda_\theta d\theta_t \tag{61}$$

$$= r_f dt + \lambda_0 dt + \frac{1}{2}(\theta - \sigma_t^2)dt + \sigma_t dB_t^x + \lambda_\sigma d\sigma_t^2 + \lambda_\theta d\theta_t \tag{62}$$

while $\lambda_0 = -\kappa_\theta\theta\lambda_\theta - \lambda(\frac{\mu_\xi}{\mu_\xi - \beta_2(u_{1-\gamma})} - \frac{\mu_\xi}{\mu_\xi - \beta_2(u_{-\gamma})}) - \frac{1}{2}\theta$ is the equity premium in this economy as the unconditional expectation of other variable terms are zero.

A.2 Q Measure

The stochastic discount factor and its dynamics in the economy are

$$M_t = \mathbb{E}_t\, u'(\tilde{x}_T)e^{-r_f t} = e^{\alpha(u_{-\gamma}) + \beta'(u_{-\gamma})X_t - r_f t}, \tag{63}$$

$$d\ln M_t = \mu_M dt - \gamma d\ln\tilde{x}_t + \beta_2(u_{-\gamma})d\sigma_t^2 + \beta_3(u_{-\gamma})d\theta_t := \mu_M dt - \eta' X_t, \tag{64}$$

$$\eta := (\gamma, -\beta_2(u_{-\gamma}), -\beta_3(u_{-\gamma}))'. \tag{65}$$

Using Theorem 2.1 in Eraker and Shaliastovich (2008), the dynamics of state variables under the equivalent measure Q are

$$
\begin{aligned}
dX_t &= (K_0^Q + K_1^Q X_t)dt + \Sigma(X_t)dB_t^Q + \xi_t^Q dN_t^Q && (66)\\
K_0^Q &= K_0 && (67)\\
K_1^Q &= K_1 - H\eta = \begin{pmatrix} 0 & -(\frac{1}{2}+\gamma) & 0 \\ 0 & -\kappa + \beta_2(u_{-\gamma})\sigma_v^2 & \kappa \\ 0 & 0 & -\kappa_\theta + \beta_3(u_{-\gamma})\sigma_\theta^2 \end{pmatrix} && (68)\\
\lambda^Q &= \lambda \frac{\mu_\xi}{\mu_\xi + \eta(2)} && (69)\\
\mu_\xi^Q &= \mu_\xi + \eta(2) && (70)
\end{aligned}
$$

Under the Q measure, state variables still follow exponential affine and thus expectations of exponential affine functions of the states can be derived semi-analytically (up to ODE's), as under the objective measure.

A.3 VIX Futures Prices

By definition we have

$$VIX_t^2 = Var_t^Q(\ln P_{t+21}). \quad (71)$$

The conditional cumulant generating function for $\ln P_{t+21}$ is given by

$$
\begin{aligned}
\Phi(u) &= \ln \mathbb{E}_t^Q\, e^{u ln P_{t+21}} && (72)\\
&= \ln \mathbb{E}_t^Q\, e^{u\lambda_X' X_{t+21}} && (73)\\
&= \alpha(u\lambda_X, t, t+21) + \beta'(u\lambda_X, t, t+21)X_t && (74)
\end{aligned}
$$

Therefore, using the property of the cumulant generating function, we see $VIX_t^2 = Var_t(\ln P_{t+21}) = a + b\sigma_t^2 + c\theta_t$, while a,b and c are second derivatives of $\alpha(\epsilon\lambda_X, t, t+21)$, $\beta_2(\epsilon\lambda_X, t, t+21)$ and $\beta_3(\epsilon\lambda_X, t, t+21)$ evaluated at $\epsilon = 0$. We numerically compute a, b and c since the analytical solution gets very messy.

We adopt the analytical formula for VIX futures up to an integral by Zhu and Lian (2011), and price VIX futures by numerical integration:

$$
\begin{aligned}
F_t^{VIX}(t+\tau) &= \mathbb{E}_t^Q \sqrt{VIX_{t+\tau}^2} \\
&= \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{1-\mathbb{E}_t^Q\, e^{-sVIX_{t+\tau}^2}}{s^{3/2}} ds \\
&= \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{1-e^{-as}\,\mathbb{E}_t^Q\, e^{u_1(s)' X_{t+\tau}}}{s^{3/2}} ds \\
&= \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{1-e^{-as} e^{\alpha(u_1(s),t,t+\tau)+\beta'(u_1(s),t,t+\tau)X_t}}{s^{3/2}} ds \\
&= \frac{1}{2\sqrt{\pi}} \int_{-\infty}^\infty e^{-0.5s}(1-e^{-ae^s} e^{\alpha(u_2(s),t,t+\tau)+\beta'(u_2(s),t,t+\tau)X_t}) ds, \qquad (75)
\end{aligned}
$$

where

$$u_1(s) = (0, -bs, -cs)' \qquad (76)$$

$$u_2(s) = (0, -be^s, -ce^s)' \qquad (77)$$

The second equality is a mathematical result using Fubini's theorem and the last equality follows by a change of variable to make the integrand bell shaped for easier numerical computation.

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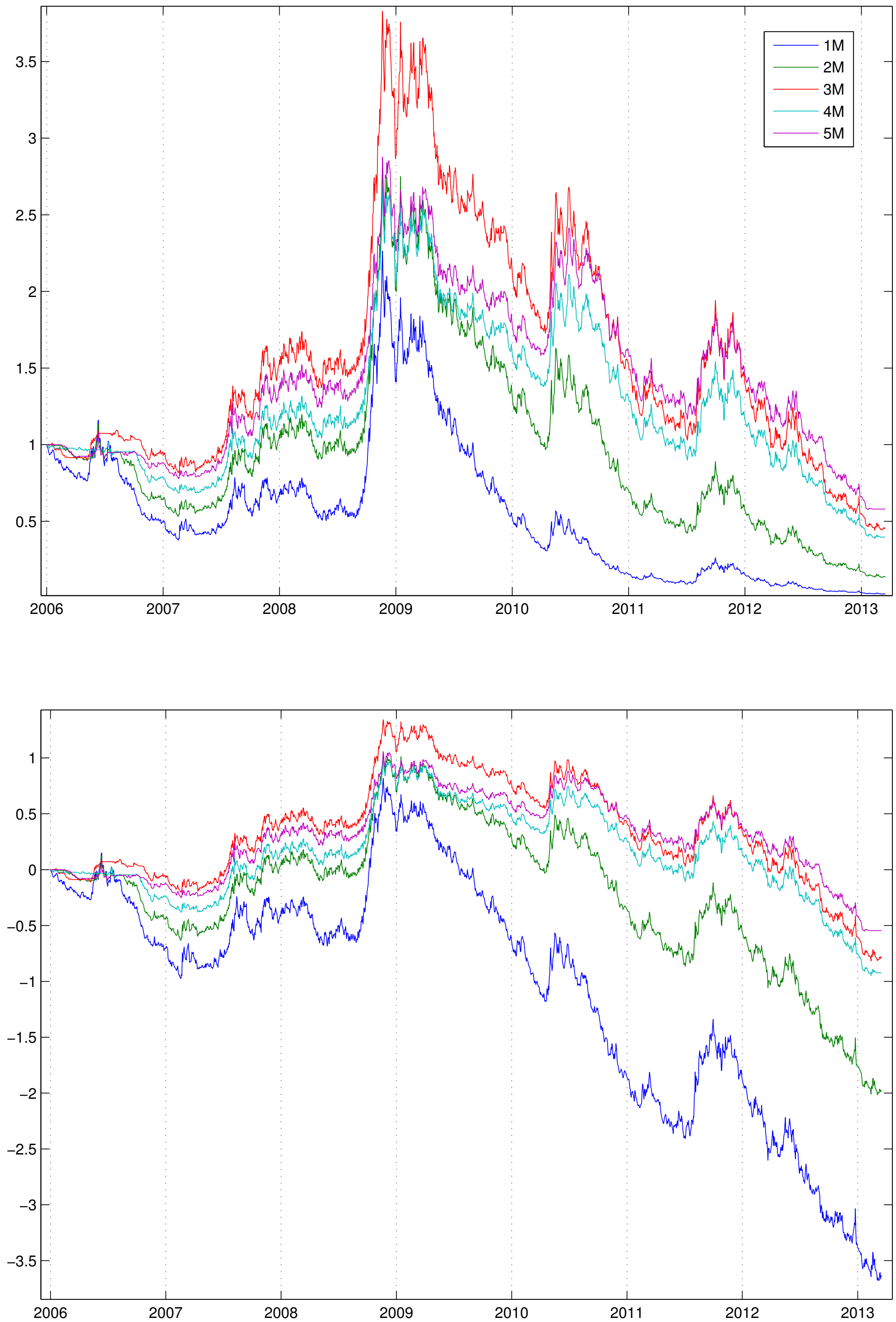

Figure 1: Value of portfolios that roll the futures contracts at closing prices.Top: Value of \$ 1 invested in Jan 2006. Bottom: Log-value of \$ 1 invested in Jan 2006.

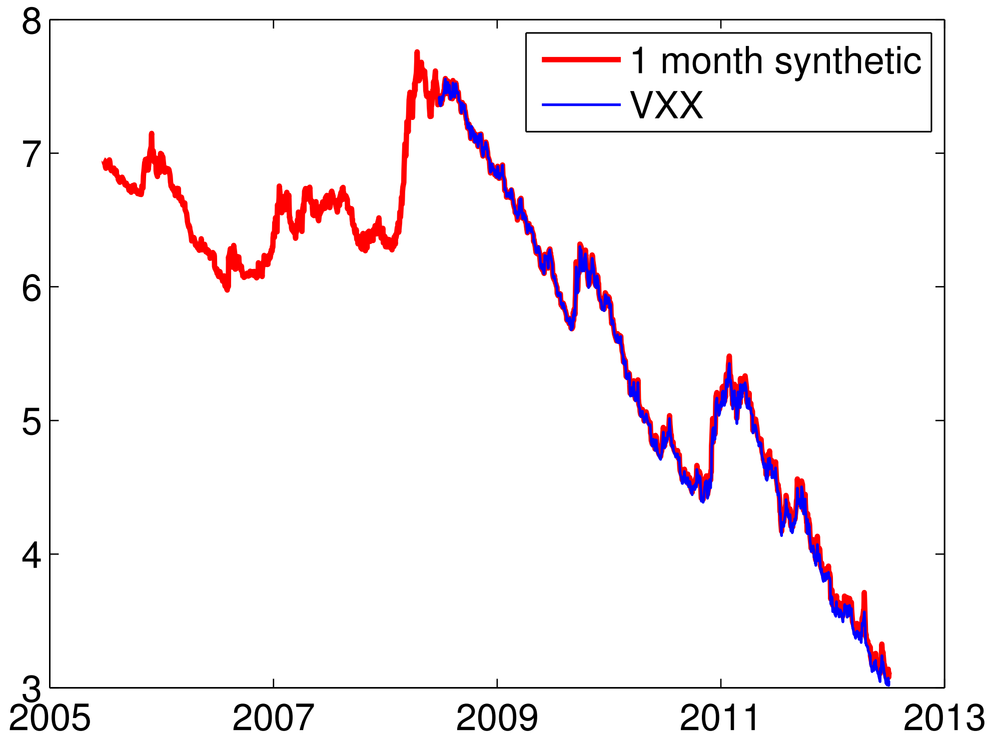

Figure 2: Log-value of the VXX and a synthetic one-month rolling VIX futures position.

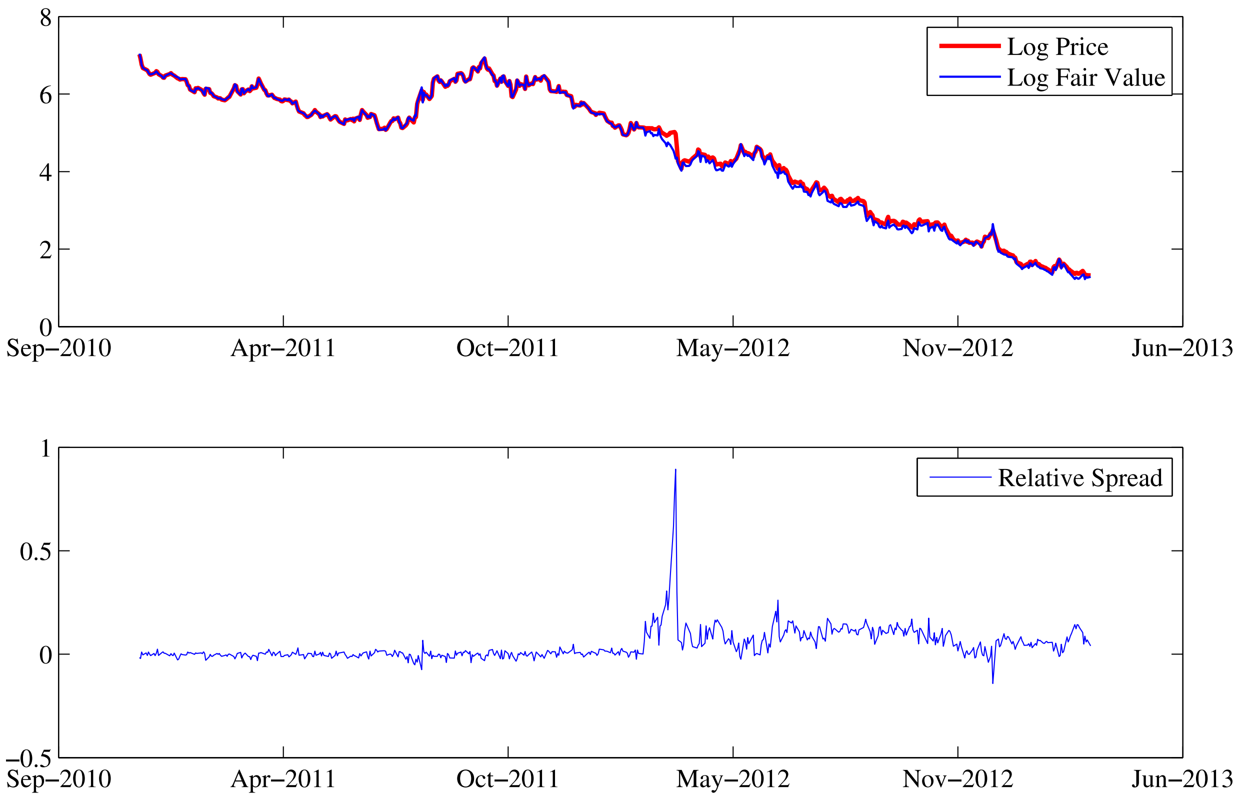

Figure 3: Price of the TVIX and its Net Asset Value (top) and the relative difference between price and NAV (bottom).

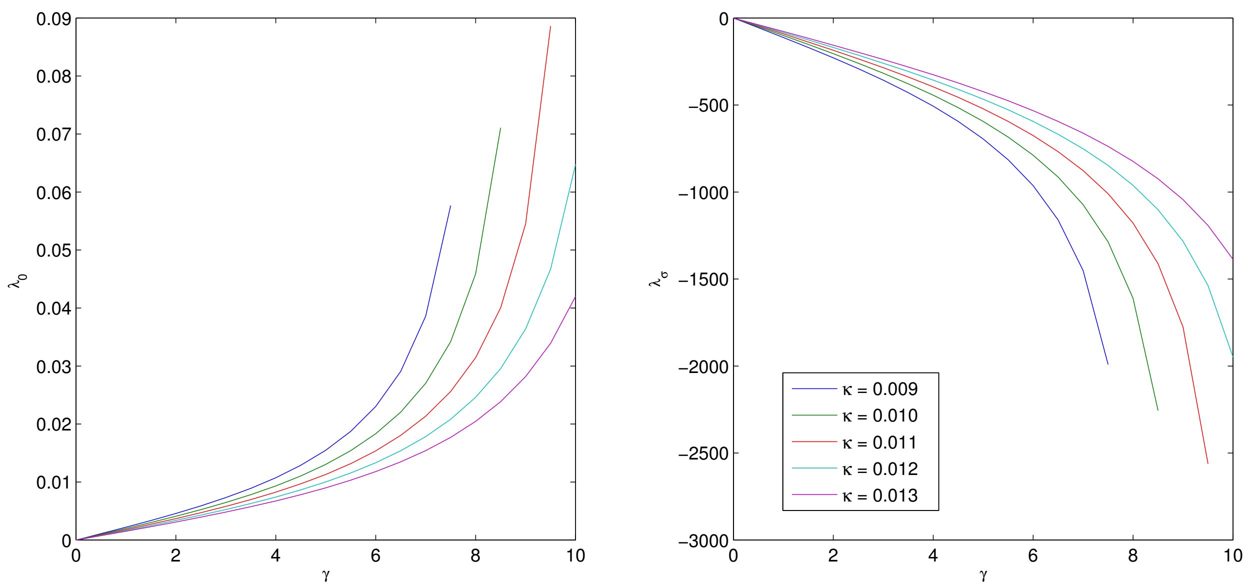

Figure 4: The equilibrium coefficients, λ_0 and λ_σ as functions of risk-aversion, γ.

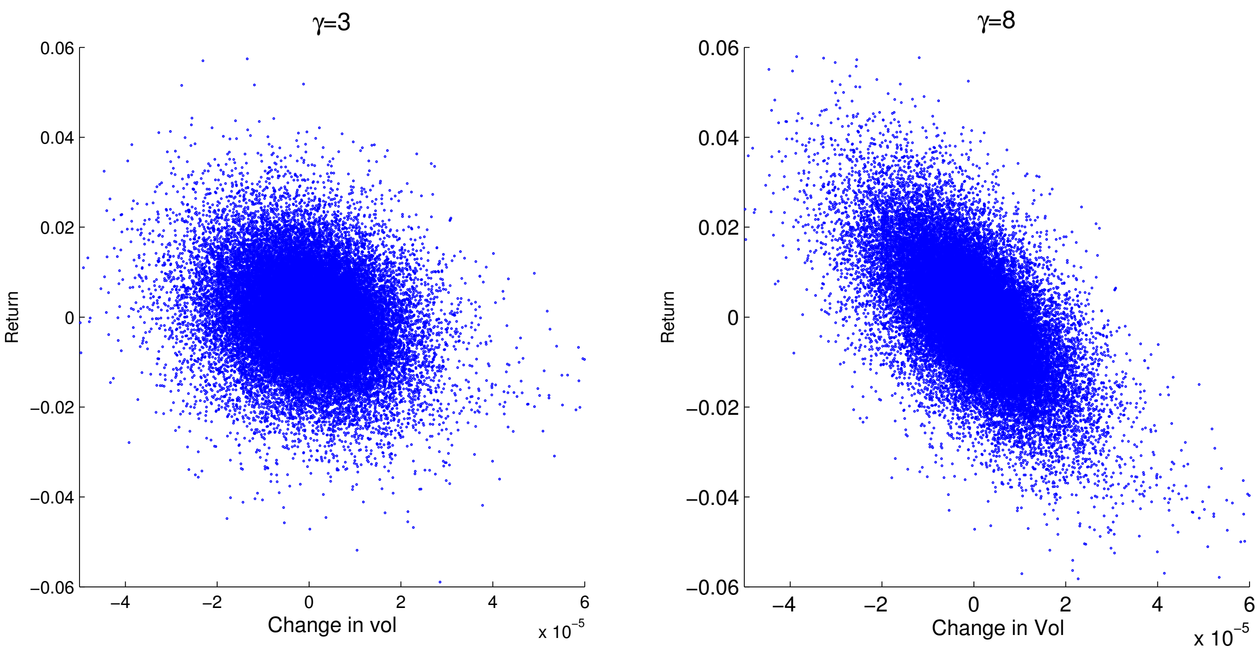

Figure 5: Scatter plots of contemporaneous daily changes in volatility and stock returns. The data are simulated from the model using $\gamma = 3$ and 8 respectively.

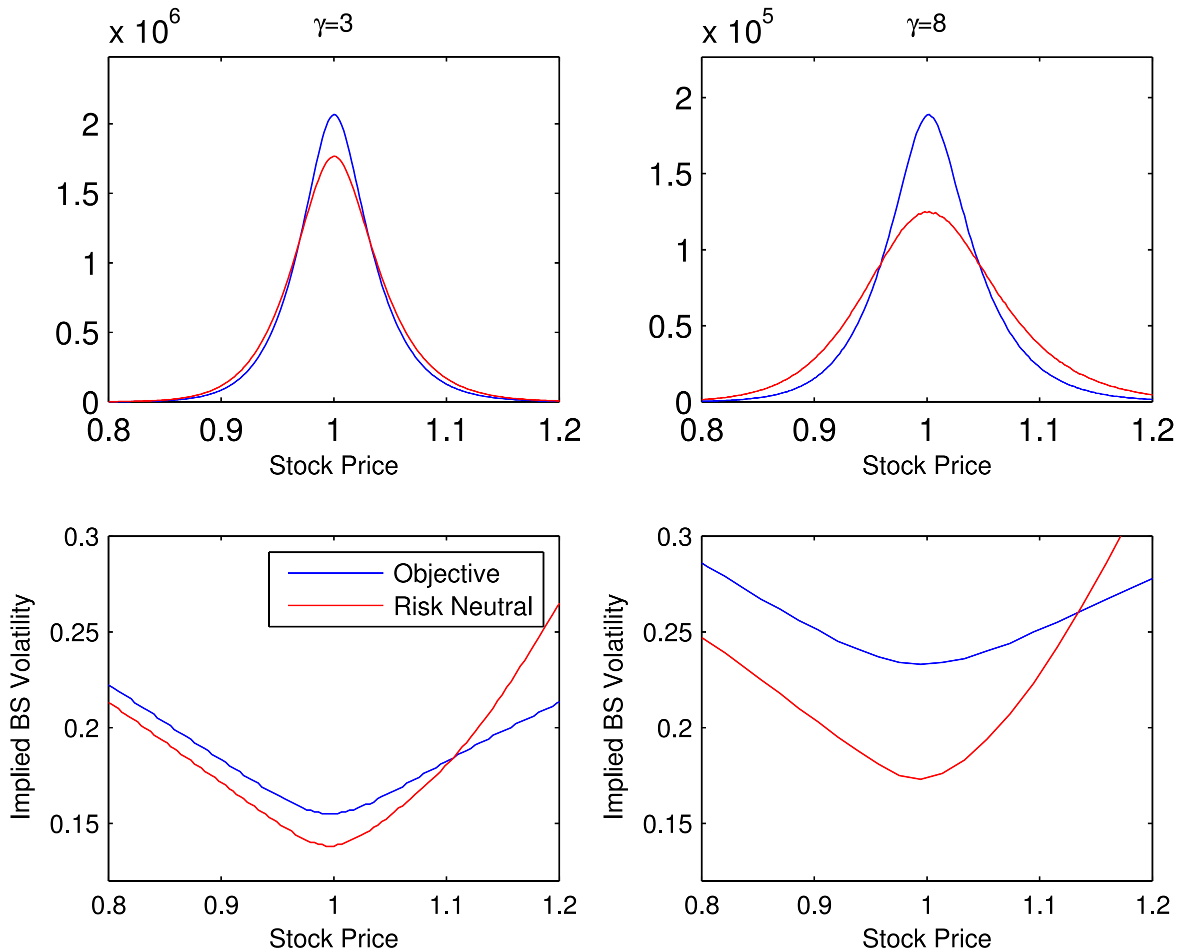

Figure 6: Top: Objective (P) and Risk Neutral (Q) one month conditional transition densities for the aggregate stock market for $\gamma = 3$ and 8. Bottom: Implied Black-Scholes volatility. The "objective measure (blue)" implied Black-Scholes volatility is computed assuming zero-volatilty and jump risk premia.

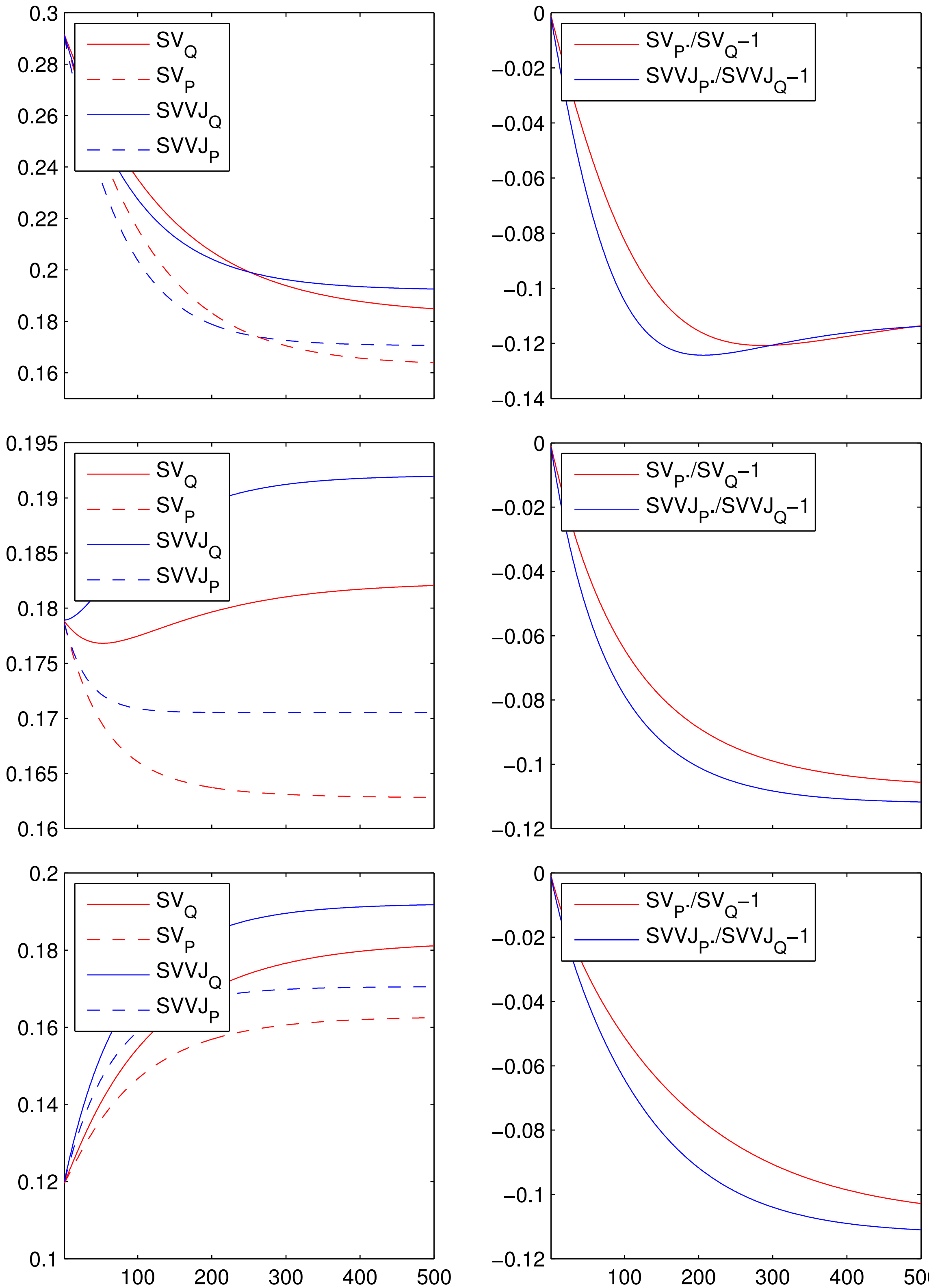

Figure 7: Left: VIX futures curve and the objective measure expected payoff in the case of high, medium (steady state), and low initial values of spot variance, σ_t^2. Right: expected holding period return, $E_t^P(VIX_{t+\tau})/E_t^Q(VIX_{t+\tau}) - 1$, to a long futures position.

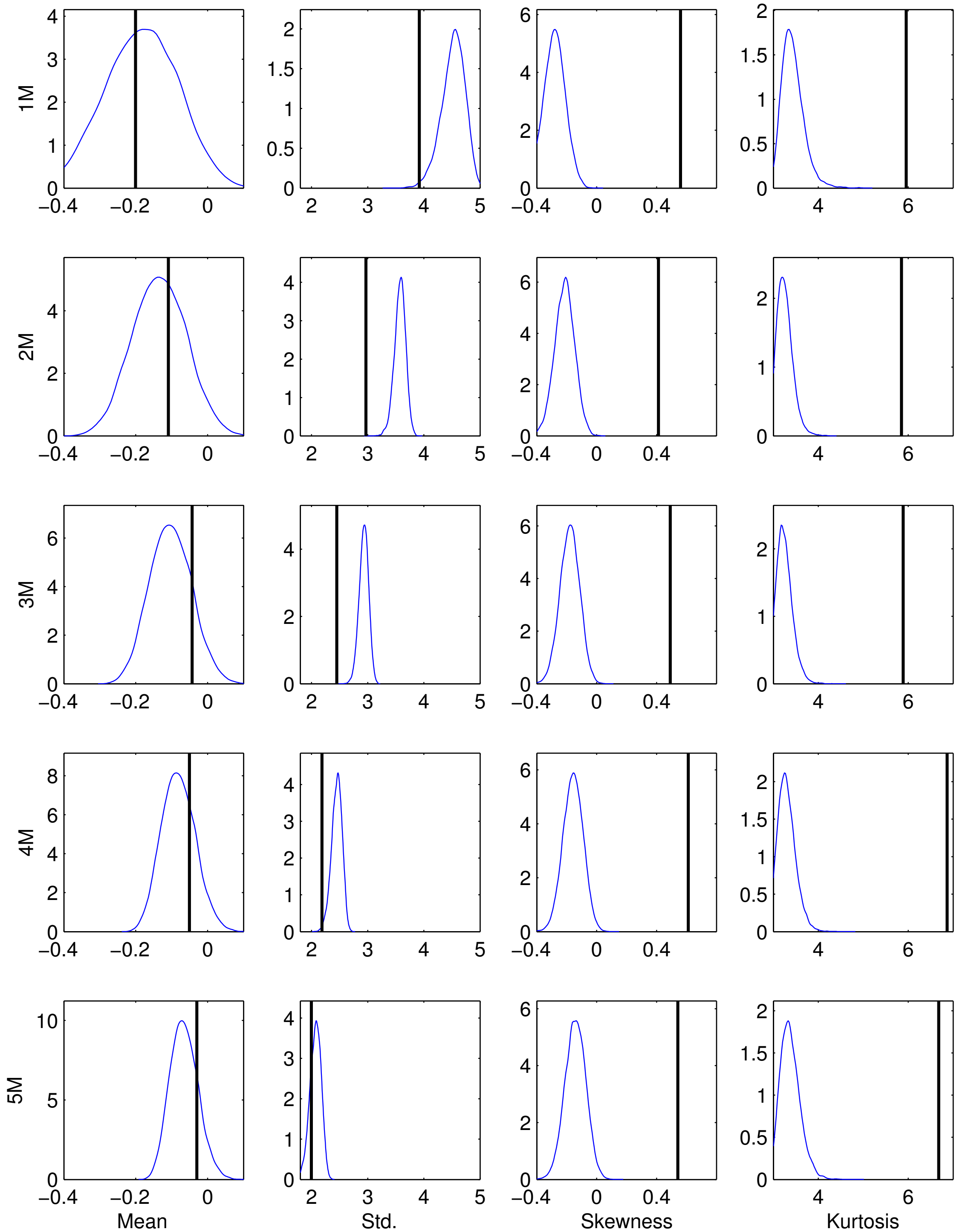

Figure 8: Sampling distributions from the SV model vs. data: We compute the sampling distributions, under the SV model, by simulating a sample for futures returns and then computing the sample moments. The small sample densities are estimated by kernel densities and compared to the sample moments in the data, shown in the vertical bars.

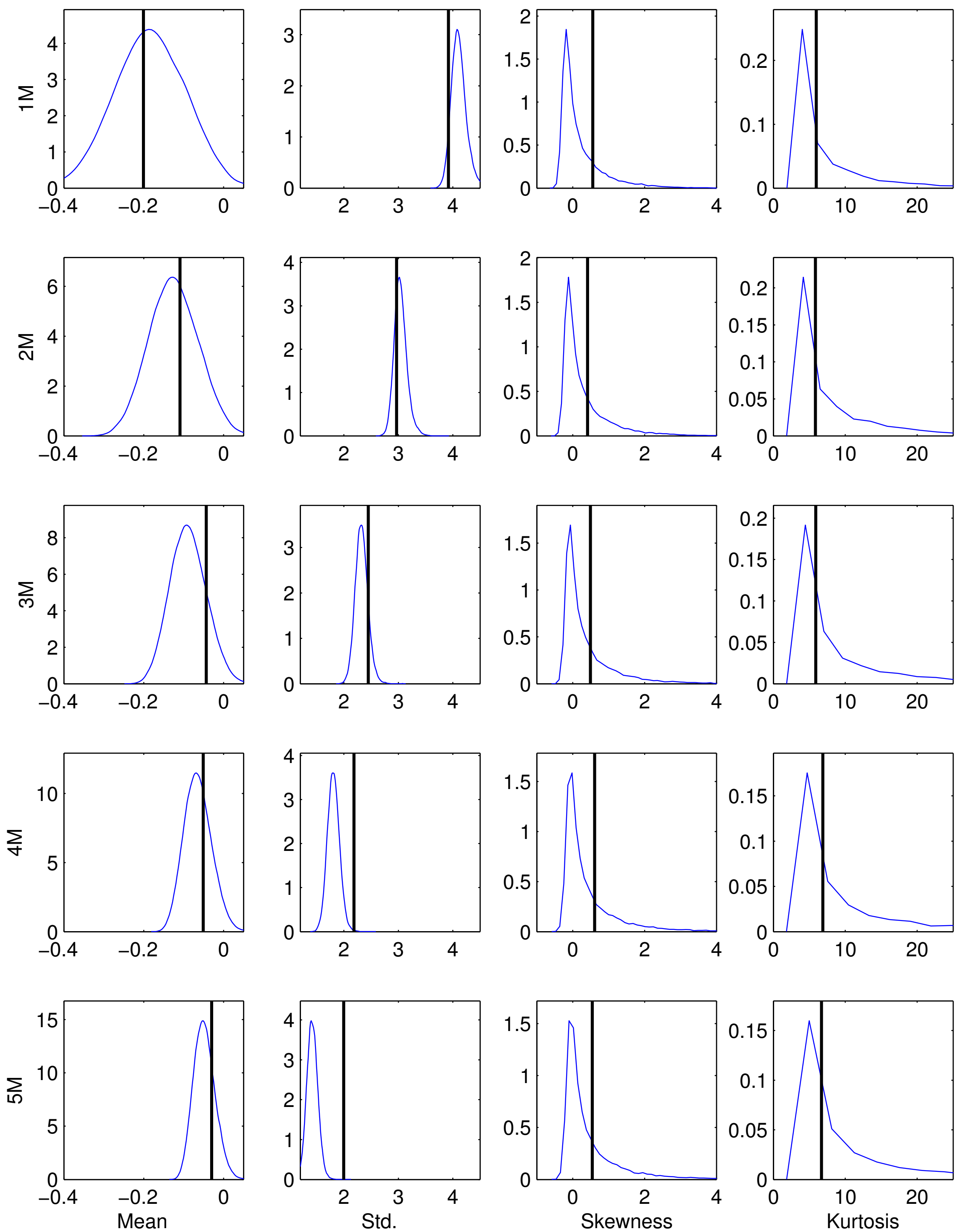

Figure 9: Sampling distributions from the SVVJ model vs. data: We compute the sampling distributions under the SVVJ model by simulating a sample for futures returns and then computing the sample moments. The small-sample densities are estimated by kernel smoothing and compared to the sample moments in the data, shown in the vertical bars.

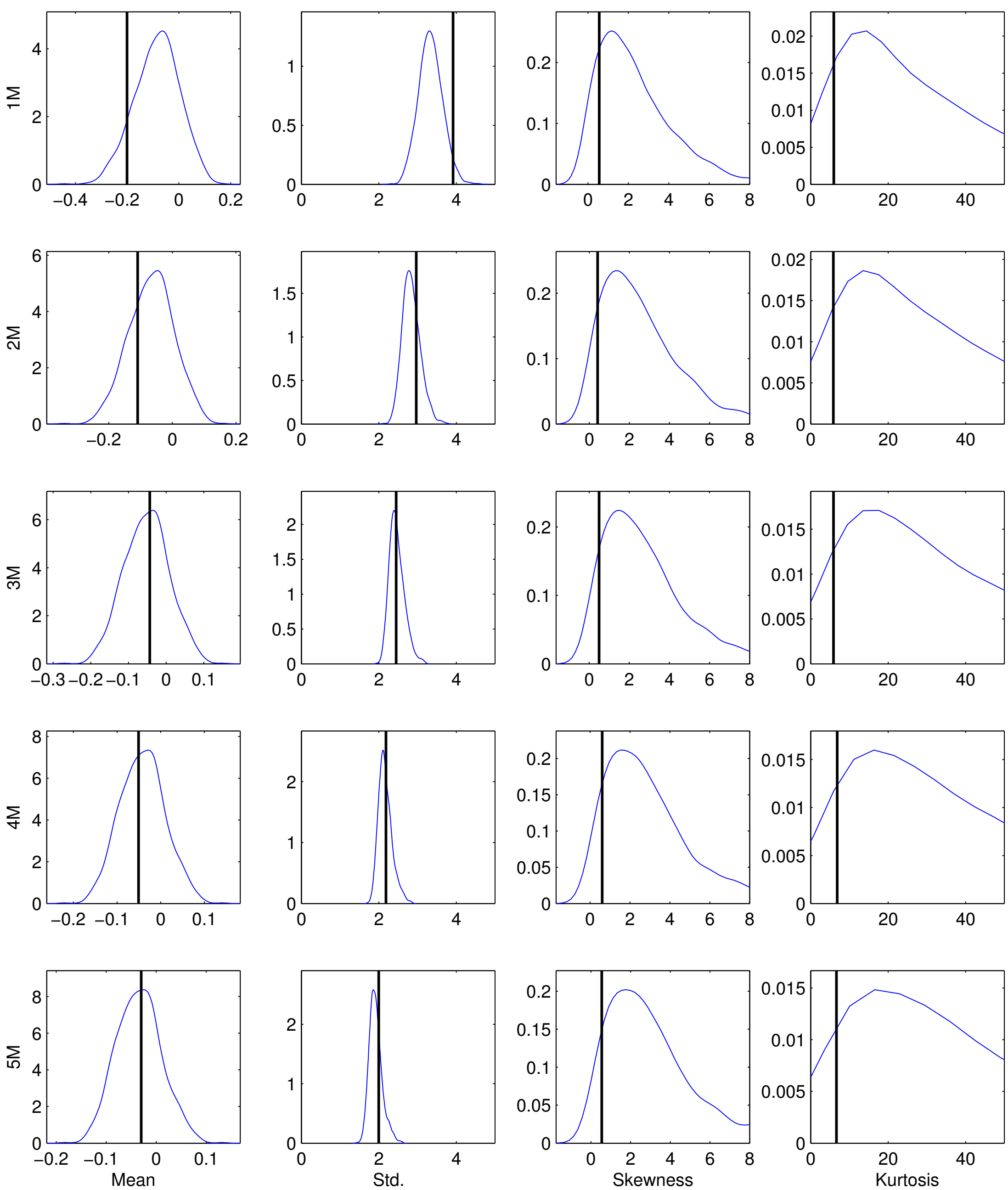

Figure 10: Sampling distributions from the two-factor volatility model vs. data: We compute the sampling distributions, under the model in eqns. (23) - (30), by simulating a sample for futures returns and then computing the sample moments. The small sample densities are estimated by kernel densities and compared to the sample moments in the data, shown in the vertical bars.

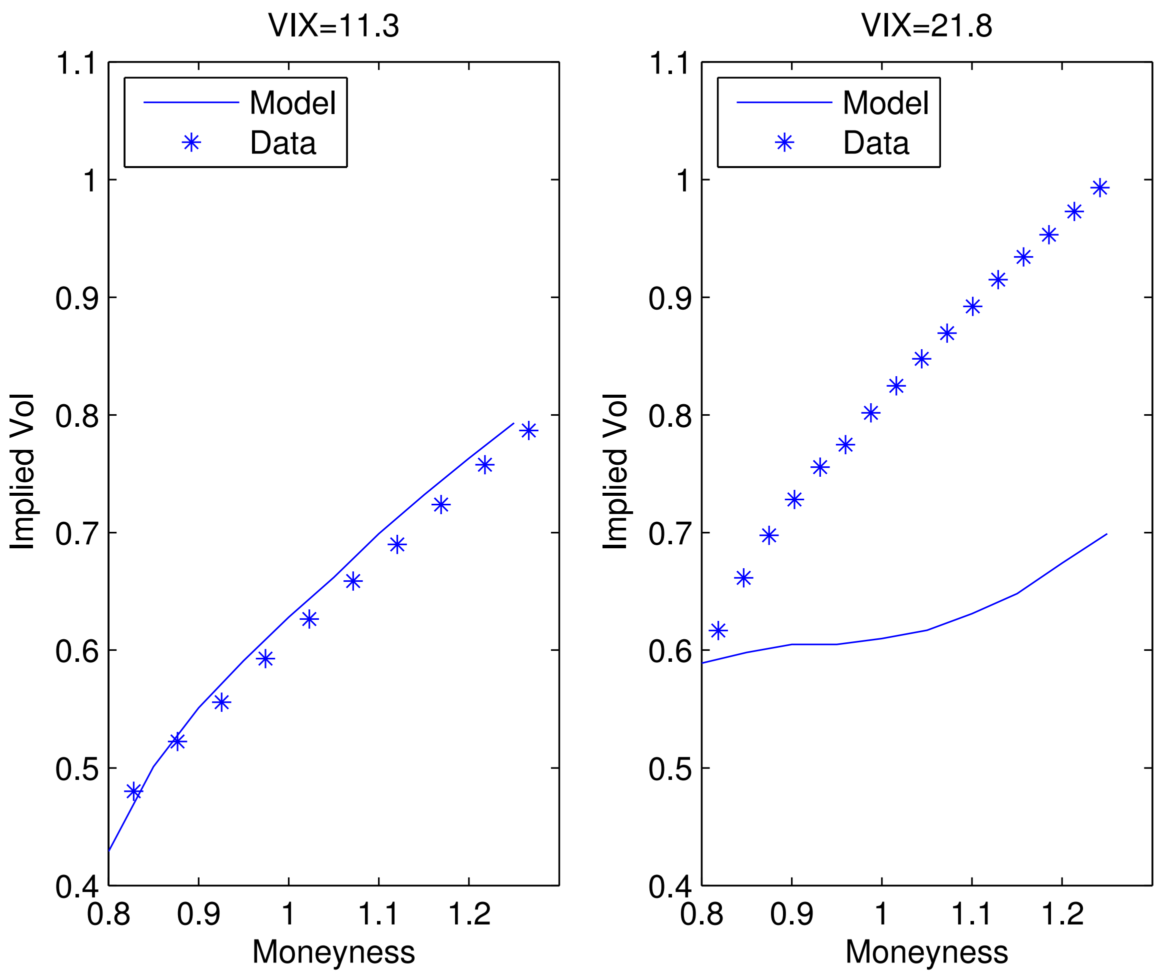

Figure 11: Model and Market implied volatility for one month VXX options: We plot the market implied volatility for two days, 3/14/2013 when the VIX was low at 11.3 (left) and 12/28/2012 when the VIX closed at 21.8 (right). The model Implied Volatility is shown in solid dark green. The violet line in the right plot shows the model price using volatility jump size equal to twice that estimated from data.

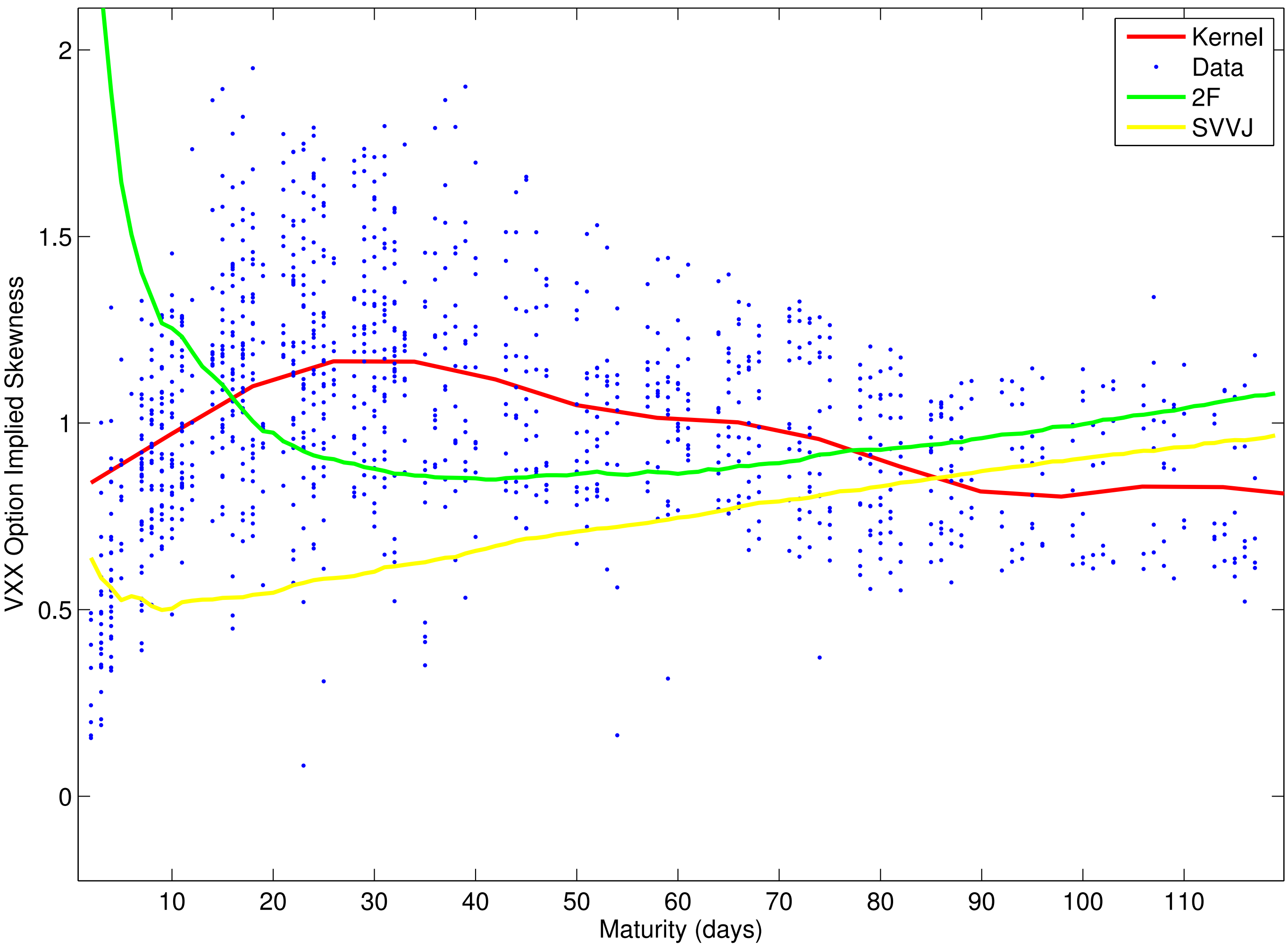

Figure 12: Option implied skewness for the VXX as a function of maturity. Option implied skewness is estimated each day from from Sept.4, 2012 until Aug.30, 2013 using end-of-day OptionMetrics data. The red line is a non-parametric regression.

Table 1: VIX Futures Return Descriptive Statistics

This table reports the summary statistics of returns to rolling positions in VIX futures. The sample data is at daily frequency from Jan 2006 to May 2013. R^1 is the daily average arithmetic return, R^2 is the daily average logarithmic return and R^3 is the average, annualized geometric return. Standard deviation, skewness and kurtosis are from daily arithmetic returns.

Maturity	R^1	R^2	R^3	Std	Skewness	Kurtosis
1 Month	-0.12	-0.20	-39.40	3.98	0.83	6.42
2 Month	-0.07	-0.11	-24.65	3.00	0.62	6.02
3 Month	-0.01	-0.04	-10.15	2.47	0.66	6.11
4 Month	-0.03	-0.05	-12.09	2.21	0.79	7.28
5 Month	-0.01	-0.03	-7.33	2.01	0.70	6.90

Table 2: Factor Regressions

Using the VIX futures logarithmic returns, this table reports factor regressions. The factors are the daily logarithmic change in the VIX, $\triangle VIX = \ln(VIX_t/VIX_{t-1})$, and the Fama-French factors MKT, SMB and HML. $t(\alpha)$ denotes a standard t-test of the null hypothesis $\alpha = 0$. θ is a test statistic for the null-hypothesis, $\alpha = 0$ (see Campbell, Lo, and Mackinlay (1997)), and $p(\theta)$ is the associated p-value.

	α	$t(\alpha)$	$\triangle VIX$	MKT	SMB	HML
			Full Model, $H_0 : \alpha = 0, \theta = 6.35, p(\theta) = 0.00$			
1 Month	-0.19	-4.51	0.36	-0.67	-0.41	-0.09
2 Month	-0.10	-2.87	0.24	-0.63	-0.18	-0.11
3 Month	-0.04	-1.16	0.18	-0.58	-0.10	-0.11
4 Month	-0.04	-1.48	0.14	-0.61	-0.05	-0.05
5 Month	-0.02	-0.81	0.13	-0.45	-0.10	-0.07
			$\triangle VIX$, MKT, $H_0 : \alpha = 0, \theta = 6.37, p(\theta) = 0.00$			
1 Month	-0.19	-4.52	0.37	-0.71		
2 Month	-0.10	-2.89	0.24	-0.67		
3 Month	-0.04	-1.17	0.18	-0.62		
4 Month	-0.04	-1.49	0.14	-0.62		
5 Month	-0.02	-0.82	0.13	-0.47		
			MKT, $H_0 : \alpha = 0, \theta = 3.99, p(\theta) = 0.00$			
1 Month	-0.17	-2.89		-2.08		
2 Month	-0.09	-1.93		-1.58		
3 Month	-0.02	-0.64		-1.30		
4 Month	-0.03	-1.01		-1.16		
5 Month	-0.01	-0.46		-0.98		
			$\triangle VIX$, $H_0 : \alpha = 0, \theta = 6.48, p(\theta) = 0.00$			
1 Month	-0.21	-4.51	0.48			
2 Month	-0.11	-2.96	0.35			
3 Month	-0.05	-1.39	0.28			
4 Month	-0.05	-1.68	0.24			
5 Month	-0.03	-1.03	0.21			

Table 3: Average VIX Futures Prices

This table reports the summary statistics of the extrapolated constant maturity (one to seven month) VIX futures. The sample data is at daily frequency from March 2004 to May 2013.

		Futures Maturity						
	Spot	1	2	3	4	5	6	7
Mean	20.57	21.48	22.10	22.45	22.70	22.94	23.14	23.29
Median	17.61	19.51	20.94	21.80	22.31	22.75	23.19	23.44
Std. Dev.	10.11	8.79	8.05	7.51	7.13	6.87	6.68	6.52
Implied Return		-40.52%	-28.93%	-17.18%	-12.44%	-11.86%	-9.89%	-7.46%
	Eigenvalues of the Correlation Matrix							
	6.85	0.14	0.01	0.00	0.00	0.00	0.00	

Table 4: VIX ETNs Investment Objectives and Average Returns

This table summarizes investment objectives and performance of VIX Exchange Traded Notes (ETNs). The investment objectives are denoted $1x$, $2x$ and $-1x$, reflecting a single long, 100% levered long, and single short position, respectively. $\bar{r}$ is the average daily arithmetic return in percent.

Ticker	First Date	Leverage	Horizon	Underlying	$\bar{r}$	$\hat{\text{Std}}(r)$
UVXY	04-Oct-2011	2x	1M	VIX-F	-1.13	8.57
TVIX	30-Nov-2010	2x	1M	VIX-F	-0.67	8.06
CVOL	15-Nov-2010	1x	3-4M	VIX-S&P	-0.47	6.96
TVIZ	30-Nov-2010	2x	5M	VIX-F	-0.38	4.20
VXX	29-Jan-2009	1x	1M	VIX-F	-0.34	3.94
VIIX	30-Nov-2010	1x	1M	VIX-F	-0.30	4.27
VIXY	04-Jan-2011	1x	1M	VIX-F	-0.26	4.34
VIXM	04-Jan-2011	1x	5M	VIX-F	-0.17	2.12
VIIZ	30-Nov-2010	1x	5M	VIX-F	-0.19	2.08
VXZ	29-Jan-2009	1x	5M	VIX-F	-0.14	2.00
XVIX	01-Dec-2010	S/L	-1&5M	VIX-F	-0.05	0.79
IVOP	19-Sep-2011	-1x	1M	VIX-F	0.22	2.95
XIV	30-Nov-2010	-1x	1M	VIX-F	0.25	4.27
SVXY	04-Oct-2011	-1x	1M	VIX-F	0.49	4.33
ZIV	30-Nov-2010	-1x	5M	VIX-F	0.15	1.99

Table 5: VIX ETN Relative Performance

This table compares the performance of the XIV and TVIX stocks to their synthetic securities using VXX returns. The returns on TVIX are related to the VXX through $r_t^{\text{TVIX}} = 2 \times r_t^{\text{VXX}}$ and the returns on XIV are $r_t^{\text{XIV}} = -r_t^{\text{VXX}}$. We use these relationships to construct synthetic TVIX and XIV and compare the prices and fair values of each security to the corresponding synthetic security. Gi is defined as the annualized geometric gain relative to the synthetic security, $Gi = (P_{T,i}/P^s_{T,i})^{252/T} - 1$ where $i = 1, 2$ representing actual trade price and net asset value respectively and $P^s_{T,i}$ denotes the ending value of the synthetic security created from VXX returns with $P^s_{0,i} = P_{0,i}$.

	G1	G2
TVIX	3.36	1.61
XIV	-1.80	-1.90

Table 6: Model Parameter Estimates

The table reports parameter estimates of the underlying model for S&P 500. The parameter estimates are obtained using stock returns only. We report posterior means and standard deviation based on a joint MCMC simulation of latent volatility, jump times and sizes.

	SVVJ	SV
$\theta \times 10000$	0.677	0.758
	(0.058)	(0.069)
κ	0.014	0.0091
	(0.002)	(0.001)
$\sigma_v \times 10000$	10.788	9.716
	(0.357)	(0.307)
λ	0.002	
	(0.000)	
$\mu_v \times 10000$	0.390	
	(0.096)	
γ	7.945	5.898
	(0.760)	(0.677)

Table 7: VIX Futures Return Moments: Data vs. Model

This table compares average returns of VIX futures positions in data and model simulations. R^1 is the daily average arithmetic return, R^2 is the daily average logarithmic return and R^3 is the average annual geometric return. Standard deviation, skewness and kurtosis are from daily logarithmic returns.

Maturity	R^1	R^2	R^3	Std	Skewness	Kurtosis
			Data			
1 Month	-0.12	-0.20	-39.40	3.92	0.56	5.95
2 Month	-0.07	-0.11	-24.65	2.97	0.41	5.85
3 Month	-0.01	-0.04	-10.15	2.45	0.49	5.88
4 Month	-0.03	-0.05	-12.09	2.18	0.61	6.86
5 Month	-0.01	-0.03	-7.33	2.00	0.54	6.67
			SVVJ			
1 Month	-0.10	-0.19	-36.14	4.15	1.00	26.33
2 Month	-0.08	-0.13	-26.29	3.06	0.94	23.10
3 Month	-0.06	-0.09	-19.60	2.34	0.91	21.85
4 Month	-0.05	-0.07	-14.84	1.82	0.89	21.26
5 Month	-0.04	-0.05	-11.37	1.43	0.88	20.97
			SV			
1 Month	-0.09	-0.19	-36.10	4.50	-0.13	3.29
2 Month	-0.07	-0.14	-28.15	3.56	-0.10	3.17
3 Month	-0.06	-0.11	-22.57	2.92	-0.08	3.20
4 Month	-0.05	-0.08	-18.41	2.44	-0.07	3.27
5 Month	-0.05	-0.07	-15.19	2.06	-0.06	3.36