Network Games and Agricultural Collectivization in China

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Abstract

This project uses the framework of network games to analyze incentive structures and behavioral dynamics in the context of Chinese agricultural collectivization during the 1960s and 1970s. Starting from a baseline model of small reciprocal production teams, we incrementally build up to more complex systems. Firstly, we expand the network by conecting different teams, allowing for inter-team interactions. Then we add signaling games to capture the information assymetries and communications among agents. Finally, we introduce external shocks to the system, such as natural disaster, to study the reactions of different network structures. And we find that ...

1. Introduction

Agricultural collectivization has been one of the most prominent and controversial policies in modern China. Existing studies (Chinn [1980], Nitzan and Schnytzer [1987]) have used game-theoretic approaches such as the Prisoner's Dilemma to model individual incentives and effort in collective settings. While insightful, these models often assume homogeneity and lack the structural complexity seen in real-world rural networks and governance.

This project expands the analytical lens by incorporating **network games** to capture how local interdependencies shape collective effort and participation decisions. Households in a commune do not interact uniformly; instead, their decisions are influenced by their neighbors' behavior, visibility, and reputational spillovers. Network games offer a natural framework to reflect this decentralized but interconnected structure.

Furthermore, to analyze the vertical interactions between local commune leaders and higher-level government officials—especially under asymmetric information and crisis conditions—we introduce **signaling games**. During natural disasters or in the context of procurement targets, local leaders often faced incentives to misreport productivity or overstate effort to secure promotions or avoid sanctions. Signaling games allow us to formally represent these strategic misalignments and the role of beliefs and incentives in shaping observed actions.

By combining network and signaling models, we aim to offer a richer, more institutionally grounded understanding of how collectivization outcomes emerged—not just from household-level decisions, but also from the complex interplay between local coordination and hierarchical governance.

2. Base Model: The Small Network within a Commune

Let $I = \{1, 2, \ldots, n\}$ denote the set of players, $n > 1$, connected by a network G. We use matrix G to track the connections in this network. We define $g_{ij} = 1 = g_{ji}$ if i and j are linked to each other, and $g_{ij} = 0$ otherwise. We also set $g_{ii} = 0$. The neighborhood of the individual i is the set given by $[N_i := j \mid g_{ij}] = 1$. Therefore, $|N_i|$, the cardinality, which is $\sum_j g_{ij}$, is called the **degree** of i.

We use the following quadratic utility function to capture the *strategy complementarity*:

$$U_i(x_i, x_{-i}, G_i) = \alpha x_i - \frac{1}{2}x_i^2 + \beta x_i \sum_j g_{ij} x_i x_j,$$

where α is the marginal return to effort x_i, and β is the strength of strategic interactions.

We assume $\beta > 0$, so that we have

$$\frac{\partial^2 U_i(x_i, x_{-i}, G_i)}{\partial x_i \partial x_j} = \beta g_{ij} > 0,$$

which reflects strategic complementarity in efforts.

In small groups with n players in a network, people know each other well, and it is reasonable to assume network G is a **complete network** and players have full information.

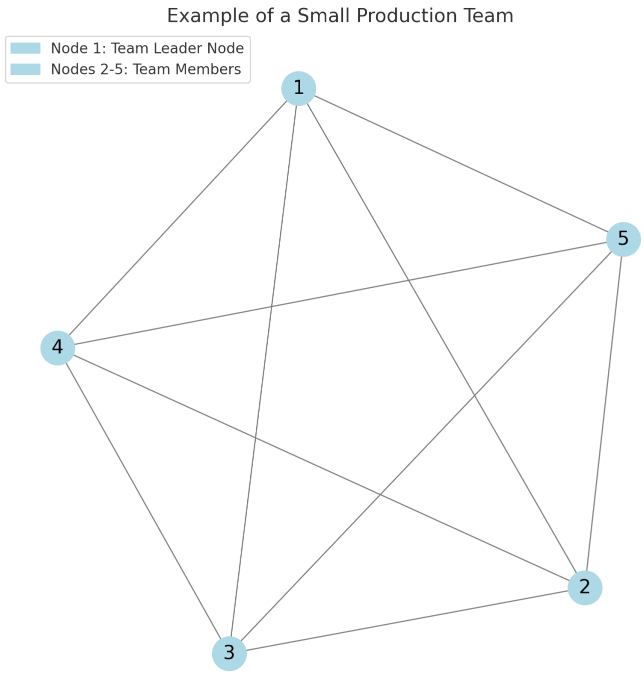

Figure 1: Example of a Small Production Team

In equilibrium, each agent maximizes their utility. By the FOC, we get:

$$x_i^* = \alpha + \beta \sum_j g_{ij} x_i x_j. \tag{2}$$

Since G is a complete network, everyone has the same degree. We assume total effort is

$$\bar{x} = \sum_i x_i,$$

and so:

$$x_i^* = \alpha + \beta \left(\sum_{j \neq i} x_j^* \right) = \alpha + \beta(n-1)\bar{x}. \tag{3}$$

Assuming symmetry, i.e., $x_i^* = \bar{x}$ for all i, we get:

$$x_i^* = \frac{\alpha}{1 - \beta(n-1)}.$$

Then, in a small group, the utility of agent i' is given by:

$$\frac{\alpha^2}{\beta + 1 - n\beta} - \frac{1}{2} \left(\frac{\alpha}{(\beta + 1 - n\beta)} \right)^2 + \beta(n-1) \left(\frac{\alpha}{\beta + 1 - n\beta} \right)^2. \tag{4}$$

3. Large Networks

3.1. Large Network with Without Signaling

Now we move to the large network setting where there are multiple small groups. And in each small complete network, there is one node that represents the leader of the small group. Between the small groups, there are indirect connections among the small group leaders through a central node the represents higher level government.
By this construction, we have three types of players with different degrees. We assume $N = nk + 1$, where n is the number of players in a small group, k is the number of small groups.

- Type 1: $(n-1)$ degrees (internal to small group)
- Type 2: k degrees (one connection to each small group leader)
- Type 3: n degrees (small group leader)

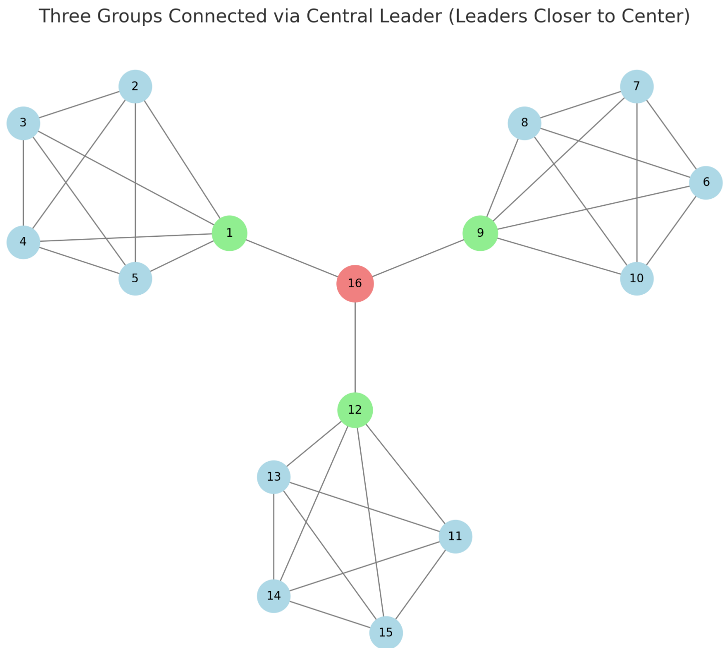

Figure 2: Example of a Large Network

Incomplete information about β without signals.
We assume β takes two possible values: $\beta = \{\beta_l, \beta_h\}$ where $\beta_l < \beta_h$.
We assume each agent holds a belief for β.
They share a common prior:

$$\mathbb{P}(\beta = \beta_l) = p, \quad \mathbb{P}(\beta = \beta_h) = 1 - p$$

Therefore we assume there is no communication between players, and the network does not affect the value of p.

Then:

$$\mathbb{E}[x_i] = \alpha - \sum_j g_{ij}\mathbb{E}[x_j] + \mathbb{E}[\beta]p_i$$

where:

$$\mathbb{E}[\beta] = p\beta_l + (1-p)\beta_h$$

Also:

$$\mathbb{E}[x_i] = x_i$$

Thus:

$$\alpha - x_i + \mathbb{E}[\beta]p_i - \sum_j g_{ij}x_j = 0$$

Thus:

$$x_i^* = \alpha + \mathbb{E}[\beta] + \sum_{j=1}^{n} g_{ij}x_j$$

where G is the network matrix.

The solution is given by:

$$x^* = \alpha(I_n - \mathbb{E}[\beta]G)^{-1}\mathbf{1}$$

where I_n is the $n \times n$ identity matrix.
G is the same network matrix as before.

We assume that:

$$\mathbb{E}[\beta] \leq \frac{1}{\lambda_{\max}(G)}$$

where $\lambda_{\max}(G)$ is the largest eigenvalue of the network G.

Moreover:

$$(I_n - \mathbb{E}[\beta]pG)^{-1} = P_G \left(I - \mathbb{E}[\beta]pD_G\right)^{-1} P_G^{-1}$$

where:

$$G = P_G D_G P_G^{-1}$$

and D_G is the diagonal matrix of eigenvalues of G:

$$D_G = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}$$

Given the structure of the network, here is the G matrix that represents the connections between players:

1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1
2	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
3	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
7	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0	0
8	0	0	0	0	0	1	1	0	1	1	0	0	0	0	0	0
9	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	1
10	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
12	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1
13	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0
14	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0
15	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
16	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0

Figure 3: Adjacency matrix G for the default 16-player network

So the 1st, 9th, and 12th rows are the leaders of the small groups, while the 16th row (the last row) is the central node that connects all the small group leaders. The rest are the members in the small groups.

Then we calculate the eigenvalues of the G matrix. (See in appendix) The largest eigenvalue is $\lambda_{\max}(G) \approx 4.1622$ and thus the upperbound for $E[\beta] = \frac{1}{\lambda_{max}(G)} \approx 0.24$.

The Effects of $E[\beta]$ on Equilibrium Efforts

We want to see how the equilibrium effort x^* changes with the value of $E[\beta]$. However, it is very diffucult to compute with the differentiation, so here we just plug in $E[\beta] = 0.2, 0.1$

Player Type	x^* **when** $E[\beta] = 0.2$	x^* **when** $E[\beta] = 0.045$
Small Group Leader	6.67	1.27
Small Group Member	5.83	1.22
Central Leader (Player 16)	5.00	1.17

Table 1: Equilibrium actions x^* under different values of β

The table above confirms that the equilibrium effort x^* is increasing in $E[\beta]$. This is because the higher the value of $E[\beta]$, the more likely the players are to believe that their neighbors are also putting in effort, which leads to a higher equilibrium effort. Besides, the results also show that group leaders have the highest equilibrium, following by group members and central leader, corresponding to the fact that the group leaders have the highest numbers of nodes connected, while the central leader has the least.

The Effects of p on Equilibrium Efforts

More importantly, we want to investigate how the equilibrium effort x^* changes with the value of p which is the probability of being a high type player. Here we return to the original equation for $E[\beta]$ which equals $p\beta_h + (1-p)\beta_l$. We assume $\beta_l = 0.03$ and $\beta_h = 0.06$. (So the $p = 0.5$ case will have the same $E[\beta]$ as above.) Again, doing differentiation is complicated so we plug in several possible values for $p \in [0, 1]$

Player Type	x^* **at** $p = 0.1$	x^* **at** $p = 0.5$	x^* **at** $p = 0.9$
Small Group Leader	1.19	1.27	1.37
Small Group Member	1.15	1.22	1.30
Central Government (Player 16)	1.12	1.17	1.23

Table 2: Equilibrium actions x^* as a function of $p = \mathbb{P}(\beta = \beta_h)$

The table above shows that the equilibrium effort x^* is increasing in p which is aligned with the strategic complementarity.

3.2. Large Network with Signaling

We move to a large network setting where each household only knows its own number of neighbors (degree) and forms beliefs over others' behavior. The model assumes strategic complements or substitutes depending on the distribution mechanism and social context.

There are $M \gg n$ players now. In this case, we treat β as important information to capture the feature that in large groups, an agent does not know their neighbors' neighborhoods well. This uncertainty introduces people's incomplete information about β.

We assume that β can only take two values, β_L or β_H. All individuals share a common prior:

$$\Pr(\beta = \beta_H) = p, \quad p \in (0, 1)$$

Individuals receive two signals which are not always correct. We define:

$$\Pr(s_i = 1 \mid \beta = \beta_H) = q_1, \quad \Pr(s_i = 1 \mid \beta = \beta_L) = q_2$$

where $s_i = 1$ and $s_i = -1$ denote the event that agent i has received signal 1 and -1, respectively.

Besides, we assume that the network is no longer a complete network. It is given by a graph

The idea is that a large network is composed of several small complete networks. And in each small complete network, there is one person who connects with leaders of the large group.

There are three types of players with different degrees. We assume $N = nk + 1$, where n is the number of players in a small group, k is the number of small groups.

- Type 1: $(n - 1)$ degrees (internal to small group)
- Type 2: k degrees (one connection to each small group leader)
- Type 3: n degrees (small group leader)

The utility function is the same for three types of players, given by:

$$u_i(x_i, x_{-i}) = \alpha_i x_i - \frac{1}{2} x_i^2 + x_i \left(\sum_j g_{ij} x_j \right)$$

By the first order condition, we get:

$$x_i^* = \alpha + \beta \sum_{j=1} g_{ij} x_j^*$$

We define:

$$b(\beta, G) = \sum_{k=0}^{\infty} \beta^k G^k I_n = (I_n - \beta G)^{-1} l_n,$$

where I_n is the identity matrix $n \times n$ and l_n is the vector $n \times 1$ of 1.

When agent i receives signal $s_i = l$, his utility function satisfies:

$$\mathbb{E}[x_i \mid s_i = l] = \alpha\, \mathbb{E}[x_i \mid s_i = l] - \frac{1}{2}\mathbb{E}[x_i^2|[s_i = l]] + \sum_{j=1}^{n} g_{ij} x_i\, \mathbb{E}[x_j \mid s_j = t]$$

$$= \alpha\, x_i(l) - \frac{1}{2} x_i(l)^2 + \sum_{j=1}^{n} g_{ij}\, x_i(l)\, \mathbb{E}\left[\beta\, x_j \mid s_j = l\right]$$

Thus:

$$x_i = \alpha_i(l) + \sum_j g_{ij} \mathbb{E}[x_j \mid s_i = l]$$

By the first order condition on $x_i(l)$, we get:

$$\alpha - x_i^*(l) + \sum_{j=1}^{n} g_{ij} \mathbb{E}[\beta x_j^* \mid s_i = l] = 0$$

When agent i receives signal $s_i = l$, for each possible j we have:

$$\mathbb{E}(\beta_j x_j \mid s_i = l) = \sum_{t=1}^{2} \sum_{m=1}^{2} \beta_{m x_j}(t) P\{\beta = \beta_m \cap s_j = l\}$$

$$= \beta_{\max} \sum_{t=1}^{2} \left(P\left(\{\beta = \beta_m\} \cap \{s_j = t\} \mid s_i = l\right) \cdot \frac{\beta_m}{\beta_{\max}} \right) x_j(t)$$

We define:

$$P_\mu = \sum_{j=1}^{2} P(\{\beta = \beta_m\} \cap \{s_j = l\} \mid s_i = l) \frac{\beta_m}{\beta_{\max}}$$

and

$$P_{ij} = \sum_t \mathbb{P}(\beta = \beta_m \cap s_j = t \mid s_i = L) \quad \text{with} \quad \frac{p_m}{p_{\max}}$$

where $\beta_{max} = \beta_h$ in our case.

P_{ll} represents individual i receives signal l and j receives signal l.

Therefore, we can rewrite the first order conditions as follows:

$$\alpha - x_i^*(l) + \beta_{max} \sum_{j=1}^{N} g_{ij} \sum_{t=1}^{2} p_{lt} * x_j(t) = 0$$

$$\alpha - x_i^*(h) + \beta_{max} \sum_{j=1}^{N} g_{ij} \sum_{t=1}^{2} p_{ht} * x_j(t) = 0$$

which can be characterized by:

$$\begin{pmatrix} x_L \\ x_H \end{pmatrix} = (-I_{2n} - \beta_{max}(\Gamma \otimes G))^{-1} \begin{pmatrix} \alpha_{1n} \\ \alpha_{2n} \end{pmatrix}$$

where Γ is the information matrix, G is the network matrix, $\otimes$ is the Kronecker product of Γ and G.

Γ is given by:

$$\Gamma = \begin{pmatrix} P_{ll} & P_{lh} \\ P_{hl} & P_{hh} \end{pmatrix}$$

Now we are going to calculate information matrix Γ.

We have:

$$\begin{aligned} P(s_i = l) &= P(s_i = l \mid \beta = \beta_l)P(\beta = \beta_l) + P(s_i = l \mid \beta = \beta_h)P(\beta = \beta_h) \\ &= q(1-p) + (1-q)p \end{aligned}$$

We have:

$$P(s_i = h) = P(s_i = h \mid \beta = \beta_h)P(\beta = \beta_h) + P(s_i = h \mid \beta = \beta_l)P(\beta = \beta_l)$$

$$= qp + (1-q)p$$

$$\begin{aligned} &P(\{\beta = \beta_l\} \cap \{s_i = l\} \mid \{s_i = l\}) \\ &= \frac{P(\{s_j = l\} \cap \{s_i = l\} \cap \{\beta = \beta_l\})}{P(\{s_i = l\})} \\ &= \frac{P(\{s_j = l\} \mid \{\beta = \beta_l\})\, P(\{s_i = l\} \mid \{\beta = \beta_l\})\, P(\{\beta = \beta_l\})}{P(\{s_i = l\})} \\ &= \frac{q^2(1-p)}{q(1-p) + (1-q)p} \end{aligned}$$

Similarly, we have

$$P(\{\beta = \beta_h\} \cap \{S_j = h\} \mid S_i = l) = \frac{q(1-p)(1-q)}{q(1-p) + (1-q)p}$$

and

$$P(\{\beta = \beta_h\} \cap \{S_j = l\} \mid S_i = l) = \frac{p(1-q)^2}{q(1-p) + (1-q)p}$$

as well as

$$P(\{\beta = \beta_h\} \cap \{S_j = h\} \mid S_i = l) = \frac{(1-q)pq}{q(1-p) + (1-q)p}.$$

Now, for $P(ll)$,

$$\begin{aligned} P(ll) &= P\left(\{\beta = \beta_h\} \cap \{S_j = l\} \mid S_i = l\right) \times \frac{\beta_h}{\beta_h} + P\left(\{\beta = \beta_l\} \cap \{S_j = l\} \mid S_i = l\right) \times \frac{\beta_l}{\beta_h} \\ &= \frac{q^2(1-p)}{q(1-p) + (1-q)p} \times \frac{\beta_l}{\beta_h} + \frac{p(1-q)^2}{q(1-p) + (1-q)p}. \end{aligned}$$

For Plh,

$$\begin{aligned} Plh &= P\left(\{\beta = \beta_h\} \cap \{S_j = h\} \mid S_i = l\right) \times \frac{\beta_h}{\beta_h} + P\left(\{\beta = \beta_l\} \cap \{S_j = h\} \mid S_i = l\right) \times \frac{\beta_l}{\beta_h} \\ &= \frac{(1-q)pq}{q(1-p) + (1-q)p} + \frac{q(1-q)(1-p)}{q(1-p) + (1-q)p} \times \frac{\beta_l}{\beta_h}. \end{aligned}$$

For Phl,

$$\begin{aligned} Phl &= P\left(\{\beta = \beta_h\} \cap \{S_j = l\} \mid S_i = h\right) \times \frac{\beta_h}{\beta_h} + P\left(\{\beta = \beta_l\} \cap \{S_j = l\} \mid S_i = h\right) \times \frac{\beta_l}{\beta_h} \\ &= \frac{qp(1-q)}{qp + (1-q)(1-p)} + \frac{(1-q)(1-p)q}{qp + (1-q)(1-p)} \times \frac{\beta_l}{\beta_h}. \end{aligned}$$

For Phh,

$$\begin{aligned} Phh &= P\left(\{\beta = \beta_h\} \cap \{S_j = h\} \mid S_i = h\right) \times \frac{\beta_h}{\beta_h} + P\left(\{\beta = \beta_l\} \cap \{S_j = h\} \mid S_i = h\right) \times \frac{\beta_l}{\beta_h} \\ &= \frac{pq^2}{qp + (1-q)(1-p)} + \frac{(1-p)(1-q)^2}{qp + (1-q)(1-p)} \times \frac{\beta_l}{\beta_h}. \end{aligned}$$

Therefore, the information matrix is given by

$$\Gamma = \begin{pmatrix} Pll & Plh \\ Phl & Phh \end{pmatrix}.$$

For simplicity, we assume that $p = 0.5$ and $\beta_l = 0.03$, $\beta_h = 0.06$ and $q = 0.6$ for now. Then the information matrix is given by:

$$\Gamma = \begin{bmatrix} 0.34 & 0.36 \\ 0.36 & 0.44 \end{bmatrix}$$

Then we can calculate the equilibrium effort x^* using the following formulas:

For low type players:

$$
\begin{aligned}
x_L^* = \alpha * [&a_{11}a_{11}^{-1}b(\lambda_1(\Gamma)\beta_{max})G \\
&+ a_{12}a_{21}^{-1}b(\lambda_2(\Gamma)\beta_{max}G) \\
&+ a_{11}a_{12}^{-1}b(\lambda_1(\Gamma)\beta_{max}G) \\
&+ a_{12}a_{22}^{-1}b(\lambda_2(\Gamma)\beta_{max}G)] * 1
\end{aligned}
$$

For high type players:

$$
\begin{aligned}
x_H^* = \alpha * [&a_{21}a_{11}^{-1}b(\lambda_1(\Gamma)\beta_{max})G \\
&+ a_{22}a_{21}^{-1}b(\lambda_2(\Gamma)\beta_{max}G) \\
&+ a_{21}a_{12}^{-1}b(\lambda_1(\Gamma)\beta_{max}G) \\
&+ a_{22}a_{22}^{-1}b(\lambda_2(\Gamma)\beta_{max}G)] * 1
\end{aligned}
$$

Where α is the marginal return to efforts, G is the network matrix (here it is the same as the one we used in the without-signaling model). In addition, a_{ij} correponds to the elements in the eigenvectors of the information matrix Γ.
b is a function that captures the effects of the network structure on the equilibrium effort. It is given by:

$$b(\lambda_i(\Gamma), \beta_{max}, G) = (I - \lambda_i(\Gamma)\beta_{max}G)^{-1}$$

Where $\lambda_i(\Gamma)$ is the i-th eigenvalue of the information matrix Γ.Besides, β_{max} is given by:

$$\beta_{max} < \frac{1}{\lambda_{max}(G)\lambda_{max}(\Gamma)}$$

where $\lambda_{max}(G)$ is the largest eigenvalue of a complete network matrix $G_{complete}$ instead of the one we had for the default network and $\lambda_{max}(\Gamma)$ is the largest eigenvalue of the information matrix Γ.

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After calculation we had the upperbound for β_{max} as 0.08848 and thus we choose 0.045 just to compare with the no-signaling case.

Table 3: Equilibrium Efforts in Default Network with Signaling

	Group Leader	**Group Member**	**Central Government**
$\beta = \beta_l$	1.1722	1.1416	1.1109
$\beta = \beta_h$	1.3723	1.3001	1.2343

We can see that high productivity type players have higher equilibrium efforts than low type players across roles. Comparing to the no-signaling case, it is clear that no-signaling equilibrium efforts lay between the two signaling equlibria. This can be attributed to . why Also, we would like to compare this result with the one we may had in the complete network case. The results are given:

Table 4: Equilibrium Efforts in Complete Network with Signaling

	Members
$\beta = \beta_l$	1.9601
$\beta = \beta_h$	2.1000

Since in the complete network, all players are connected to each other, there is no difference between the roles of players; that is, everyone has the same equilibrium effort. The results show that the equilibrium effort is much higher than the one in the default network, aligined with the intuition that peer effects are stronger in a complete network.

The Effects of q on Equilibrium Efforts

As we have investigated the effects of p on equilibrium efforts in the previous section, we would like to turn to q and analyze its impacts on the equilibrium efforts. Again, for the sake of simulation, we plug in $q = \{0.1, 0.5, , 0.9\}$.

Table 5: Equilibrium Actions x^* as a Function of Signal Precision q in the Default Network for Low Productivity Players

Player Type	x^* **at** $q = 0.1$	x^* **at** $q = 0.5$	x^* **at** $q = 0.9$
Group Leader	1.25	1.19	1.14
Group Member	1.21	1.16	1.11
Higher-Level Leader	1.16	1.12	1.09

Table 6: Equilibrium Actions x^* as a Function of Signal Precision q in the Default Network for High Productivity Players

Player Type	x^* **at** $q = 0.1$	x^* **at** $q = 0.5$	x^* **at** $q = 0.9$
Group Leader	1.14	1.19	1.25
Group Member	1.11	1.16	1.21
Higher-Level Leader	1.09	1.12	1.16

The results are actually symmetric: for low type players, as q increases, they have lower equilibrium efforts because they are more assure that they are low type players. On the contrary, for high type players, as q increases, they have contribute more because they are more assure that they are high type players.

4. Natural Disaster Shocks

4.1. Shocks in small network

4.2. Shocks in large network

5. Conclusion

Summary of insights from each model extension. Discussion of historical plausibility, policy relevance, and potential directions for further research.

References

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