# Econometrics Cheat Sheet (Page 1)

## 1. Simple Linear Regression

**Model:**

$$y_i = \alpha + \beta x_i + u_i, \quad i = 1, \ldots, N.$$

**OLS Estimators:**

$$\hat{\beta} \;=\; \frac{\mathrm{Cov}(x, y)}{\mathrm{Var}(x)}, \quad \hat{\alpha} \;=\; \bar{y} - \hat{\beta}\,\bar{x}.$$

**Residuals:**

$$\hat{u}_i \;=\; y_i - \hat{y}_i, \quad \sum_{i=1}^{N} \hat{u}_i = 0, \quad \sum_{i=1}^{N} x_i \hat{u}_i = 0.$$

**SSR and $R^2$:**

$$\mathrm{SSR} = \sum (y_i - \hat{y}_i)^2, \quad R^2 = 1 - \frac{\mathrm{SSR}}{\mathrm{SST}}.$$

**Special cases:**

$$\text{(No regressor) } \hat{\alpha} = \bar{y}; \quad \text{(No intercept) } \hat{\beta} = \frac{\sum x_i y_i}{\sum x_i^2}.$$

## 2. Multiple Regression

**Model:**

$$y_i = x_i' \beta + u_i, \quad x_i = (1, x_{i1}, x_{i2}, \ldots, x_{iK})'.$$

**OLS Estimator (Matrix Form):**

$$\hat{\beta} = (X'X)^{-1} X'y, \quad X = \begin{pmatrix} x_1' \\ x_2' \\ \vdots \\ x_N' \end{pmatrix}.$$

**Predictions and Residuals:**

$$\hat{y} = P_X y, \quad \hat{u} = y - \hat{y}, \quad P_X = X(X'X)^{-1}X', \quad M_X = I - P_X.$$

**ANOVA Decomposition:**

$$\mathrm{SST} = \mathrm{SSE} + \mathrm{SSR}.$$

**Adjusted $R^2$:**

$$R^2_{\text{adj}} = 1 - \frac{\mathrm{SSR}/(N - K - 1)}{\mathrm{SST}/(N - 1)}.$$

**Dummy Variables:**

- Represent categories by 0/1 indicators.
- Omit one reference category to avoid perfect collinearity.

## 3. Hypothesis Testing

**General Form:**

$$H_0 : \theta = \theta_0 \quad \text{vs.} \quad H_1 : \theta \neq \theta_0.$$

**t-test (single parameter):**

$$t = \frac{\hat{\theta} - \theta_0}{\mathrm{SE}(\hat{\theta})}.$$

**F-test (joint hypotheses):**

$$F = \frac{(\mathrm{SSR}_R - \mathrm{SSR}_{UR})/q}{\mathrm{SSR}_{UR}/(N - K - 1)} \quad \text{(for } q \text{ restrictions).}$$

**Wald (Chi-square) Test:**

$$W = (R\hat{\beta} - r)' \big(R(X'X)^{-1}R'\big)^{-1} (R\hat{\beta} - r) \sim \chi^2(q).$$

**Type I & II Errors:**

- Type I: Rejecting $H_0$ when it's true.
- Type II: Failing to reject $H_0$ when it's false.

# Econometrics Cheat Sheet (Page 2)

## 4. Asymptotic Theory

**Convergence:**

- $X_n \xrightarrow{d} X$: Convergence in distribution (CDFs converge pointwise).
- $X_n \xrightarrow{p} X$: Convergence in probability ($P(|X_n - X| > \varepsilon) \to 0$).
- $X_n \xrightarrow{a.s.} X$: Almost sure convergence ($P(\lim_{n\to\infty} X_n = X) = 1$).

**Law of Large Numbers (LLN):**

$$\frac{1}{N}\sum_{i=1}^{N} X_i \xrightarrow{p} E[X_i], \quad \frac{1}{N}\sum_{i=1}^{N} X_i \xrightarrow{a.s.} E[X_i].$$

**Central Limit Theorem (CLT):**

$$\sqrt{N}\big(\bar{X} - \mu\big) \xrightarrow{d} N(0, \sigma^2).$$

**Slutsky's Theorem:**

- If $X_n \xrightarrow{d} X$ and $a_n \xrightarrow{p} a$, then $X_n + a_n \xrightarrow{d} X + a$.
- Multiplication: $X_n a_n \xrightarrow{d} aX$, etc.

**Delta Method:** If $\sqrt{N}(X_n - \mu) \xrightarrow{d} N(0, \Sigma)$ and $f$ is differentiable,

$$\sqrt{N}\big[f(X_n) - f(\mu)\big] \xrightarrow{d} N\big(0,\ J(\mu)\,\Sigma\,J(\mu)'\big),$$

where $J(\mu)$ is the Jacobian of $f$ at $\mu$.

## 5. OLS Asymptotics

**Consistency of OLS:**

$$\hat{\beta} \xrightarrow{p} \beta_0 \quad \text{if } E[x_i u_i] = 0 \text{ and } \operatorname{rank}(X'X) = K + 1.$$

**Asymptotic Normality:**

$$\sqrt{N}(\hat{\beta} - \beta_0) \xrightarrow{d} N\big(0,\ \sigma^2 (X'X)^{-1}\big).$$

(Under homoskedasticity and normal errors.)

## 6. Maximum Likelihood Estimation (MLE)

**General Steps:**

- Likelihood: $L(\theta) = \prod_{i=1}^{N} f(y_i; \theta)$.
- Log-likelihood: $\ell(\theta) = \sum_{i=1}^{N} \ln f(y_i; \theta)$.
- $\hat{\theta}_{MLE} = \arg\max_\theta \ell(\theta)$.

**Normal Linear Model MLE:**

$$\hat{\beta}_{MLE} = \hat{\beta}_{OLS}, \quad \hat{\sigma}^2_{MLE} = \frac{\text{SSR}}{N}.$$

## 7. Heteroskedasticity

**Definition:** $\operatorname{Var}(u_i \mid X_i) = \sigma_i^2$ (not constant). **Implications:**

- OLS remains unbiased if $E[x_i u_i] = 0$.
- Variances of OLS estimates are misestimated if uncorrected.

**Robust (White) Standard Errors:**

$$\widehat{\operatorname{Var}}(\hat{\beta}) = (X'X)^{-1}\Big(\sum_{i=1}^{N} \hat{u}_i^2 x_i x_i'\Big)(X'X)^{-1}.$$