TA Session 5 for Macro 2

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1 Monetary Policy Shock

The model is characterized by these four equations:

1. Euler equation:

$$c_t = \mathbb{E}_t[-\frac{1}{\gamma}(r_t - \pi_{t+1}) + c_{t+1}] \tag{1}$$

2. New Keynesian Phillips Curve:

$$\pi_t = \frac{(1-\lambda)(1-\lambda\beta)}{\lambda}\xi\Big(c_t - c_t^f\Big) + \beta\, E_t\Big(\pi_{t+1}\Big) \tag{2}$$

3. Monetary Policy Rule:

$$r_t = \phi_\pi\, \pi_t + v_t \tag{3}$$

4. Output Gap:

$$c_t - c_t^f = c_t - \left(\psi + \frac{1}{\psi+\gamma}\, a_t\right) \tag{4}$$

5. Exogenous Shock:

$$v_t = \rho v_{t-1} + \varepsilon_t^v \tag{5}$$

1.1 Method of undertermined coefficients

The method on undetermined coeffcient, also known as guess and verify, consists in guessing a functional form for the solution. We know from the simple monetary model that a solution of the model is to express the endogenous variables as a function of the structural shocks. Guess that the solution takes the form:

$$c_t = \phi_c \times v_t$$

and

$$\pi_t = \phi_\pi \times v_t$$

Then, verify:
For the Euler equation, we have:

$$c_t = \mathbb{E}_t[-\frac{1}{\gamma}(r_t - \pi_{t+1}) + c_{t+1}]$$

Using the monetary policy rule,

$$r_t = \phi_\pi \, \pi_t + v_t = \phi_\pi \, \psi_\pi \, v_t + v_t,$$

and noting that

$$E_t(\pi_{t+1}) = E_t(\psi_\pi v_{t+1}) = E_t(\psi_\pi(\rho v_t + \varepsilon^v_{t+1})) = \psi_\pi \, \rho \, v_t,$$

Also,

$$E_t(c_{t+1}) = E_t(\psi_c v_{t+1}) = E_t(\psi_c(\rho v_t + \varepsilon^v_{t+1})) = \psi_c \rho v_t$$

Substitute these equations into the Euler equation, we have:

$$\psi_c \, v_t = -\frac{1}{\gamma}\Big[\big(\phi_\pi \, \psi_\pi \, v_t + v_t\big) - \psi_\pi \, \rho \, v_t\Big] + \psi_c \, \rho \, v_t.$$

Dividing by v_t (assuming $v_t \neq 0$) gives:

$$\psi_c = -\frac{1}{\gamma}\Big[\phi_\pi \, \psi_\pi + 1 - \rho \, \psi_\pi\Big] + \rho \, \psi_c.$$

Collecting terms, we have:

$$\psi_c \, (1-\rho) = -\frac{1}{\gamma}\Big[1 + \psi_\pi \, (\phi_\pi - \rho)\Big],$$

or

$$\psi_c = -\frac{1}{\gamma(1-\rho)}\Big[1 + \psi_\pi \, (\phi_\pi - \rho)\Big]. \tag{A}$$

For the New Keynesian Phillips Curve, we have:

$$\pi_t = \frac{(1-\lambda)(1-\lambda\beta)}{\lambda}\xi\Big(c_t - c_t^f\Big) + \beta \, E_t\Big(\pi_{t+1}\Big).$$

With $c_t^f = 0$, and $\kappa \equiv \frac{(1-\lambda)(1-\lambda\rho)}{\lambda}\xi$, the Phillips curve becomes:

$$\pi_t = \kappa \, c_t + \beta \, E_t(\pi_{t+1}).$$

Substitute $c_t = \psi_c \, v_t$ and $\pi_t = \psi_\pi \, v_t$ (with $E_t(\pi_{t+1}) = \psi_\pi \, \rho \, v_t$):

$$\psi_\pi \, v_t = \kappa \, \psi_c \, v_t + \beta \, \psi_\pi \, \rho \, v_t.$$

Cancel v_t:

$$\psi_\pi = \kappa \, \psi_c + \beta \, \psi_\pi \, \rho.$$

Thus,

$$\psi_\pi \, (1-\beta\rho) = \kappa \, \psi_c, \quad \text{or} \quad \psi_\pi = \frac{\kappa \, \psi_c}{1-\beta\rho}. \tag{B}$$

Solving for the system of equations (A) and (B):
Substitute (B) into (A):

$$\psi_c = -\frac{1}{\gamma(1-\rho)}\left[1 + \frac{\kappa\,\psi_c}{1-\beta\rho}\,(\phi_\pi - \rho)\right].$$

Multiply both sides by $\gamma(1-\rho)$:

$$\gamma(1-\rho)\,\psi_c = -\left[1 + \frac{\kappa\,(\phi_\pi - \rho)}{1-\beta\rho}\,\psi_c\right].$$

Collecting terms in ψ_c:

$$\psi_c\left[\gamma(1-\rho) + \frac{\kappa\,(\phi_\pi - \rho)}{1-\beta\rho}\right] = -1.$$

Thus,

$$\psi_c = -\frac{1}{\gamma(1-\rho) + \dfrac{\kappa\,(\phi_\pi - \rho)}{1-\beta\rho}}.$$

Writing the denominator as a single fraction,

$$\gamma(1-\rho) + \frac{\kappa\,(\phi_\pi - \rho)}{1-\beta\rho} = \frac{\gamma(1-\rho)(1-\beta\rho) + \kappa\,(\phi_\pi - \rho)}{1-\beta\rho},$$

so that

$$\psi_c = -\frac{1-\beta\rho}{\gamma(1-\rho)(1-\beta\rho) + \kappa\,(\phi_\pi - \rho)}.$$

Substituting back into (B), we obtain

$$\psi_\pi = \frac{\kappa\,\psi_c}{1-\beta\rho} = -\frac{\kappa}{\gamma(1-\rho)(1-\beta\rho) + \kappa\,(\phi_\pi - \rho)}.$$

Determining the effect of a monetary policy shock:

- $\gamma > 0$ (the coefficient of relative risk aversion),
- ρ is the coefficient on shock from the previous period to smooth monetary policy and $\rho \in [0, 1)$ so that $1 - \rho > 0$,
- β is again the discounted factor for the future period, $\beta \in (0, 1)$ implying $1-\beta\rho > 0$,
- We also have $0 \leq \lambda \leq 1$ which is the percentage of firms that cannot adjust prices flexibly
- $\xi = \psi+\gamma$, recall that ψ is the coefficient of responsiveness of labor supply to changes in real wages and $\psi \geq 0$, so $\xi > 0$,
- $\kappa \equiv \frac{(1-\lambda)(1-\lambda\rho)}{\lambda}\xi$, $\kappa > 0$ by its definition, and
- ϕ_π is the coefficient on inflation in the monetary policy rule, and it is usually assumes that $\phi_\pi > \rho$, so that $\phi_\pi - \rho > 0$.

Thus, both ψ_π and ψ_c coefficients are all negative. Therefore, a positive (negative) monetary policy shock will lead to a decrease (increase) in consumption c_t and inflation π_t.