TA5 Zero Lower Bound (ZLB)

Consider the simple New Keynesian model with a Zero Lower Bound (ZLB) characterized by:

Euler equation (with preference shock):

$$c_t = E_t\left[-\frac{1}{\gamma}(\epsilon^d_{t+1} - \epsilon^d_t + r_t - \pi_{t+1}) + c_{t+1}\right]$$

New Keynesian Phillips curve:

$$\pi_t = \kappa c_t + \beta E_t[\pi_{t+1}]$$

Monetary policy rule (ZLB constraint):

$$r_t = \max\{-\ln(R),\ \phi_\pi \pi_t\}$$

According to the last equation, the monetary authority follows a Taylor rule as long as the implied net nominal interest rate is non-negative and the monetary authority sets the net nominal interest rate to zero otherwise.
Guess there is an equilibrium satisfying the conditions below:
(a) Consumption, inflation and the nominal interest rate are constant over time until the preference shock reverts permanently back to 0.
(b) The economy is in the non-stochastic steady-state with zero inflation thereafter.
This implies that in period t, the economy is hit by a negative preference shock $\epsilon^d_t < 0$ and that afterwards, the preference shock keeps its current value with probability μ and reverts permanently back to 0 with probability $1 - \mu$.

2.1 Expectations

Since when the preference shock reverts to 0, consumption, inflation and the nominal interest rate are constant over time, so the deviation of them in the steady states are 0, $c^{ss} = \pi^{ss} = r^{ss} = 0$

$$E_t(c_{t+1}) = \mu c_t + (1-\mu)\cdot c^{ss} = \mu c_t$$

$$E_t(\pi_{t+1}) = \mu \pi_t + (1-\mu)\cdot \pi^{ss} = \mu \pi_t$$

$$E_t(\epsilon^d_{t+1}) = \mu \epsilon^d_t + (1-\mu)\cdot 0 = \mu \epsilon^d_t$$

Euler equation explicitly at time t: Expanding the expectation of the Euler equation at time t:

$$c_t = -\frac{1}{\gamma}\left[(E_t(\epsilon^d_{t+1}) - \epsilon^d_t + r_t - E_t(\pi_{t+1}))\right] + E_t(c_{t+1})$$

Then we can substitute the expectations of c_{t+1} and π_{t+1} into the equation:

$$c_t = -\frac{1}{\gamma}\left[\mu\epsilon_t^d - \epsilon_t^d + r_t - \mu\pi_t\right] + \mu c_t$$

Then we isolate c_t:

$$(1-\mu)c_t = -\frac{1}{\gamma}\left[\mu\epsilon_t^d - \epsilon_t^d + r_t - \mu\pi_t\right]$$

Then we move γ to the left side and collect ϵ on the right hand side:

$$\gamma(1-\mu)c_t = (1-\mu)\epsilon_t^d - r_t + \mu\pi_t$$

2.2 Phillips Curve

Using expectations, rewrite the Phillips curve:

$$\pi_t = \kappa c_t + \beta E_t[\pi_{t+1}] \quad \Rightarrow \quad \pi_t = \kappa c_t + \beta\mu\pi_t$$

Solve explicitly for inflation:

$$\pi_t = \frac{\kappa}{1-\beta\mu}c_t$$

Note here that $c_t^f = 0$ again because the preference shock like monetary policy shock does not affect a_t.

2.3 ZLB not binding

(i) Solve for c_t (when $r_t = \phi_\pi\pi_t$)

When the ZLB is not binding, substitute $r_t = \phi_\pi\pi_t$ into the Euler equation:

$$\gamma(1-\mu)c_t = (1-\mu)\epsilon_t^d - \phi_\pi\pi_t + \mu\pi_t$$

Substitute $\pi_t = \frac{\kappa}{1-\beta\mu}c_t$:

$$\gamma(1-\mu)c_t = (1-\mu)\epsilon_t^d + (\mu-\phi_\pi)\frac{\kappa}{1-\beta\mu}c_t$$

Isolate c_t:

$$\left[(\gamma(1-\mu)) - \frac{(\mu-\phi_\pi)\kappa}{1-\beta\mu}\right]c_t = (1-\mu)\epsilon_t^d$$

Move the coefficient to the right side:

$$c_t = \frac{(1-\mu)\epsilon_t^d}{(\gamma(1-\mu)) - \frac{(\mu-\phi_\pi)\kappa}{1-\beta\mu}}$$

Divided both the numberator and denominator by $\gamma(1-\mu)$

$$c_t = \frac{\frac{1}{\gamma}\epsilon_t^d}{1 - \frac{\frac{1}{\gamma}}{1-\mu}\frac{(\mu-\phi_\pi)\kappa}{1-\beta\mu}}$$

Which is the equation appears on the slides.

(ii) Interpretation

Not sure about why the guess is verified.

2.4 ZLB binding

(i) Solve for c_t (when $r_t = -\ln(R)$)

Euler equation at ZLB $r_t = -\ln(R)$:

$$\gamma(1-\mu)c_t = (1-\mu)\epsilon_t^d + \ln(R) + \mu\pi_t$$

Using the Phillips curve $\pi_t = \frac{\kappa}{1-\beta\mu}c_t$, we get:

$$\gamma(1-\mu)c_t = (1-\mu)\epsilon_t^d + \ln(R) + \frac{\mu\kappa}{1-\beta\mu}c_t$$

Isolate c_t:

$$\left[(\gamma(1-\mu)) - \frac{\mu\kappa}{1-\beta\mu}\right]c_t = (1-\mu)\epsilon_t^d + \ln(R)$$

Move the coefficient to the right side:

$$c_t = \frac{(1-\mu)\epsilon_t^d + \ln(R)}{(\gamma(1-\mu)) - \frac{\mu\kappa}{1-\beta\mu}}$$

Again, divided both the numberator and denominator by $\gamma(1-\mu)$:

$$c_t = \frac{\frac{1}{\gamma}\epsilon_t^d + \frac{\frac{1}{\gamma}}{1-\mu}ln(R)}{1 - \frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}$$

Which is the same as the equation appears on the slides.

(ii) Interpretation

Again, not sure about why the guess is verified.