

Introduction to Biomedical Imaging

Project 2: Magnetic Resonance Diffusion Tensor Imaging

In MR Diffusion Tensor Imaging (DTI), water diffusivity can be modeled as a symmetric tensor with 6 independent elements in a 3×3 matrix

$$\mathbf{D} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yx} & D_{zz} \end{bmatrix}$$

where \mathbf{D} is the diffusion tensor, $D_{xx}, D_{yy}, D_{zz}, D_{xy}, D_{xz}, D_{yz}$ are tensor elements.

The diffusion weighted signal can be expressed as

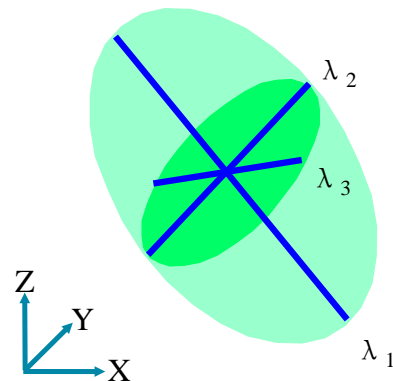
$$S = S_0 \exp \left(-b \begin{bmatrix} \hat{g}_x & \hat{g}_y & \hat{g}_z \end{bmatrix} \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{bmatrix} \begin{bmatrix} \hat{g}_x \\ \hat{g}_y \\ \hat{g}_z \end{bmatrix} \right)$$

where S and S_0 are signals with and without diffusion weighting respectively ; b is the diffusion weighting strength, b-factor, which can be treated a constant (e.g. 1000); $\hat{g}_x \ \hat{g}_y \ \hat{g}_z$ are XYZ components of the unit vector of the diffusion direction.

Diffusion tensor elements D_{ij} ($i, j \in x, y, z$) can be obtained on a pixel-by pixel basis by solving the system of linear equations. As D has 6 independent variables only, obtaining a unique solution requires diffusion to be measured along at least 6 non-coplanar encoding directions. Therefore, the minimum DT-MRI dataset would consist of 7 MRI acquisitions, including the image volume without diffusion weighting. The simplest and most widely used diffusion weighting direction scheme is:

X	Y	Z
1	0	1
-1	0	1
0	1	1
0	1	-1
1	1	0
-1	1	0

Positive defined 3×3 symmetrical tensors can be viewed as an ellipsoid with eigenvalues of $\lambda_1, \lambda_2,$



λ_3 . The principal eigenvector corresponding to the largest eigenvalue λ_1 is usually treated as local tissue orientation.

There is a need to easily get the most important characteristics of diffusion tensors in a straightforward and quantitative way. For example, it is helpful to have simple measurements to depict the degree of anisotropy derived from the diffusion tensor. This leads to many parameters derived from the diffusion tensor. Among them, the indexes defined with scalar rotational invariant properties are mostly widely used. For example, fractional anisotropy, FA, which represents the fraction of the anisotropic diffusion in a diffusion tensor, is defined as

$$FA = \sqrt{\frac{3((\lambda_1 - \bar{\lambda})^2 + (\lambda_2 - \bar{\lambda})^2 + (\lambda_3 - \bar{\lambda})^2)}{2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}},$$

where $\bar{\lambda} = (\lambda_1 + \lambda_2 + \lambda_3) / 3$

Data Format: dtidata.mat – matlab MAT file in 7 x 128 x 128 x 75.

dtidata(1, :, :, :) – the volume without diffusion weighting

dtidata(2, :, :, :) – the volume diffusion weighted in the direction of (1 0 1)

... ..

dtidata(7, :, :, :) – the volume diffusion weighted in the direction of (-1 1 0)

Task 1 (80 points): For given DTI images acquired from an MRI scanner, process the data, calculate diffusion tensors, decompose diffusion tensors into eigenvectors and eigenvalues and calculate FA values. Your report should contain a map of a selected slice for each tensor elements, and a map of FA for the same slice.

Task 2 (20 points): Overlay the regions with FA > 0.25 on the image volume without diffusion weighting and find a proper method to visualize the regions in high FA values.

Project Report: Submit two files to Blackboard. 1) Report: The report should include descriptions of implementation, the results and conclusions. 2) Source code: The code should be sufficiently commented. Do NOT append the source code to the report.