

# Decoding Algorithm with Fast Hadamard Transform for Channel Quality Indication (CQI) in 3GPP-LTE

Minliang Li, Dahai Han, Songwei Ma

Key Laboratory of Information Photonics and Optical Communications (Beijing University of Posts and Telecommunications), Ministry of Education, 10 Xitucheng Road, Haidian District, Beijing 100876, China

**Abstract**—Channel quality indication (CQI) report is an important element of LTE and has significant impact on the system performance. The encoding method for CQI in 3GPP-LTE is Reed-Muller (RM) code and the transmission quality of CQI can be improved by channel coding with RM code. On the receiver, the contradiction between low latency which is one of goals of LTE and reliability of communication system can be eased up by fast decoding algorithms, among which Fast Hadamard Transform (FHT) is described in this paper. Performance analysis as well as computer simulation of the whole process of CQI coding is also presented.

**Keywords**—CQI; 3GPP-LTE; Reed-Muller; FHT

## I. INTRODUCTION

As an essential control signaling of the uplink (UL) in evolved UMTS terrestrial radio access (E-UTRA), channel quality indication (CQI) report provides the remote connection (e.g. base station) with channel quality information. According to the 3GPP standard [1], the granularity of CQI report can be classified into three levels: wideband, UE selected sub-band, and higher layer configured sub-band. For efficient transmission of CQI report, Reed-Muller (RM) [2] code that presented by the standard is used for channel coding of CQI. Through fast and efficient decoding algorithm, receiver can retrieve the CQI report with lower bit error rate (BER). Fast Hadamard Transform (FHT) that used for decoding RM code can gain lower latency and meet the goals of system evolving.

RM coding is beneficial to performance improvement for CQI transmission while FHT is useful to the decoding efficiency of encoded CQI. The theory and process of decoding for encoded CQI together with the verification for the decoding algorithm through simulation are given in the paper. The paper is organized as follows. In Section II, an overview of RM code and encoder for CQI is presented. In Section III, decoding algorithm with fast Hadamard transform (FHT) for CQI is described in detail. In Section IV, the computer simulation is presented and the result is discussed. Finally, conclusions about the algorithm are drawn in Section V.

## II. RM CODE AND CQI ENCODER

### The Structure of RM Code

RM codes allow more flexibility in the size of code word and the number of correctable errors per code word. They are simple in construction and rich in structural properties [3]. For any integer  $m$  and  $r$  with  $0 \leq r \leq m$ , there exists a binary

$r^{\text{th}}$ -order RM code, denoted by  $RM(r, m)$ , with the following parameters:

Code length:  $n = 2^m$

$$k(r, m) = 1 + \binom{m}{1} + \binom{m}{2} + \dots + \binom{m}{r}$$

Dimension:

$$\text{Minimum distance: } d_{\min} = 2^{m-r}$$

Where  $\binom{m}{i} = \frac{m!}{(m-i)!i!}$  is the binomial coefficient. For

$1 \leq i \leq m$ , let  $\mathbf{v}_i$  be a  $2^m$ -tuple over  $GF(2)$  of the following formula:

$$\mathbf{v}_i = (\underbrace{0 \dots 0}_{2^{i-1}}, \underbrace{1 \dots 1}_{2^{i-1}}, \underbrace{0 \dots 0}_{2^{i-1}}, \dots, \underbrace{1 \dots 1}_{2^{i-1}}), \quad (1)$$

which consists of  $2^{m-i+1}$  alternating all-zero and all-one  $2^{i-1}$ -tuples. The  $r^{\text{th}}$ -order RM code,  $RM(r, m)$ , of length  $2^m$  is generated (or spanned) by the following set of independent vector:

$$G_{RM}(r, m) = \{v_0, v_1, v_2, \dots, v_m, v_1v_2, v_1v_3, \dots, v_{m-1}v_m, \dots \text{ to } r^{\text{th}} \text{ stage}\} \quad (2)$$

If the vector in  $G_{RM}(r, m)$  are arranged as rows of a matrix, then the matrix is the generator matrix of  $RM(r, m)$  code.

### The Encoder of CQI

The number of CQI bits may vary between 5 and 10 depending on whether wideband or narrowband CQI reports are transmitted. If the number of CQI bits is less than seven, then the encoding matrix  $(32 \times 6)$  is a standard RM  $(1, 5)$  code which is processed with interleaver. In the standard, they are the vectors from  $M_0$  to  $M_5$ . Otherwise, the encoding matrix  $(32 \times 11)$  consists of above RM vectors and five masks. The masks are listed as follows:

$$M_6 : 00100110011100011011100011001110$$

$$M_7 : 00001101101011110010001011010110$$

$$M_8 : 00110111000110000100001110111110$$

$$M_9 : 01100010111011011000010110110010$$

$$M_{10} : 11111111111111110000111101000010$$

The encoded CQI block is denoted by:

$$b_0, b_1, b_2, b_3, \dots, b_{B-1}, \quad (3)$$

where  $B = 32$  and

$$b_i = \sum_{n=0}^{O-1} (o_n \cdot M_{i,n}) \bmod 2, \quad (4)$$

where  $i = 0, 1, 2, \dots, B-1$ .

The output bit sequence  $q_0, q_1, q_2, q_3, \dots, q_{Q_{CQI}-1}$  is obtained by circular repetition of the encoded CQI block as follows:

$$q_i = b_{(i \bmod B)} \text{ where } i = 0, 1, 2, \dots, Q_{CQI}-1. \quad (5)$$

### III. DECODING ALGORITHM FOR CQI

Because the encoder of CQI is based on RM code, many decoding algorithms for RM codes have been researched in [4], [5], and [6]. These algorithms include hard-decision decoding and soft-decision decoding. Correlations between Hadamard matrix and Reed-Muller codes, and using Hadamard transform to RM codes have been presented in [7]. As for decoding for CQI, the bipolarity change for codeword, interleaving and eliminating infection of the masks should be considered before decoding codeword with FHT.

#### A. Decoding for Long CQI Report

When the number of bit in each encoding block is larger than

The Original Sequence of Codeword

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	High 16 bits
17	18	19	20	21	22	23	23	25	26	27	28	29	30	31	32	Low 16 bits

The Sequence after Interleaving

32	1	21	2	3	22	4	5	23	6	7	24	8	9	10	25	High 16 bits
20	26	11	12	13	14	27	28	15	16	29	17	18	19	30	31	Low 16 bits

Interleaving

From High bit to low bit

Figure 1. The Order of Interleaving for Codeword

six, the length of codeword through CQI encoder is 32 bits. On the receiver, infection caused by mask sequences

$(M_7, M_8, M_9, M_{10}, M_{11})$  in encoding should be eliminated first. This infection can be observed from the encoding process, i.e. each bit in the codeword is associated with the linear combination of corresponding bit of all mask sequences. There are 32 kinds of linear combination for these five mask sequences in the encoder matrix, and every linear combination can be obtained through the following steps:

Let the value of  $i$  be an integer from 0 to 31.

1)  $a_0, a_1, a_2, a_3, a_4$  are the five bits of binary number of  $i$ .

i.e.  $a_k = F(i) \quad k = 0, 1, 2, 3, 4$ .

The function  $F(i)$  transforms  $i$  from decimal to binary. The linear combination is:

$$v_n = a_{i,0} \cdot M_7 + a_{i,1} \cdot M_8 + a_{i,2} \cdot M_9 + a_{i,3} \cdot M_{10} + a_{i,4} \cdot M_{11}, \quad (6)$$

where  $n = 1, 2, \dots, 32$  and  $i = 0, 1, 2, \dots, 31$ .

Then, the mask matrix, denoted by  $M_D$ , that consist of vectors which are transformed with mode 2 operation and bipolar transformation for above 32 linear combinations. So,  $M_D = [v_1, v_2, \dots, v_{32}]^T$  and each row in  $M_D$  is  $v_n, n = 1, 2, \dots, 32$ . When the value of  $i$  is 31,  $a_{i,0}, a_{i,1}, a_{i,2}, a_{i,3}, a_{i,4}$  are all '1'. In this case, the linear combination of five mask sequences forms  $v_{32}$ . Received word that processed by interleaving (Fig.1. shows the order of interleaving for codeword) and bipolar changing multiply with  $v_{32}$  can eliminate the infection of mask sequences.

The prerequisite for decoding on the basis of the maximum FHT matrix is that the first five columns in encoder must be standard Rademacher sequences, i.e. they are a series of cyclical square wave with the relationship of half-divided

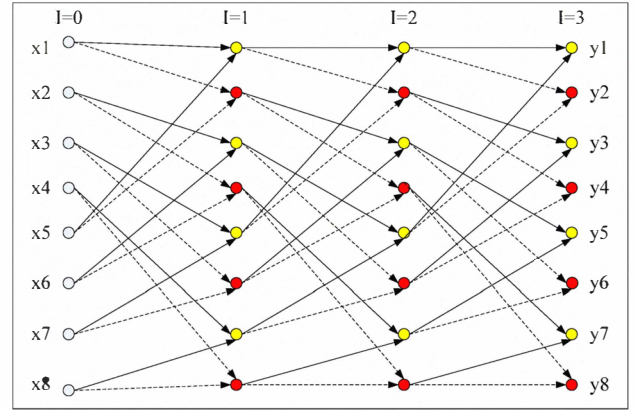


Figure 2. The Structure of Fast Hadamard Transform

frequency. From the structure of encoder, the first five columns aren't standard Rademacher sequences. It is necessary to interleave for received words and mask matrix, i.e. reorder the bits of the received block, and reorder the rows of the mask matrix  $M_D$  with the same order. According to coding matrix, we can easily know the principle of codeword interleaving of CQI report.

Fig.1. illustrates the principle of interleaving process. In the codeword with 32 bits, the 32th bit moves to the position of the first bit, the first bit moves to the position of the second, and so on. For the mask matrix, the same principle is used for row transformation.

After interleaving and bipolar transformation for received word and mask matrix, FHT can be used to decode the received block and the structure of FHT is shown in Fig.2. of eight elements' FHT. Every circle represents a number. Solid lines represent addition for tow numbers while dotted lines represent subtraction for tow numbers. The number of circulation is  $\log_2 M$ , where  $M$  the length of data transformation for FHT is. Fig.2. shows the transformation of 8 data and the cycle number is 3. In the decoding process for CQI, the length of data in each row of the decoding matrix is 32, so the cycle number is 5.

Assuming that the received word is  $\mathbf{c} = (c_1, c_2, \dots, c_{32})$ , then the whole decoding procedure is given as follows:

- 2) Bipolar transformation for the decision block:

$$\tilde{c}_i = 1 - 2 \times c_i \quad \text{where } i=0,1,2,\dots,32. \quad (7)$$

We donate the result as  $\tilde{\mathbf{c}}$ .

- 3) According to the order of Fig.1.,  $\tilde{\mathbf{c}}$  for interleaving.  
 4) Constructing mask matrix  $M_D$  based on the mask sequences in the CQI encoder, where  $M_D = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_{32}]^T$  and  $\mathbf{v}_n = a_{i,0} \cdot M_7 + a_{i,1} \cdot M_8 + a_{i,2} \cdot M_9 + a_{i,3} \cdot M_{10} + a_{i,4} \cdot M_{11}$   
 5) Bipolar transformation for each element of  $M_D$ . Then changes the row order according to Fig.1. and the result is the decoding matrix  $\bar{M}_D$ .  
 6) Multiply every bit of  $\tilde{\mathbf{c}}$  with corresponding bit of each row in  $\bar{M}_D$ . Each row is a vector of  $1 \times 32$ .  
 7) Making FHT for each vector in (5). Then the results can make up a matrix of  $32 \times 32$ , which is denoted by  $D_{32 \times 32}$ .  
 8) Finding out the element, donated as K, whose magnitude is the largest in  $D_{32 \times 32}$ . Then decodes the word according to the number of row and column corresponds with K.  
 9) Transforming the row and column of K to five binary bits separately. These bits are then decoded to  $O_7, O_8, O_9, O_{10}, O_{11}$  and  $O_2, O_3, O_4, O_5, O_6$  respectively. Then obtaining  $O_1$  according to the sign of K, thus, if K is positive number,  $O_1$  is '0' and  $O_1$  is '1' otherwise.

#### B. Decoding for Short CQI Report

When the number of bit in every encoding block is less than seven, there is no mask sequences inflection for the codeword. So the procedure of decoding for received word is simplified compared to above situation.

Interleaving and bipolar transformation process is only needed for the received word while not for the mask matrix. Find the element in the word with largest magnitude after FHT and then decode them on the basis of the column number and the sign of it.

#### IV. COMPUTER SIMULATION AND DATA DISCUSS

Bit error rate is a significant index to judge the performance of a error correction coding. The simulation is carried out on the additive white Gaussian noise channel (AWGN) with different SNR and the simulation result for the long and short CQI report coding are given in Fig.3. and Fig.4. respectively. Each figure shows the relationship between BER and SNR for the condition without RM code (red circles) and with RM code (blue triangles). The range for SNR is from 2dB to 14dB. In the process of simulation, one million blocks for each SNR were used. If we compare the performance of long CQI report coding with that of short CQI report coding, we can see that the BER of short CQI report coding is lower than that of long CQI report coding

for same SNR, e.g. when SNR is 3, the BER of long CQI report coding is 8 times of that of short CQI report coding. CQI performance with varied coding schemes and advanced receiver is given in [8].

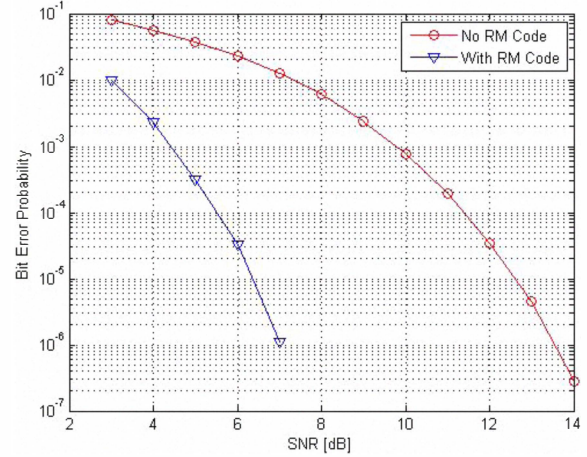


Figure 3. Simulation for Long CQI Report Coding

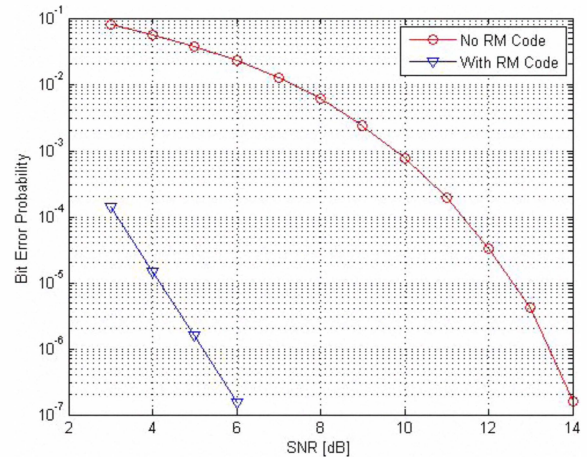


Figure 4. Simulation for Short CQI Report Coding

The performance of the decoding algorithm for CQI is simulated by Matlab. The whole Matlab program is implemented through a nested loop, and the variable of the external loop is SNR while that of the internal loop is the number of code blocks for each SNR. In the simulation, one million code blocks are used for each SNR and coding, modulation, adding noise, demodulation as well as decoding are operated in the internal loop. After each internal loop, the BER without channel coding and that with RM channel coding is calculated for each corresponding SNR respectively. From the simulation result we find that CQI coding with RM code can help to improve its transmission performance while FHT eases the contradiction between real-time property and reliability effectively.

## V. CONCLUSION

Decoding the CQI report word with fast Hadamard transform, the received word and mask matrix are needed to be interleaved to standard Rademacher sequences. When decoding long CQI report, generating linear combination of mask sequences is a key step for decoding. FHT is an efficient decoding method for CQI report. In this paper, decoding process has been introduced in detail and the simulation also shows the performance of decoding algorithm. It is clearly seen from the simulation result that RM code can improve the reliability of CQI transmission distinctly. Meanwhile, the detailed description of the decoding algorithm provides theoretical support for the way to improve CQI transmission performance and the way to enhance decoding efficiency at the receiver.

## ACKNOWLEDGMENT

This work has been supported in part by 973 program (2010CB328204), 863 program (2009AA01Z255), NSFC project (60772022), PCSIRT program (IRT0609) and 111 Project (B07005) of China and BUPT Excellent Ph.D. Students Foundation

## REFERENCE

- [1] 3GPP TS 36.212 3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Multiplexing and channel coding (Release 9)
- [2] Othman O.Khalifa, Aisha-Huassan Abdullah, N.Suriyana, "Reed-Muller Codec Simulation Performance"[J] Journal of Computer Science 4 (10):792-798,2008
- [3] Shu Lin, Daniel J.Costello, "Error Control Coding"[M] 1983,2004 by Pearson Education, Inc.
- [4] David Chase, "A Class of Algorithms for Decoding Block Codes With Channel Measurement Information",IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. IT-8, NO.1, JANUARY 1972
- [5] Kenneth G. Paterson, Alan E. Jones, "Efficient Decoding Algorithm for Generalized Reed-Muller Codes", IEEE TRANSACTION ON COMMUNICATIONS, VOL.48,NO.8,AUGUST 2000
- [6] Ilya Dumer, Kirill Shabunov, "Soft-Decision of Reed-Muller Codes Recursive Lists",IEEE TRANSACTIONS ON INFORMATION THEORY, VOL.52,NO.3,MARCH 2006
- [7] Scott A. Vanstone, Paul C.van Oorschot, vAn Introduction to Error Correcting Codes with Applications", 1989 by Kluwer Academic Publishers
- [8] Amitava Ghosh, Rapeepat Ratasuk, Weimin Xiao, "Uplink Control Channel Design for 3GPP LTE", The 18th Annual IEEE International Symposium on Personal, Indoor and Mobile Radio Communications