CHAPTER TWO

2.0 FLOWS THROUGH PIPES

2.1 INTRODUCTION

The flow of ~~[illegible]~~ Newtonian fluid can be classified as

(i) Laminar flow
(ii) Turbulent flow

2.1.1 LAMINAR FLOW; It is one in which the paths taken by the individual particles do not cross one another and move along well defined paths. It is also called STREAMLINE FLOW or VISCOUS FLOW

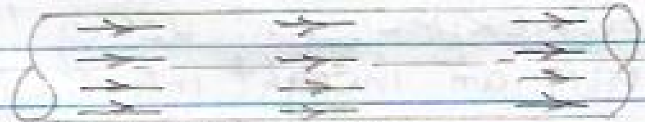


Fig 2.1: Laminar flow

Example of laminar flow includes

1. Flow through a capillary tube
2. Flow of blood in veins and arteries
3. Flow of oil in measuring instrument
4. Ground water flow
5. Rise of water in plants through their roots

Characteristics of laminar flow

1. There is 'no slip' at the boundary
2. Flow is rotational
3. There is shear stress in fluid layers due to viscosity
4. Continous dissipation of energy as a result of viscous shear
5. Energy loss depends on velocity and viscosity
6. There is no mixing between different fluid layers except by molecular motion

2.1.2 TURBULENT FLOW: It is one in which the fluid particles ~~[illegible]~~ have their layers cross one another without any defined pattern. It is a zig-zag motion observed in a pipe

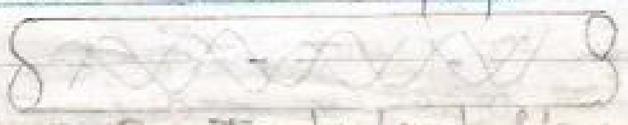


Fig 2.2 Turbulent f[illegible]

Examples of turbulent flow are

1. High velocity flow in a conduit of large size

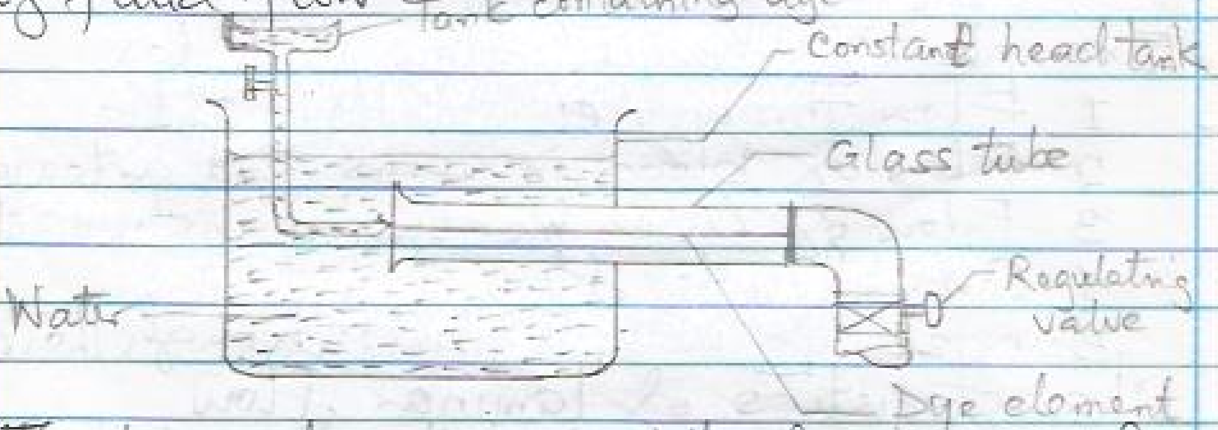
Characteristics of turbulent flow

1. Random, irregular and haphazard movement of fluid particles
2. The velocity gradient near the boundary is quite large resulting in more shear
3. The pressure distribution fluctuates with time about a mean value

~~The Reynolds~~

2·2 CRITERION OF FLOW

The Reynolds experiment using the Reynolds apparatus gave an insight into the two categories of fluid flow



The two categories are the laminar and turbulent flow. Laminar and turbulent flow are characterized on the basis of Reynold's number

$$\text{Reynolds number, } Re = \frac{\rho v d}{\mu} = \frac{vd}{\nu} \qquad (2\cdot1)$$

where ρ = fluid density
v = velocity of flow
d = charateristic ~~diameter~~ of pipe
μ = dynamic viscosity
ν = kinematic viscosity

Note that $\nu = \frac{\mu}{\rho}$ . . . . (2·2)

Inertia force on fluid particles

F_i = mass x acceleration

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$$F_i = \text{density} \times \text{volume} \times \text{acceleration}$$
$$= \rho V a$$
$$= \rho l^3 \times \frac{v}{l/v} = \rho l^2 v^2 \quad \text{----} (2\cdot3)$$

Viscous force on fluid particles

F_v = Area over which shear stress acts × Dynamic viscosity × $\frac{\text{Rate of relative movement}}{\text{Distance perpendicular to relative movement}}$

$$= l^2 \times \mu \times \frac{v}{l}$$
$$= l \mu v \quad \text{--------} (2\cdot4)$$

Reynolds number, $Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{F_i}{F_v}$

$$= \frac{\rho l^2 v^2}{l \mu v} = \frac{\rho v l}{\mu}$$

If $l = d$, then

$$Re = \frac{\rho v d}{\mu}$$

Laminar flow occurs when Re is low ⇒ the viscous force (F_v) predominate

Turbulent flow occurs when Re is high ⇒ The inertia force (F_i) predominate

Reynolds number, Re, from the relation above depends on
1. the diameter of the pipe
2. the density of the fluid
3. the viscosity of the fluid
4. the velocity of fluid-flow.

Experimental results show that,

For Reynolds number (Re) < 2000, the flow is Laminar

For Reynolds number (Re) > 4000, the flow is Turbulent

For Re between 2000 and 4000, the flow is unpredictable. It is at the transition stage which could be either be laminar or turbulent

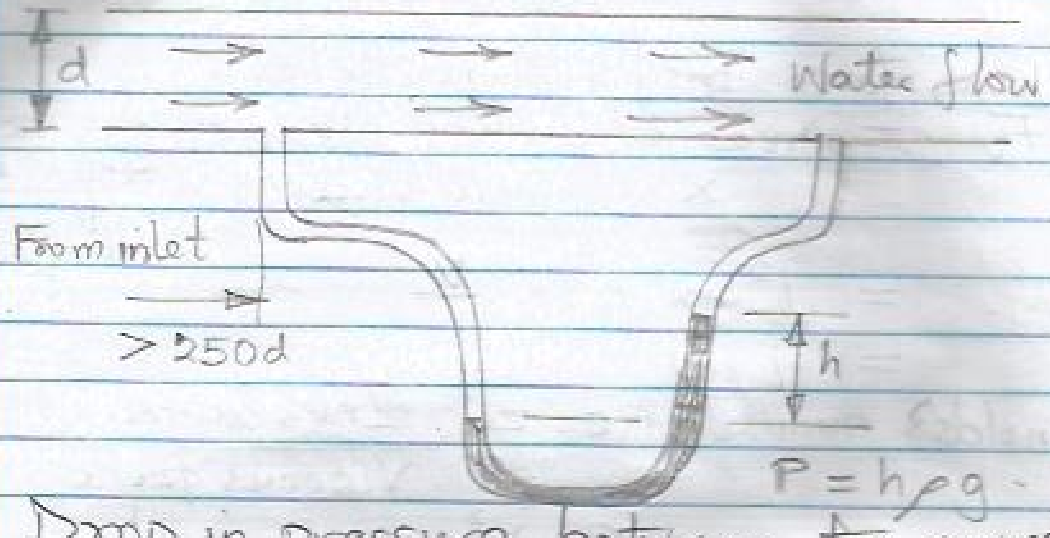
Also, it should be noted that the loss of pressure varies directly with the velocity for laminar flow, while the loss of pressure head varies approximately directly as the square of velocity i.e

$$h_f \propto v \quad (\text{for laminar flow})$$

$h_f \propto V^n$ (for turbulent flow)

where $n = 1.75$ to 2.0

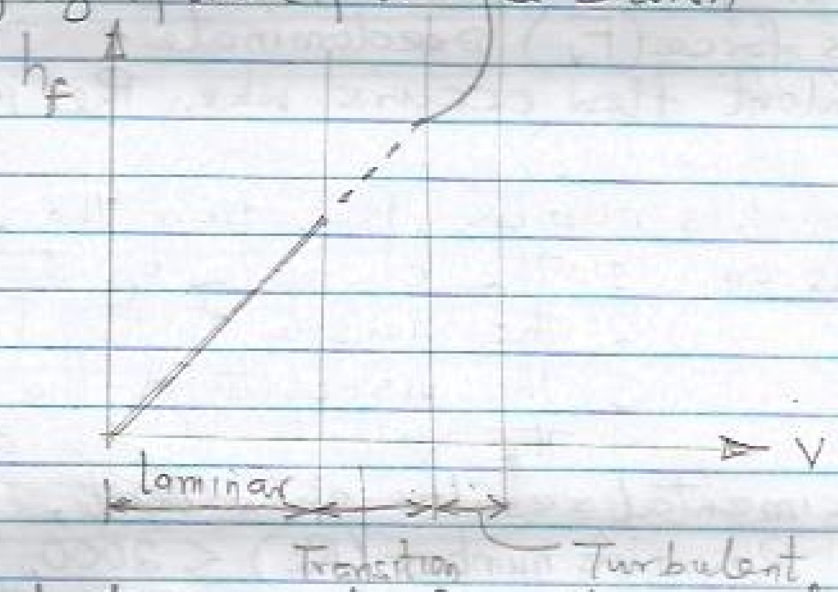
2.3 PRESSURE DROP IN PIPE FLOW



Drop in pressure between two cross-sections is due to friction loss

$$z_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} = z_2 + \frac{V_2^2}{2g} + \frac{P_2}{\rho g} + h_f \quad (2.5)$$

The graph of pressure head, h_f with the velocity of fluid flow is shown



The deductions made from the graph above shows that

1. At low velocities, the graph is a straight line indicating that the loss of head (h_f) is directly proportional to the velocity i.e. $h_f \propto V$. The flow in this region is inevitably Laminar flow
2. At high velocities, the graph is a parabolic curve indicating that the loss of head (h_f) is

directly proportional to the velocity of index n

i.e. $h_f \propto V^n$ where $n = 1.75$ to 2.00

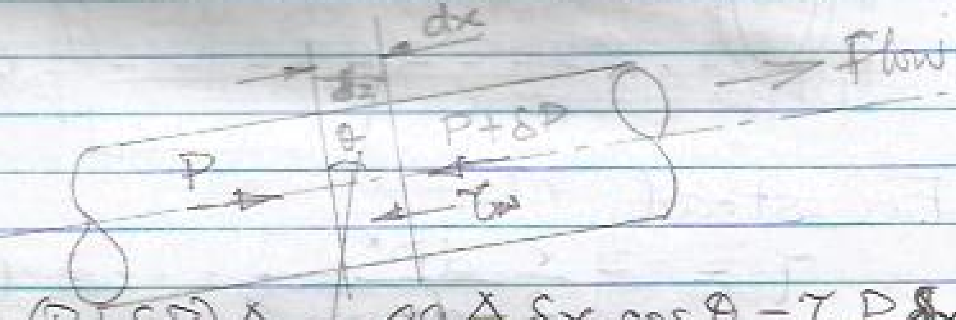
Hence, the flow in this region is inevitably Turbulent flow.

3. Between the straight line and the parabolic curve is dotted line called the Transition region. At this region, the flow is unpredictable. Hence, it could be either laminar or turbulent flow.

2.4. DISTRIBUTION OF THE SHEAR STRESS IN A CIRCULAR PIPE

Loss of mechanical energy in the fluid due to frictional resistance results in decrease in piezometric pressure $P + \rho g z$

This decrease is related to shear stress at the wall



$$PA - (P+\delta P)A - \rho g A \delta x \cos\theta - \tau_w P \delta x = 0 \quad --(2.6)$$

and $$\delta x \cos\theta = \delta z \quad ------(2.7)$$

Substituting (2.7) in eqn (2.6) and rearranging then

$$\tau_w P dx = -A(\delta P + \rho g \delta z) \quad ----(2.8)$$

For incompressible flow, ρ = constant and the right hand side of equation (2.8) can be written as $A\delta P^*$

where $P^* = P + \rho g z$

As $\delta x \to 0$, $$\tau_w = -\frac{A}{P'}\frac{dP^*}{dx} \quad ------(2.9)$$

Since τ_w is positive, it implies that P^* reduces in the direction of flow

For a circular conduit

$$\tau_w = -\frac{\pi R^2}{2\pi R} \cdot \frac{dP^*}{dx}$$

$$= -\frac{R}{2}\frac{dP^*}{dx} \quad ---------(2.10)$$

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Applying equation (2·10) ~~in eq~~ to a smaller concentric cylinder, then

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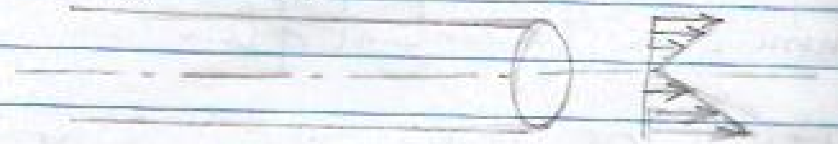
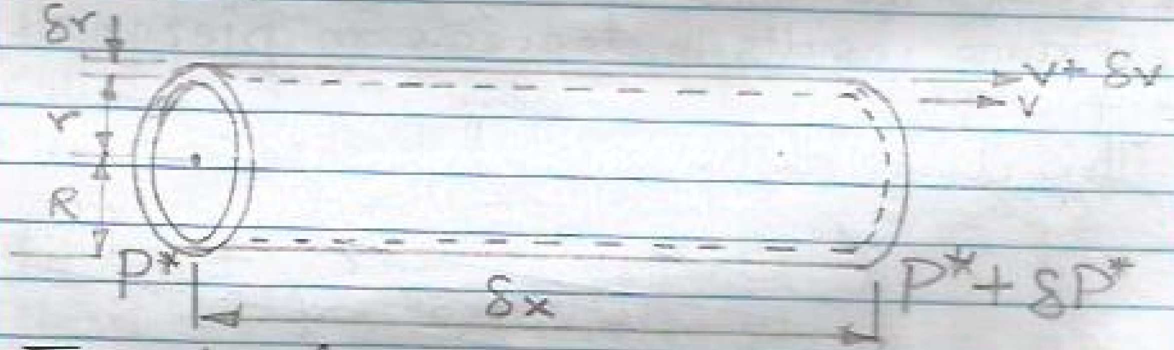
$$\tau = -\frac{r}{2}\frac{dP^*}{dx} \quad \text{------} \quad (2\cdot11)$$

Thus, τ varies with r according to

$$\frac{\tau}{\tau_w} = \frac{r}{R} \quad \text{--------} \quad (2\cdot12)$$



2·5 STEADY LAMINAR FLOW IN CIRCULAR PIPE



For steady flow,

$$\tau = -\frac{r}{2}\frac{dP^*}{dx} \quad \text{-----} \quad (2\cdot13)$$

Shear stress in laminar flow is

$$\tau = \mu\frac{\partial v}{\partial r} = \mu\frac{dv}{dr} \quad \text{---} \quad (2\cdot14)$$

Since V varies in r direction only for fully developed laminar flow, equate (2·13) to (2·14), then

$$\mu\frac{dv}{dr} = -\frac{r}{2}\frac{dP^*}{dx}$$

$$\Rightarrow \frac{dv}{dr} = -\frac{r}{2\mu}\frac{dP^*}{dx}$$

Integrating w.r.t. r, then

$$V = -\frac{r^2}{4\mu}\frac{dP^*}{dx} + A$$

where $\frac{dv}{dr}$ = velocity gradient

V = velocity of flow

r = internal radius of pipe

R = external radius of pipe

P^* = pressure of flow at one end
$P^* + \delta P$ = pressure of flow at other end
τ = shear stress
For boundary condition when $r = R$ and $v = 0$, then

$$\boxed{v = \frac{1}{4\mu}(R^2 - r^2)\frac{dP^*}{dx}} \quad \text{------} (2\cdot15)$$

The discharge through an annular space between r and $r + \delta r$

$$dQ = v\,dA \quad \text{(continuity equation)}$$

$$dQ = v \cdot 2\pi r\,dr \quad \text{--------------} (2\cdot16)$$

Put $(2\cdot15)$ in $(2\cdot16)$, then

$$dQ = \frac{1}{4\mu}(R^2 - r^2)\frac{dP^*}{dx} \cdot 2\pi r\,dr$$

Simplifying the foregoing equation, then

$$dQ = \frac{\pi}{2\mu}\frac{dP^*}{dx}(R^2 r - r^3)dr$$

Integrating both sides of the above equation, then

$$\int dQ = \frac{\pi}{2\mu}\frac{dP^*}{dx}\int_0^R (R^2 r - r^3)dr$$

$$\boxed{Q = \frac{\pi R^4}{8\mu}\frac{dP^*}{dx}} \quad \text{----------} (2\cdot17)$$

For a length of the pipe over which P^* drop from P_1^* to P_2^*

$$Q = \frac{\pi R^4}{8\mu L}(P_1^* - P_2^*) \quad \text{--------} (2\cdot18)$$

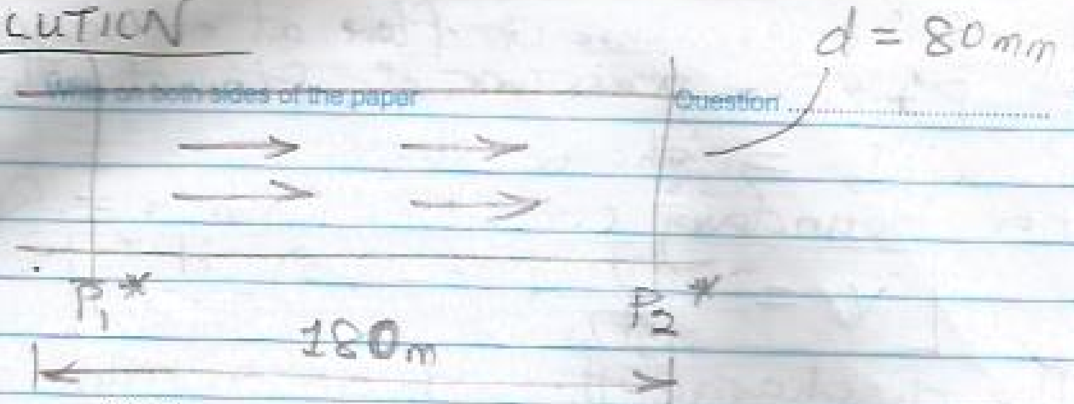
In terms of diameter

$$Q = \frac{\pi d^4}{128\mu L}(P_1^* - P_2^*) \quad \text{--------} (2\cdot19)$$

WORKED EXAMPLE 2-2

A fluid of dynamic viscosity 2·18 poise flows through a uniform pipe, 180m long of diameter s 80mm. If the pressure ~~at the~~ drop ~~along~~ along the pipe is 45 kPa, calculate the
(i) discharge through the pipe
(ii) mass flow rate of the fluid
(Density of fluid = 998 kgm^{-3}

SOLUTION



$d = 80mm = 0.08m$

$L = 180m$

$\mu = 2.18 \text{ poise} = 2.18 \times 0.1 = 0.218$

$= 0.218 \, Nsm^{-2}$

$\Delta P^* = 45 kPa = 45 \times 10^3 Pa$

(i) Required: Dischage through the pipe, Q

$$Q = \frac{\pi d^4}{128 \mu L}(P_1^* - P_2^*)$$

$$= \frac{\pi d^4}{128 \mu L} \Delta P^*$$

$$= \frac{22}{7} \times \frac{(0.08)^4}{128 \times 0.218 \times 180} \times 45000$$

$$= 1.153 \times 10^{-3} m^3 s^{-1} = 0.001153 m^3 s^{-1}$$

(ii) Required: Mass flow rate of fluid, $\dot{m}$

~~Note~~ The Mass flow rate $\dot{m}$ is given by

$$\dot{m} = \rho Q = 998 kgm^{-3} \times 0.001153 m^3 s^{-1}$$

$$= 1.151 kgs^{-1}$$

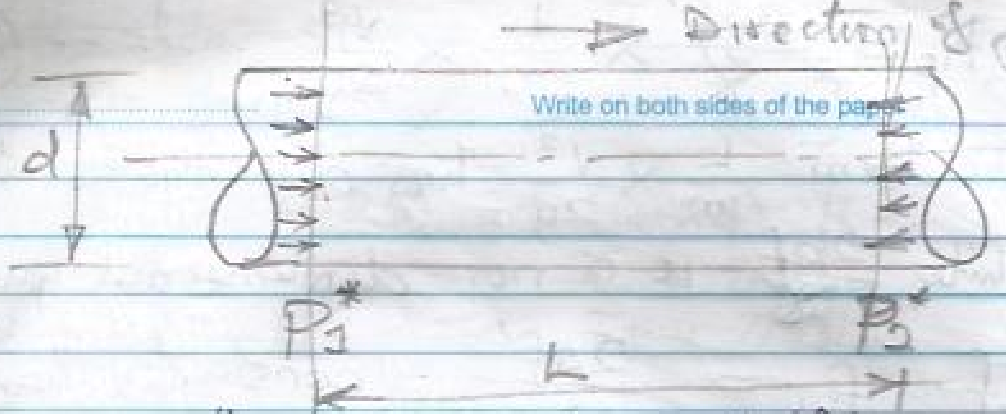
2.6 TURBULENT FLOW IN CIRCULAR PIPE

For a fully developed turbulent flow in a circular pipe of uniform cross-sectional area and roughness. The Darcy's experimental result shows that the viscous frictional effects associated with fluid are proportional to the

(i) length of the pipe, L

(ii) wetted perimeter, C

(iii) average velocity, v, of flow of index n whose range is $1.5 < n < 2.0$



The propelling force on the flowing fluid between two sections is

$$= (P_1 - P_2)A$$

The frictional resistance force on the flow is

$$= f' C L V^2$$

where P_1 = intensity of pressure at section 1
P_2 = intensity of pressure at section 2
A = cross-sectional area of the pipe
L = length of the pipe
d = diameter of the pipe
C = perimeter of the pipe
v = average velocity of flow
f' = non-dimensional friction factor (depends on material and nature of pipe surface)

At equilibrium,

Propelling force = Frictional resistance force

$$(P_1 - P_2)A = f' C L V^2$$

Divide both sides by γA, then

$$\frac{(P_1 - P_2)A}{\gamma A} = \frac{f' C L V^2}{\gamma A}$$

$$h_f = \frac{f'}{\gamma}\left(\frac{C}{A}\right) L V^2 \quad \text{-----} \quad (2.20)$$

Multiply both numerator and denominator by $2g$, then

$$h_f = \frac{2gf'}{\gamma}\left(\frac{C}{A}\right)\frac{LV^2}{2g} = \frac{2gf'}{\gamma}\left(\frac{1}{m}\right)\frac{LV^2}{2g}$$

where γ = weight density (specific weight) = ρg

The ratio $\frac{A}{C} = m$ is called the HYDRAULIC RADIUS or HYDRAULIC MEAN DEPTH

Simplifying the relation above, then

$$h_f = \frac{2gf'}{\gamma} \times \frac{L}{m} \times \frac{v^2}{2g} \quad \text{------} (2\cdot21)$$

The term $\frac{L}{m} \times \frac{v^2}{2g}$ has dimensions of h_f while $\frac{2gf'}{\gamma}$ is a non-dimensional quantity

If $f = \frac{2gf'}{\gamma}$ = constant, then

$$h_f = f \times \frac{L}{m} \times \frac{v^2}{2g} \quad \text{------} (2\cdot22)$$

For circular pipe, the hydraulic radius, m is

$$m = \frac{A}{C} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$$

Substituting this in equation (2·22) then

$$h_f = f \times \frac{L}{d/4} \times \frac{v^2}{2g} = \frac{4fLv^2}{d \times 2g}$$

$$\boxed{h_f = \frac{4fLv^2}{d \times 2g}} \quad \text{------} (2\cdot23)$$

Equation (2·23) above is called the DARCY-WEISBACH equation for all types of flow.

f = Darcy coefficient of friction

Equation (2·22) can be simplified as

$$h_f = \frac{f_1 L v^2}{d \times 2g}$$

where $f_1 = 4f$ is called the FRICTION FACTOR.

The friction factor is a function of

(i) relative roughness, $\frac{\epsilon}{2}$

(ii) Reynolds' number, Re

i.e. $f_1 = \phi\left(\frac{\epsilon}{2}, Re\right)$

The relative roughness of the pipe is the average height of the "bumps" on surface pipe diameter

Considering the shear stress at the pipe wall:

Force due to shear stress $= (P_1 - P_2)A$ ---(2·24)

If the shear stress is τ_0 at pipe wall, then

Force due to shear stress $= \tau_0 \times$ Surface area

$$= \tau_0 \times \pi d L \quad \text{- - - (2.25)}$$

Put equation (2.25) into (2.24) Then

$$(P_1 - P_2) A = \tau_0 \times \pi d L$$

$$(P_1 - P_2) \frac{\pi}{4} d^2 = \tau_0 \times \pi d L$$

Simplifying, then

$$P_1 - P_2 = \frac{4 \tau_0 L}{d} \quad \text{- - - - - - - (2.26)}$$

But $h_f = \dfrac{P_1 - P_2}{\gamma} = \dfrac{4 f L v^2}{d \times 2g}$ (from 2.23)

$$P_1 - P_2 = \frac{4 f L v^2}{d \times 2g} \times \gamma \quad \text{- - - - (2.27)}$$

Compare equation (2.27) with equation (2.26), then

$$\frac{4 \tau_0 L}{d} = \frac{4 f L v^2}{d \times 2g} \times \gamma$$

$$\tau_0 = \frac{f v^2}{2g} \gamma = \frac{f v^2}{2g} \times \rho g$$

$$\tau_0 = \frac{f \rho v^2}{2}$$

$$\Rightarrow f = \frac{2 \tau_0}{\rho v^2} = \frac{\tau_0}{\frac{1}{2} \rho v^2} \quad \text{- - - - (2.28)}$$

Equation (2.28) is the coefficient of friction in terms of the **shear stress**

2.7 LOSS OF ENERGY (HEAD) IN PIPES

Two categories of head loss are considered in pipes. These are

1. Major energy (head) losses
2. Minor energy (head) losses

2.7.1 MAJOR HEAD LOSSES

The major head losses are the losses due to friction in pipes. These losses are determined in two ways:

2.7.1.1 The Darcy-Weisbach approach - using the formula derived earlier for head loss i.e. equation (2.23)

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$$h_f = \frac{4fLv^2}{d \times 2g}$$

where h_f = head loss due to friction
and f = coefficient of friction (a function of Reynolds' number)

~~For laminar (viscous) flow,~~

$$\cancel{h_f = \frac{16}{Re}}$$

Rearranging the above formula, then

$$f = \left(\frac{d \times 2g}{4Lv^2}\right) h_f$$

* For laminar (viscous) flow

$$h_f = \frac{\Delta P}{\rho g} = \frac{8QL\mu}{\pi R^4 \rho g}$$

$$= \frac{8vL\mu}{\rho g R^2}$$

Since $Q = vA = v \cdot \pi R^2$

$$f = \left(\frac{d \times 2g}{4Lv^2}\right)\left(\frac{8vL\mu}{\rho g R^2}\right)$$

$$= \left(\frac{d \times 2g}{4Lv^2}\right)\left(\frac{32vL\mu}{\rho g d^2}\right)$$

$$= \frac{16\mu}{\rho v d}$$

But $Re = \frac{\rho v d}{\mu}$, hence

$$f = \frac{16}{Re} \quad \text{for } Re < 2000 \cdots (2.29)$$

~~* For turbulent flow~~

The variation of f with Re and relative roughness $\left(\frac{\epsilon}{d}\right)$ is shown in the Moody chart. In laminar flow, f is independent of wall roughness unless k/d is so high to constitute appreciable diameter change

* For turbulent flow

$$f = \frac{0.0791}{(Re)^{1/4}} \quad \text{for } Re > 4000 \cdots (2.30)$$

The Darcy-Weisbach formula holds for turbulent for Re varying from 4000 to 10^6

2.7.1.2 The Chezy's approach – using the formula derived earlier for head loss i.e. equation (2.20)

$$h_f = \frac{f'}{\gamma}\left(\frac{C}{A}\right) L V^2$$

where h_f = head loss due to friction

f' = non-dimensional factor

C, A = wetted perimeter and cross-sectional area of flow

From the equation above, the mean velocity, V is

$$V = \sqrt{\frac{\gamma}{f'}} \times \sqrt{\frac{A}{C} \times \frac{h_f}{L}} \quad \text{------} (2.31)$$

where the factor, $\sqrt{\frac{\gamma}{f'}}$ is called the CHEZY's ~~constant~~ constant, k

The ratio $\frac{A}{C}$ is the hydraulic radius or hydraulic mean depth while $\frac{h_f}{L}$ is called head loss per unit length of pipe.

Equation (2.31) above can be compressed into

$$\boxed{V = k\sqrt{mi}} \quad \text{---------} (2.32)$$

where V = Mean velocity

$k = \sqrt{\frac{\gamma}{f'}}$ = Chezy's constant

$m = A/C$ = hydraulic radius

$i = h_f/L$ = slope

Equation (2.32) is called the CHEZY's FORMULA.

Note: Darcy-Weisbach formula for head loss is generally used for pipe flow while Chezy's formula for head loss is generally used for open channels flow.

WORKED EXAMPLE 2-3

Water flows through a ~~pipe~~ uniform circular pipe of diameter, 120mm and 110m long at a velocity of $2.5ms^{-1}$. Find the head loss due to friction using (i) Darcy-Weisbach formula

(ii) Chezy's formula for which $k = 56$

(Kinematic viscosity of water = 0.012 stoke)

SOLUTION

Diameter of pipe, $d = 120mm = 0.12m$

Length of pipe, $L = 110m$

Mean velocity of flow, $v = 2.5ms^{-1}$

Kinematic viscosity, $\nu = 0.012$ stoke

$= 0.012 \times 10^{-4} m^2s^{-1}$

(i) Required: Head loss, h_f using Darcy-Weisbach formula.

The Darcy-Weisbach formula for head loss is

$$h_f = \frac{4fLv^2}{d \times 2g}$$

f = friction coefficient which is a function of Re

$$Re = \frac{\rho vd}{\mu} = \frac{vd}{\nu} = \frac{2.5ms^{-1} \times 0.12m}{0.012 \times 10^{-4} m^2s^{-1}}$$

$$= 25 \times 10^4 = 250,000 > 4000,$$

hence the flow is turbulent

For turbulent flow, $f = \frac{0.0791}{(Re)^{1/4}} = \frac{0.0791}{(250,000)^{1/4}}$

$$f = \frac{0.0791}{22.4} = 0.00353125$$

Therefore, head loss due to friction is

$$h_f = \frac{4fLv^2}{d \times 2g} = \frac{4 \times 0.00353 \times 110m \times (2.5)^2}{0.12m \times 2 \times 9.81}$$

$$= 4.12459 \approx \underline{4.125m}$$

(ii) Required: Head loss, h_f using Chezy's formula

The mean flow velocity through pipe is

$$v = k\sqrt{mi}$$

$k = 56$ (Chezy's constant)

$$m = \frac{A}{C} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4} = \frac{0.12m}{4} = 0.03m$$

$v = 2.5ms^{-1}$

Therefore, using the formula,

$$2.5 = 56\sqrt{0.03i}$$

$$\sqrt{0.03i} = \frac{2.5}{56}$$

$$0.03i = \left(\frac{2.5}{56}\right)^2 = (0.0446428 5714)^2$$

$$0.03i = 0.0019929 8469$$

$$i = \frac{0.0019929846 9}{0.03} = 0.066433$$

But $i = \frac{h_f}{L} \Rightarrow 0.066433 = \frac{h_f}{110m}$

$$h_f = 0.066433 \times 110m = 7.3076$$

$$h_f = \underline{7.31m}$$

WORKED EXAMPLE 2-4

Oil flows through a circular pipe of diameter 240mm and 500m long at the flow rate of $0.56m^3s^{-1}$. Determine the

(a) head loss due to friction;

(b) power needed to maintain the flow.

(Relative density of oil = 0.8, kinematic viscosity of oil = 0.3 stoke)

SOLUTION

Diameter of pipe, $d = 240mm = 0.24m$

Length of pipe, $L = 500m$

Relative density of oil = 0.8

Kinematic viscosity of oil, $\nu = 0.3$ stoke $= 0.3 \times 10^{-4} m^2s^{-1}$

Discharge, $Q = 0.56m^3s^{-1}$

Density of oil $= 0.8 \times 1000 = 800kgm^{-3}$.

(a) Required: Head loss due to friction, f

Discharge, $Q = VA$

$$Q = V\frac{\pi d^2}{4}$$

$$0.56m^3s^{-1} = V \times \frac{22}{7} \times \frac{(0.24m)^2}{4}$$

Velocity, $V = \frac{0.56m^3s^{-1} \times 7 \times 4}{22 \times 0.0576m^2} = 12.37ms^{-1}$

Reynolds' number, $Re = \frac{\rho Vd}{\mu} = \frac{Vd}{\nu}$

$$Re = \frac{12.37ms^{-1} \times 0.24m}{0.3 \times 10^{-4}m^2s^{-1}} = 9.9 \times 10^4$$

$Re > 4000$, hence it is turbulence, then Coefficient of friction, $f = \frac{0.0791}{Re^{1/4}} = \frac{0.0791}{(9.9 \times 10^4)^{1/4}}$

$$f = \frac{0.0791}{17.74} = 0.00446$$

Head loss due to friction, h_f is

$$h_f = \frac{4fLv^2}{d \times 2g} = \frac{4 \times 0.00446 \times 500m \times (12.3}{0.24m \times 2 \times 9.81ms^{-2}}$$

$$h_f = 289.8638 \approx \underline{289.9m}$$

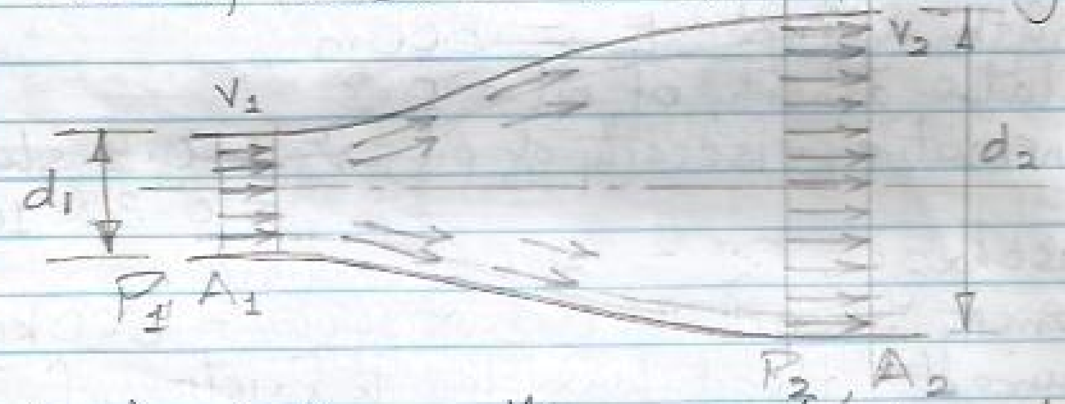
(b) Required: Power needed to maintain the flow, P

$$P = \gamma Q h_f = \rho g Q h_f$$
$$= 800 kgm^{-3} \times 9.81 ms^{-2} \times 0.56 m^3 s^{-1} \times 28$$
$$= 1,273,916.698 W$$
$$= \underline{1.274 MW}$$

2.7.2 MINOR HEAD LOSSES

Minor head losses in pipes include the following

2.7.2.1 Loss of head due to sudden enlargement



Applying Bernoulli's equation to sections (1) and (2) then

$$z_1 + \frac{v_1^2}{2g} + \frac{P_1}{\rho g} = z_2 + \frac{v_2^2}{2g} + \frac{P_2}{\rho g} + h$$

where h is the head loss due to sudden enlargement

For horizontal pipe, $z_1 = Z_2$, then

$$\frac{v_1^2}{2g} + \frac{P_1}{\rho g} = \frac{v_2^2}{2g} + \frac{P_2}{\rho g} + h$$

Therefore,

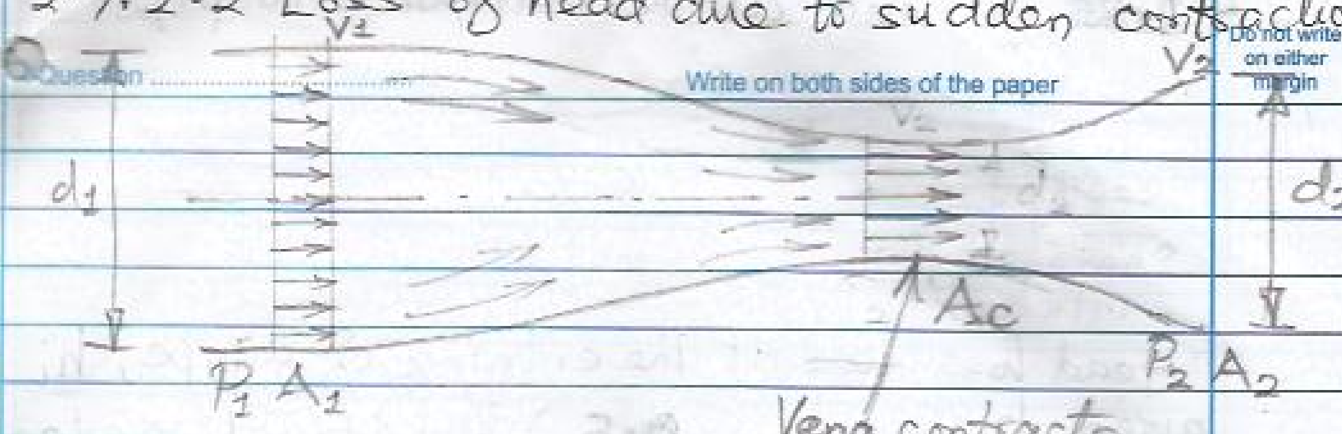
$$h = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + \left(\frac{v_1^2}{2g} - \frac{v_2^2}{2g}\right) \quad \text{---} (2.33)$$

Force on liquid in pipe flow is

$$F = (P_1 - P_2)A = P_1 A - P_2 A$$

At the two sections, $F = P_1 A_1 - P_2 A_2$ --- (2.34)

2.7.2.2 Loss of head due to sudden contraction



A_c = Cross-sectional area of vena contracta

Loss of head due to sudden contraction
= Loss up to vena contracta + Loss due to sudden enlargement beyond vena contracta

$$h_c = \text{Negligible loss} + \frac{(V_c - V_2)^2}{2g}$$

But $A_c V_c = A_2 V_2$ and $C_c = \frac{A_c}{A_2}$

$$\frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{A_c/A_2} = \frac{1}{C_c}$$

$$h_c = \frac{\left(\frac{V_2}{C_c} - V_2\right)^2}{2g} = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

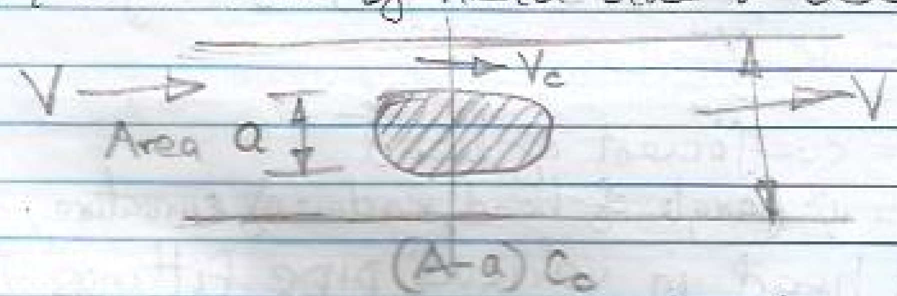
But $K = \left(\frac{1}{C_c} - 1\right)^2$

Hence, $h_c = K\frac{V_2^2}{2g}$

Experimental result shows that $K = 0.5$

Therefore, $h_c = 0.5\frac{V_2^2}{2g}$ ---------- (2.35)

2.7.2.3 Loss of head due to obstruction in pipe

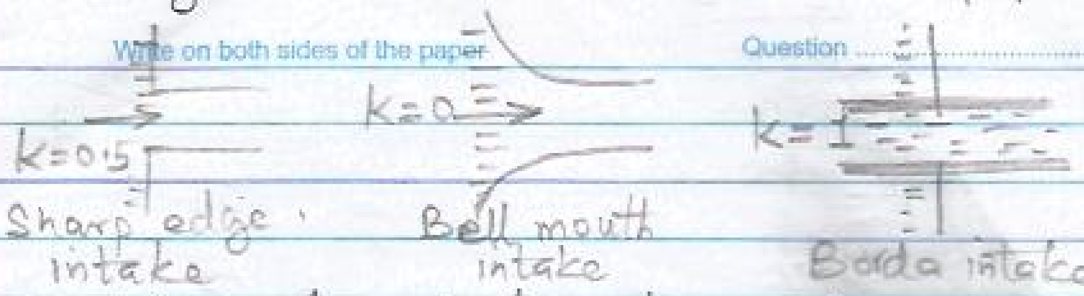


Head loss due to an obstruction, h_{obs} is given by

$$h_{obs} = \left[\frac{A}{C_c(A-a)}\right]^2 \frac{V^2}{2g}$$

Where A = cross-sectional area of the pipe
a = maximum area of obstruction
V = velocity of liquid in pipe

2.7.2.4 ~~Loss~~ Loss of head at the entrance to pipe



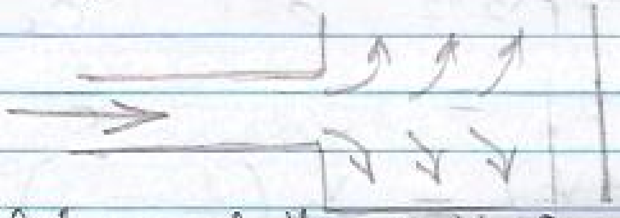
Head loss ~~due~~ at the entrance of a pipe, h_i is given by

$$h_i = 0.5\frac{V^2}{2g} \quad \text{(for sharp edge intake)}$$

$$h_i = \frac{V^2}{2g} \quad \text{(for ~~bell mouth~~ Borda intake)}$$

where V = velocity of liquid in pipe

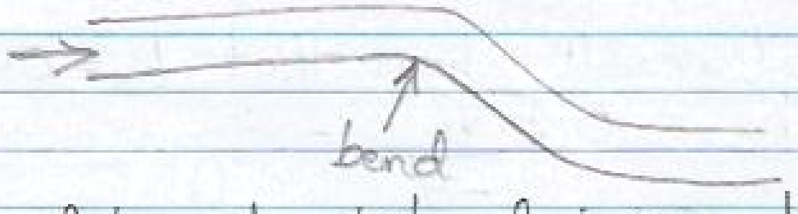
2.7.2.5 Loss of head at the exit of a pipe



Head loss at the exit of a pipe, h_o is given by

$$h_o = \frac{V^2}{2g}$$

2.7.2.6 Loss of head due to bend in the pipe



Head loss due to bend in pipe, h_b is given by

$$h_b = K\frac{V^2}{2g}$$

where K = coefficient of bend

$K = \phi$ (angle of bend, radius of curvature, diameter of pipe)

2.7.2.7 Loss of head in various pipe fittings

All pipe fittings include valves, coupling, orifice, plate e.t.c. cause head losses because of eddies generated in the flow.

Head loss in pipe fittings, $h_{fittings}$ is

$$h_{fittings} = K\frac{V^2}{2g}$$ K depends on fitting.