# Storage and Prices: Australian Potatoes

## Background

Potatoes are an important horticultural crop in Australia. Due to concerns over disease transmission, imports of fresh potatoes are largely banned. Exports of fresh potatoes from Australia have averaged less than three percent of production over the past three years. The majority of potatoes grown in Australia are processed (e.g., french fries). Over the past five years, Australian exports and imports of processed potatoes have averaged to 6.6 percent and 4.8 percent respectively. These trade data come from the ComTrade database.

The above data suggests that the Australian potatoe market can well approximated as a closed market. In the analysis below we will assume that all of the potatoes produced in Australia are consumed domestically, either fresh or processed. We will not distinguish between fresh and processed potatoes, and we will further assumed that the monthly demand schedule is constant throughout the year.

Most Australian potatoes are grown in New Southwales, South Australia and Tazmania (see <https://silo.tips/download/the-potato-industry-in-new-south-wales>). Potatoes are harvested in many different months depending on the type of potato being produced (e.g., early versus mid versus late season) and the region. We will simplify by using the March harvest date for the midseason potatoes in the Riverina District as the representative harvesting time period.

## Data

According to FAOstat, the average level of Australian potato production for 2015 - 2019 was (in metric tonnes)

Q\_year <- 1160760

The average annual value over this same time period was (in Australian dollars)

Value <- 717000000

Divide annual value by annual production to obtain an estimate of the average price per tonne:

(P <- Value/Q\_year)

## [1] 617.6987

Divide annual production by 12 to obtain an estimate of monthly consumption (in tonnes):

(x <- Q\_year/12)

## [1] 96730

## Demand Curve

The inverse demand schedule can be expressed as . Invert this to obtain . The elasticitity of demand evaluated at the calculated level of monthly consumption and average price is given by . Substitute in and then solve for to obtain . Finally, solve for and substitute in to obtain

Assume that

E <- -0.5

This gives

(b <- -1/E\*P/x)

## [1] 0.01277161

and

(a <- (1-1/E)\*P)

## [1] 1853.096

Let’s double check by substituting x into the inverse demand curve: we should see monthly consumption.

a - b\*x

## [1] 617.6987

## Intertemporal Law-of-One-Price (LOP)

To keep things simple, assume that the Australian potato producers can rent potato storage facilities at a cost of dollars per tonne per month. If the potatoes are stored instead of sold then then the opportunity cost of capital must also be considered. Assume that the foregone interest earnings are percent per month. This means that the total cost of storage per tonne for month is . Storage will only take place if the additional revenue from storing the potatoes, , is greater than or equal to the total cost of storage, .

As more and more potatoes are stored in the market, will be driven up and will be driven down. Arbitrage in the cash market for selling the commodity and in the market for storage will ensure that the in equilibrium which emerges, the marginal revenue from storing will equal to the marginal cost of storing. That is, .

If arbitrage in the cash and storage markets is costless then the intertemporal LOP is similar to the spatial LOP.

**Intertemporal LOP**

1. If Storage is positive then prices in successive periods must satisfy .
2. If there are zero stocks in storage then .

## Simlated Prices with Guess Value for P0

You may recognize the equilibrium pricing equation, , as a linear first order difference equation with constant coefficients. The solution to this equation (see <https://mjo.osborne.economics.utoronto.ca/index.php/tutorial/index/1/fod/t>) is given by:

If we knew the starting price, , then we would have the full pricing solution. For now we will assume

P0 <- 600

We can use the above equation along with to generate a price series for the 12 months “between” harvesting periods. To keep things simple we assume that all potatoes are harvested on March 1 and so the first month is March, the second month is April, etc.

We begin by assigning values to the remaining parameters of the model:

m <- 1.5  
r <- 0.002

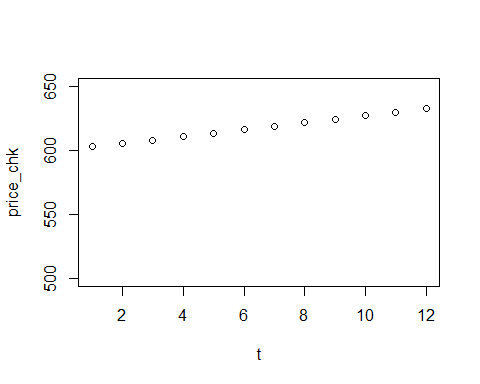
It will be tedious to write the above equations 12 times and so we will use an R loop.

price\_chk <- numeric(12)  
for(t in 1:12){  
price\_chk[t]<- (1+r)^t/r\*(r\*P0+m)-m/r  
}  
price\_chk

## [1] 602.7000 605.4054 608.1162 610.8324 613.5541 616.2812 619.0138 621.7518  
## [9] 624.4953 627.2443 629.9988 632.7588

You should verify that the 12 simulated prices satisfy the LOP equation: . A graph of the 12 prices looks as follows:

t <- 1:12  
plot(t,price\_chk, ylim=c(500,650))

 The plot looks linear but you can see from the equation that it is slightly non-linear.

## Solution Value for P0

To solve for the unknown value of begin by substituting into the inverse demand schedule, and then solve for . This gives

Let denote the size of the beginning stockpile, which consists of stocks brought in, plus the March 1 harvest, . Let denote the level of stocks which are carried over to the next year. That is, is the same as if the problem was to be solved again, one year later. Assume initially that and . In this case,

S\_in <- 0  
S\_out <- 0  
H <- Q\_year  
(S\_0 <- S\_in + H)

## [1] 1160760

Market clearing requires where for the current problem since there are 12 months. After substituting in the previous expression for this market clearing condition can be rewritten as

where . Solve this equation for to get

The variable is a standard finite sum of a geometric sequence (see <https://mathworld.wolfram.com/GeometricSeries.html>). It can be shown that

To derive the solution value for note that

N <- 12  
(Z <- (1+r)/r\*((1+r)^N-1))

## [1] 12.15715

It now follows that

(P0\_star <- (a\*r+m)\*N/(Z\*r) - m/r - b/Z\*(S\_0-S\_out))

## [1] 600.0192

## Graph of Price and Stocks over Time

It is useful to observe how potatoe stocks gradually deplete over the 12 months. After harvest is complete, stocks evolve according to . Recall that . Thus, . We can now use an R loop to observe price and stocks over the 12 month period.

t2 <- 1:12  
stocks <- numeric(12)  
price <- numeric(12)  
price[1] <- (1+r)^1/r\*(r\*P0\_star+m)-m/r  
stocks[1] <- S\_0 - a/b +1/b\*price[1]  
for(t in 2:12){  
price[t]<- (1+r)^t/r\*(r\*P0\_star+m)-m/r  
stocks[t]<- stocks[t-1]-a/b +1/b\*price[t]  
}  
price

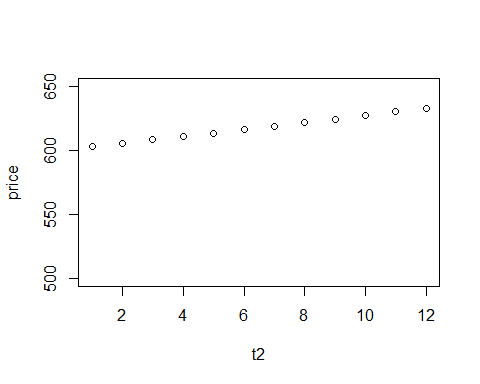
## [1] 602.7192 605.4246 608.1355 610.8517 613.5735 616.3006 619.0332 621.7713  
## [9] 624.5148 627.2638 630.0184 632.7784

stocks

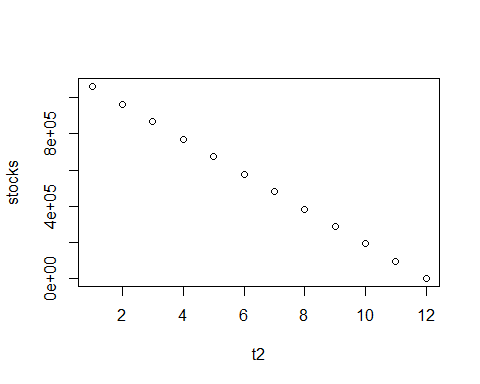
## [1] 1.062857e+06 9.651661e+05 8.676873e+05 7.704212e+05 6.733682e+05  
## [6] 5.765287e+05 4.799032e+05 3.834921e+05 2.872957e+05 1.913147e+05  
## [11] 9.554928e+04 -9.604264e-10

Graph the price and stocks data points.

plot(t2,price, ylim=c(500,650))



plot(t2, stocks)



## Two Identical Successive Years

Suppose harvest at the beginning of a second year was identical to the first year harvest. Moreover, there were no stocks carried into or out of the second year. This means that the set of prices in year 2 will be identical to the set of prices in year 1.

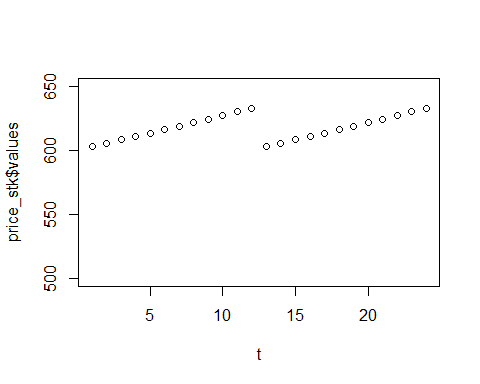
To program this in R we create a matrix with the first set of prices in the first column and the second set of prices in the second column. We then convert to a dataframe and use the Stack function to create a single set of values

price\_2 <- cbind(price, price)  
price\_2 <-as.data.frame(price\_2)  
price\_stk <- stack(price\_2)  
price\_stk

## values ind  
## 1 602.7192 price  
## 2 605.4246 price  
## 3 608.1355 price  
## 4 610.8517 price  
## 5 613.5735 price  
## 6 616.3006 price  
## 7 619.0332 price  
## 8 621.7713 price  
## 9 624.5148 price  
## 10 627.2638 price  
## 11 630.0184 price  
## 12 632.7784 price  
## 13 602.7192 price.1  
## 14 605.4246 price.1  
## 15 608.1355 price.1  
## 16 610.8517 price.1  
## 17 613.5735 price.1  
## 18 616.3006 price.1  
## 19 619.0332 price.1  
## 20 621.7713 price.1  
## 21 624.5148 price.1  
## 22 627.2638 price.1  
## 23 630.0184 price.1  
## 24 632.7784 price.1

Graph the new (double) set of prices

t <- 1:24  
plot(t,price\_stk$values, ylim=c(500,650))



## Stockout and the Intertemporal LOP

It should be obvious that potato merchants have no incentive to store from year 1 to year 2 because doing so will result in a lower selling price and positive storage costs. When zero storage from one year to the next is optimal we call this a market *stockout*.

It should be obvious that if beginning stocks for year 1 are less than the year 2 harvest then the market will stockout. In this case prices in year 1 will uniformly be above prices in year 2, in which case merchants have an even stronger negative incentive to carry stocks from year 1 to year 2.

When the market stocks out, prices jump down with the arrival of the new harvest, and then gradually rise back up. If there are repeated stockouts (e.g., 5 - 6 years) then the pricing pattern looks like a “saw tooth” (see <https://en.wikipedia.org/wiki/Sawtooth_wave>).

It is important to keep in mind that when the market stocks out the intertemporal LOP continues to hold when transitioning from year 1 to year 2. Recall the second component of the LOP, which states: If there are zero stocks in storage then . This equation allows for the outcome where the price one day after the new harvest is lower than the price one day before the new harvest.

## Positive Carry Over

The more interesting case is when year 1 beginning stocks are large relative to the year 2 harvest. In this case the month 12 price of year 1 might be below the month 1 price of year 2. If the price difference is large enough then merchants will have an incentive to carry over potatoes from year 1 to year 2.

For example, suppose beginning potato stocks for year 1 equal 1.2 million tonnes. This value is about 3.4 percent higher than the year 2 harvest, which is assumed to equal the long term average of 1,160,760 tonnes. If the pricing model specified above is re-run with and not carryout stocks (i.e., ) the following set of (approximate) equilibrium monthly prices emerge.

559 561 564 566 569 572 574 577 580 582 585 588

In year 2, the 12 monthly (approximate) prices are: 603 605 608 611 614 616 619 622 625 627 630 633

It should be clear that the year 1 month 12 price of $588/tonne is well below the year 2 month 1 price of $603/tonne. The intertemporal LOP no longer holds. Some stock from year 1 should be carried over to year 2.

As stock is shifted from year 1 to year 2, the year 1, month 12 price will increase and the year 2, month 1 price will decrease. Stocks will continue to shift until the intertemporal LOP holds when transitioning from year 1 to year 2.

## Equilibrium Level of Carry Over Stocks

What remains to be determined is the actual amount which is carried over from year 1 to year 2. Let denote the month 12 price in year 1 and let denote the month 1 price in year 2. If carry over from year 1 to year 2 is positive then according to the LOP equation, we must have = (1+r)P\_{12}^1 + m$.

We can substitute the previous equations into this formula and solve for , which is the amount of stock carried over from year 1 to year 2. The steps involved are somewhat complex, and so we will jump to the final solution. Assuming that are beginning stocks for year 1 and is the level of harvest for year 2, we have:

Suppose is equal to 1,160,760 tonnes, which is the 2015 - 2019 average level of potato production in Australia. Further suppose that is equal to 1.2 million tonnes. We can use the previous formula to identify that amount of potatoes that will be carried over from year 1 to year 2 (presumably in a processed form). The formula will be build using three separate components so that it is easier to follow.

H2 <- Q\_year  
S0 <- 1200000  
S\_out\_A <- (a\*r+m)\*N/(b\*r)\*(1-(1+r)^12)  
S\_out\_B <- (1+r)^12\*S0 - H2  
S\_out\_C <- (1+r)^12 + 1  
(S\_out <- (S\_out\_A+S\_out\_B)/S\_out\_C)

## [1] 4450.514

You might expect the above formula to return a value of zero if year 1 beginning stocks are low relative to the size of the year 2 harvest. What you will see instead is a negative value for . More sophisticated programming is required if want the restriction that stocks can’t be negative to be imposed on the model. You must keep an eye on the value of . If it is negative then zero carry over stocks are optimal and the market should be viewed as stocked out.

## Price Path with Positive Carry Over Stocks

Lets continue with the previous examply by solving the model for 24 months of potato prices. We will first solve the 12 month model from the perspective of year 1 with beginning stocks equal to 1.2 million tonnes and tonnes of potatoes carried over from year 1 to year 2. We will then solve the model for year 2, assuming a normal harvest and tonnes of potatoes added to the year 2 stockpile.

For year 1 we have:

(P0\_star\_yr1 <- (a\*r+m)\*N/(Z\*r) - m/r - b/Z\*(S0-S\_out))

## [1] 563.4713

stocks\_yr1 <- numeric(12)  
price\_yr1 <- numeric(12)  
price\_yr1[1] <- (1+r)^1/r\*(r\*P0\_star\_yr1+m)-m/r  
stocks\_yr1[1] <- S0 - a/b +1/b\*price\_yr1[1]  
for(t in 2:12){  
price\_yr1[t]<- (1+r)^t/r\*(r\*P0\_star\_yr1+m)-m/r  
stocks\_yr1[t]<- stocks\_yr1[t-1]-a/b +1/b\*price\_yr1[t]  
}  
price\_yr1

## [1] 566.0982 568.7304 571.3679 574.0106 576.6587 579.3120 581.9706 584.6345  
## [9] 587.3038 589.9784 592.6584 595.3437

stocks\_yr1

## [1] 1099229.749 998665.595 898307.951 798157.230 698213.845 598478.212  
## [7] 498950.745 399631.861 300521.978 201621.513 102930.885 4450.514

And for year 2 we have:

(P0\_star\_yr2 <- (a\*r+m)\*N/(Z\*r) - m/r - b/Z\*(S\_0+S\_out))

## [1] 595.3437

stocks\_yr2 <- numeric(12)  
price\_yr2 <- numeric(12)  
price\_yr2[1] <- (1+r)^1/r\*(r\*P0\_star\_yr2+m)-m/r  
stocks\_yr2[1] <- S\_0 +S\_out - a/b +1/b\*price\_yr2[1]  
for(t in 2:12){  
price\_yr2[t]<- (1+r)^t/r\*(r\*P0\_star\_yr2+m)-m/r  
stocks\_yr2[t]<- stocks\_yr2[t-1]-a/b +1/b\*price\_yr2[t]  
}  
price\_yr2

## [1] 598.0344 600.7305 603.4319 606.1388 608.8511 611.5688 614.2919 617.0205  
## [9] 619.7545 622.4940 625.2390 627.9895

stocks\_yr2

## [1] 1.066941e+06 9.688822e+05 8.710352e+05 7.734000e+05 6.759773e+05  
## [6] 5.787673e+05 4.817705e+05 3.849874e+05 2.884184e+05 1.920639e+05  
## [11] 9.592425e+04 -7.057679e-10

The most important result from this last simulation is that the price continues to rise over all 24 months, despite the fact that new harvest arrives in month 13. It seems counterintuitive that the price rises when new supply becomes available. However, if you think about this for awhile there can be no other outcome. A merchant is willing to carry inventory from year 1 to year 2 only if price increase is sufficient to satisfy her storage costs. The LOP tells us that as long as storage is positive, the price must rise according to .