**NATIONAL UNIVERSITY OF SINGAPORE**

**MA3111 - COMPLEX ANALYSIS I**

(Semester 2 : AY2018/2019)

Time allowed : 2 hours

---

**INSTRUCTIONS TO CANDIDATES**

1. Please write your matriculation/student number only. Do not write your name.
2. This examination paper contains **6** questions and comprises **2** printed pages.
3. Answer **ALL** questions.
4. Please start each question on a new page.
5. This is a CLOSED BOOK (with helpsheet) examination.
6. Students are allowed to use one handwritten, A4 size, double-sided helpsheet.
7. Calculators are not necessary and not allowed.

Some formulas that may be useful: $\pi/2 = 1.57\ldots$, $\sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$.

1. (10 marks) Express all possible values of

$$\mathrm{Log}((2i)^{5/4})$$

in Cartesian form.

2. Consider the function

$$f(z) = \frac{z^3 - 1}{z^2 + 3z - 4}.$$

(a) (10 marks) Find the first three nonzero terms of the Taylor series of $f(z)$ at $z_0 = 0$, and find the radius of convergence of the Taylor series of $f(z)$ at $z_0 = 0$.

(b) (10 marks) Find the Laurent series of $f(z)$ in the annulus $2 < |z + 1| < 3$.

3. Consider the function

$$f(z) = \frac{e^z}{z^2 \sin(\pi z)}.$$

(a) (10 marks) Find all isolated singular points of $f(z)$, tell if they are removable singular points, essential singular points, or poles, and determine their orders for all poles.

(b) (10 marks) Compute the contour integral

$$\int_\gamma f(z)dz,$$

where $\gamma(t) = \frac{\pi}{2}e^{2\pi it}$, $0 \le t \le 1$.

4. (10 marks) Let $a$ be a real number. Evaluate the integral

$$\int_0^\infty \frac{\cos(ax)}{x^4 + 16}dx.$$

5. Let $n$ be a positive integer and $r$ be a positive real number.

(a) (10 marks) Compute

$$\int_C \frac{\exp(rz^n)}{z}dz,$$

where $C$ is the unit circle $\{z \in \mathbb{C} \mid |z| = 1\}$ with positive orientation.

(b) (10 marks) Compute

$$\int_0^{2\pi} \exp(r\cos(n\theta))\cos(r\sin(n\theta))d\theta.$$

6. Let $f(z)$ be an entire function, and $u(x, y)$ and $v(x, y)$ be the real and imaginary parts of $f(x + iy)$. Let $C$ be a positive real number.

(a) (10 marks) Suppose

$$u(x, y) + v(x, y) < C, \quad \text{for all } x, y \in \mathbb{R}.$$

Show that $f(z)$ is a constant function on $\mathbb{C}$.

(b) (10 marks) Suppose $u(x, y)$ satisfies the inequality

$$u(x, y) < u(-y, x) + C, \quad \text{for all } x, y \in \mathbb{R}.$$

Show that $f(z) = f(iz)$ for all $z \in \mathbb{C}$.

***END OF PAPER***