TUTORIAL QUESTIONS

1. Which of the following equations are linear equations in x, and z?
   If they are not linear, give reason
   (a) x− y− z = 3 (b) $\sqrt{x}$ + y + z = 6 (c) $e^{(x+y+z)}$ = 1 (d) x − 2y + 5z = $\sqrt{3}$ (e)$\sqrt{2}$ x + $\frac{1}{2}$ y + z = 0

2. Plot the graphs of these linear equations and decide on the number of solutions for each system. If there are any solutions, find them
   (a) 2x + y = 3 (b) 2x + y = 3 (c) 2x + y = 3 (d) 3x − 2y = 3 (e) 5x − 2y − 5 = 0
   x − y = 7 8x + 4y = 12 2x + y = 5 3x − 2y = 6 3x − 2y − 3 = 0

3. How many numbers of solutions does the following system possess:
   (a) 7x + y = 10 (b) 12x + 4y = 16 (c) 2x − y − z = 3
   x − y = 7 8x + 4y = 16 4x – 2y – 2z = 3

4. Obtain the Echelon form of the system

$$\begin{aligned} x - 3y + 5z &= -9 \\ 2x - y - 3z &= 19 \\ 3x + y + 4z &= -13 \end{aligned}$$

5. Give the Reduced Row Echelon form of the system

$$\begin{pmatrix} 1 & 5 & -3 & -9 \\ 0 & -13 & 5 & 37 \\ 0 & 0 & 5 & -15 \end{pmatrix}$$

6. Give the reduced Row Echelon form of the system

$$\begin{pmatrix} 1 & 1 & 1 & -2 \\ 2 & -1 & -1 & -4 \\ 4 & 2 & -3 & -3 \end{pmatrix}$$

7. Let u = $\begin{pmatrix} x-3 \\ 2y+1 \\ -z+x \end{pmatrix}$ and v = $\begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$,
   If u = v, then determine the real numbers x, y, and z.

8. Classify the variables in the system, as free variable and lead variables

$$\begin{aligned} x + y - 2z + 4t &= 5 \\ 2x + 2y - 3z + t &= 3 \\ 3x + 3y - 4z - 2t &= 1 \end{aligned}$$

9. Given u = 5i − 3j + 6k and v = i − 6j + 4k
   Compute (i) u × v (ii) u.v (iii) |u × v| and (iv) angle θ between u and v

10. Let u = (-3, 0, 4), v = (2, -7, 1) and (5, -8, 0)
    Find (i) u – 4v (ii) 3u - 4v + 5w

11. Express the vector p = (1, -6, 5) as a linear combination of u = (1, 2, 3), v = (2, 5, 8) and w = (3,2, 3)

12. Write the vector u = (2, 3, -5) as a linear combination of v_1 = (1,2, -3), v_2 = (2, -1, -4) and v_3 = (1,7, -5)

13. Which of these pair of vectors are perpendicular u = (5, 4, 1), v = (3, -4, 1) and w = (1, -2, 3)

14. Determine whether the following vectors u,v,w are linearly independent, and if not express one of them as a linear combination of the others
    (a) u = (1, 0, 1), v = (1, 2, 3), w = (3, 2, 5)
    (b) u = (1, 0, 1), v = (1, 1, 1), w = (0, 1, 1)
    (c) u = (1, 0, 0, 1), v = (0, 1, 2, 1), w = (1,2, 3, 4)
    (d) u = (1, 0, 0, 1), v = (0, 1, 2, 1), w = (1, 2, 4, 3)

15. Find 3A – 2B, where A = $\begin{pmatrix} 1 & -2 & 3 \\ 4 & 5 & -6 \end{pmatrix}$ and B = $\begin{pmatrix} 3 & 0 & 2 \\ -7 & 1 & 8 \end{pmatrix}$

16. If A = $\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 4 & 2 & -3 \end{pmatrix}$, compute A^T. A

17. Find the transpose of the following matrices:

    (i) A= $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ (ii) C = $\begin{pmatrix} -1 & 3 & 4 \\ 7 & 9 & 0 \end{pmatrix}$

18. Show that the given system of linear equation is inconsistent

    x + y + 2z = 3
    -x + 3y – 5z = 7
    2x -2y + 7z = 1

19. Show that the matrix Q is orthogonal

    Q = $\begin{pmatrix} cos\,\theta & sin\,\theta \\ sin\,\theta & -cos\,\theta \end{pmatrix}$

20. Compute determinant of matrix

    B = $\begin{pmatrix} 2 & -10 & 11 \\ 5 & 3 & -4 \\ 7 & 9 & 12 \end{pmatrix}$

21. Let P_2 be the vector space of polynomials of degree 2 or less. Decide whether the following vectors in P_2 are linearly independent or dependent
    p = $6t^2 + 8t + 2$ and q = $3t^2 + 4t + 1$
    p = $2t^2 + 3t + 2$ and q = $t^2 + t + 1$
    p = $t^2 + 3t – 1$, q = $2t^2 + 7t + 5$ and r = 7