Fisher Scoring Algorithm Example

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Trying to implement Fisher Scoring for linear regression instead of basic Ordinary Least Square Estimation (OLE). The followings are the Notation Set up:
Consider the following simple linear regression model:

$$y = X\beta + \epsilon, \text{ where } \epsilon \sim N(0, \sigma^2)$$

The loglikelihood for σ^2 and β is given by:

$$loglikelihood = -\tfrac{N}{2}ln(2\pi) - \tfrac{N}{2}ln(\sigma^2) - \tfrac{1}{2\sigma^2}(y - X\beta)'(y - X\beta)$$

Computing the score function $S(\theta)$, where θ is the vector of parameter $(\beta, \sigma^2)'$. Taking the first derivatives with respect to β and σ^2.

$$\frac{\partial L}{\partial \beta} = \frac{1}{\sigma^2}(y - X\beta)'X$$
$$\frac{\partial L}{\partial \sigma^2} = -\frac{N}{\sigma^2} + \frac{1}{2\sigma^4}(y - X\beta)'(y - X\beta)$$

Then the Fisher Scoring Algorithm is implemented as:

$$\theta_{j+1} = \theta_j - (S(\theta_j))(S(\theta_j))'S(\theta_j)$$