

Newton-Raphson Method

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Example: Censored exponentially distributed observations
Suppose that $T_i \sim \text{Exp}(\theta)$ and the censored times

$$Y_i = \begin{cases} T_i & \text{if } T_i \leq C \\ C & \text{otherwise} \end{cases}$$

Let m be the number of uncensored observations. Then $l_n(\theta|Y) = m \log(\theta) - \theta \sum_{i=1}^n Y_i$ with first and second derivative:

- $\frac{\partial \ln(\theta|Y)}{\partial \theta} = \frac{m}{\theta} - \sum_{i=1}^n Y_i$
- $\frac{\partial^2 \ln(\theta|Y)}{\partial \theta^2} = -\frac{m}{\theta^2}$

The MLE can be obtained by the Newton-Raphson iteration:

$$\hat{\theta}^{(k+1)} = \hat{\theta}^{(k)} + \frac{(\hat{\theta}^{(k)})^2}{m} \left(\frac{m}{\hat{\theta}^{(k)}} - \sum_{i=1}^n Y_i \right)$$

Choose the starting value is 0.4. If you choose the initial value far away from the true, Nan will be produced. (Ex: 0.6)