Simulation of Continuous Systems

A Pure Pursuit Problem

- We have a **target**, which moves along a pre-determined path, and there is a **pursuer**, who follows the target.
- Example of target: intruder aircraft
- Example of pursuer: defender aircraft
- The defender aircraft will fly pointing towards the intruder
- In our simulation, we have to find whether our defender is able to destroy the intruder or not.

Path of the target

Time	0	1	2	3	4	5	6	7	8	
x(t)	100	110	120	129	140	149	158	168	179	188
y(t)	0	3	6	10	15	20	26	32	37	34

Time	10	11	12	13	14	15
x(t)	198	209	219	226	234	240
y(t)	30	27	23	19	16	14

Path of the fighter

- Suppose the speed of the defender is fixed S=20km/min
- Starting position of the defender is at (0,60)
- Suppose the defender is able to destroy the intruder if the distance is less than 10 km
- The pursuit ends if the pursuer is not able to destroy the target with in 12 minutes

Target and the pursuer

- Distance between the target and the pursuer, dist(t)=
$$\sqrt{\left(xt(t)-xd(t)\right)^2+\left(yt(t)-yd(t)\right)^2}$$
- Suppose, ϴ is the angle between x axis and the line between the target and the pursuer.
- sin(ϴ)=(yt(t)-yd(t))/dist(t) and cos(ϴ) = (xt(t)-xd(t))/dist(t) --- > how?
- Then the position of the defender at time t+1 is determined as, xd(t+1)=xd(t)+S cosϴ and yd(t+1)=yd(t)+S sinϴ

Simulation results

Time	xt(t)	yt(t)	xd(t)	yd(t)	dist(t)
0	100	0	0	60	116.61
1	110	3	17.14	49.71	103.93
2	120	6	35.01	40.72	91.80
3	129	10	53.53	33.15	78.94
4	140	15	72.65	27.29	68.46
5	149	20	92.32	23.70	56.79
6	158	26	112.28	22.39	45.85
7	168	32	132.22	23.96	36.66
8	179	37	151.73	28.34	28.60
9	188	34	170.79	34.39	17.20
10	198	30	190.79	33.93	8.21
11	**BOMB!**				

Simulation of a Chemical Reaction

A chemical reaction

- Suppose there are two reagents ch1 and ch2 that react together to produce another reagent ch3
- So there will be one forward reaction and a backward reaction
- Forward Reaction: ch1+ch2 → ch3 (rate kf)
- Backward Reaction: ch3 →ch1+ch3 (rate kb)

Chemical Equilibria

- Initially the reaction starts with only ch1 and ch2 and produces ch3
- As soon as the reaction makes progress, the backward reaction also produces ch1 and ch2
- In equilibrium, the rate of changes for all the reagents becomes close to zero.
- We have to simulate until the reaction reaches the equilibrium condition

Rate of changes

- We need three equations for the rate of changes for each of the reagents.
- We suppose c1, c2 and c3 are the mounts of three reagents at any time instance t.

$$\frac{dc1}{dt} = kb * c3 - kf * c1 * c2$$

$$\frac{dc2}{dt} = kb * c3 - kf * c1 * c2$$

$$\frac{dc3}{dt} = 2\mathrm{kf} * c1 * c2 - 2\mathrm{kb} * c3$$

Amount of Reagent

- We need three equations for the amount of any reagent at any time instance t+Δt

$$c1(t+\Delta t)=c1(t)+\frac{dc1(t)}{dt}*\Delta t$$

$$c2(t+\Delta t)=c2(t)+\frac{dc2(t)}{dt}*\Delta t$$

$$c3(t+\Delta t)=c3(t)+\frac{dc3(t)}{dt}*\Delta t$$

Starting the simulation

- We set initial values, c1(0)=50, c2(0)=25 and c3(0)=0, and rates, kf=0.025 and kb=0.01
- Now if we set Δt=0.1 s,

$$c1(t+\Delta t)=c1(t)+[kb*c3(t)-kf*c1(t)*c2(t)]*\Delta t$$

$$c2(t+\Delta t)=c2(t)+[kb*c3(t)-kf*c1(t)*c2(t)]*\Delta t$$

$$c3(t+\Delta t)=c3(t)+[2*kf*c1(t)*c2(t)-kb*c3(t)]*\Delta t$$

Simulation Results

Time	**c1**	**c2**	**c3**
0.000	50.00	25.00	0.00
0.1	46.88	21.88	6.25
0.2	44.32	19.32	11.36
0.3	42.19	17.19	15.62
0.4	38.86	15.39	19.22
0.5	37.53	13.86	22.29
0.6	36.38	12.53	24.93
0.7	35.37	11.38	27.24
0.8	34.49	10.37	29.25
0.9	33.80	9.49	31.03
1.0	32.37	8.70	32.60
1.1	31.81	8.00	34.00
And so on	...	...	...

A serial Chase Problem

A serial chase problem

- Similar to the pure pursuit problem
- But now we have a number of moving objects
- Suppose we have 4 objects A,B,C and D
- We have the following scenerio:
  - D moves freely
  - D is chased by C
  - C is chased by B and
  - B is chased by A
  -

A serial chase problem

- Each of them have their own speed and starting locations
- The objects do not have any idea of whether they are being chased or not
- At each time instance, a chaser or follower adjusts the direction of its chase towards the moving target (similar to the pure pursuit problem)
- Simulation ends as soon as a hit occurs

Initial conditions

- Speed of each object
  - Va=35km/hr
  - Vb=25km/hr
  - Vc=15km/hr
  - Vd=10km/hr
- Starting locations
  - xa(0)=10, ya(0)=10
  - xb(0)=30, yb(0)=10
  - xc(0)=30, yc(0)=30
  - xd(0)=10, yd(0)=30
- Suppose D moves according to a straight line parallel to x axis
- A hit occurs if distance is less than 0.005
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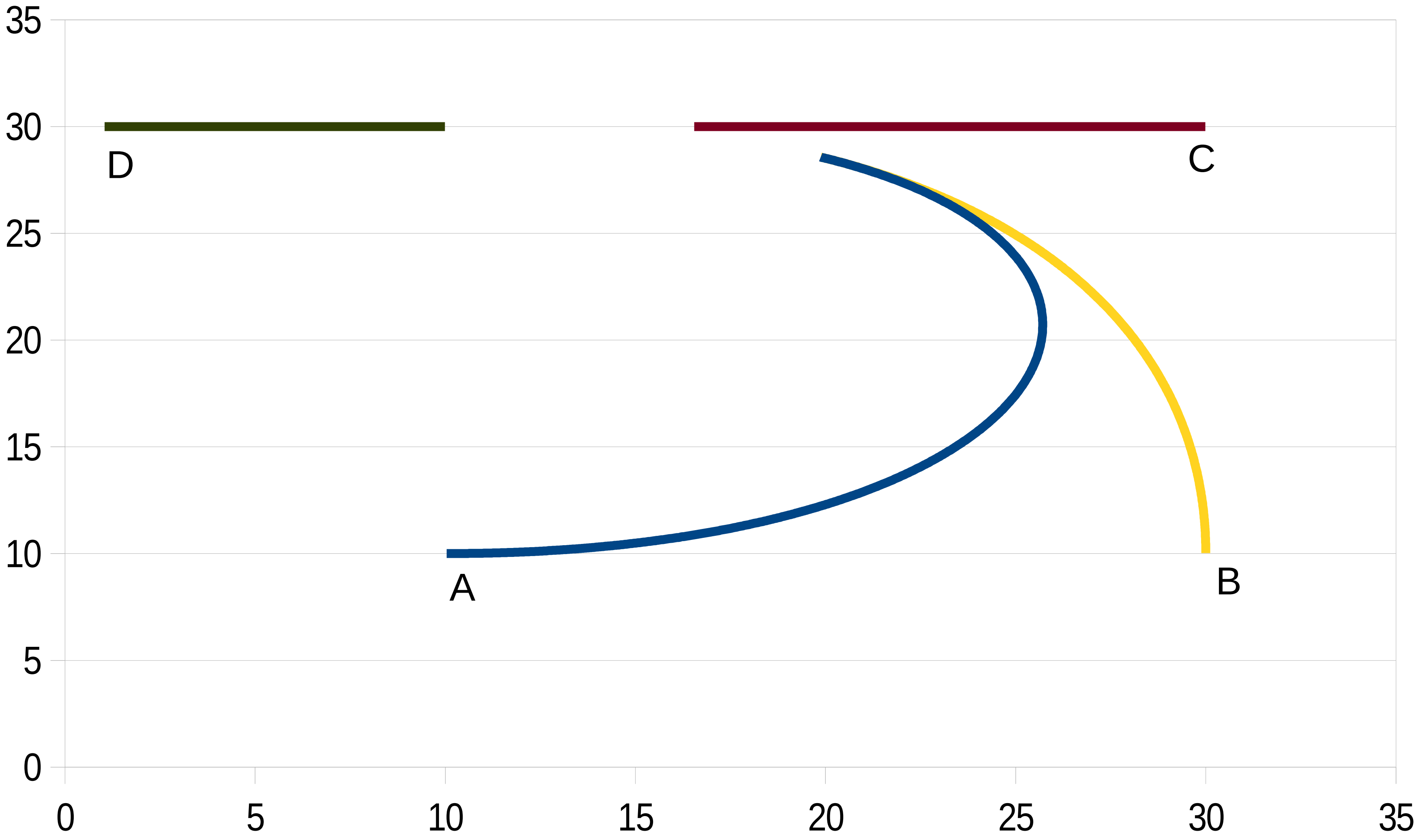
Simulation results



Another variant

- What happens if, A is chased by D?