HW6 Kunal Ninawe Applied Matrix Theory N12512995 kvn238

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1) Transform the following Hermitian matrix

$$H = \begin{pmatrix} 0 & 2 & -1 \\ 2 & 5 & -6 \\ -1 & -6 & 8 \end{pmatrix}$$

into a diagonal form.

To convert this matrix in a diagonal form.

We equate $|A - \lambda I| = 0$ and solve.

$$\rightarrow \quad A - \lambda I = \begin{bmatrix} 0-\lambda & 2 & -1 \\ 2 & 5-\lambda & -6 \\ -1 & -6 & 8-\lambda \end{bmatrix}$$

$$|A - \lambda I| = (-\lambda)[(5-\lambda)(8-\lambda) - 36] - 2[16 - 2\lambda - 6] - 1[-12 + 5 - \lambda]$$

$$= \lambda^3 - 13\lambda - \lambda + 13 = 0 \qquad (\because |A - \lambda I| = 0)$$

$$\therefore \lambda_1 = 13, \quad \lambda_2 = 1, \quad \lambda_3 = -1$$

Now, we put all λ values in the equ. $A - \lambda I$

for $\lambda_1 = 13$;

$$\begin{bmatrix} -13 & 2 & -1 \\ 2 & 5-13 & -6 \\ -1 & -6 & 8-13 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} -13 & 2 & -1 & 0 \\ 2 & -8 & -6 & 0 \\ -1 & -6 & -5 & 0 \end{array}\right]$$

Now we will use Row wise operations and solve.

to get

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1/5\, x_3 \\ -4/5\, x_3 \end{bmatrix}$$

to get $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1/5\ x_3 \\ -4/5\ x_3 \\ 1\ x_3 \end{bmatrix}$

for $\lambda = 1$, $A - \lambda I = 0 \Rightarrow \left[\begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 2 & 4 & -6 & 0 \\ -1 & -6 & 7 & 0 \end{array}\right]$

solving this again using row wise operations to get :

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1x_3 \\ x_3 \\ x_3 \end{bmatrix}$ //

for $\lambda = -1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & 6 & -6 & 0 \\ -1 & -6 & 9 & 0 \end{array}\right]$

& after solving, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3x_3 \\ 2x_3 \\ 1x_3 \end{bmatrix}$

eigenvectors $\Rightarrow \underset{V_1}{\begin{bmatrix} -1/5 \\ -4/5 \\ 1 \end{bmatrix}}, \underset{V_2}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}, \underset{V_3}{\begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}}$

We can see that distinct eigenvalues have distinct eigen vectors.

$(u_1, u_2, u_3) \Rightarrow \left(\frac{V_1}{\|V_1\|}, \frac{V_2}{\|V_2\|}, \frac{V_3}{\|V_3\|} \right)$

$$(u_1, u_2, u_3) \Rightarrow \left(\frac{v_1}{\|v_1\|}, \frac{v_2}{\|v_2\|}, \frac{v_3}{\|v_3\|}\right)$$

$\|v_1\| =$

$$\left(\sqrt{\frac{1+16+25}{25}}\right) = \sqrt{\frac{42}{25}} = \frac{\sqrt{42}}{5}$$

$\|v_2\| =$

$$\left(\sqrt{1+1+1}\right) = \sqrt{3}$$

$\|v_3\| =$

$$\sqrt{9+4+1} = \sqrt{14}$$

$$\Rightarrow u_1 = \left(-\frac{1}{\sqrt{42}}, \frac{-4}{\sqrt{42}}, 1\right)$$

$$\Rightarrow u_2 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow u_3 = \left(\frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right)$$

$$\Rightarrow u \Rightarrow [u_1^T, u_2^T, u_3^T]$$

2) If a (real) Hermitian matrix H is positive definite, prove that $H = P^2$, for a positive definite matrix P.

$$\Rightarrow H = U \overbrace{D}^{\text{Diagonal matrix}} U^*$$

Assuming $P = U\sqrt{D}\,U^*$

$\Rightarrow P^2 = U\sqrt{D}\,U^*\,U\sqrt{D}\,U^*$

$P^2 = U D U^*$

$\Rightarrow \underline{\underline{P^2 = H}}$.