1. For matrix, $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A - \lambda I = \begin{bmatrix} 1-\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix}$

$|A - \lambda I| = (-\lambda + 1)^2 (-\lambda + 2) = 0$

(This is the characteric polynomial).

algebriac multiplicity is : 2

$\therefore \lambda = 1, 2$

for $\lambda = 1$, null space = null $(A - \lambda I)$

$$= \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right], \text{ i.e.} \rightarrow A\bar{x} = \bar{0}$$

$\rightarrow$ Augmented Matrix

we can see from above, null space is :

$$\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right\}$$

$\Rightarrow (A - \lambda I) V_1 = 0$.

$\Rightarrow V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

similarly, $(A - \lambda I) V_2 = V_1$

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Here v_2 is the principal vector of grade 2 and v_1 is the eigen vector.

Jordan Basis is : $\{v_1, v_2\}$

$$= \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

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for $\lambda = 2$, $A - \lambda I$ (augmented) $= \left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right]$

i.e $\rightarrow A\bar{x} = \bar{0}$

calculating null space from above matrix

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\} = v_1 \Big\} \text{ eigenvector.}$$

Using similar steps, $(A - \lambda I) v_2 = v_1$

$$\left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{array}\right]$$

we can see, $v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

This v_2 is the principal vector of grade 2.

Jordan basis is : $\{v_1, v_2\}$

$$= \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Q3. $A = \begin{bmatrix} \lambda & \lambda & \lambda \\ 0 & \lambda & \lambda \\ 0 & 0 & \lambda \end{bmatrix}$, $\lambda \neq 0$.

Characteric polynomial for matrix A is
$C_A(x) = (x-d)(x-d)(x-d) = (x-d)^3$

where d is eigenvalue of A with geometric multiplicity 3
Now, we find minimal poly of A, we know that minimal poly of A is the factor of $(x-d)^3$.

Now $m_A(x)$ will be $(x-d)$, $(x-d)^2$ or $(x-d)^3$

$$A - \lambda I = \begin{bmatrix} \lambda & \lambda & \lambda \\ 0 & \lambda & \lambda \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \lambda & \lambda \\ 0 & 0 & \lambda \\ 0 & 0 & 0 \end{bmatrix} \neq \text{zero matrix.}$$

$$(A - \lambda I)^2 = \begin{pmatrix} 0 & \lambda & \lambda \\ 0 & 0 & \lambda \\ 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} 0 & \lambda & \lambda \\ 0 & 0 & \lambda \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & \lambda^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \neq \text{zero matrix}$$

$$(A - \lambda I)^3 = \begin{bmatrix} 0 & \lambda & \lambda \\ 0 & 0 & \lambda \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \lambda^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \text{zero matrix}$$

$\Rightarrow$ minimal polynomial for A, $m_A(x) = (x-d)^3$

$\Rightarrow$ Jordan form of $A = \begin{bmatrix} \lambda & \lambda & \lambda \\ \cancel{\phi} & \cancel{\phi} & \cancel{\phi} \\ 0 & \lambda & \lambda \\ 0 & 0 & \lambda \end{bmatrix}$

will be : $\begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}$,

Yes, it can be Transformed into Jordan Form.

Q2. $x = \left.\begin{bmatrix} \sqrt{2} \\ -9 \\ 84 \end{bmatrix}\right\}$ we'll solve for this.

let $x = w_1 z_1 + w_2 z_2 + w_3 z_3$.

$$[z_1 \ z_2 \ z_3] \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{2} \\ -9 \\ 84 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$w_1 = \sqrt{2}$, $w_2 = 84$, $w_3 = -9$

$$\therefore \begin{bmatrix} \sqrt{2} \\ -9 \\ 84 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} \\ 84 \\ -9 \end{bmatrix}$$

$\rightarrow$ let $x = \begin{bmatrix} 0 \\ 9.3 \\ 0 \end{bmatrix}$, $w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 9.3 \end{bmatrix}$

$$\begin{bmatrix} 0 \\ 9.3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 9.3 \end{bmatrix}$$