Applied Matrix Theory HW 4 Kunal Ninawe kvn238 N12512995

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1.
$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix}$$

$$|A - \lambda I| = (1-\lambda)^2 - 1$$
$$= \lambda^2 + \cancel{1} - 2\lambda - \cancel{1}$$
$$= \lambda^2 - 2\lambda$$
$$\lambda^{\cancel{2}} = 2\cancel{\lambda}$$
$$\lambda = 2, 0$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

<u>Row wise operations</u>

$R_2 \to R_2 + R_1$, $R_2 \to R_1 - R_2$

$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

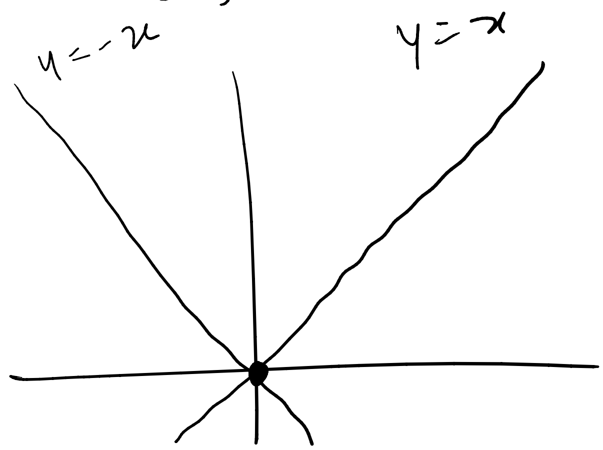
$-x_1 + x_2 = 0$ $\quad x_1 + x_2 = 0$

$x_2 = x_1$ $\quad x_1 = -x_2$

$x + y = 0$ $\quad x = y$

$$e.\bar{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

→ Plotting graph.



as from graph we can see both eigenvectors are intersecting at origin, hence They are orthogonal & independent.

2. To show A and A^T have same eigenvalues:

2. To show, A and A^T have same eigenvalues.

→ 1) using an example:

let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{bmatrix}$

$|A - \lambda I| = 4 - \lambda - 4\lambda + \lambda^2 - 6$

$= \lambda^2 - 5\lambda - 2$

$= \frac{1}{2}(5 - \sqrt{33})$,

$\frac{1}{2}(5 + \sqrt{33})$

$\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$, $A I = \begin{bmatrix} 1-\lambda & 3 \\ 2 & 4-\lambda \end{bmatrix}$

$|A - \lambda I| = \lambda^2 - 5\lambda - 2$

$= \frac{1}{2}(5 - \sqrt{33})$, $\frac{1}{2}(5 + \sqrt{33})$

As we can see, A^T & A have same eigenvalues

Proof: $Ac = d_1 c$ $\qquad$ $A^T c = d_1 c$

$\det(A - d_1 I) = 0$ $\qquad$ $\det(A^T - d_1 I) = 0$

$I = I^T$

$\det(A - d_1 I) = 0$ $\qquad$ $\det(A^T - d_1 I^T) = 0$

$\det(A - d_1 I) = 0$ $\qquad$ $\det((A - d_1 I)^T) = 0$

$\det(A) = \det(A^T)$

$\det(A - d_1 I) = 0$

→ A ∧ ∣

$$\det(A - d_1 I) = 0$$

$$d = d_1$$

Eigen Values

c) We know that $AB = B^{-1}(AB)B$,

similarly, $BA = A^{-1}(AB)A$

$\because$ both are similar, both of them would have the same eigenvalues & hence the same characteristics equation.

4) $|A - \lambda I| = 0$

$$A = \begin{bmatrix} a_1 - 2 & a_2 \\ a_3 & a_4 - 2 \end{bmatrix}$$

$$(a_1 - 2)(a_4 - 2) - a_2 a_3 = 0$$

$$(a_1 - 1)(a_4 - 1) - a_2 a_3 = 0$$

$$(a_1 - 2)(a_4 - 2) = (a_1 - 1)(a_4 - 1)$$

$$a_1 \cancel{a_4} + \cancel{4} - 2a_1 - 2a_4 = a_1 \cancel{a_4} + \cancel{1} - \cancel{a_1} - \cancel{a_4}$$

$$\left.\begin{array}{l} a_1 + a_4 = 3 \\ (a_1 - 1)(a_4 - 1) = a_2 a_3 \end{array}\right\}$$

$(a_1-1)(a_4-1) = a_2 a_3$

let $a_1 = 6$, $a_4 = -3$

$5 \times -4 = a_2 a_3$

$-20 = a_2 a_3$

let $a_2 = 5$, $a_3 = -4$

then $\boxed{A = \begin{bmatrix} 6 & 5 \\ -4 & -3 \end{bmatrix}}$, another soln. ↓

let $a_1 = 9$, $a_4 = -6$, $(a_1-1)(a_4-1) = a_2 a_3$

then $a_2 a_3 = 8, -7 = -56$

then $a_2, a_3 = 8, -7$, $\therefore A = \begin{bmatrix} 9 & 8 \\ -7 & -6 \end{bmatrix}$