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1. Given: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

(Here A is singular)

→ Using row operations to reduce $Ax = b$ to row echelon form. on $\begin{bmatrix} 1 & 2 & 3 & b_1 \\ 4 & 5 & 6 & b_2 \\ 7 & 8 & 9 & b_3 \end{bmatrix}$

- $R_2 \to -4R_1 + R_2 \Rightarrow \begin{bmatrix} 1 & 2 & 3 & b_1 \\ 4 & 5 & 6 & -4b_1 + b_2 \\ 7 & 8 & 9 & b_3 \end{bmatrix}$

- $R_3 \to -7R_1 + R_3 \Rightarrow \begin{bmatrix} 1 & 2 & 3 & b_1 \\ 0 & -3 & -6 & -4b_1 + b_2 \\ 0 & -6 & -12 & -7b_1 + b_3 \end{bmatrix}$

- $R_2 \to -\frac{1}{3}R_2 \Rightarrow \begin{bmatrix} 1 & 2 & 3 & b_1 \\ 0 & 1 & 2 & 4/3\, b_1 - 1/3\, b_2 \\ 0 & -6 & -12 & -7b + b_3 \end{bmatrix}$

- $R_3 \to 6R_2 + R_3 \Rightarrow \begin{bmatrix} 1 & 2 & 3 & b_1 \\ 0 & 1 & 2 & 4/3\, b_1 - 1/3\, b_2 \\ 0 & 0 & 0 & 8b_1 - 2b_2 + b_3 \end{bmatrix}$

- $R_1 \to -2R_2 + R_1 \Rightarrow \begin{bmatrix} 1 & 0 & -1 & -5/3\, b_1 - 2/3 b_2 + b_1 \\ 0 & 1 & 2 & 3\, b_1 - 1/3\, b_2 \\ 0 & 0 & 0 & 8b_1 - 2b_2 + b_3 \end{bmatrix}$

If $Ax = b$ has soln. then $8b_1 - 2b_2 + b_3 = 0$.

$$x_1 - x_3 = -\frac{5}{3}b_1 - \frac{2}{3}b_2 + b_1 \rightarrow ①$$

$$x_2 + 2x_3 = 3b_1 - \frac{1}{3}b_2 \rightarrow ②$$

$$0 = 8b_1 - 2b_2 + b_3 \rightarrow ③$$

$\Rightarrow$ Solving all equations we get.

$$b = \begin{array}{c} b_1 \\ b_2 \\ b_3 \end{array} \begin{bmatrix} \frac{1}{4}b_2 - \frac{1}{8}b_3 \\ 4b_1 + b_3/2 \\ 2b_2 - 8b_1 \end{bmatrix}$$

2. **Problem 2**. Is the following statement true or false? Justify your answer with the fullest possible details.

If $A \in \mathbb{R}^{n \times n}$ is invertible, there always exists an invertible upper triangular matrix B such that AB is orthogonal and an invertible lower triangular matrix C such that CA is orthogonal.

$\rightarrow$ let above statements be true. AB is orthogonal and AB is orthogonal. i.e. $AB = I$

and CA is orthogonal, i.e. $CA = I$

(Here I is an identity matrix)

then $AB = I$

$$CAB = C$$

$$B = A^{-1} = C$$

But we know that B is an upper triangular matrix.

Hence, what we assumed is false.

Hence, Statement is False.

3. Given : $A = xy^*$, $\lambda = y^* x$, $x, y \neq 0$

To show that λ is an eigenvalue of A.

→ Now, while computing an eigenvector, we

we know that $Ax = \lambda x$.

where λ is the eigenvalue of A &

x is the corresponding eigenvector

$$\Rightarrow Ax = \lambda x \Rightarrow (A - \lambda)x = 0.$$

i.e → $Ax - \lambda x = 0$.

⇒ substituting given values of A & λ.

$$xy^*(x) - y^* x(x) = 0.$$

which indeed is zero.

which indeed is zero.

$\therefore$ λ is scalar.

Now since given is $x \neq 0$, we can only see that λ is indeed the eigenvalue of A.

(b) If $\lambda \neq 0$, the rank of $A = xy^*$ is 1

& $\dim(\text{Ker}(A)) = n-1$

so there are atleast $n-1$ zero eigenvalues because A can have atmost $\underline{\underline{n}}$ eigenvalues

If $z \in \mathbb{C}^n$ (or $\mathbb{R}^n$) is any vector orthogonal to y

then $y^* z = 0$

$\Rightarrow x y^* z = 0$

$\Rightarrow Az = 0$

$\Rightarrow z \in N(A)$

Since G.M of $\lambda = 0$ is $n-1$,

A would be diagonizable $\underline{\underline{\text{iff}}}$ it has a non-zero eigenvalue.

Also we know that when any two vectors are orthogonal, then their dot-product is zero.

i.e $\rightarrow$ $\vec{u} \cdot \vec{y} = 0$. } If orthogonal.

Since orthogonal complement of a rowspace of A is the nullspace of A,

we also know that y lies in the nullspace of A.

Then, any vector orthogonal to y must also lie in the left null space of A.