Kunal Ninawe, N12512995, KVN238

H/W 2 EL5253

1. Consider matrix, $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

(a) <u>Null space of A</u>:

To get the null space of a matrix, we need to perform row operations on it and reduce it to row echelon form.

$$A = \left[\begin{array}{ccc|c} 1 & 4 & 7 & 0 \\ 2 & 5 & 8 & 0 \\ 3 & 6 & 9 & 0 \end{array}\right], \text{ i.e. } A \cdot \bar{x} = \bar{0}$$

$\bar{x}$ ↓ we need to find this.

using row wise operations,

$R_3 - R_2 \rightarrow R_3$
$R_2 - R_1 \rightarrow R_2$
$$\begin{bmatrix} 1 & 4 & 7 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_3 - R_2 \rightarrow R_3$
$R_1 - R_2 \rightarrow R_1$
$$\begin{bmatrix} 0 & 3 & 6 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1/3 \rightarrow R_1$
$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 - R_1 \rightarrow R_2$
$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

equations are :

$x_2 + 2x_3 = 0$, $x_2 = -2x_3$

$x_1 - x_3 = 0$, $x_1 = x_3$

, $x_3 = x_3$

$$\therefore \bar{x} = \begin{bmatrix} x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}_{3\times1} = N(A)$$

This is the null space of A . $(Dim(N(A))) = 3\times1$

b) Rank of A :

Rank of a matrix is given by :

(3) , if 'A' is non singular & $|A| \neq 0$

(2) , if 'A' is singular, $(|A| = 0)$ & one of its submatrix A_1 , $|A_1| \neq 0$.

(1) , if A is singular $(|A| = 0)$

(0) , if A is a null matrix .

$$|A| = 1.(45-48) - 4(18-24) + 7(12-15)$$
$$= -3 - (-24) + (-21)$$
$$= -3 + 24 - 21$$
$$= 0$$ (2) cond 1 satisfied .

Check cond 2

$|A_1| = \begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix}$, $|A_1| = 12 - 15 = -3 \neq 0$

cond (2) satisfied , $\boxed{Rank(A) = 2}$

Q2. For any $n \times n$ matrices $A \times B$, S.T. $|AB| = |BA| = |A| \cdot |B|$

→ Lets take one eg. first.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}_{2\times 2}$$

$|A| = 4 - 6 = -2$, $|B| = 40 - 42 = -2$

$$A \cdot B = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}, \quad B \cdot A = \begin{bmatrix} 19 & 43 \\ 22 & 50 \end{bmatrix}$$

$|A \cdot B| = 4$ $\quad |BA| = 4$ $\quad |A| \cdot |B| = 4$

$|A \cdot B| = |BA| = |A| \cdot |B|$.

which is true.

Now, proofing: case 1 → A is invertible.

∴ A is a product of elementary row matrices.

Let $A = \mathcal{E}_K, \mathcal{E}_{K-1}, \ldots \mathcal{E}_1$

$|A| = |(\mathcal{E}_K, \mathcal{E}_{K-1}, \ldots \mathcal{E}_1 B)|$

$= |\mathcal{E}_K| \cdot |\mathcal{E}_{K-1}| \cdots |\mathcal{E}_1| \cdot |B|$

$= |A| \cdot |B|$

case 2: If A is not invertible, $|A| = 0$

∵ A is not invertible, then AB will also be N.I

∴ $|AB| = 0$.

∴ $|AB| = |A| \cdot |B|$

3. Give examples to show $A \cdot B \neq B \cdot A$.

$\rightarrow$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 3 \\ 3 & -2 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} -1+6 & 3-4 \\ -3+12 & 9+8 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 9 & 1 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} -1+9 & -2+12 \\ 3-6 & 6-8 \end{bmatrix} = \begin{bmatrix} 8 & 10 \\ -3 & -2 \end{bmatrix}$$

We can clearly see $A \cdot B \neq B \cdot A$.

4. Consider equations of the form:

$x_1 + 2x_2 + 3x_3 + 4x_4 = 0 \rightarrow (1)$

$2x_1 + 4x_2 + \lambda_1 x_3 + \lambda_2 x_4 = 0 \rightarrow (2)$

What is range of param. (λ_1, λ_2) for which equ. (1) & (2) have non-zero soln

find all non-zero soln.

$$\bar{x} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \cdot y \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & \lambda_1 & \lambda_2 \end{bmatrix} = \bar{0}.$$

we can rewrite y as: $y/2$

$$y = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & \lambda_1/2 & \lambda_2/2 \end{bmatrix}$$

$$x_1 + 2x_2 = -3x_3 - 4x_4 \rightarrow (3)$$

$$x_1 + 2x_2 = -\frac{\lambda_1}{2} x_3 - \frac{\lambda_2}{2} x_4 \rightarrow (4)$$

equating (3) & (4)

$$-3x_3 - 4x_4 = -\frac{\lambda_1}{2} x_3 - \frac{\lambda_2}{2} x_4$$

$$\frac{\lambda_1}{2} = 3, \quad \frac{\lambda_2}{2} = 4.$$

$$\lambda_1 = 6, \quad \lambda_2 = 8$$

Scanned with CamScanner

$$\overset{\bar{y}}{A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \end{bmatrix}} \cdot \overset{\bar{x}}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}} = \overset{\bar{0}}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}$$

$$\Downarrow$$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 0.$$

$\underline{\text{Rank} = 1}$, $\underline{\text{Nullity} = 3}$