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1. Prove :

$(A+B)^T = A^T + B^T$

suppose order of both A and B is $m \times n$.
order of A + B would be $m \times n$.

order of $(A+B)^T$ would be $n \times m$. (Transpose)

Now, check order of R.H.S. of equation
$(A+B)^T = A^T + B^T$

order of both A^T, & B^T is $n \times m$.
then order of $A^T + B^T$ would be $n \times m$.

$\therefore$ order of both the sides of equ. is same.
→ Now prove equal corresponding elements.
$(j,i)^{th}$ element of A + $(i,j)^{th}$ element of B = $(j,i)^{th}$ element of A^T + $(j,i)^{th}$ element of B^T
= $(j,i)^{th}$ element of $(A^T + B^T)$

$\because$ corresponding elements of $(A+B)^T$ & $(A^T + B^T)$ are equal & order is same.
$\therefore$ $(A+B)^T = A^T + B^T$

2. S.T. AB is not necessarily symmetric, if A & B are symmetric.

→ Consider two symmetric matrices A and B as:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$$ which isn't symmetric.

This shows that AB need not be necessarily symmetric if A & B are symmetric.

3. If $A + jB$ is Hermitian, A, B real then $A^T = A$; $B^T = -B$

→ let $A + jB = \overline{(A+jB)}^T$

$$(A+jB)_{i,j} = A_{ij} + j\,B_{ij} \longrightarrow (1)$$

$$(A+jB)^T_{i,j} = A^T + j(B)^T$$

$$= A_{j,i} + j\,B_{j,i}$$

$$\overline{(A+jB)}^T = A_{j,i} - j\,B_{j,i} \longrightarrow (2)$$

$$(1) \rightarrow A_{ij} + j\,B_{i,j} = A_{j,i} - j\,B_{j,i}$$

We can see from equ ① & ②.

$A_{i,j} = A_{j,i}$, i.e $A = A^T$ ✓ proved.

$B_{ij} = -B_{j,i}$, i.e → $B = -B^T$ ✓ proved.

4. for any square matrix $A = \begin{bmatrix} A_1 & * \\ 0 & A_2 \end{bmatrix}$
with A_1 & A_2 being square submatrices.
S.T. $\det A = \det A_1 \cdot \det A_2$.

We know that for above matrix (square) A,
$d(A) = \det(A_1) \cdot \det(A_2) - \det(0) \cdot \det(*)$

and we know that
$\det(A) = \det(A^T)$
$A^T = \begin{bmatrix} A_1 & 0 \\ * & A_2 \end{bmatrix}$, Here $\det(A^T) =$
$\det(A_1) \cdot \det(A_2) - \det(0) \cdot \det(*)$
which is: $\det(A^T) = \det(A_1) \cdot \det(A_2)$.
∴ $\det(A) = \det(A^T)$
we can say $\det(A) = \det(A_1) \cdot \det(A_2)$
for any square matrix A.

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