

Answer: Given: $\dot{x} = Ax + g(x)$, $x(0) = x_0 \in \mathbb{R}^n$ $\frac{\|g(x)\|}{\|x\|} \rightarrow 0$
 when $\|x\| \rightarrow 0$

we can say, $\|x_0\|$ is sufficiently small.

Apply laplace transformation on $\dot{x} = Ax + g(x)$
 $x(0) = x_0$

$$s \cdot X(s) - s x_0 = AX(s) + g(s)$$

$$s(X(s)) - x_0 = AX(s) + g(s)$$

$$s(X(s)) = AX(s) + g(s) + x_0$$

$$X(s) = \frac{g(s) + x_0}{s - A}$$

laplace transformation is as follows:

$$F(s) = \int_0^{\infty} f(t) e^{-st} \cdot t'$$

$\{ s \rightarrow \text{complex number}$

$\{ t \rightarrow \text{real number} \geq 0$

$\{ t' \rightarrow \text{first derivative of } f(t) \}$

$$L'(F(s-a)) = e^{at} f(t)$$

\therefore By laplace transform:

$$x(t) = x_0 \cdot e^{-At} + e^{-At}$$

where $t \rightarrow \infty$

$$x(t) \rightarrow 0$$

Hence solved!