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1. Solve the initial-value problem

$$\dot{x}_1 = x_2 + e^{-t}, \qquad x_1(0) = 1,$$

$$\dot{x}_2 = 6(t+1)^{-2} x_1 + \sqrt{t}, \quad x_2(0) = 2.$$

→ The given func. is :

$$\dot{x}_1 = x_2 + e^{-t}$$

$$\frac{dx_1}{dt} = x_2 + e^{-t}$$

$$x_1(0) = 1$$

Solving the equations as :

$$\frac{dx_1}{dt} = x_2 + e^{-t}$$

$$dx_1 = (x_2 + e^{-t})\,dt$$

$$x_1(t) = x_2 t - e^{t} + c \qquad \longrightarrow (1)$$

Substituting the initial condition,

$$x_1(t) = x_2(t) - e^{t} + c$$

$$x_1(0) = x_2(0) - e^{0} + c$$

$$\Rightarrow \quad c = 2$$

substitute in equation 1

$$\boxed{x_1(t) = x_2(t) - e^t + 2}$$ } soln. for the 1st given equ.

Next : Given: $\dot{x}_2 = 6(t+1)^{-2} x_1 + \sqrt{t}$

i.e: $\dfrac{dx_2}{dt} = 6(t+1)^{-2} x_1 + \sqrt{t}$

$$x_2(0) = 2$$

Solving the equations as:

$$\frac{dx_2}{dt} = 6(t+1)^{-2} x_1 + \sqrt{t}$$

$$dx_2 = (6(t+1)^{-2} x_1 + \sqrt{t})\,dt$$

$$x_2(t) = -6(t+1)^{-1} x_1 + \frac{2t\sqrt{t}}{3} + c \rightarrow 2$$

substituting $x_2(0) = 2$ in 2

$$x_2(0) = 6(1)^{-1} x_1 + 0 + c$$

$x_2(0) = -6(1)^{-1} x_1 + 0 + c$

$2 = -6x_1 + c$

$c = 2 + 6x_1$

Sustitute in equ ②

$x_2(t) = -6(t+1)^{-1} x_1 + \frac{2t\sqrt{t}}{3} + 2 + 6x_1$

$\boxed{x_2(t) = [t(t+1)^{-1}]6x_1 + \frac{2t\sqrt{t}}{3} + 2}$ } soln. for 2nd equ.

2. Solve the initial-value problem

$$\frac{dy(t)}{dt} = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 & 3 \end{pmatrix} y(t), \quad y(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}.$$

$\to \quad y' = Ay$

we know $|A - \lambda I| = 0$

$$\Rightarrow A - \lambda I = \begin{bmatrix} 2-\lambda & 0 & 0 & 0 & 0 \\ 1 & 2-\lambda & 0 & 0 & 0 \\ 0 & 0 & 3-\lambda & 0 & 0 \\ 0 & 0 & 1 & 3-\lambda & 0 \\ 0 & 0 & 0 & 1 & 3-\lambda \end{bmatrix}$$

$$|A - \lambda I| = (2-\lambda)^2 (3-\lambda)^3 = 0$$

i.e – $\lambda = 2$ & $\lambda = 3$

for $\lambda = 2 \Rightarrow$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ Geometric multiplicity $= 1$

for $\lambda = 3 \Rightarrow A - \lambda I = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Similarly G.M $= 1$

$\therefore$ we have $H^{-1} A H = J$

$$H = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad J = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

General form $\Rightarrow$

$$y(t) = c_1 e^{2t} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_2 e^{2t} \left(t \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$+ c_3 e^{3t} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + c_4 e^{3t} \left[t \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right] + c_5 e^{3t}$$

& $y(t)$ when $t = 0$ $\qquad y(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = \begin{array}{ccccccccc} c_1 & + & c_2 & + & c_3 & + & c_4 & + & c_5 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 1 & & 2 & & 3 & & 4 & & 5 \end{array}$$

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