Applied Matrix Theory HW5 Kunal Ninawe kvn238 N12512995

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1) To find a set of mutually orthonormal vectors using gram schmidt's process. based on :

$$x^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \quad x^2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad x^3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

→ We will first try to find an orthogonal basis.

$$\boxed{V_1 = X_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\boxed{V_2 = x_2 - \mathrm{proj}_{x_1} \cdot x_2} \Rightarrow x_2 - \frac{x_2^T \cdot V_1}{V_1^T \cdot V_1} \cdot V_1$$

$$V_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} - \left(\frac{\begin{bmatrix} 0 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}}{\begin{bmatrix} 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} - \frac{1}{2} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 1/2 \\ 0 \\ 0 \\ -1/2 \end{bmatrix}$$

$$- \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad 2 \quad \begin{bmatrix} \\ -1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ -1/2 \end{bmatrix}$$

$$\boxed{v_2 = \begin{bmatrix} -1/2 \\ 1 \\ 0 \\ -1/2 \end{bmatrix}}, \quad \boxed{v_3 = x_3 - \text{proj}_{v_1} \cdot x_3 - \text{proj}_{v_2} \cdot x_3}$$

$$\boxed{v_3 = x_3 - \frac{x_3^T v_1}{v_1^T v_1} \cdot v_1 - \frac{x_3^T v_2}{v_2^T v_2} \cdot v_2}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} - \left(\frac{[0 \ 0 \ 1 \ -1] \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}}{[1 \ 0 \ 0 \ -1] \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \right)$$

$$- \left(\frac{[0 \ 0 \ 1 \ -1] \begin{bmatrix} -1/2 \\ 1 \\ 0 \\ -1/2 \end{bmatrix}}{[-1/2 \ 1 \ 0 \ -1/2] \begin{bmatrix} -1/2 \\ 1 \\ 0 \\ -1/2 \end{bmatrix}} \cdot \begin{bmatrix} -1/2 \\ 1 \\ 0 \\ -1/2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} -1/2 \\ 1 \\ 0 \\ -1/2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 1/2 \\ 0 \\ 0 \\ -1/2 \end{bmatrix} - \begin{bmatrix} -1/6 \\ 1/3 \\ 0 \\ -1/6 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ 0 \\ 1 \\ -1/2 \end{bmatrix} - \begin{bmatrix} -1/6 \\ 1/3 \\ 0 \\ -1/6 \end{bmatrix}$$

$$\boxed{v_3 = \begin{bmatrix} -1/3 \\ -1/3 \\ 1 \\ -1/3 \end{bmatrix}}$$

now, our orthogonal basis is:

$$w = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1/2 \\ 1 \\ 0 \\ -1/2 \end{bmatrix}, \begin{bmatrix} -1/3 \\ -1/3 \\ 1 \\ -1/3 \end{bmatrix} \right\}$$

$$\{ v_1 , \quad v_2 , \quad v_3 \}$$

To get Orthonormal basis, we find magnitudes of

$$\|\vec{v_1}\| , \quad \|\vec{v_2}\| , \quad \|\vec{v_3}\|$$

$$\Downarrow$$

$$\sqrt{1^2 + 0 + 0 + (-1)^2} , \quad \sqrt{(-1/2)^2 + 1^2 + 0 + (-1/2)^2} , \quad \sqrt{\frac{1}{9} + \frac{1}{9} + 1 + \frac{1}{9}}$$

$$= \sqrt{2} \qquad = \sqrt{\frac{1}{4} + 1 + \frac{1}{4}} \qquad = \sqrt{\frac{12}{}}$$

$= \sqrt{2}$ $\quad = \sqrt{\frac{1}{4}+1+\frac{1}{4}}$ $\quad = \sqrt{\frac{12}{9}}$

$= \sqrt{3/2}$ $\quad = \sqrt{4/3}$

now, divide each orthogonal vector by its magnitude to get orthonormal vector. basis

$$\beta = \{ v_1 / \|\vec{v_1}\| \; , \; v_2 / \|\vec{v_2}\| \; , \; v_3 / \|\vec{v_3}\| \}$$

$$= \left\{ \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ -1/\sqrt{2} \end{bmatrix} , \begin{bmatrix} \frac{-1}{2\sqrt{3/2}} \\ \frac{1}{\sqrt{3/2}} \\ 0 \\ \frac{-1}{2\sqrt{3/2}} \end{bmatrix} , \begin{bmatrix} \frac{-1}{3\sqrt{4/3}} \\ \frac{-1}{3\sqrt{4/3}} \\ 1/\sqrt{4/3} \\ \frac{-1}{3\sqrt{4/3}} \end{bmatrix} \right\}$$