ECE-GY 5253 Final **Fall 2020**

Due Monday 12/21, 4:30pm US Eastern time

Problem 1.

Consider the initial-value problem

$$\dot{x}(t) = \begin{pmatrix} 3 & 2 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} x(t) \triangleq Ax(t), \text{ with } x(0) = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}.$$

1.1) Transform the matrix A into a Jordan form.

1.2) On the basis of the Jordan form, solve the initial-value problem for all $t \geq 0$.

Problem 2.

Suppose the n Gersgorin discs of an $n \times n$ matrix A are mutually disjoint. Show that every eigenvalue of A is real, if A is real.

Problem 3.

Consider $A = \begin{pmatrix} 1 & 1 \\ -1.5 & 2 \end{pmatrix}$ and show that $\rho(A) < \min \left\| D^{-1}AD \right\|_\infty$ over all $D = \begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix}$, with $p_1 > 0$ and $p_2 > 0$. (Recall that $\|M\|_\infty$ stands for the *maximum row sum matrix norm* of $M \in \mathbb{R}^{n \times n}$, *i.e.* $\|M\|_\infty := \max_{1 \le i \le n} \sum_{j=1}^{n} |a_{ij}|$.)

Notes:

- Provide sufficient details for partial credits.
- You should submit your completed final to the course site at NYU Classes by **Monday 12/21, 4:30pm US Eastern time**
- Feel free to contact the TAs in case you have trouble in submitting your final. As the last resort, you can send a copy of your final in the PDF format by email to the TAs and cc Professor Jiang.