Applied Matrix Theory Homework 3 (Kunal Ninawe kvn238 N12512995)

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1. Give all the solutions of the system

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} x = \begin{pmatrix} 10 & 13 \\ 11 & 14 \\ 12 & 15 \end{pmatrix}.$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 & x_4 \\ x_2 & x_5 \\ x_3 & x_6 \end{bmatrix} = \begin{bmatrix} 10 & 13 \\ 11 & 14 \\ 12 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 & x_4 \\ x_2 & x_5 \\ x_3 & x_6 \end{bmatrix} = \begin{bmatrix} 10 & 13 \\ 11 & 14 \\ 12 & 15 \end{bmatrix}$$

Solving 1.

$$\begin{bmatrix} 1x_1 + 2x_2 + 3x_3 & 1x_4 + 2x_5 + 3x_6 \\ 4x_1 + 5x_2 + 6x_3 & 4x_4 + 5x_5 + 6x_6 \\ 7x_1 + 8x_2 + 9x_3 & 7x_4 + 8x_5 + 9x_6 \end{bmatrix} = \begin{bmatrix} 10 & 13 \\ 11 & 14 \\ 12 & 15 \end{bmatrix}$$

$x_1 + 2x_2 + 3x_3 = 10 \to (1)$

$4x_1 + 5x_2 + 6x_3 = 11 \to (2)$

$7x_1 + 8x_2 + 9x_3 = 12 \to (3)$

$x_4 + 2x_5 + 3x_6 = 13$

$4x_4 + 5x_5 + 6x_6 = 14$

$7x_4 + 8x_5 + 9x_6 = 15$

Taking (1) & (2) & (1) x 2 (7)

$$\begin{aligned} 2x_1 + 4x_2 + 6x_3 &= 20 \\ -\ 4x_1 + 5x_2 + 6x_3 &= -11 \\ \hline -2x_1 - x_2 &= 9 \end{aligned}$$

$$\boxed{x_2 = -2x_1 - 9}$$

using similar method,

$$\boxed{x_3 = x_1 + \frac{28}{3}}$$

$\boxed{x_5 = -2x_4 - 12}$ $\boxed{x_6 = x_4 + \frac{37}{3}}$

take $x_1 = 0$, $x_2 = 1$

$$X = \begin{bmatrix} x_1 & x_4 \\ -2x_1 - 9 & -2x_4 - 12 \\ x_1 + \frac{28}{3} & x_4 + 37/3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 \\ -9 & -14 \\ \frac{28}{3} & \frac{40}{3} \end{bmatrix} \rightarrow \text{soln. 1}$$

soln 2: $x_1 = 1$, $x_2 = 0$

$$\begin{bmatrix} 1 & 0 \\ -11 & -12 \\ \frac{31}{3} & 37/3 \end{bmatrix}$$

soln. 3: $x_1 = 0$, $x_2 = 0$

$$\begin{bmatrix} 0 & 0 \\ -9 & -12 \\ \frac{28}{3} & \frac{37}{3} \end{bmatrix}$$

soln 4: $x_1 = 1$, $x_2 = 1$

$$\begin{bmatrix} 1 & 1 \\ -11 & -14 \\ \frac{31}{3} & 40/3 \end{bmatrix}$$

2: 2. Prove that the following eq. has no solution:

$$\begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix} x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

Solving: $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$1x_1 + 3x_2 = 1 \rightarrow (1)$

$2\cancel{x_1} + 6\cancel{x_2} = 3 \rightarrow (2)$

$- \quad 2x_1 + 6x_2 = 2 \rightarrow (1) \times 2$

$0 \neq 1$ ∴ It is not equating

$0 \neq 1$ $\}$ $\therefore$ It is not equating
soln. is 0.

3. 3. Find a least-squares fit

$$b = x_0 + x_1 a^1 + x_2 a^2$$

for the data:

$$b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad a^1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad a^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

<u>L.S.F:</u>

$$(A^T \cdot A)^{-1} \cdot (A^T \cdot b) = x$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \\ 1 & 4 & 1 \end{bmatrix}_{4\times 3}$$

$$b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3\times 4}, \quad A^T \cdot A = \begin{bmatrix} 4 & 10 & 1 \\ 10 & 30 & 4 \\ 1 & 4 & 1 \end{bmatrix}_{3\times 3}$$

$$A^T \cdot b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad (A^T \cdot A)^{-1} = \begin{bmatrix} \frac{7}{3} & -1 & \frac{5}{3} \\ -1 & 1/2 & -1 \\ 5/3 & -1 & 10/3 \end{bmatrix}$$

$$(A^T \cdot A)^{-1} \cdot (A^T \cdot b) = \begin{bmatrix} 4/3 \\ -1/2 \\ 2/3 \end{bmatrix}$$

8i

4.

4. Find independent eigenvectors for

$$A = \begin{pmatrix} 1 & 2 \\ -3 & 1 \end{pmatrix}.$$

Can you express $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ as a linear combination of these eigenvectors of A?

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix}_{2\times 2}$$

$$A \cdot x = \lambda I x$$

multiply λ with $I_{2\times 2}$ $\Rightarrow$ $\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}_{2\times 2}$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ -3 & 1-\lambda \end{bmatrix} \Rightarrow |A - \lambda I| = (1-\lambda)^2 + 6$$

$$= 1 + \lambda^2 - 2\lambda + 6$$

$$= \lambda^2 - 2\lambda + 7 \rightarrow (1)$$

equate (1) to zero. $\Rightarrow \lambda^2 - 2\lambda + 7 = 0$ & solve for λ. , Solving it gives complex values:

$$x = 1 + \sqrt{6}\, i \quad , \quad 1 - \sqrt{6}\, i \; \} \text{ eigenvalues.}$$

eigenvectors: putting eigenvalues in $A - \lambda I$

$$= \begin{bmatrix} 1-1-\sqrt{6}\, i & 2 \\ -3 & 1-1-\sqrt{6}\, i \end{bmatrix} \rightarrow 1 + \sqrt{6}\, i$$

$$= \begin{bmatrix} -\sqrt{6}\, i & 2 \\ -3 & -\sqrt{6}\, i \end{bmatrix}$$ $\rightarrow$ performing row wice op. to reduce it.

$$\begin{bmatrix} -3 & -\sqrt{6}i \end{bmatrix}$$ to reduce it.

$$\left[\begin{array}{cc|c} -\sqrt{6}i & 2 & 0 \\ -3 & -\sqrt{6}i & 0 \end{array}\right]$$

$\frac{\sqrt{6}\times\sqrt{6}}{2}$ $\frac{\sqrt{6}}{2i}$

$R_2 \to \frac{\sqrt{6}}{2i} R_1 + R_2$

$$\left[\begin{array}{cc|c} -\sqrt{6}i & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$-\frac{\sqrt{6}}{i} + -\frac{\sqrt{6}i^2}{i}$

$\frac{-\cancel{\sqrt{6}} + \cancel{\sqrt{6}}}{i}$ 0

$\sqrt{6}i x_1 - 2x_2 = 0$

for $x_1 = 1$

$x_2 = \frac{\sqrt{6}i x_1}{2}$, $\vec{x} = \begin{bmatrix} 1x_1 \\ \frac{\sqrt{6}i x_1}{2} \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{\sqrt{6}i}{2} \end{bmatrix}$

now putting $1-\sqrt{6}i \Rightarrow \begin{bmatrix} \cancel{1}-\cancel{1}+\sqrt{6}i & 2 \\ -3 & \cancel{1}-\cancel{1}+\sqrt{6}i \end{bmatrix}$

$\Rightarrow \left[\begin{array}{cc|c} \sqrt{6}i & 2 & 0 \\ -3 & \sqrt{6}i & 0 \end{array}\right]$ $R_2 \to \sqrt{6}/2i\, R_1 + R_2$

$\Rightarrow \left[\begin{array}{cc|c} \sqrt{6}i & 2 & 0 \end{array}\right.$ $\Rightarrow \sqrt{6}i x_1 + 2x_2 = 0$

$x_2 = -\sqrt{6} i x_1$

$$\left[\begin{array}{cc|c} \sqrt{6}i & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad x_2 = \frac{-\sqrt{6}\, i\, x_1}{2}$$

$$\therefore \vec{x} = \begin{bmatrix} 1 x_1 \\ -\frac{\sqrt{6}i}{2} x_1 \end{bmatrix} \text{ using } x_1 = 1 \Rightarrow \begin{bmatrix} 1 \\ -\frac{\sqrt{6}i}{2} \end{bmatrix}$$

$$\text{eigenvectors} = \begin{bmatrix} 1 \\ \frac{\sqrt{6}i}{2} \end{bmatrix}, \begin{bmatrix} 1 \\ -\frac{\sqrt{6}i}{2} \end{bmatrix}$$

for linearCombination of eigenvectors:

$$\vec{x}_1 = \begin{bmatrix} 1 \\ \sqrt{6}i/2 \end{bmatrix} \cdot \vec{x}_2 = \begin{bmatrix} 1 \\ -\sqrt{6}i/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1, & 3 \end{bmatrix} \text{ which is } \neq \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$