ECE-GY 5253 Final

Fall 2022

Due: Dec. 19th. Monday, 7:30 pm, US Eastern Time.

1 Problem 1

Are the following statements true or false? If true, prove the statement. If false, give a counterexample.

1. Let $A \in \mathbb{R}^{3\times 3}$. If each eigenvalue of A is zero, then $A^2 = \mathbf{0}$. (Note: $\mathbf{0}$ denotes the zero matrix $\in \mathbb{R}^{3\times 3}$)
2. Let $A \in \mathbb{R}^{n\times n}$ be a real symmetric matrix, and $\lambda_1, \lambda_2, ..., \lambda_n$ the eigenvalues of A, then

$$\|A\|_2 = \max_i |\lambda_i|$$

   where $|\lambda_i|$ denotes the absolute value of λ_i for $i = 1, ..., n$.
3. Let $A \in \mathbb{R}^{n\times n}$ and a_{ij} denote the element in i^{th} row and j^{th} column. If $0 \leq a_{ij} < 1 \ \forall i \in \{1, ..., n\}, j \in \{1, ..., n\}$, and $\sum_{j=1}^{n} a_{ij} < 1$ for $i = 1, ..., n$, then $\rho(A) < 1$ where $\rho(A)$ is the spectral radius of A.

2 Problem 2

Let $A = \alpha\beta^T$ where $\alpha, \beta \in \mathbb{R}^n$ and $\beta^T\alpha \neq 0$.

1. When will the linear system $\dot{x} = Ax$ be asymptotically or marginally stable?
2. Compute e^{At}.