EL5253 HW 10 Kunal Ninawe kvn238 N12512995

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1) Consider $\dot{x} = Ax + g(x)$, $x(0) = x_o \in \mathbb{R}^n$, where

- A is a stable matrix.
- $\|g(x)\| / \|x\| \to 0$, as $\|x\| \to 0$.
- $\|x_o\|$ is sufficiently small.

Can you try to prove that the solution $x(t)$ of the nonlinear equation converges to 0, as $t \to \infty$?

→ Given: $\dot{x} = Ax + g(x)$, $x(0) = x_0 \in \mathbb{R}^n$

$$\frac{\|(g(x))\|}{\|x\|} \to 0 \quad \text{when} \quad \|x\| \to 0$$

we can say, $\|x_0\|$ is sufficiently small.

Apply laplace transformation on $\dot{x} = Ax + g(x)$

$$x(0) = x_0$$

$$S \cdot x(s) - S(x_0) = A\,x(s) + g(s)$$

$$s(x(s)) - x_0 = A\,x(s) + g(s)$$

$$s(x(s)) = A\,x(s) + g(s) + x_0$$

$$x(s) = \frac{g(s) + x_0}{S - A}$$

Laplace transform is as follows:

$$F(s) = \int_0^{\infty} f(t)\, e^{-st} \cdot t'$$

$\hookrightarrow$ Laplace transform.

$s \rightarrow$ complex number

$t \rightarrow$ real no. ≥ 0

$t' \rightarrow$ first derivative of $f(t)$

$$L'(F(s-a)) = e^{at} \boxed{f(t)} \rightarrow \text{Inverse transform of } F(s)$$

$\therefore$ By Laplace transform:

$$x(t) = x_0 \cdot e^{-At} + e^{-At}$$

where $t \rightarrow \infty$

$$x(t) \rightarrow 0$$

Hence Proved.