ECE-GY 5253 Midterm

Fall 2022

Due: Monday, October 31, 7:30 pm (New York Time)

1 Problem 1

Are the following statements true or false? If true, prove the statement. If false, give a counterexample.

1. Let $A \in \mathbb{R}^{n\times n}$. If $A^2 = A$, then eigenvalues of A are either 0 or 1.
2. Let $A \in \mathbb{R}^{m\times n}$. If $A^T A = \mathbf{0}$, then $A = \mathbf{0}$. (Note: $\mathbf{0} \in \mathbb{R}^{m\times n}$ denotes zero matrix with only zeros as its elements)
3. Let $A, B \in \mathbb{R}^{n\times n}$ be two real symmetric matrices. If $x^T Ax = x^T Bx$ for all $x \in \mathbb{R}^n$, then $A = B$.

2 Problem 2

If $A \in \mathbb{R}^{n\times n}$ has n distinct eigenvalues and commutes with a given matrix $B \in \mathbb{R}^{n\times n}$, show that

1. B is diagonalizable.
2. There exist coefficients $a_0, a_1, ..., a_{n-1}$ such that

$$B = a_{n-1}A^{n-1} + a_{n-2}A^{n-2} + ... + a_1 A + a_0 I$$

where I is the identity matrix.

3 Problem 3

For the following matrix $A = \begin{pmatrix} -2 & 2 & 1 \\ -7 & 4 & 2 \\ 5 & 0 & 0 \end{pmatrix}$, is the matrix A diagonalizable? If yes, explain. If not, what is its Jordan form?