A is a stable matrix, so $A^T P + PA = -I$ has a solution P which is positive.

Because $\|g(x)\|/\|x\| \to 0$ as $\|x\| \to 0$,

$\forall \varepsilon$. $\exists \delta$ such that if $\|x\| < \delta(\varepsilon)$ $\|g(x)\|/\|x\| \le \varepsilon$.

Thus $\|g(x)\| \le \varepsilon \|x\|$ when $\|x\| < \delta(\varepsilon)$

$$V = x^T P x$$

$$\begin{aligned}
\dot V &= \dot x^T P x + x^T P \dot x \\
&= (Ax + g(x))^T P x + x^T P (Ax + g(x)) \\
&= x^T (A^T P + PA) x + 2x^T P g(x) \\
&= -x^T x + 2x^T P g(x) \\
&\le -x^T x + 2\|Px\| \cdot \|g(x)\| \\
&\le -\|x\|^2 + 2\varepsilon \|P\| \|x\|^2
\end{aligned}$$

when $\varepsilon \le \frac{1}{4\|P\|}$ and $\|x\| < \delta(\varepsilon)$

$$\begin{aligned}
\dot V &\le -\frac{1}{2}\|x\|^2 \\
&\le -\frac{x^T P x}{2\lambda_{max}(P)} \\
&= -\frac{V}{2\lambda_{max}(P)} \\
&= -\mu V
\end{aligned}$$

Thus. $\dot V(t) \le e^{-\mu t} V(0) \Rightarrow \lim_{t \to \infty} V(t) = 0$

$\Rightarrow \lim_{t \to 0} x(t) = 0$ when $\|x_0\| < \delta(\varepsilon)$ and $\varepsilon = \frac{1}{4\|P\|}$