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For the matrix

$A=\begin{bmatrix}1&0&1\\0&2&0\\0&0&1\end{bmatrix}$ identify the spaces $P_g(\lambda)$ and the principal vectors of grade 2

Answer:

$\det(\lambda I-A)=\det\begin{bmatrix}\lambda-1&0&1\\0&\lambda-2&0\\0&0&\lambda-1\end{bmatrix}=0$ so $\lambda_1=1$, $\lambda_2=2$

according to the definition of $P_g(\lambda)$, we have

$(\lambda_i I-A)^g P=0$

① $\lambda=1$

$(I-A)^2P=0$

$\begin{bmatrix}0&0&1\\0&-1&0\\0&0&0\end{bmatrix}\begin{bmatrix}0&0&1\\0&-1&0\\0&0&0\end{bmatrix}P=0$

$\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}\begin{bmatrix}P_1\\P_2\\P_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

② $\lambda=2$

$(2I-A)P=0$

$\begin{bmatrix}\phi&0&1\\0&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}P_1\\P_2\\P_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$P_1(\lambda_2=2)=\{p^2 \mid p^2=\mathrm{col}(0,\beta,0)\}$

$P_2(\lambda_1=1)=\{p^1 \mid p^1=\mathrm{col}(\alpha,0,\gamma)\}$

Here, $p^1=\begin{bmatrix}\alpha\\0\\\gamma\end{bmatrix}$, $p^2=\begin{bmatrix}0\\\beta\\0\end{bmatrix}$

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2. $x = \begin{bmatrix} \sqrt{2} \\ -9 \\ 84 \end{bmatrix}$, $x = \begin{bmatrix} 0 \\ 9.3 \\ 0 \end{bmatrix}$ Express the following vectors as unique representation of principa vectors found in problem 1

Answer:
we know that $p^1 = \begin{bmatrix} 0 \\ \beta \\ 0 \end{bmatrix}$ $p^2 = \begin{bmatrix} \alpha \\ 0 \\ \gamma \end{bmatrix}$ from problem 1

$$x = \sqrt{2}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - 9\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 84\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 0 \\ 9.3 \\ 0 \end{bmatrix}$$

$$x = 0\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 9.3\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 0\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 9.3\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Where $p^2 = \begin{bmatrix} 0 \\ \beta \\ 0 \end{bmatrix}$, and $\beta = 1$

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(3) $A = \begin{bmatrix} \lambda & \lambda & \lambda \\ 0 & \lambda & \lambda \\ 0 & 0 & \lambda \end{bmatrix} \lambda \neq 0$ transform the following matrix into a Jordan form.

$$\lambda I - A = \begin{vmatrix} 0 & -\lambda & -\lambda \\ 0 & 0 & -\lambda \\ 0 & 0 & 0 \end{vmatrix}$$

$\text{null}(P_1) = \langle 1, 0, 0 \rangle$

$$(\lambda I - A)^2 = \begin{pmatrix} 0 & -\lambda & -\lambda \\ 0 & 0 & -\lambda \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -\lambda & -\lambda \\ 0 & 0 & -\lambda \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \lambda^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\text{null}(P_2) = \langle 1, 0, 0 \rangle ; \langle 0, 1, 0 \rangle$

$$(\lambda I - A)^3 = \begin{pmatrix} 0 & 0 & \lambda^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -\lambda & -\lambda \\ 0 & 0 & -\lambda \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\text{null}(P_3) = \langle 1, 0, 0 \rangle ; \langle 0, 1, 0 \rangle ; \langle 0, 0, 1 \rangle$

$$V_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad V^2 = (A - \lambda I) V^1 = \begin{pmatrix} 0 & \lambda & \lambda \\ 0 & 0 & \lambda \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{bmatrix} \lambda \\ \lambda \\ 0 \end{bmatrix}$$

$$V^3 = (A - \lambda I) V^2 = \begin{pmatrix} 0 & \lambda & \lambda \\ 0 & 0 & \lambda \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda \\ \lambda \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda^2 \\ 0 \\ 0 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & \lambda & \lambda^2 \\ 0 & \lambda & 0 \\ 1 & 0 & 0 \end{pmatrix} ; \quad P^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & \frac{1}{\lambda} & 0 \\ \frac{1}{\lambda^2} & -\frac{1}{\lambda^2} & 0 \end{pmatrix}$$

$$P^{-1} A P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & \frac{1}{\lambda} & 0 \\ \frac{1}{\lambda^2} & -\frac{1}{\lambda^2} & 0 \end{pmatrix} \begin{pmatrix} \lambda & \lambda & \lambda \\ 0 & \lambda & \lambda \\ 0 & 0 & \lambda \end{pmatrix} \begin{pmatrix} 0 & \lambda & \lambda^2 \\ 0 & \lambda & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \lambda & 0 & 0 \\ 1 & \lambda & 0 \\ 0 & 1 & \lambda \end{pmatrix}$$