

1. Demonstrate what happens when we insert the keys 10, 22, 35, 12, 1, 21, 6, 15, 36, 33 into a hash table with collisions resolved by chaining.

h(k)	keys
0	36
1	1->10
2	
3	21->12
4	22
5	
6	33->15->6
7	
8	35

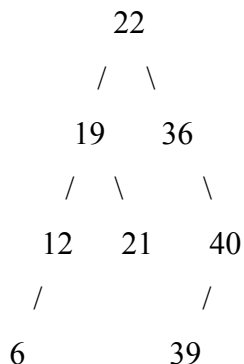
2. Exercise 11.2-1 in CLRS Textbook.

Under the assumption of simple uniform hashing, we will use linearity of expectation to compute this. Suppose that all the keys are totally ordered  $\{k_1, \dots, k_n\}$ . Let  $X_i$  be the number of  $1 > k_i$  so that  $h(1) = h(k_i)$ . Note, that this is the same thing as  $\sum_{j>i} \Pr(h(k_j) = h(k_i)) = \sum_{j>i} \frac{1}{m} = \frac{(n-i)}{m}$ . Then,

by linearity of expectation, the number of collisions is the sum of the number of collisions for each possible smallest element in the collision.

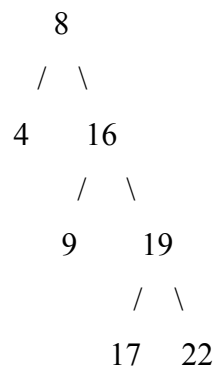
The expected number of collisions is  $\sum_{i=1}^n \frac{n-i}{m} = \frac{n^2 - \frac{n(n+1)}{2}}{m} = \frac{n^2 - n}{2m}$ .

3. Given a binary search tree in pre-order as  $\{22, 19, 12, 6, 21, 36, 40, 39\}$ , draw this BST and determine if this BST is the same as one described in post-order as  $\{6, 12, 21, 19, 36, 40, 39, 22\}$ .



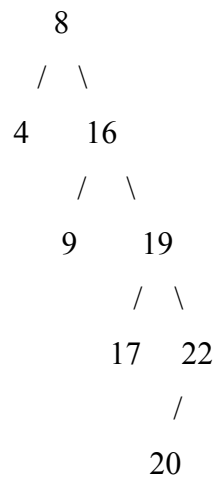
The BST described in post-order is  $\{6, 12, 21, 19, 39, 40, 36, 22\}$ , so they are not the same one.

4. For the following binary search tree, show the result of following operations (Please follow the algorithm from the lecture/textbook):

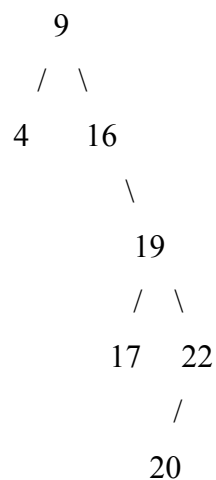


- Insert key 20;
- Delete 8 from the result of a);
- Delete 19 from the result of b);
- Delete 16 from the result of c).

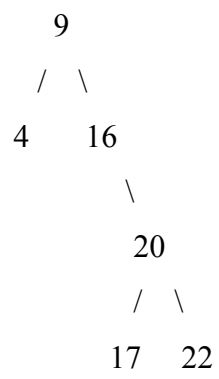
a)



b)



c)



d)

