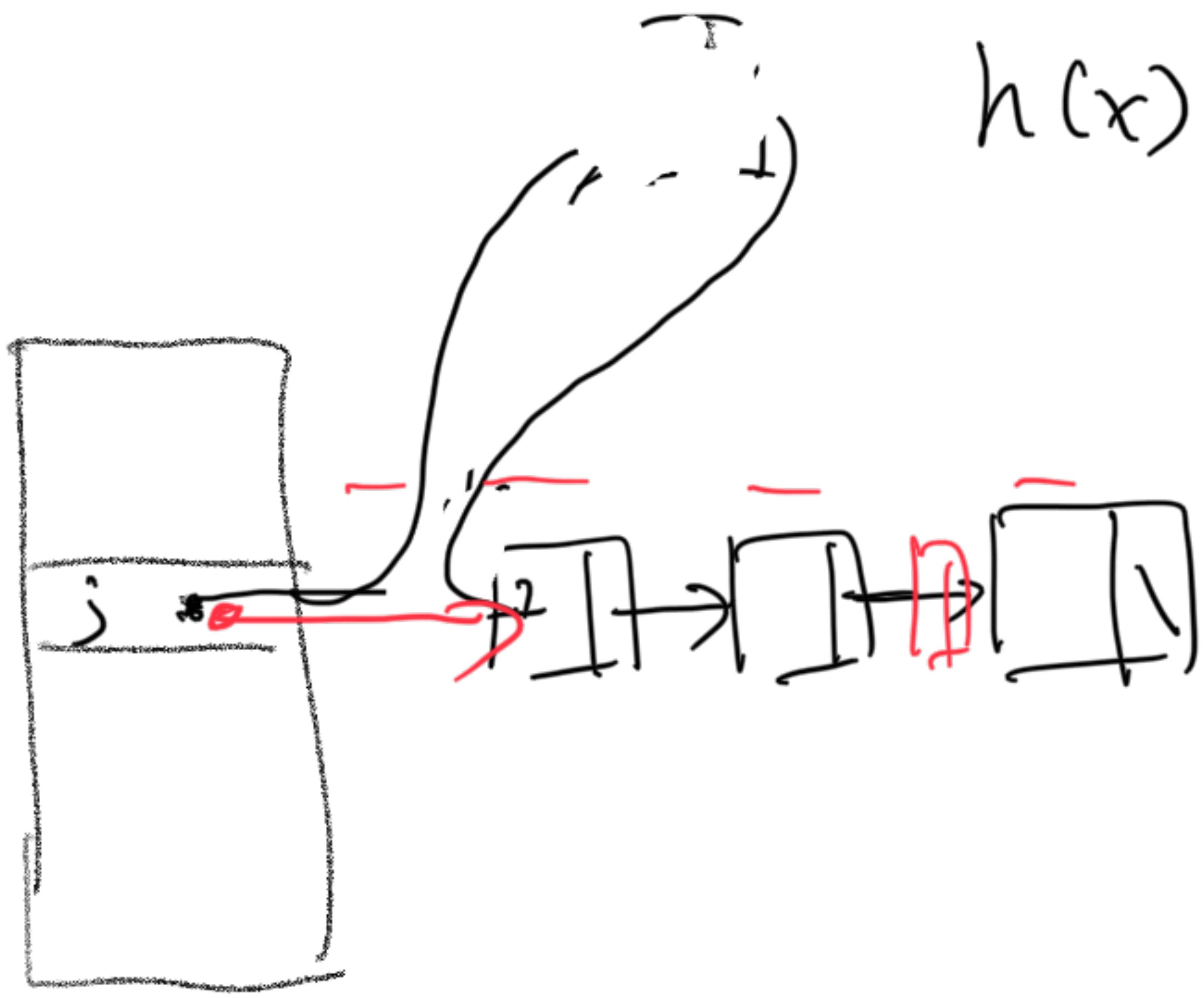
# Spring 2022, Lecture 6

$h(x) = j$




$$Pr(h(x) = i) = \frac{1}{m}$$

$x, Y,$ $Pr(h(x) = h(Y)) = ?$

$$Pr(h(x) = i = h(Y)) = Pr(h(x) = i) \times Pr(h(Y) = i)$$

$$= \frac{1}{m^2}$$

$$Pr(h(x) = h(Y)) = \sum^{m-1} Pr(h(x) = i = h(Y))$$

$$\Pr(h(x)=h(y)) = \sum_{i=0} \Pr(h(x)=h(y)=i)$$

$$= m \times \frac{1}{m^2} = \frac{1}{m}$$

$n_j$ is the number of keys hashed to position $j$

$n_j$ follows binomial distribution

$$\Pr(n_j = k) = \binom{n}{k} \left(\frac{1}{m}\right)^k \left(1 - \frac{1}{m}\right)^{n-k}$$

$$E(n_j) = E\left(\sum_{i=1}^{n} \mathbb{1}\left(\text{key } i \text{ hashed to position } j\right)\right)$$

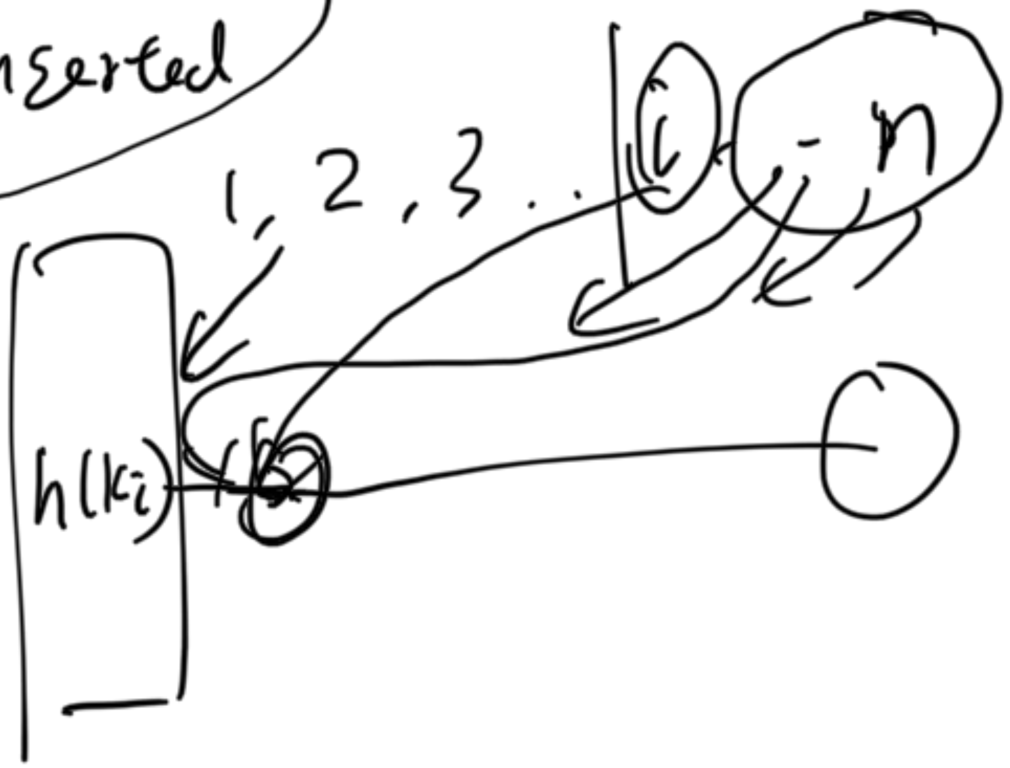
$$= \sum_{i=1}^{n} E\left[\mathbb{1}(\ )\right]$$

$$= \sum_{i=1}^{n} P(\text{key } i \text{ hashed to } j)$$

$$= \frac{n}{m}$$

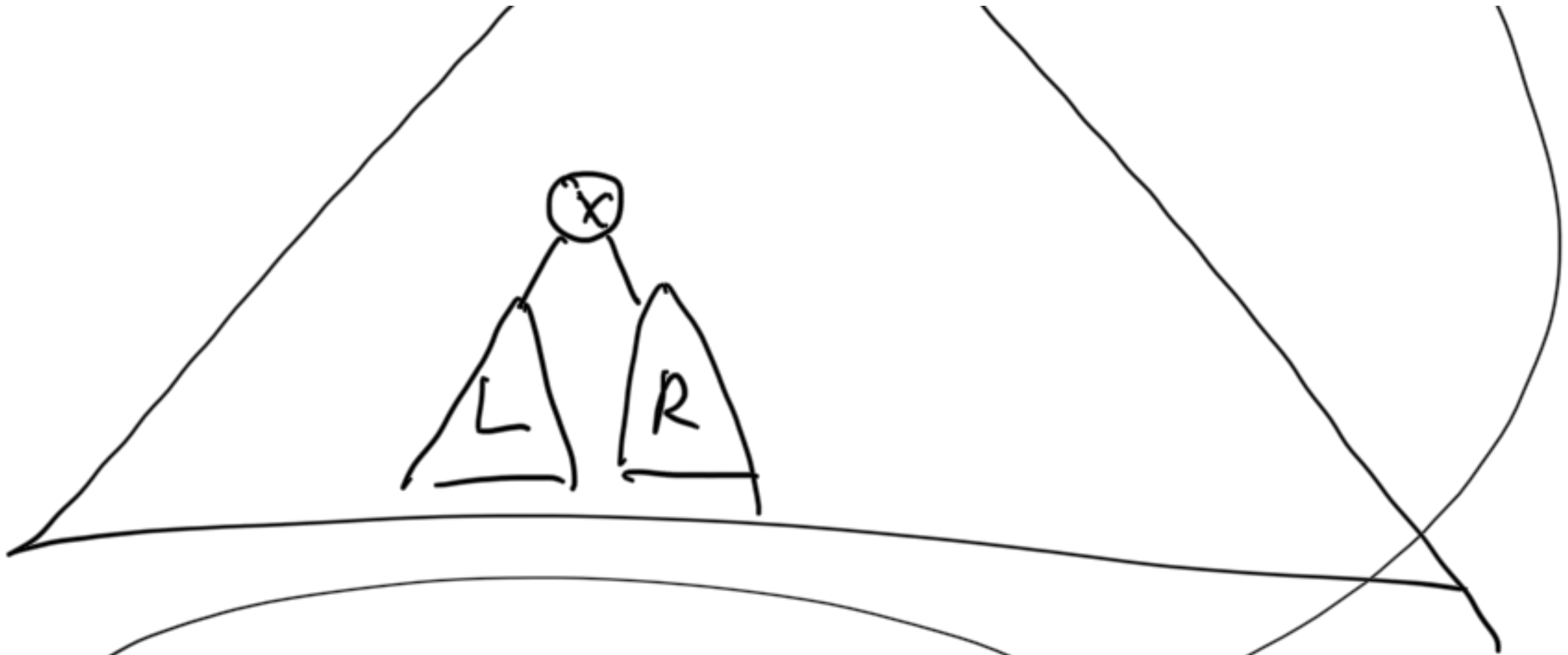
position of the i-th inserted item

= # items inserted after i that are hashed to $h(K_i)$




n - i items

$(n-i) \times \frac{1}{m}$




$\text{key}(L) \le \text{key}(x) \le \text{key}(R)$


$\text{key}(L) \le \text{key}(\text{root}) \le \text{key}(R)$