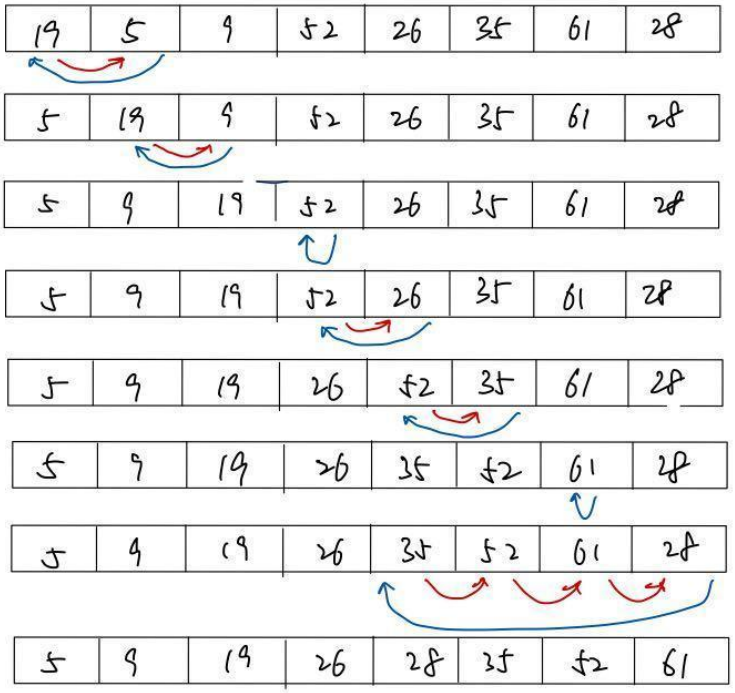
Spring 2022 Homework 3 Solution

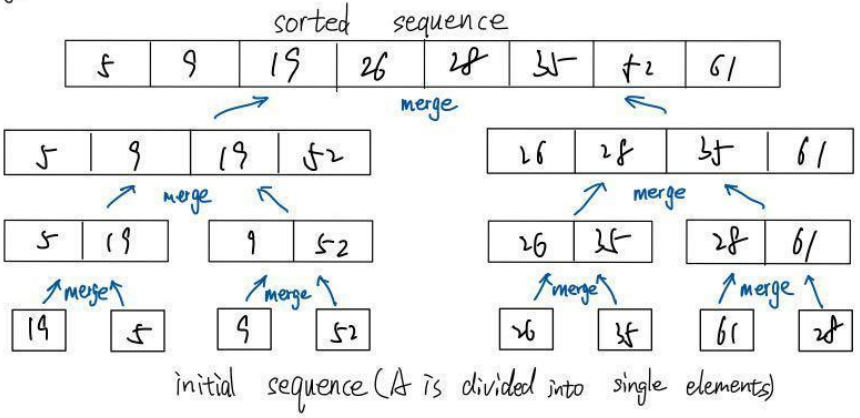
1. a. T
   b. T
   c. T
   d. F
   e. F

2.

a. insertion sort:



b. merge sort:



3.

a. SELECTION-SORT(A)

```
n = A.length
for j = 1 to n - 1
        smallest = j
        for i = j + 1 to n
                if A[i] < A[smallest]
                        smallest = i
        exchange A[j] with A[smallest]
```

b. Loop invariant:

At the start of each iteration of the outer **for** loop, the subarray $A[1..j-1]$ consists of the smallest j - 1 elements in array A[1...n], and this subarray is in sorted order.

c. After the first n - 1 elements, the subarray $A[1...n-1]$ contains the smallest n - 1 elements, sorted, and therefore element $A[n]$ must be the largest element.

d. The running time of the algorithm is $\Theta(n^2)$ for all cases.

4.

```
PRINT(origin, destination):
print("Move the top disk from rod", origin, "to rod", destination)

MOVE(n, start, end):
mid_rod = 6 - start - end                    # mid_rod is the remaining rod
if n == 1:
   PRINT(start, end)
else:
   MOVE(n - 1, start, mid_rod)
   MOVE(1, start, end)
   MOVE(n - 1, mid_rod, end)
```

a. output:

Move the top disk from rod 1 to rod 2

Move the top disk from rod 1 to rod 3

Move the top disk from rod 2 to rod 3

Move the top disk from rod 1 to rod 2

Move the top disk from rod 3 to rod 1

Move the top disk from rod 3 to rod 2

Move the top disk from rod 1 to rod 2

Move the top disk from rod 1 to rod 3

Move the top disk from rod 2 to rod 3

Move the top disk from rod 2 to rod 1

Move the top disk from rod 3 to rod 1

Move the top disk from rod 2 to rod 3

Move the top disk from rod 1 to rod 2

Move the top disk from rod 1 to rod 3

Move the top disk from rod 2 to rod 3

b. 31; $2^n - 1$