EL9343 Homework 1

(Due February 4nd, 2022)

No late assignments accepted

All problem/exercise numbers are for the third edition of CLRS text book

---

1.Prove the following properties of asymptotic notation:

(a) $n = \omega(\sqrt{n})$;

(b) If $f(n) = \Omega(g(n))$, and $h(n) = \Theta(g(n))$, then $f(n) = \Omega(h(n))$.

(c) $f(n) = O(g(n))$ *if and only if* $g(n) = \Omega(f(n))$ (*Transpose Symmetry* property)

(a)

$n = \omega(\sqrt{n})$

For any constant $c > 0$, There exist $n_0 = c^2 + 1 \geq 0$, such that $n > c\sqrt{n}$ for all $n \geq n_0$

Therefore, $n = \omega(\sqrt{n})$

(b)

If $f(n) = \Omega(g(n))$, and $h(n) = \Theta(g(n))$, then $f(n) = \Omega(h(n))$.

$f(n) = \Omega(g(n)) \Leftrightarrow$ there exist constants $c_1 > 0$ and $n_1 \geq 0$ such that $f(n) \geq c_1 g(n)$ for all $n \geq n_1$

$h(n) = \Theta(g(n)) \Rightarrow$ there exist constants $c_2 > 0, n_2 > 0$ such that $h(n) \leq c_2 g(n) \Leftrightarrow g(n) \geq \frac{1}{c_2} h(n)$ for all $n \geq n_2$

Therefore, there exist $c_3 = \frac{c_1}{c_2} > 0$, $n_3 = max(n_1, n_2) > 0$, such that for all $n \geq n_3$, $f(n) \geq c_1 g(n) \geq c_1(\frac{1}{c_2} h(n)) = c_3 h(n)$

Therefore, $f(n) = \Omega(h(n))$.

(c)

f(n) = Θ(g(n)) ==> g(n) = Θ(f(n))

Suppose that f(n) = Θ(g(n)). We got f(n) = O(g(n)) and f(n) = Ω (g(n)).

Then, there exist A, B > 0 such that Ag(n) ≤ f(n) ≤ Bg(n) for sufficiently large n.

Since f(n) ≤ Bg(n) ==> (1/B) f(n) ≤ g(n) and Ag(n) ≤ f(n) ==> g(n) ≤ (1/A) f(n),

we have (1/B) g(n) ≤ g(n) ≤ (1/A) g(n) for sufficiently large n.

Since 1/A, 1/B > 0, We got g(n) = O(f(n)) and g(n) = Ω (f(n)). we conclude that g(n) = Θ(f(n)).

g(n) = Θ(f(n)) ==> f(n) = Θ(g(n))

Can proof using the same way we proof f(n) = Θ(g(n)) ==> g(n) = Θ(f(n)).

We can conclude that f(n) = Θ (g(n)) if and only if g(n) = Θ (f(n)).

2. Problem 3-2 in CLRS Text book.

Problem 3-2

A	B	O	o	Ω	ω	Θ
$\lg^k n$	n^ϵ	yes	yes	no	no	no
n^k	c^n	yes	yes	no	no	no
$\sqrt{n}$	$n^{\sin n}$	no	no	no	no	no
2^n	$2^{n/2}$	no	no	yes	yes	no
$n^{\log c}$	$c^{\log n}$	yes	no	yes	no	yes
$\log(n!)$	$\log(n^n)$	yes	no	yes	no	yes

3. You have 5 algorithms, A1 took $O(n)$ steps, A2 took $\Theta(n\ log\ \ n\ )$ steps, and A3 took $\Omega(n)$ steps, A4 took $O(n^3)$ steps, A5 took $o(n^2)$ steps. You had been given the exact running time of each algorithm, but unfortunately you lost the record. In your messy desk you found the following formulas:

(a) $4(5^{2\log_5 n}) + 6n + 9527$

(b) $\sqrt[3]{3n!}$

(c) $(\frac{4^{\log_{15} n}}{6})^2 + 4n + 17$

(d) $3nlog_2n\ + (log_2n)^2$

(e) $log_2log_2n\ \ + 6$

(f) 2^{3log_2n}

(g) $(log_2n)^3 + log_2log_2n$

For each algorithm write down all the possible formulas that could be associated with it.

Solution

(a) $= \Theta\left(5^{2\log_5 n} + n\right) = \Theta(n^2 + n) = \Theta(n^2)$

(b) $by\ stirling's\ approximation\ n! = \sqrt{2\pi n}(\frac{n}{e})^n,\ so\ \sqrt[3]{3n!} = \sqrt[6]{2\pi n}(\frac{n}{e})^{\frac{n}{3}} = \Omega(n^n)$

(c) $= \Theta\left(n^{\log_{15} 16} + n\right) = \ \Theta\left(n^{\log_{15} 16}\right) \approx\ \Theta\left(n^{1.17}\right)$

(d) $= \Theta(n\ log\ \ n\ )$

(e) $= \Theta(\log log\ \ n\ )$

(f) $=\ \Theta(n^3)$

(g) $= \Theta(\log^3{}_2\, n)$

A1: e g

A2: d

A3: a b c d f

A4: a c d e f g

A5: c d e g

4. For the following algorithm: Show what is printed by the following algorithm when called with MAXIMUM(A, 1, 5) where A = [9, 12, 15, 5, 2]? Where the function PRINT simple prints its arguments in some appropriate manner.

```
MAXIMUM(A, l, r)
if (r − l == 0)
    return A[r]

lmax = MAXIMUM(A, l, ⌊(l + r)/2⌋)
rmax = MAXIMUM(A, ⌊(l + r)/2⌋ + 1, r)
PRINT(rmax, lmax)
if rmax < lmax
    return lmax
else
    return rmax
```

Solution:

12, 9

15, 12

2, 5

5, 15