ECE-GY 6303, Probability & Stochastic Processes

Homework # 10

Prof. Pillai

Fall 2021

Problem 1

Given system $H(\omega)$ with input $X(t)$ and output $Y(t)$, show that

a.) if $X(t)$ is W.S.S. and $R_{XX}(\tau) = e^{j\alpha\tau}$, then

$$R_{YX}(\tau) = e^{j\alpha\tau}H(\alpha), \quad \text{and} \quad R_{YY}(\tau) = e^{j\alpha\tau}|H(\alpha)|^2.$$

b.) if $R_{XX}(t_1, t_2) = e^{j(\alpha t_1 - \beta t_2)}$, then

$$R_{YX}(t_1, t_2) = e^{j(\alpha t_1 - \beta t_2)}H(\alpha), \quad \text{and} \quad R_{YY}(t_1, t_2) = e^{j(\alpha t_1 - \beta t_2)}H(\alpha)H^*(\beta).$$

Problem 2

Show that

a.) if $R_{XX}[m_1, m_2] = q[m_1]\delta[m_1 - m_2]$ and $S = \sum_{n=0}^{N} a_n X[n]$, then $E[S^2] = \sum_{n=0}^{N} a_n^2 q[n]$.

b.) if $R_{XX}(t_1, t_2) = q(t_1)\delta(t_1 - t_2)$ and $S = \int_0^T a(t)X(t)dt$, then $E[S^2] = \int_0^T a^2(t)q(t)dt$.

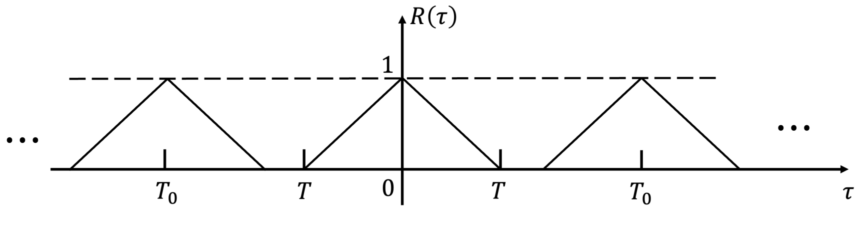
Problem 3

Find the power spectrum of the following autocorrelation functions

a.) $e^{-\alpha|\tau|}\cos(\omega_0\tau)$;

b.) $e^{-\alpha\tau^2}\cos(\omega_0\tau)$;

c.) $R(\tau)$ is a periodic function with period $T_0 > 2T$ as shown below



Problem 4

Let $X(t)$ be W.S.S. with auto-correlation function $R_{XX}(\tau)$ and power spectrum $S_{XX}(\omega)$. Let $Y(t) = X(t+a) - X(t-a)$.

a.) Find the auto-correlation function of $Y(t)$.

b.) Find the power spectral density of $Y(t)$.

Problem 5

Let

$$Y(t) = e^{j(\pi X(t)+\theta)},$$

where $X(t)$ is a Poisson process with parameter λt. Here, $\theta \sim U(0, 2\pi)$ and independent of $X(t)$.

a.) Is $Y(t)$ a W.S.S.?

b.) If so, find its power spectral density.

Problem 6

$X(t)$ is a wide sense stationary zero mean Gaussian process with auto-correlation function $R_{XX}(\tau)$ and power spectrum $S_{XX}(\omega)$. Consider the process

$$Z(t) = X(t)\cos(\omega_0 t + \theta) + X^2(t-T)\sin(\omega_0 t + \theta)$$

and $\theta \sim U(0, 2\pi)$ independent of $X(t)$. Find the power spectrum of $Z(t)$ and express it in terms of $S_{XX}(\omega)$ and ω_0.