HW # 12

Probability and Stochastic Processes

ECE-GY 6303 (Prof. Pillai)

Final Exam (Dec. 18, 2019)

1. Two data sets are given as follows:

$$\text{(i)}\quad x_1 = s + n, \quad x_2 = n + w, \qquad\qquad \text{(ii)}\quad x_1 = s + n, \quad x_2 = s + w.$$

Here, s, n, and w are zero mean independent random variables with variances σ_s^2, σ_n^2, and σ_w^2 respectively.

a. Find the minimum mean square error linear estimator for s using each of the data sets above.

b. To estimate s, which data set is better in terms of the mean square error? [25]

2. Let

$$Y(t) = e^{j\left(X(t)+\theta\right)}(\backslash\mathtt{pi}),$$

where $X(t)$ is a Poisson process with autocorrelation function

$$R_{XX}(t_1, t_2) = t_1 t_2 + \min(t_1, t_2)\ ,$$

and $\theta \sim U(0, 2\pi)$ is independent of $X(t)$.

a. Is $X(t)$ W.S.S.?

b. If so, find its power spectral density. [25]

3. $X(t)$ is a wide sense stationary (W.S.S.) zero mean stochastic process with autocorrelation function $R_{XX}(\tau)$ and power spectrum $S_{XX}(\omega)$. Consider the process

$$Z(t) = X(t+T)\cos(\omega_0 t + \theta) + X(t-T)\sin(\omega_0 t + \theta),$$

where $\theta \sim U(0, 2\pi)$ and independent of $X(t)$.

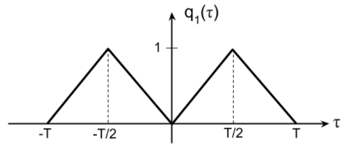
a. Find the power spectrum of $Z(t)$ in terms of $S_{XX}(\omega)$, T, and ω_0.

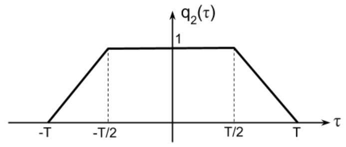
b. Find the power spectrum of the following autocorrelation function and sketch it:

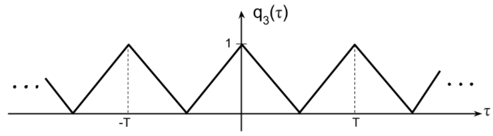
$$R_{XX}(\tau) = e^{-\alpha|\tau|}\cos(\omega_1 \tau) + e^{-\beta\tau^2}\cos(\omega_2 \tau), \quad \omega_2 > \omega_1.$$

[25]

4. Which of the following represent autocorrelation functions? Give simple reasons to justify your answer. [25]

i. 

ii. 

iii. 

iv. $q_4(t_1, t_2) = t_1^2 t_2^2$

v. $q_5(t_1, t_2) = R_1(t_1, t_2) R_2(t_1, t_2)$, where $R_1(t_1, t_2)$ and $R_2(t_1, t_2)$ are two valid autocorrelation functions.