ECE-GY 6303, Probability & Stochastic Processes

Homework # 8

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Problem 1

a.) Find the correlation function of

$$X(t) = \sum_{k=1}^{n} a_k \cos(\omega_k t + \phi_k),$$

where a_k and ω_k are constants and ϕ_k are independent random variables that are uniformly distributed in $(0, 2\pi)$.

b.) Show that $R(t_1, t_2) = \min(t_1, t_2)$ is an auto-correlation function.

Problem 2

$X(t) = n(0, t)$ is a Poisson process with parameter λt, i.e., k arrivals in $(0, t)$ is governed by

$$P[X(t) = k] = e^{-\lambda t}\frac{(\lambda t)^k}{k!}.$$

a.) Show that "the duration of the first arrival" is an exponential random variable;

b.) Show that "the duration of the n-th arrival" is a Gamma distributed random variable. Find its parameters.

Problem 3

Let

$$X(t) \;=\; e^{j(\omega_0 t + W(t))},$$

where $W(t)$ is a Wiener process and ω_0 is a constant. Calculate the mean and auocorrelation function of $X(t)$.

Problem 4

Show that

a.) all strict sense stationary processe are wide sense stationary;

b.) wide sense stationarity implies strict sense stationarity for Gaussian process.

Problem 5

Let $X(t)$ be a zero-mean real stochastic process with

$$E[(X(t) - X(s))^2] = |t - s|, \quad \text{and} \quad X(0) = 0.$$

a.) Show that

$$R_{XX}(t, s) = E[X(t)X(s)] = \frac{1}{2}(|t| + |s| - |t - s|) = \min(t, s).$$

b.) Define

$$Z_n(t) = n\left[X\left(t + \frac{1}{n}\right) - X(t)\right], \quad n = 1, 2, ...$$

Given n, find the auto-correlation function of $Z_n(t)$ and plot it. Is $Z_n(t)$ W.S.S.?

c.) What happens to $Z_n(t)$ when $n \to \infty$? (Does it appear to have the characteristics of any known stochastic process?)