ECE-GY 6303, Probability & Stochastic Processes

Homework # 11

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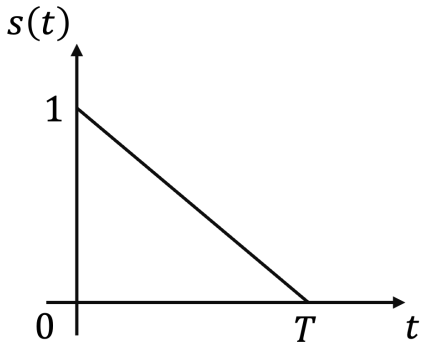
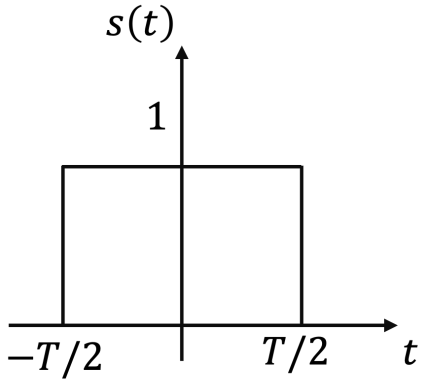
Problem 1

A signal $s(t)$ is contaminated by white noise $n(t)$ with spectral height σ^2.

a.) Find the best filter $h(t)$ that maximizes the output SNR at $t = T$ and sketch it for the following case:



b.) Find the best filter $h(t)$ that maximizes the output SNR at $t = T$ and sketch it for the following case:



c.) Find the maximum value of output SNR for those two cases.

Problem 2

Consider the dataset:

$$Y_1 = s + n, \quad Y_2 = sn,$$

Here s and n are independent with $E[n] = \mu \neq 0$, $E[s] = 0$, $\text{Var}(s) = \sigma_s^2$ and $\text{Var}(n) = \sigma_n^2$.

a.) Find the best linear estimator $\hat{s}$ for s based on Y_1 and Y_2.

b.) Find the minimum mean square error for $\hat{s}$.

Problem 3

Let $X_n = X(nT)$ represent a discrete-time W.S.S real stochastic process with auto-correlation function $r_k = E[X_{n+k}X_n]$. Suppose X_0 is known, and X_1 and X_2 are unknown.

a.) Determine the best estimator for the unknown X_1. Find the associated minimum mean square error σ_1^2.

b.) Determine the best estimator for the unknown X_2. Find the associated minimum mean square error σ_2^2.

c.) Show that $\sigma_1^2 \geq \sigma_2^2$ in some cases and $\sigma_1^2 < \sigma_2^2$ in other cases. Show an example where σ_2^2 is smaller than σ_1^2.

Problem 4

Consider different versions of data

$$\begin{cases} Y_1 = X + n, \\ Y_2 = X + w, \end{cases} \quad \text{versus} \quad \begin{cases} Y_1 = X + n, \\ Y_2 = n + w. \end{cases}$$

Here, X, n, w are zero mean uncorrelated random variables with variances σ_X^2 , σ_n^2 and σ_w^2 respectively. X is the desired unknown.

a.) Find the best estimator for X using Y_1 and Y_2 in the both versions of data.

b.) Find the mean square error in the both cases above. Which set of data is preferable for estimating X?

Problem 5

Let X, Y, Z be zero mean correlated random variables with common correlation coefficient equal to -1/2 and all of the variances are equal to 1.

a.) Find the best linear estimate for Z in terms of X and Y .

b.) Find the best linear estimate for X in terms of Y and Z.

c.) What are the minimum mean square estimation errors in the above cases?

Problem 6

Given the datasets

$$Y_1 = s + n, \quad \text{and} \quad Y_2 = s + w,$$

where s, n, w are zero mean independent random variables with variances σ_s^2, σ_n^2 and σ_w^2.

a.) Find the best estimator $\hat{s}_1$ for s using Y_1 and find the corresponding minimum mean square error σ_1^2.

b.) Find the best estimator $\hat{s}_2$ for s using Y_1 and Y_2, and find the corresponding minimum mean square σ_2^2.

c.) Which one is smaller, σ_1^2 or σ_2^2?