Problem 1

(a) Let A = "a ball drawn randomly from Box#2 is green", B = "the ball that transferred from Box#1 to Box#2 is green".

$$
\begin{aligned}
P(A) &= P(A|B)P(B) + P(A|\bar{B})P(\bar{B} \\
&= \frac{4}{10} \times \frac{5}{10} + \frac{3}{10} \times \frac{5}{10} \\
&= 0.35
\end{aligned}
$$

(b) Let C = "the ball drawn from Box#1 is not white", D = "the ball that transferred from Box#1 to Box#2 is white"

$$
\begin{aligned}
P(C) &= P(C|D)P(D) + P(C|\bar{D})P(\bar{D}) \\
&= \frac{8}{9} \times \frac{2}{10} + \frac{7}{9} \times \frac{8}{10} \\
&= 0.8
\end{aligned}
$$

Problem 2

(a) Let R = "a lot is rejected"

$$
\begin{aligned}
P(\bar{R}) &= \sum_{i=0}^{1} \tbinom{10}{i} (10^{-3})^i (1 - 10^{-3})^{10-i} = 0.99996 \\
P(R) &= 1 - P(\bar{R}) = 0.00004
\end{aligned}
$$

So the probability that a lot is rejected is 0.00004

(b) Let A_i = "the ith lot is retained", we have $P(A_i) = 0.99996$. So $P(A_1A_2A_3A_4A_5A_6) = 0.99996^6 = 0.99976$
The probability that at least 60 microprocessors are retained is 0.99976

Problem 3

(a) Let S_k^n = "obtaining k 'head' in n trials", $P(S_k^n) = \tbinom{n}{k} p^k (1-p)^{n-k}$

$$
\begin{aligned}
P(S_{k+1}^n) - P(S_k^n) &= p^k (1-p)^{n-k-1} (\tbinom{n}{k+1} p - \tbinom{n}{k} (1-p)) \\
&= \frac{p^k (1-p)^{n-k-1} n!}{k!(n-k-1)!} (\frac{p}{k+1} - \frac{1-p}{n-k})
\end{aligned} \tag{1}
$$

Let $\frac{p}{k+1} - \frac{1-p}{n-k} \geq 0$, we get $k \leq (n+1)p - 1$; $\frac{p}{k+1} - \frac{1-p}{n-k} \leq 0$, we get $k \geq (n+1)p - 1$.
So when $k \leq (n+1)p - 1$, $S_k^n \leq S_(k+1)^n$; when $k \geq (n+1)p - 1$, $S_k^n \geq S_(k+1)^n$.
So we now have $(n+1)p - 1 \leq k_0 \leq (n+1)p$, which means $\frac{n+1}{n}p - \frac{1}{n} \leq \frac{k_0}{n} \leq \frac{n+1}{n}p$.
When $n \to \infty$, $\frac{n+1}{n} \to 1$, $\frac{1}{n} \to 0$, So $\frac{k_0}{n} \to p$