EL-6303: Probability Theory and Stochastic Processes

Midterm Exam. Fall 2014. (Answer all four problems.)

Use the first answer book for Problems 1 and 2.

1. Consider the following functions of two random variables X and Y:
   (i) $2X$; (ii) Y^2; (iii) $X+Y$; (iv) $X-Y$; (v) $\min(X,Y)$; (vi) $\max(X,Y)$.

   a). Suppose X and Y above are independent Poisson random variables with common parameter λ. Which among the above functions represent a Poisson random variable? Determine its parameter. (To show that something is *not* Poisson, for example, you can simply check its moment properties etc.)

   b). Suppose X and Y above are jointly Gaussian random variables. Which among the above functions represent a Gaussian random variable? Can you identify two random variables from the above set [(i)-(vi)] that are jointly Gaussian? Justify your answers.

(25)

2. X and Y are independent exponential random variables with common parameter $\lambda = 1$. Thus

$$f_{X,Y}(x,y) = \begin{cases} e^{-(x+y)}, & x > 0,\ y > 0 \\ 0, & \text{Otherwise} \end{cases}.$$

Let $Z = X + \max(X, Y)$. Find the p.d.f of Z.

(25)

See next sheet for Problems 3 and 4.

EL6303: Use the second answer book for Problems 3 and 4.

3. X and Y are zero mean jointly Gaussian random variables with equal variance σ^2 and correlation coefficient ρ. Thus the joint p.d.f. of X and Y is given by

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} e^{-\frac{1}{2\sigma^2(1-\rho^2)}(x^2-2\rho xy+y^2)}, \quad -\infty < x,\ y < \infty, \quad |\rho| < 1.$$

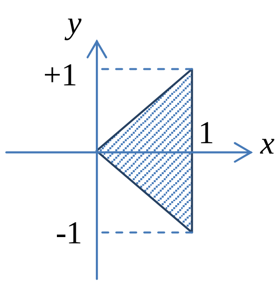
Define $\quad \theta = \tan^{-1}\left(\frac{Y}{X}\right).$

a) Find the p.d.f $f_\theta(\theta)$ of θ and plot it. (Make sure the area under $f_\theta(\theta)$ is unity).
Hint: Set up an auxiliary random variable such as $R = \sqrt{X^2+Y^2}$ or $R = X$ and proceed.

b) Discuss the special case when $\rho = 0$.

(25)

4. $$f_{X,Y}(x,y) = \begin{cases} \frac{3}{2}x, & \text{shaded area} \\ 0, & \text{otherwise} \end{cases}.$$



a) Find the conditional p.d.f. $f_{X|Y}(x\,|\,y)$ of X given Y and plot it.
b) Find $E\{X\,|\,Y=y\}$
c) Find the correlation coefficient ρ_{xy} between X and Y.

or

Write MATLAB code to generate n-dimensional vectors X and Y, such that for every i, $[x(i), y(i)]$ are distributed with the above distribution. Hint: Generate Y from $f_Y(y)$ and then generate X from $f_{X|Y}(x\,|\,y)$.

You may do either part in (c) above. (25)

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