ECE-GY 6303, Probability & Stochastic Processes

Homework # 6

Prof. Pillai

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Problem 1

Let

$$f_{XY}(x,y) = \begin{cases} 2e^{-(x+y)} & 0 < x < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

Define

$$Z = X + Y \qquad W = Y/X$$

a.) Find $f_{ZW}(z,w)$.

b.) Are Z and W independent random variables? Prove your answer.

Problem 2

Given the joint density function

$$f_{XY}(x,y) = \begin{cases} 2e^{-(2x-y)} & 0 < y < x < \infty, \\ 0 & \text{otherwise,} \end{cases}$$

and the two functions

$$Z = 2X - Y, \qquad W = Y/X.$$

a.) Find $f_{ZW}(z,w)$.

b.) Are Z and W independent random variables? Prove your answer.

Problem 3

The joint p.d.f of X and Y is given by

$$f_{XY}(x,y) = \begin{cases} 2xye^{-(x+y)} & 0 < y < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

Define

$$Z = X + Y \quad W = X/Y$$

a.) Find $f_{ZW}(z, w)$.

b.) Are Z and W independent random variables?

c.) Are Z and W uncorrelated random variables?

Problem 4

The joint p.d.f of X and Y is given by

$$f_{XY}(x, y) = \begin{cases} \frac{3}{4}(x+y)^2 & 0 < x < 1, -1 < y < 1, \\ 0 & \text{otherwise} \end{cases}$$

Define

$$Z = X + Y \quad W = X - Y$$

a.) Find $f_{ZW}(z, w)$, $f_Z(z)$ and $f_W(w)$.

b.) Are Z and W independent random variables?

c.) Are Z and W uncorrelated random variables?

d.) Are Z and W orthogonal random variables ($E[ZW] = 0$)? Prove your answers.

Problem 5

X, Y are independent, idential geometric random variables with common parameter p, i.e., with $q = 1 - p$,

$$P(X = k) = P(Y = k) = pq^k, \quad k = 0, 1, 2, ...$$

a.) $Z = X + Y$, $W = \min\{X, Y\}$, find $f_{ZW}(z, w)$, $f_Z(z)$ and $f_W(w)$.

b.) $Z = \min\{X, Y\}$, $W = X - Y$, find $f_{ZW}(z, w)$, $f_Z(z)$ and $f_W(w)$.

Problem 6

X and Y are independent Geometric random variables with common parameter p, i.e., $P(X = k) = P(Y = k) = pq^k$ with $q = 1 - p$. Define

$$Z = X + Y, \qquad W = |X - Y|.$$

Find

a.) $P(Z = m, W = k)$

b.) $P(Z = m)$

c.) $P(W = k)$