

ECE-GY 6303, PROBABILITY & STOCHASTIC PROCESSES
HOMEWORK # 1

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Problem 1

Box 1 contains 3 red balls, 5 green balls and 2 white balls. Box 2 contains 5 red balls, 3 green balls and 1 white ball. One ball of unknown color is transferred from Box 1 to Box 2.

- a. What is the probability that a ball drawn at random from Box 2 is green?
- b. What is the probability that a ball drawn from Box 1 is not white?

Problem 2

In a batch of microprocessors, the probability that a microprocessor is defective is 10^{-3} . In one draw an assembly machine picks 10 microprocessors from this batch and tests each. It rejects the entire lot of 10 microprocessors if 2 or more of them are defective, else all the 10 are retained.

- a. Find the probability that a lot is rejected.
- b. If the machine draw 6 times, what is the probability that at least 60 microprocessors are retained.

Problem 3

- a. Toss a coin n times, Let ' p ' represent the probability of obtaining a "Head" in any toss. Show that the most likely number of "Heads" k_0 in n trials is given by

$$(n+1)p - 1 \leq k_0 \leq (n+1)p$$

$$\text{and hence } \frac{k_0}{n} \rightarrow p$$

- b. In a book of 200 pages long, it is not unreasonable to expect 20 misprints. Find the probability that a given page will contain
 - i) two misprints
 - ii) two or less prints
 - iii) two or more misprints.

Problem 4

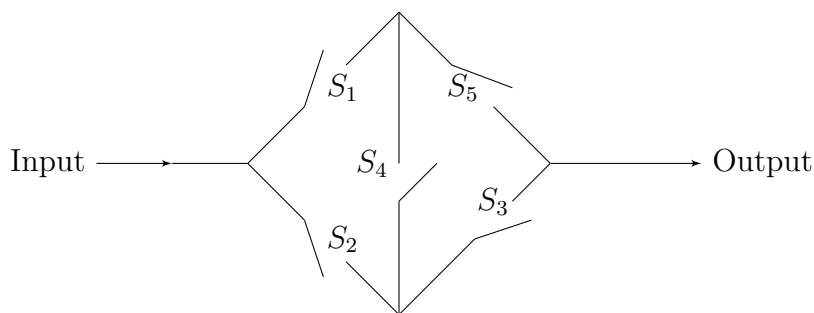
The pdf of a continuous random variable X is given by

$$f_X(x) = \begin{cases} \frac{1}{7} & -2 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

Find

- (i) $P(X^2 > 1)$
- (ii) $P(\sin(\pi X) \leq 0)$

Problem 5



The five switches in the figure operate independently. Each switch is closed with probability p and open with probability $(1 - p)$.

- a. Find the probability that the signal at the input will **not** be received at the output.
- b. Find the conditional probability that the switch S_4 is open given that the signal is received at the output.

Problem 6

Among a certain group of people 5 % are (professional) liars. A lie detector test on a liar is found to be positive with a probability of 0.94. If the test is positive for a non-liar, it is positive with a probability of 0.08. Given that the test is positive for a randomly picked person from that group, what is the probability that he is a liar.

Problem 7

We have two sealed boxes. In the first box we have 125 white and 75 black marbles. The second box contains 60 white and 90 black marbles. You pick a marble randomly from a box. For any given pick, the probability of picking from Box i , $P(B_i) = 0.5$.

- a. What is probability that the marble drawn is black?
- b. The marble picked turned to be black. What is the probability that it is picked out of Box 2?