ECE-GY 6303, Probability & Stochastic Processes

Homework # 7

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Problem 1

The random variables X and Y are jointly distributed over the region $0 < x < y < 1$ as

$$f_{XY}(x,y) = \begin{cases} kx & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

for some k.

a.) Determine k.

b.) Find the variances of X and Y.

c.) What is the covariance between X and Y?

Problem 2

The random variables X and Y are jointly distributed over the region $0 < x < y < \infty$ as

$$f_{XY}(x,y) = \begin{cases} 2xye^{-(x+y)} & 0 < x < y < \infty \\ 0 & \text{otherwise.} \end{cases}$$

a.) Determine $E[X|Y]$ and $E[Y|X]$.

b.) Determine the correlation coefficient ρ bwteen X and Y.

Problem 3

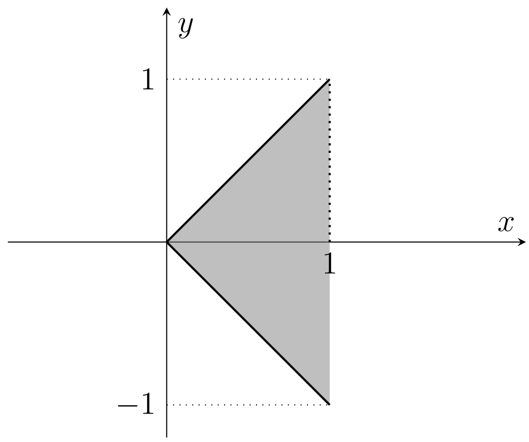
For any two random variables X and Y with $E[X^2], E[Y^2] < \infty$, show that

$$\text{Var}(X) = \text{Var}(E[X|Y]) + E[\text{Var}(X|Y)].$$

Problem 4

The random variables X and Y are jointly distribute

$$f_{XY}(x,y) = \begin{cases} \frac{3}{2}x & (x,y) \in \text{ shaded area,} \\ 0 & \text{otherwise.} \end{cases}$$



a.) Find $E[X|Y=y]$.

b.) Find the correlation coefficient ρ_{XY} between X and Y.

c.) Write MATLAB code to generate n-dimensional vectors i, $[x(i), y(i)]$, are distributed with the above distribution. (*Hint: Generate Y from $f_y(y)$, then generate X from $f_{X|Y}(x|y)$.*)

Problem 5

a.) Suppose X is a Geometric random variable with parameter p. Show that $P(X > m+n|X > m)$ is not a function of m.

b.) Suppose X and Y are zero mean jointly normal random variables with equal variances σ^2, and correlation coefficient $\rho \neq 0$.

i.) Is there a value for the coefficient a for which the random variables $aX + Y$ and $X - Y$ are independent?

ii.) Find the variance of $Z = \alpha X^2 + \beta Y^2$, where α and β are constants.