ECE-GY 6303, Probability & Stochastic Processes

Homework # 9

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Problem 1

Which among the following represent auto-correlation function of a stochastic process?

a.) $\max(t_i, t_j)$;

b.) $t_i^2\, t_j^2$;

c.) $t_i + t_j$;

d.) $1/(t_i + t_j)$.

Problem 2

Given

$$R_1 = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad \text{and} \quad R_2 = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix},$$

define

$$R = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix},$$

where $c_{ij} = a_{ij} b_{ij}$. Show that if R_1 and R_2 are positive definite, then R is also positive definite.

Problem 3

a.) $X(t)$ is a W.S.S. process. Define

$$Y(t) = X(t) + aX(t-T) + bX(t+T).$$

Is $Y(t)$ W.S.S.?

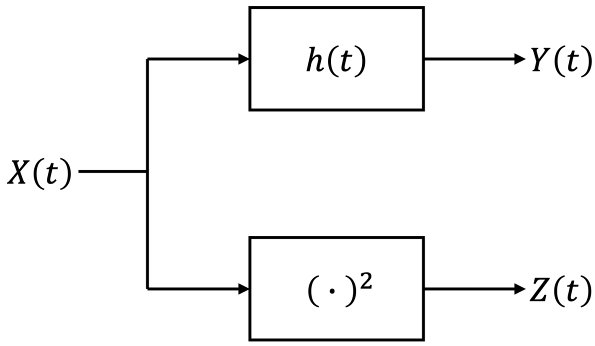
b.) $X(t)$ is a zero mean Gaussian process with auto-correlation function $R_{XX}(t_1 - t_2)$. Let

$$Y(t) = X^2(t) + X(t-T).$$

Find $R_{YY}(t_1, t_2)$. Is $Y(t)$ stationary in any sense? Is $Y(t)$ Gaussian?

Problem 4

$X(t)$ is a zero mean stationary Gaussian process with auto-correlation function $R_{XX}(\tau)$.



a.) Find $R_{YY}(t_1, t_2)$ and $R_{ZZ}(t_1, t_2)$.

b.) Is $Y(t)$ or $Z(t)$ stationary in any sense?

Problem 5

Suppose X_n conditional on X_{n-1} is Poisson distributed with parameter λX_{n-1}. Let

$$\mu_n = E[X_n], \quad \sigma_n^2 = \text{Var}(X_n).$$

Find μ_n and σ_n^2 in terms of λ, given $\mu_1 = 1$ and $\sigma_1^2 = 1$.

Problem 6

X(t) is a zero mean Gaussian process with auto-correlation function $R_{XX}(t_i, t_j) = \min(t_i, t_j)$. Define

$$Y(t) \; = \; e^{j(X(t)+\theta)},$$

where $\theta \sim U(-\pi, \pi)$ and independent of $X(t)$.

a.) Find the mean and auto-correlation function of $Y(t)$.

b.) Is $Y(t)$ W.S.S.?