ECE-GY 6303, Probability & Stochastic Processes

Homework # 5

Prof. Pillai

Fall 2021

Problem 1

The joint p.d.f of X and Y is given by

$$f_{XY}(x,y) = \begin{cases} e^{-y} & 0 < x < y < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

Find the p.d.f of

a.) $Z = X + Y$.

b.) $Z = X - Y$.

c.) $Z = X/Y$.

Problem 2

X and Y are independent and uniform in the interval $(0, a)$. Find the p.d.f. of $Z = X - Y$.

Problem 3

X and Y are independent exponential random variables with parameters α and β respectively, i.e.,

$$f_{XY}(x,y) = f_X(x) f_Y(y) = \begin{cases} \alpha\beta e^{-(\alpha x + \beta y)} & x \geq 0, y \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Define $Z = \min(X, 3Y)$. Show that Z is also an exponential random variable, and find the value of corresponding exponential parameter.

Problem 4

Given the joint density function

$$f_{XY}(x,y) = \begin{cases} xye^{-(x+y)} & x > 0, y > 0, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$Z = \frac{\min(X,Y)}{\max(X,Y)}.$$

Determine the p.d.f of Z.

Problem 5

X and Y are independent random variables with geometric p.m.f

$$P(X = k) = pq^k, k = 0, 1, 2, ...,$$
$$P(Y = m) = pq^m, m = 0, 1, 2.....$$

Find the p.m.f. of $Z = X + Y$.

Problem 6

X and Y are random variables with joint p.d.f.

$$f_{XY}(x, y) = \begin{cases} ke^{-(x+y)} & 0 < y < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

Find the p.d.f. of $Z = X - Y$.