Friction and backlash measurement and identification method for robotic arms

Lőrinc Márton and Béla Lantos

***Abstract*— The paper presents a friction and a backlash identification method for robotic arms that are based on measurements performed during controlled robot motion. A modified friction model is proposed making use of which the linear least squares method can be applied for friction identification. The influence of the robot nonlinearities were also taken into consideration during the friction identification. The backlash gap was measured when the robot arm changes its direction taking into consideration the velocity and the drive motor current. A robot velocity control algorithm is also proposed for friction and backlash measurement. Experimental measurements are presented to show the applicability of the theoretical results.**

***Index Terms*— Friction, backlash, speed control, response measurement, robot identification**

I. Introduction

In many robotic systems, especially in the cases when low weight-to-payload ratio is necessary and the robot segments are made of light weight materials, the friction and backlash has a great impact on the control system performances. The friction is a non-linear phenomenon that is universally present in the motion of bodies in contact. To describe the friction phenomena both static and dynamic [1] models were introduced. The static models describe the friction force in function of velocity, the dynamic ones are introduced to explain some very low velocity friction induced phenomena, such as presliding displacement. However even in early works it was shown that an accurate static friction model can describe the behavior of the friction with 90% confidelity in the velocity domain where the robotic system generally works [2]. The backlash appears in such cases when the moving parts may temporarily loose the contact [3]. Due to these nonlinearities limit cycles can appear in the control system response and the control precision may also deteriorate [4].

For effective compensation of these non-smooth nonlinearities it is necessary to know the parameters that describe the friction phenomena and the backlash gap size. With known friction model, the robot control algorithm can simply be extended to compensate the effect of the friction [5]. With the size of the backlash gap known, the control algorithm can be extended with an inverse backlash element that compensates de backlash-induced effects in the control system [6].

The problem of friction identification was discussed in many studies. In the paper [7] a wavelet network based friction estimation method is proposed. Frequency domain friction identification algorithm is proposed in [8]. The frequency domain identification methods are also popular for backlash identification [9]. Time domain identification method for backlash was proposed in [10].

The frictional parameters may change in time, they depend on external factors such as temperature, humidity, dwell time (the time interval a junction spends in the stuck, when the machine is not moving). The friction parameter variation is a slow process but it could affect the performance of the control system in time. Hence it is not enough to determine these parameters once, before the robot is putted in operation, the friction should be remeasured periodically and the friction compensation part of the control algorithm retuned. It is why the friction identification should be a practical algorithm, that uses only sensors necessary for the control of the robotic system.

The friction and backlash are generally arise in the gear transmissions that link the drive motor to the robot joint. The typical robot sensors are mounted around the drive motor of the joint. These sensors measure the joint position and velocity (on the motor side) and the motor current, which is necessary to sense the joint overload and for cascade type motor controllers.

In this paper the friction characteristics and backlash parameter of the robot joint are measured based only on motor side position, velocity and current measurement. Both the friction and the backlash are measured during a common measurement procedure. It is considered that the other parameters of the robot arm (length, mass, inertia, position of the center of gravity of the segment) are known.

The rest of the paper is organized as follows: Section II presents the model of the robotic arm with gear drive in which the friction and backlash cannot be neglected. In Section III the measurement and identification procedures for joint friction and backlash are described in detail. In Section IV experimental measurements are presented. Finally Section V sums up the conclusions of this study.

II. Robot arm model with backlash and friction

The dynamics of an n degrees of freedom rigid manipulator is described by the following relation:

$$H(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{B}(\tau) \qquad (1)$$

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where $\mathbf{q}$ denote the joint position vector, the vector τ contains the control torques at the outputs of the gear-boxes. The inertial matrix $H(\mathbf{q})$ is symmetric and positive definite, the vector $\mathbf{C}$ incorporate the effect of centrifugal and Coriolis forces, the effect of gravitational force is incorporated in the vector $\mathbf{G}$.

The backlash function $\mathbf{B}$ introduces a discontinuity in the robot model, since the moving parts in the gear transmission may temporarily loose the direct contact. For the ith joint:

$$B_i = \begin{cases} \tau_i \ in \ Contact \ Mode \\ 0 \ in \ Backlash \ Mode \end{cases} \tag{2}$$

Due to the backlash, the load side position of the ith joint will not be equal with the motor position q_{Mi} that drives the joint. In order to determine the condition for contact mode denote with $\frac{N_2}{N_1}$ the gear ratio and with β the backlash gap size. For contact mode the difference between the motor shaft position and joint position modified with the gear ratio, should be equal with the backlash gap size if the machine moves in negative direction. Otherwise the position difference should be equal with the negative of the gap.

$$\begin{aligned} & Contact \ mode \ (CM): \\ & \quad if \ \left((\dot{q}_{Mi} > 0) \ and \ (q_i \frac{N_2}{N_1} - q_{Mi} = -\beta) \right) \\ & \quad\quad or \ \left((\dot{q}_{Mi} < 0) \ and \ (q_i \frac{N_2}{N_1} - q_{Mi} = \beta) \right) \\ & Backlash \ mode \ (BM): \\ & \quad otherwise \end{aligned} \tag{3}$$

The control torque in the ith joint (τ_i) is generated by the motor drive. A part of the motor generated torque (which is proportional with the motor current) should compensate the friction force that appears in the gear transmission. Assuming that the joint is driven by a direct current servo motor the control torque at the output of the gearbox can be obtained as:

$$N_2 \tau_{Mi} = N_1 \left(\tau_i + \tau_{fi}(\dot{q}_{Mi})\right)$$

$$\tau_{Mi} = K_\tau i_i \tag{4}$$

$$\tau_i = \frac{N_2}{N_1} K_\tau i_i - \tau_{fi}(\dot{q}_{Mi}) \tag{5}$$

where τ_{Mi} is the torque developed by the motor, i_i is the current in the ith joint motor, K_τ is the constant of the joint drive.

The relation above describes the connection between the robot arm dynamics, joint friction and the drive motor current.

Friction modeling: The nonlinear behavior of friction is accentuated in the low velocity regime. If fluid lubrication is applied, decreasing friction with increasing velocities can be expected in the low velocity regime (Stribeck phenomena). In the high velocity regime the friction force slowly increases

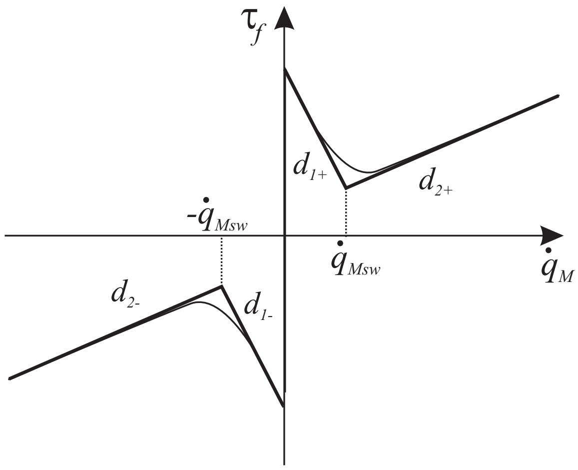

Fig. 1. Linearization of exponential friction model

with the velocity. To describe this phenomena many models were introduced, such as the exponential model:

$$\tau_{fi} = (F_C + (F_S - F_C)e^{-|\dot{q}_{Mi}|/\dot{q}_{MS}}) sign(\dot{q}_{Mi}) + F_V \dot{q}_{Mi} \tag{6}$$

with F_C is the Coulomb friction coefficient, F_S is the static friction coefficient, F_V is the viscous friction coefficient, $\dot{q}_{MS}$ is the Stribeck velocity.

The identification algorithm was developed based on the model (6). For the simplicity, only the positive velocity domain is considered, but similar study can be made for the negative velocities.

Consider a linear approximation for the exponential curve represented by two lines: d_{1_+} which cross through the $(0, \tau_{fi}(0))$ point and it is tangent to curve and d_{2_+} which passes through the $(\dot{q}_{Mimax}, \tau_{fi}(\dot{q}_{Mimax}))$ point and tangential to curve. (see Figure 1.) These two lines meet each other at the $\dot{q}_{Misw}$ velocity. In the domain $0 \dots \dot{q}_{Misw}$ the d_{1_+} can be used for the linearization of the curve and d_{2_+} is used in the domain $\dot{q}_{Msw} \dots \dot{q}_{Mimax}$.

Thus the linearization of the exponential friction model with bounded error can be described by two lines in the $0 \dots \dot{q}_{Mmax}$ velocity domain:

$$\begin{aligned} d_{1_+} \quad &: \quad \tau_{L1f_+}(\dot{q}_{Mi}) = a_{1+} + b_{1+}\dot{q}_{Mi}, \\ & \quad\quad for \ 0 \ \le \dot{q}_{Mi} \ \le \ \dot{q}_{Misw} \\ d_{2_+} \quad &: \quad \tau_{L2f_+}(\dot{q}_{Mi}) = a_{2+} + b_{2+}\dot{q}_{Mi}, \\ & \quad\quad for \ \dot{q}_{Misw} \ \le \dot{q}_{Mi} \ \le \ \dot{q}_{Mimax} \end{aligned} \tag{7}$$

The value of $\dot{q}_{Misw}$ can easily be determined from linearization (7):

$$\dot{q}_{Misw} = \frac{a_{1+} - a_{2+}}{b_{2+} - b_{1+}} \tag{8}$$

Same study can be made for negative velocities. Based on linearization, the friction can be modeled as follows:

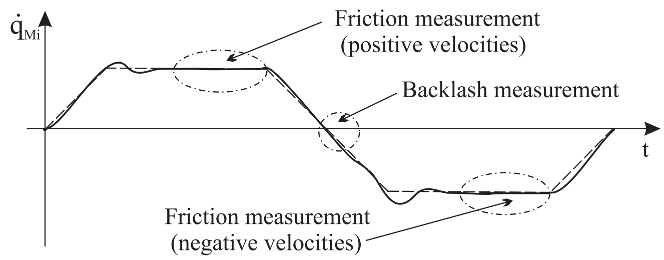

Fig. 2. Reference velocity for friction and backlash identification

$$\tau_{Lfi}(\dot{q}_{Mi}) = \begin{cases} a_{1+} + b_{1+}\dot{q}_{Mi}, & if\ \dot{q}_{Mi} \in (0,\ \dot{q}_{Misw}] \\ a_{2+} + b_{2+}\dot{q}_{Mi}, & if\ \dot{q}_{Mi} \in (\dot{q}_{Misw},\ \dot{q}_{Mimax}] \\ a_{1-} + b_{1-}\dot{q}_{Mi}, & if\ \dot{q}_{Mi} \in [-\dot{q}_{Misw},\ 0) \\ a_{2-} + b_{2-}\dot{q}_{Mi}, & \\ \quad if\ \dot{q}_{Mi} \in [-\dot{q}_{Mimax},\ -\dot{q}_{Misw}) & \end{cases} \tag{9}$$

This model can also effectively be used for adaptive friction compensation [11] in positioning systems.

III. Friction and backlash identification

The friction depends on joint velocity, hence it should be measured for different constant velocity values. In the other hand the frictional parameters differ for positive and negative velocity domains, the measurement should be performed in both velocity domains. The backlash gap can be captured when there is a transition from positive to negative velocity domain or vice versa. Accordingly, to capture one measurement point, the joint motion should be planned in such way to have constant velocity regimes both for positive and negative domain and a controlled transition from positive to negative velocities (see Figure 2). Note that these types of velocity profiles (acceleration-constant velocity regime - deceleration) are also extensively used for robot trajectory planning. Hence the non-smooth nonlinearities will be identified in such conditions that characterize the robot's motion during normal operation.

A. Friction measurements

Assuming that the joint moves in the constant velocity regime ($\ddot{q}_i = 0$) and only the ith joint moves, the motion of the joint is given by:

$$C_{ii}\dot{q}_i^2 + G_i(q_i) = \tau_i \tag{10}$$

Since the other joints of the robot do not move, the centrifugal term C_{ii} is constant.

Based on (10) and (5), the relation among the velocity friction force and current is given by:

$$\tau_{fi}(\dot{q}_{Mi}) = \frac{N_{2i}}{N_{1i}} K_{\tau i} i_i - C_{ii}\dot{q}_i^2 - G_i(q_i) \tag{11}$$

For friction identification consider that a measurement point is given by the average of the measured velocity and measured current in the constant velocity regime after the transients: $(\dot{q}_{Mi}, i_i)$.

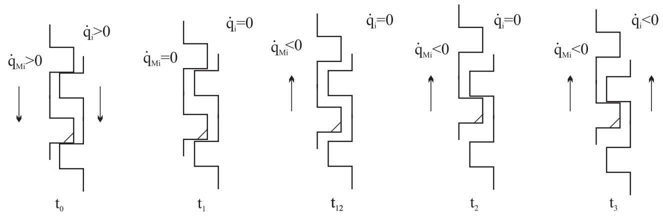

Fig. 3. Motion in joint gear transmission during direction change

The velocity dependent friction characteristic can be obtained by repeating the measurement systematically for many velocity values in the operating range.

The parameters of the friction model (9) can be obtained using the least squares method. With the frictional parameter and centrifugal term unknown, the vector of the estimated parameters and the regressor vector for the i'th joint can be written as:

$$\widehat{\theta}_i = (a_{ij}\ b_{ij}\ C_{ii})^T \tag{12}$$

$$\xi_i = (1\ \dot{q}_{Mi}\ \dot{q}_i^2)^T \tag{13}$$

For the parts of the measurements, where the value of the subindex j will be 1, if the friction decreases with velocity, otherwise $j = 2$. The value of the parameters may also differ in the negative and positive velocity regime, see (9).

The cost function for the least squares problem is given by:

$$J = \sum_{i=1}^{No\_meas} \left(\widehat{\theta}_i^T \xi_i - \left(\frac{N_{2i}}{N_{1i}} K_{\tau i} i_i - G_i(q_i) \right) \right)^2 \tag{14}$$

By solving the optimization problem $min_\theta J$ the first two elements in the vector $\widehat{\theta}$ give the unknown frictional parameters.

B. Backlash measurement

The parameter, that characterizes the backlash is the size of the backlash gap, the size of the clearance between mating components in the gear transmission that links the driving motor to the robot segment.

The motion of the robot joint transmission during backlash mode is showed in Figure 3. If the machine changes its direction both in motor and load side the joint velocity will be zero (time moment t_1). In order to achieve zero velocity both on motor and load side during direction change, the joint should be in such configuration that the direction of motion is perpendicular on gravitational force. The inertia driven motion of the the load side (after the motor side stops moving) is avoided due to uniform deceleration, with low velocity rate change value. However it is beneficial if the backlash measurement is performed in the low velocity regime, when the first points of the friction characteristic are measured.

There will be a time period during which only the motor side load moves. It reaches the load side at the time moment

t_2 when the drive motor will suffer a shock since the inertia of the mechanics abruptly increases.

To measure the value of the backlash gap the measured velocity and current signals during direction change will be used. When the machine changes its direction, the motor current will have a peak and after that its value starts to decrease. Note that firstly the robot moves in backlash mode. When the motor shaft reaches the load (at the end of the backlash gap) the value of the current will increase again, it will have a local maximum, since the motor starts to drive the robot segment as well.

Two time instants will be captured:

- t_1 - the time instant when the machine velocity changes its sign (zero cross detection for the velocity signal).

- t_2 - the first moment after t_1 when the current value will have a local minimum (minimum detection for current signal).

The size of the backlash gap can be calculated by integrating the velocity from t_1 to t_2:

$$\beta = \int_{t_1}^{t_2} \dot{q}_{Mi}(\sigma) d\sigma \tag{15}$$

The integration can be performed using numerical methods. The precision of the numerical integration depends on the chosen control period. If the size of the backlash gap is small, higher sampling frequency should be applied.

C. Speed control for identification

For friction measurement the velocity controller should guarantee zero steady state velocity error in the constant velocity regime and it has to deal with modeling uncertainties, since the friction and backlash are considered unknown during the measurements.

To handle the modeling uncertainties a robust, sliding mode controller is developed. To assure zero steady state error for constant reference velocity signal, an integral term is introduced in the controller.

To solve the velocity tracking problem define the following error metric:

$$\mathbf{S} = \dot{\mathbf{e}} + \Lambda \int_0^t \dot{\mathbf{e}}(\sigma) d\sigma \ where \ \dot{\mathbf{e}} = \dot{\mathbf{q}} - \dot{\mathbf{q}}_{\mathbf{d}} \tag{16}$$

$\Lambda = diag(\lambda_1 \ \lambda_2 \ ... \ \lambda_n)$, $\lambda_i > 0$

Consider that in the robot model during the contact mode the effect of friction and backlash appears as an additive bounded uncertainty:

$$H(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) + \mathbf{d} = \tau \tag{17}$$

The vector $\mathbf{d}$ represents the uncertainty due to unknown friction and backlash. It is assumed that the elements of $\mathbf{d}$ are bounded: $|d_i| \leq D_{Mi}$.

The control law is formulated as:

$$\tau = H(\mathbf{q})(\ddot{\mathbf{q}}_d - \Lambda\dot{\mathbf{e}}) + C(\mathbf{q}, \dot{\mathbf{q}})(\dot{\mathbf{q}}_d - \Lambda \int_0^t \dot{\mathbf{e}}(\sigma) d\sigma) + \\ + \mathbf{G}(\mathbf{q}) - K_S sat(\mathbf{S}/\Phi) \ where \ K_{Si} > D_{Mi} \tag{18}$$

Φ denotes the prescribed velocity tracking precision, $sat(\mathbf{S}/\Phi)$ denotes a diagonal matrix with the elements: $sat(S_i/\Phi)$, where $sat(\cdot)$ is the saturation function.

$K_S = diag(K_{S1} \ K_{S2} \ ... \ K_{Sn})$, with $K_{Si} > D_{Mi}$.

To analyze the proposed control law, consider the following Lyapunov like function:

$$V = \frac{1}{2}\mathbf{S}^T H(\mathbf{q})\mathbf{S} \tag{19}$$

The time derivative of V is given by:

$$\dot{V} = \mathbf{S}^T H(\mathbf{q})\dot{\mathbf{S}} + \frac{1}{2}\mathbf{S}^T \dot{H}(\mathbf{q})\mathbf{S} \tag{20}$$

With the control law (18) the tracking error dynamics is given by:

$$H(\mathbf{q})\dot{\mathbf{S}} = -C(\mathbf{q}, \dot{\mathbf{q}})S - K_S sat(\mathbf{S}/\Phi) + \mathbf{d} \tag{21}$$

By substituting (21) in (20) and applying that $\dot{H}(\mathbf{q}) - 2C$ is skew symmetric, the time derivative of Lyapunov function is given by:

$$\dot{V} = -\mathbf{S}^T K_S sat(\mathbf{S}/\Phi) + \mathbf{S}^T \mathbf{d} \tag{22}$$

Outside the boundary layer Φ ($|S_i| > \Phi$) we have:

$$\dot{V} = -\mathbf{S}^T K_S sign(\mathbf{S}/\Phi) + \mathbf{S}^T\mathbf{d} = -|\mathbf{S}|^T K_S + \mathbf{S}^T\mathbf{d} \tag{23}$$

With $K_{Si} > D_{Mi}$ the time derivative of the Lyapunov function is always negative, hence with the proposed control law the velocity tracking error metric will converge inside the boundary layer with Φ.

Since during the measurements only one joint moves, the control law (18) will have a simple form that can easily be implemented.

IV. Experimental measurements

A SCARA type robotic arm was used to test the performances of the developed identification methods (see Figure 4). The robot has four Degrees Of Freedom, three rotational and one translational joint. Each joint is equipped with a 12 V DC servo motor. Each joint is equipped with a 500 Pulses Per Turn incremental encoder mounted directly on the drive motor shaft. The joint position and velocity are calculated based on the measured frequency of the incremental encoder signals. Each motor drives the robot segment through a gear head. The main component of the frictional force and the backlash appears in this gear head.

The robot is controlled by a personal computer that was equipped with three data acquisition cards. A card with analogue outputs is used for motor control, the second one that contains counters is applied for incremental encoder frequency reading. The motor current is measured using an analogue input card.

The motors are driven through H bridges. The motor current is measured using a resistor placed between the lower transistors of the bridge and the ground. With this current sensing method the absolute value of the current

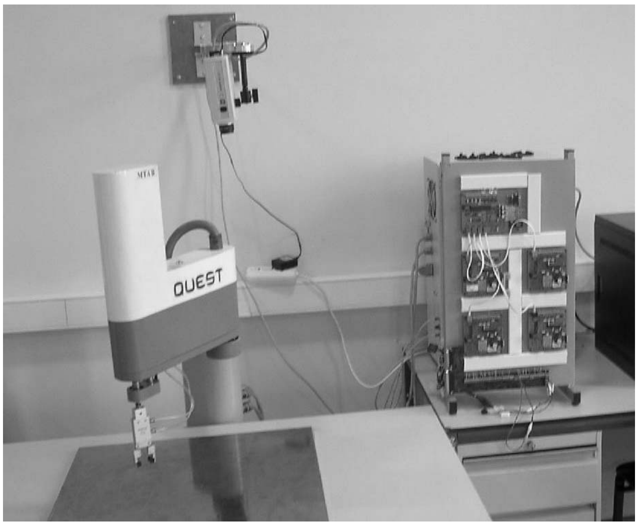

Fig. 4. Robotic arm used for experimental measurements

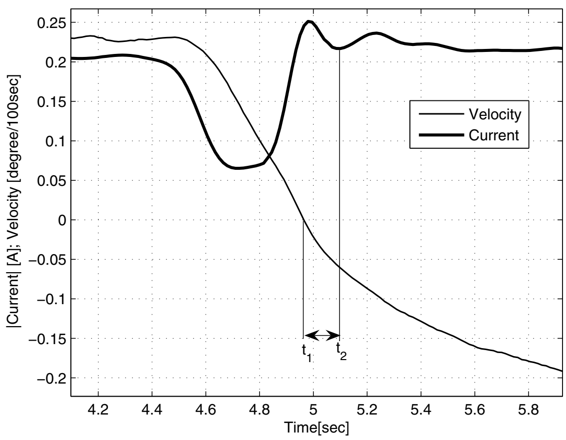

Fig. 5. Backlash identification in joint 1

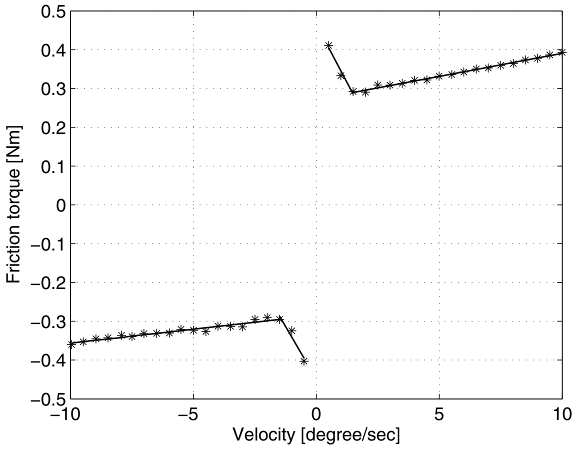

Fig. 6. Measured and identified friction in joint 1

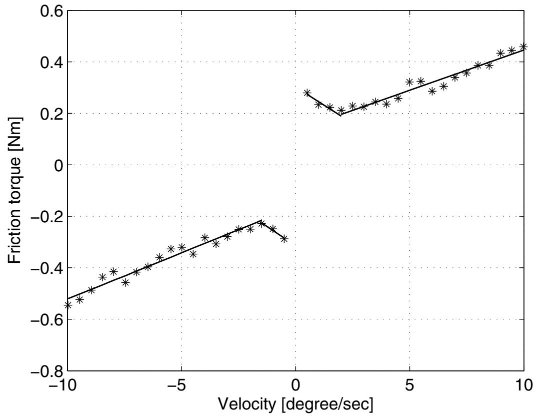

Fig. 7. Measured and identified friction in joint 2

can be obtained, which is enough for friction and backlash measurement. With the known value of the resistor the motor current can easily be calculated ($i_i = U_{INi}/R_I$). The control torque developed by the motor can be calculated by simply multiplying the current value with the torque constant which is a motor catalogue data ($\tau_i = K_{Ti} \cdot i_i$). Hence in constant velocity regime the torque developed by the motor can easily determined.

During the experiments the friction and backlash were identified in the first and the second joint. The gear ratio in the first joint is $N_2/N_1 = 86/16$, in the second joint is $N_2/N_1 = 40/12$. Both joints are driven by identical motors with torque constant $K_T = 0.023Nm/A$.

During the experiments the trajectory was planned such as the joint will move in its entire angle domain (± 160 $degree$ for the first joint, ± 135 $degree$ for the second joint). The friction was identified in the low velocity regime (± 10 $degree/second$). The reference velocity was increased with 0.5 $degree/second$ in each measurement step. A measurement point was determined by simply calculating the mean of the current and the velocity in the constant velocity regime in steady state. The frictional parameters were identified based on the measurements using the Least Squares method. The identification results can be seen in the Figures 6 and 7. It can be seen that with the proposed identification method the friction model fits well the noisy experimental measurements. The obtained friction models for the first and the second joints are:

$$\tau_{1Lf}(\dot{q}_1) = \begin{cases} 0.4645 - 0.119\dot{q}_1, & if\ \dot{q}_1 \in (0,\ 1.5] \\ 0.2718 + 0.0106\dot{q}_1, & if\ \dot{q}_1 \in (1.5,\ 10.0] \\ -0.4493 - 0.1085\dot{q}_1, & if\ \dot{q}_1 \in [-1.5,\ 0) \\ -0.2847 + 0.0072\dot{q}_1, & if\ \dot{q}_1 \in [-10.0,\ -1.5) \end{cases}$$

$$\tau_{2Lf}(\dot{q}_2) = \begin{cases} 0.3013 - 0.0562\dot{q}_2, & if\ \dot{q}_2 \in (0,\ 1.8] \\ 0.1334 - 0.0312\dot{q}_2, & if\ \dot{q}_2 \in (1.8,\ 10.0] \\ -0.3141 - 0.0594\dot{q}_2, & if\ \dot{q}_2 \in [-1.9,\ 0) \\ -0.1623 + 0.0359\dot{q}_2, & if\ \dot{q}_2 \in [-10.0,\ -1.9) \end{cases}$$

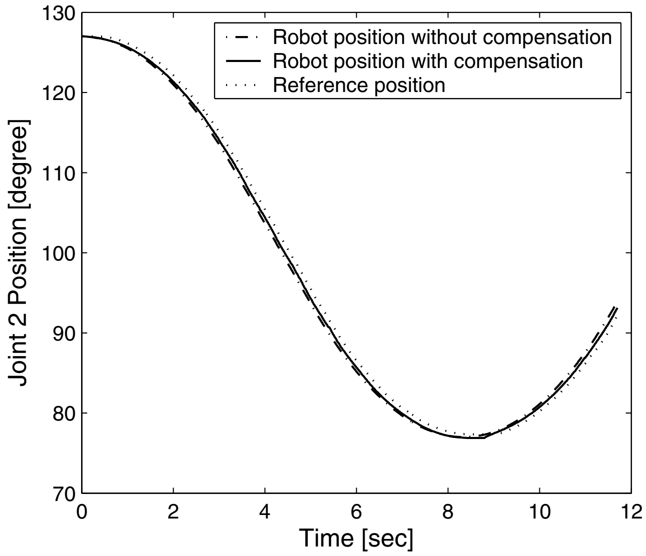

Fig. 8. Tracking performances with and without compensation

J1 no compensation	J1 with compensation	J2 no compensation	J2 with compensation
1.7313^o	0.9670^o	1.1805^o	0.7600^o

TABLE I

TRACKING PERFORMANCES

In the relations above the dimension of the friction torque is Nm, the dimension of the velocity is $degree/second$.

The backlash measurement was performed in the last 10 measurement steps. Firstly the time moments t_1 and t_2 were captured and afterward the velocity was integrated between these time instants using the trapezoidal rule.

In the first joint the mean of the calculated values gives the backlash gap $\beta_{Mean} = 0.47\ degree$. The mean deviation of the obtained value from the measured values was also calculated using the formula $\Delta\beta = \sum_{i=1}^{10} |\beta_{Mean} - \beta_{iMeasured}|/10$. Its value was found as $\Delta\beta = 6.3\%$. In the second joint the following values were found: $\beta_{Mean} = 0.35\ degree$; $\Delta\beta = 11.9\%$.

In order to show the applicability of the identification results a model based robot control algorithm was implemented. Two experiments were performed. In the first experiment the friction and backlash was not taken into consideration in the control algorithm. In the second experiment the control algorithm was extended with friction and backlash compensation terms as it was described in [12] and [6] respectively. The reference trajectory was chosen in such way that during the motion both joints were in constant acceleration regime. Direction change also was introduced into the trajectory. The tracking performances for the second joint can be seen in the Figure 8. The mean value of the absolute tracking error was also calculated for both experiments. The results are presented in the Table I.

V. CONCLUSIONS

A friction and backlash measurement method is proposed for robotic systems, that can be described with nonlinear model, when velocity and drive motor current measurements are available. The friction is measured for different constant velocities using a robust speed controller with integral term. The size of the backlash is measured by capturing the backlash mode - contact mode transition time instant based on the current signal. To identify the frictional parameters a piecewise linearly parameterized friction model is proposed. It was shown that the robot nonlinearities (gravitational, centrifugal effects) cannot be neglected during friction identification. It was shown experimentally that by applying the identification results in model based robot control algorithms the tracking performances can be improved significantly.

ACKNOWLEDGMENTS

The first author research was supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences. The research was also supported by the Hungarian National Research program under grant No. OTKA K 71762 and by the Control System Research Group of the Hungarian Academy of Sciences.

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