② $y = \frac{x^2+5x}{x^2+2x+1}$

1° D: $x^2+2x+1 \neq 0$

$D = 2^2 - 4\cdot 1\cdot 1 = 0$

$x_{1,2} \neq \frac{-2 \pm 0}{2} = \frac{-2}{2} \neq -1$

$x_1 = x_2 \neq -1$

$x \in (-\infty, -1) \cup (-1, +\infty)$

2° Parnost:

$f(-x) = \frac{(-x)^2+5(-x)}{(-x^2)+2(-x)+1} = \frac{x^2-5x}{x^2-2x+1}$

$\Rightarrow$ Ni parna ni neparna

3° Nule funkcije:

$\frac{x^2+5x}{x^2+2x+1} = 0$

$x^2+5x=0$

$x(x+5)=0$

$|x=0|$ $x+5=0$

$|x=-5|$

Presjek sa y-osom:

$x=0 \Rightarrow y = \frac{0^2+5\cdot 0}{0^2+2\cdot 0+1} = \frac{0}{1} = 0$

$(0,0)$

6°

4° Znak funkcije:

	$-\infty$	-5	-1	0	$+\infty$
x					
x^2+5x	+	0 −	−	0	+
x^2+2x+1	+	+	0 +	+	
y	+	−	−	+	

5° Asimptote:

H.A. $\lim\limits_{x\to\infty} \frac{x^2+5x}{x^2+2x+1} \begin{matrix}/:x^2\\=\\/:x^2\end{matrix} 1$ H.A $y=1 \Rightarrow$ nema K.A.

V.A. $x=-1$

$$\lim_{x\to -1_-} \frac{x^2+5x}{x^2+2x+1} = \frac{(-1)^2+5(-1)}{0} = \frac{-4}{0_+} = -\infty$$

$$\lim_{x\to -1_+} \frac{x^2+5x}{x^2+2x+1} = \frac{-4}{0_+} = -\infty$$

$\Rightarrow$ V.A $\boxed{x=-1}$

6° Prvi izvod i nultočke:

$$y' = \left(\frac{x^2+5x}{x^2+2x+1}\right)' = \frac{(2x+5)(x^2+2x+1)-(x^2+5x)(2x+2)}{(x^2+2x+1)^2}$$

$$= \frac{2x^3+5x^2+4x^2+10x+2x+5-2x^3-10x^2-2x^2-10x}{(x^2+2x+1)^2} =$$

$$= \frac{-3x^2+2x+5}{(x^2+2x+1)^2}$$

$y'=0 \Rightarrow -3x^2+2x+5=0$

$D=4-4\cdot(-3)\cdot 5$

$D=64$

$x_{1,2} = \frac{-2\pm 8}{-6}$

$x_1=-1 \notin D$ $\quad x_2=\frac{5}{3}\in D$

x	$-\infty$		-1		$\frac{5}{3}$		$+\infty$
$-3x^2+2x+5$		$-$	0	$+$	0	$-$	
y'		$-$		$+$		$-$	
y		↘	$\notin D$	↗		↘	

$x_{max}=\frac{5}{3}$

$$y_{max} = \frac{\left(\frac{5}{3}\right)^2+5\cdot\frac{5}{3}}{\left(\frac{5}{3}\right)^2+2\cdot\frac{5}{3}+1} = \frac{25}{16}$$

$T_{max}\left(\frac{5}{3}, \frac{25}{16}\right)$

$\approx$ $\quad 1{,}67 \quad 1{,}56$

7° Drugi izvod i prinudjena:

$$y'' = \left(\frac{-3x^2+2x+5}{(x^2+2x+1)^2}\right)' = \frac{(-6x+2)(x^2+2x+1)^2 - (3x^2+2x+5)\cdot 2(x^2+2x+1)(2x+2)}{(x^2+2x+1)^4}$$

$$= \frac{(x^2+2x+1)\left[(-6x+2)(x^2+2x+1) - (-3x^2+2x+5)\cdot 2(2x+2)\right]}{(x^2+2x+1)^{4\,3}}$$

$$= \frac{-6x^3 - 12x^2 - 6x + 2x^2 + 4x + 2 + 12x^3 - 8x^2 - 20x + 12x^2 - 8x - 20}{(x^2+2x+1)^3}$$

$$= \frac{6x^3 - 6x^2 - 30x - 18}{(x^2+2x+1)^3}$$

$y'' = 0 \Rightarrow 6x^3 - 6x^2 - 30x - 18 = 0 \mid :6$

$x^3 - x^2 - 5x - 3 = 0$

$*6 \quad \frac{x-3}{(x^2+2x+1)^2} \qquad x-3=0$

$x=3$

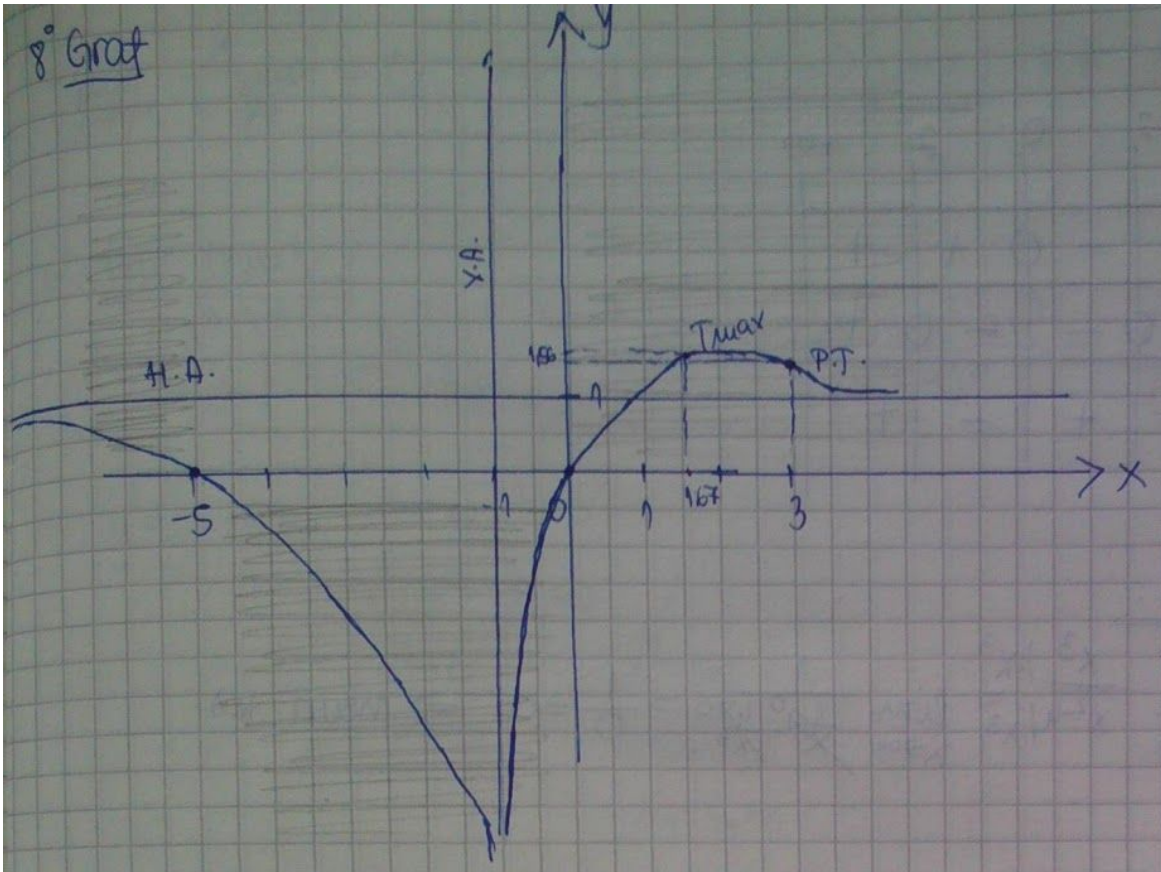
	$-\infty$		3		$+\infty$
x					
$x-3$		$-$	0	$+$	
y''		$-$		$+$	
y	$\cap$			$\cup$	

P.T. $x \in 3 \in D$

$y_{P.T.} = \frac{3^2+5\cdot 3}{3^2+2\cdot 3+1} = \frac{24}{16} = \frac{3}{2}$

P.T. $\left(3, \frac{3}{2}\right)$

8° Graf



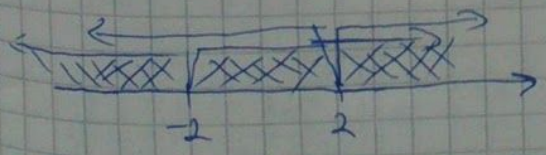
③ $y = \frac{x^3}{x^2-4}$

1° D: $x^2-4 \neq 0$

$x^2 \neq 4$

$x \neq \pm\sqrt{4}$

$x \neq \pm 2$



$x \in (-\infty, -2) \cup (-2, 2) \cup (2, +\infty)$

2° Parnost:

$f(-x) = \frac{(-x)^3}{(x)^2-4} = \frac{-x^3}{x^2-4} = -\frac{x^3}{x^2-4}$

Neparna!

3° Nule fje:

$x^3 = 0$

$x = 0$

Presjek sa y-osom:

$x = 0 \Rightarrow y = 0$

4° Znak fje:

x	$-\infty$		-2		0		2		$+\infty$
x^3		$-$		$-$	0	$+$		$+$	
x^2-4		$+$	0	$-$		$-$	0	$+$	
y		$-$		$+$		$-$		$+$	

5° Asimptote:

H.A. $\lim\limits_{x\to\infty} \frac{x^3 \,/:x^3}{x^2-4\,/:x^3} = \lim\limits_{x\to\infty} \frac{1}{\frac{1}{x}-\frac{4}{x^3}} = \frac{1}{0} = \infty \Rightarrow$ nema H.A.

V.A. $x=-2 \qquad x=2$

$x=-2$ $\lim\limits_{x\to -2-} \frac{x^3}{x^2-4} = \frac{-8}{0_+} = -\infty$

$\lim\limits_{x\to -2+} \frac{x^3}{x^2-4} = \frac{-8}{0_-} = +\infty$ } V.A. $x=-2$

$x=2$ $\lim\limits_{x\to 2-} \frac{x^3}{x^2-4} = \frac{8}{0_-} = -\infty$

$\lim\limits_{x\to 2+} \frac{x^3}{x^2-4} = \frac{8}{0_+} = +\infty$ } V.A. $x=2$

K.A. $y=kx+n$

$k = \lim\limits_{x\to\infty} \frac{\frac{x^3}{x^2-4}}{\frac{x}{1}} = \lim\limits_{x\to\infty} \frac{x^3 \,/:x^3}{x^3-4x\,/:x^3} = 1$

$n = \lim\limits_{x\to\infty} \left[\frac{x^3}{x^2-4} - 1\cdot x\right] = \lim\limits_{x\to\infty} \frac{x^3-x^3+4x}{x^2-4} = \lim\limits_{x\to\infty} \frac{4x\,/:x^2}{x^2-4\,/:x^2} =$

$$= \lim_{x\to\infty} \frac{\frac{4}{x}\to 0}{1-\frac{4}{x^2}\to 0} = 0 \Rightarrow y = 1\cdot x + 0 \Rightarrow \underline{y = x} \quad \text{K.A.}$$

6° Prvi izvod:

$$y' = \left(\frac{x^3}{x^2-4}\right)' = \frac{3x^2(x^2-4)+x^3\cdot 2x}{(x^2-4)^2} = \frac{3x^4-12x^2-2x^4}{(x^2-4)^2} = \frac{x^4-12x^2}{(x^2-4)^2}$$

$y' = 0 \Rightarrow x^4 - 12x^2 = 0$

$x^2(x^2-12) = 0$

$x^2 = 0 \qquad x^2 = 12$

$\underline{x = 0} \qquad x = \pm\sqrt{12}$

$x_1 \approx 3{,}46$

$x_2 \approx -3{,}46$

x	$-\infty$		$-3{,}46$		0		$3{,}46$		$+\infty$
x^2	+		+	0	+		+		
x^2-12	+	0	−		−	0	+		
y'	+		−		−		+		
y	↗		↘		↘		↗		

$x_{max} = -3{,}46 \qquad x_{min} = 3{,}46$

$y_{max} = \frac{(-3{,}46)^3}{(-3{,}46)^2-4} \approx -5{,}19 \qquad y_{min} = \frac{(3{,}46)^3}{(3{,}46)^2-4} = 5{,}19$

$T_{max}(-3{,}46; -5{,}19) \qquad T_{min}(3{,}46; 5{,}19)$

°Drugi izvod:

$$y'' = \left[\frac{x^4-12x^2}{(x^2-4)^2}\right]' = \frac{(4x^3-24x)(x^2-4)^2 - (x^4-12x^2)\cdot 2\cdot(x^2-4)\cdot 2x}{(x^2-4)^4} =$$

$$= \frac{(x^2-4)\left[(4x^3-24x)(x^2-4) - (x^4-12x^2)\cdot 4x\right]}{(x^2-4)^3} =$$

$$= \frac{4x^5-16x^3-24x^3+96x-4x^5+48x^3}{(x^2-4)^3} = \frac{8x^3+96x}{(x^2-4)^3} = \frac{8x(x^2+12)}{(x^2-4)^3}$$

$y'' = 0$

$8x(x^2+12) = 0$

$8x = 0 \qquad x^2 + 12 = 0$

$x = 0 \qquad x^2 = -12$ // nemoguće

x	$-\infty$		-2		0		2		$+\infty$
$8x$		$-$		$-$	0	$+$		$+$	
$(x^2-4)^3$	$+$	0	$-$		$-$	0	$+$		
y''		$-$		$+$		$-$		$+$	
y	$\cap$		$\cup$		$\cap$		$\cup$		

$\notin D$ P.T. $\notin D$

P.T. $x = 0$

$y = \frac{0^3}{0^2-4} = 0$ P.T (0,0)

8° Graf: