

## Rješenja TESTA 1

ZADACI: Grupa B

1. Date su matrice:

$$A = \begin{bmatrix} 1 & -3 \\ 0 & 7 \end{bmatrix} \quad \text{i} \quad B = \begin{bmatrix} 4 & -6 \\ 2 & 1 \end{bmatrix}.$$

Odrediti  $B \cdot A = ?$

Rješenje:

$$B \cdot A = \begin{bmatrix} 4 & -6 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 4 \cdot 1 - 6 \cdot 0 & 4 \cdot (-3) - 6 \cdot 7 \\ 2 \cdot 1 + 1 \cdot 0 & 2 \cdot (-3) + 1 \cdot 7 \end{bmatrix} = \begin{bmatrix} 4 & -54 \\ 2 & 1 \end{bmatrix}$$

2. Riješiti sistem algebarskih jednačina Kramerovim pravilom:

$$2x - 3y + z = -9$$

$$5x + y - 2z = 12$$

$$x - 2y - 3z = 1$$

Rješenje:

$$D = \begin{vmatrix} 2 & -3 & 1 \\ 5 & 1 & -2 \\ 1 & -2 & -3 \end{vmatrix} = 2 \begin{vmatrix} 1 & -2 \\ -2 & -3 \end{vmatrix} + 3 \begin{vmatrix} 5 & -2 \\ 1 & -3 \end{vmatrix} + 1 \begin{vmatrix} 5 & 1 \\ 1 & -2 \end{vmatrix} = \textcolor{red}{-64}$$

$$= 2(-3-4) + 3(-15+2) + (-10-1) = -64$$

$D = -64 \neq 0$  sistem ima rješenje.

$$D_x = \begin{vmatrix} -9 & -3 & 1 \\ 12 & 1 & -2 \\ 1 & -2 & -3 \end{vmatrix} = -9 \begin{vmatrix} 1 & -2 \\ -2 & -3 \end{vmatrix} + 3 \begin{vmatrix} 12 & -2 \\ 1 & -3 \end{vmatrix} + \begin{vmatrix} 12 & 1 \\ 1 & -2 \end{vmatrix} = \textcolor{red}{-64}$$

$$\textcolor{red}{-64} - 9(-3-4) + 3(-36+2) + (-24-1) = -64$$

$$D_y = \begin{vmatrix} 2 & -9 & 1 \\ 5 & 12 & -2 \\ 1 & 1 & -3 \end{vmatrix} = 2 \begin{vmatrix} 12 & -2 \\ 1 & -3 \end{vmatrix} + 9 \begin{vmatrix} 5 & -2 \\ 1 & -3 \end{vmatrix} + \begin{vmatrix} 5 & 12 \\ 1 & 1 \end{vmatrix} = -192$$

$$D_z = \begin{vmatrix} 2 & -3 & -9 \\ 5 & 1 & 12 \\ 1 & -2 & 1 \end{vmatrix} = 2 \begin{vmatrix} 1 & 12 \\ -2 & 1 \end{vmatrix} + 3 \begin{vmatrix} 5 & 12 \\ 1 & 1 \end{vmatrix} - 9 \begin{vmatrix} 5 & 1 \\ 1 & -2 \end{vmatrix} = 128$$

Rješenje sistema:

$$x = \frac{D_x}{D} = \frac{-64}{-64} = 1$$

$$y = \frac{D_y}{D} = \frac{-192}{-64} = 3$$

$$z = \frac{D_z}{D} = \frac{128}{-64} = -2$$

3. Odrediti vrijednost  $\det A - \det B$  ako su date determinante:

$$\det A = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 4 \\ 5 & 7 & 3 \end{vmatrix} \quad \det B = \begin{vmatrix} 2 & 0 & 0 \\ 7 & 1 & 2 \\ 0 & 3 & 4 \end{vmatrix}$$

$$\det A = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 4 \\ 5 & 7 & 3 \end{vmatrix} = 1 \begin{vmatrix} 1 & 4 \\ 7 & 3 \end{vmatrix} - 2 \begin{vmatrix} 0 & 4 \\ 5 & 3 \end{vmatrix} + 0 \begin{vmatrix} 0 & 1 \\ 5 & 7 \end{vmatrix} = 3 - 28 - 2 * (-20) = 15$$

$$\det B = \begin{vmatrix} 2 & 0 & 0 \\ 7 & 1 & 2 \\ 0 & 3 & 4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} + 0 \begin{vmatrix} 7 & 2 \\ 0 & 4 \end{vmatrix} + 0 \begin{vmatrix} 7 & 1 \\ 0 & 3 \end{vmatrix} = 2(4 - 6) = -4$$

$$\det A - \det B = 15 - (-4) = 19$$

4. Odrediti inverznu matricu matrice:

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 2 & 0 \\ 4 & 0 & 3 \end{bmatrix}$$

Rješenje:

$$A^{-1} = \frac{1}{\det A} * A^{\textcolor{red}{i}}$$

$$\det A = \begin{vmatrix} -2 & 1 & 0 \\ 1 & 2 & 0 \\ 4 & 0 & 3 \end{vmatrix} = -2 \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} - 1 \begin{vmatrix} 1 & 0 \\ 4 & 3 \end{vmatrix} + 0 \begin{vmatrix} 1 & 2 \\ 4 & 0 \end{vmatrix} = -2 * 6 - 3 = -15$$

$$a_{11} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6$$

$$a_{12} = - \begin{vmatrix} 1 & 0 \\ 4 & 3 \end{vmatrix} = -3$$

$$a_{13} = \begin{vmatrix} 1 & 2 \\ 4 & 0 \end{vmatrix} = -8$$

$$a_{21} = - \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = -3$$

$$a_{22} = \begin{vmatrix} -2 & 0 \\ 4 & 3 \end{vmatrix} = -6$$

$$a_{23} = - \begin{vmatrix} -2 & 1 \\ 4 & 0 \end{vmatrix} = 4$$

$$a_{31} = \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} = 0$$

$$a_{32} = - \begin{vmatrix} -2 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$a_{33} = \begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix} = -5$$

$$A^{\textcolor{red}{i}} = \begin{bmatrix} 6 & -3 & -8 \\ -3 & -6 & 4 \\ 0 & 0 & -5 \end{bmatrix}^T = \begin{bmatrix} 6 & -3 & 0 \\ -3 & -6 & 0 \\ -8 & 4 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} * A^{\textcolor{red}{i}} = \frac{1}{-15} \begin{bmatrix} 6 & -3 & 0 \\ -3 & -6 & 0 \\ -8 & 4 & -5 \end{bmatrix} = \begin{bmatrix} \frac{-2}{5} & \frac{1}{5} & 0 \\ \frac{1}{5} & \frac{2}{5} & 0 \\ \frac{8}{15} & \frac{4}{15} & \frac{1}{3} \end{bmatrix}$$