Derivacije i pravila za deriviranje - zadaci za vježbu

Definicija derivacije:

Ako je zadana funkcija $y=f(x)$ *njena derivacija se defininira kao:*

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

Tablica osnovnih derivacija:

$(c)'=0$, *gdje je c konsta*nta

$(x^n)'=nx^{n-1}$, *posebno za* $n=\frac{1}{2}$ $\left(\frac{1}{\sqrt{x}}\right)'=\frac{1}{2\sqrt{x}}$ *i za n*=1 $x'=1$

$(\sin x)'=\cos x$, $(\cos x)'=-\sin x$

$(tgx)'=\frac{1}{\cos^2 x}$, $(ctgx)'=-\frac{1}{\sin^2 x}$

$(\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$, $(\arccos x)'=-\frac{1}{\sqrt{1-x^2}}$

$(arctgx)'=\frac{1}{1+x^2}$, $(arcctgx)'=-\frac{1}{1+x^2}$

$(a^x)'=a^x \ln a$ *posebno* $(e^x)'=e^x$

$(\log_a x)'=\frac{1}{x \ln a}$ *posebno* $(\ln x)'=\frac{1}{x}$

Osnovna pravila za deriviranje

Derivacija zbroja: $(u+v)'=u'+v'$

Derivacija razlike: $(u-v)'=u'-v'$

Derivacija umnoška: $(u\cdot v)'=u'v+uv'$

Derivacija funkcije pomnožene s konstantom: $(Cu)'=Cu'$

Derivacija kvocijenta: $\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}$

ZADACI

1.

$$y = x^5 - 4x^3 + 2x - 3$$

$$y' = (x^5 - 4x^3 + 2x - 3)' = (x^5)' - (4x^3)' + (2x)' - 3' = 5x^4 - 4\cdot 3x^2 + 2\cdot 1 - 0 = 5x^4 - 12x^2 + 2$$

2.

$$y = \frac{1}{4} - \frac{1}{3}x + x^2 - 0{,}5x^4$$

$$y' = \left(\frac{1}{4}\right)' - \frac{1}{3}x' + (x^2)' - 0{,}5(x^4)' = 0 - \frac{1}{3} + 2x - 0{,}5\cdot 4x^3 = -\frac{1}{3} + 2x - 2x^3$$

3.

$$y = ax^2 + bx + c \qquad \{ y' = 2ax + b¿$$

4.

$$y = -\frac{5x^3}{a} \qquad y' = \left(-\frac{5}{a}x^3\right)' = -\frac{5}{a}\cdot 3x^2 = -\frac{15}{a}x^2$$

5.

$$y = at^m + bt^{m+n} \qquad \{ y' = amt^{m-1} + b(m+m)t^{m+n-1}¿$$

6.

$$y = \frac{ax^6 + b}{\sqrt{a^2 + b^2}} \qquad \{ y' = \left(\frac{a}{\sqrt{a^2+b^2}}x^6 + \frac{b}{\sqrt{a^2+b^2}}\right)' = \frac{a}{\sqrt{a^2+b^2}}\cdot 6x^5 = \frac{6ax^5}{\sqrt{a^2+b^2}}¿$$

7.

$$y = \frac{\pi}{x} + \ln 2 \qquad \{ y' = (\pi\cdot x^{-1} + \ln 2)' = \pi(-1)x^{-2} = -\frac{\pi}{x^2}¿$$

Drugi način po pravilu za deriviranje kvocijenta:

$$y = \frac{\pi}{x} + \ln 2 \qquad \{ y' = \frac{\pi' x - \pi x'}{x^2} + 0 = -\frac{\pi}{x^2}¿$$

8.

$$y=3x^{\frac{2}{3}}-2x^{\frac{5}{2}}+x^{-3} \qquad \{ y'=3\cdot\frac{2}{3}x^{\frac{2}{3}-1}-2\cdot\frac{5}{2}x^{\frac{5}{2}-1}+(-3)x^{-3-1}=2x^{-\frac{1}{3}}-5x^{\frac{3}{2}}-3x^{-4}\text{¿}$$

9.

$$y=x^2\sqrt[3]{x^2} \qquad y=x^2x^{\frac{2}{3}}=x^{\frac{8}{3}} \qquad \{ y'=\frac{8}{3}x^{\frac{8}{3}-1}=\frac{8}{3}x^{\frac{5}{3}}=\frac{8}{3}\sqrt[3]{x^5}=\frac{8}{3}x\sqrt[3]{x^2}\text{¿}$$

10.

$$y=\frac{a}{\sqrt[3]{x^2}}-\frac{b}{x\sqrt[3]{x}} \qquad y=ax^{-\frac{2}{3}}-bx^{-\frac{4}{3}} \qquad \{ y'=a\left(-\frac{2}{3}\right)x^{-\frac{5}{3}}-b\left(-\frac{4}{3}\right)x^{-\frac{7}{3}}=-\frac{2}{3}\frac{a}{\sqrt[3]{x^5}}+\frac{4}{3}\frac{b}{\sqrt[3]{x^7}}\text{¿}$$

11.

$$y=\frac{a+bx}{c+dx} \qquad \{ y'=\frac{(a+bx)'(c+dx)-(a+bx)(c+dx)'}{(c+dx)^2}=\frac{b(c+dx)-(a+bx)d}{(c+dx)^2}=\text{¿}=\frac{bc+bdx-ad-bdx}{(c+dx)^2}=\frac{bc-ad}{(c+dx)^2}\text{¿¿}$$

12.

$$y=\frac{2x+3}{x^2-5x+5} \qquad \{ y'=\frac{(2x+3)'(x^2-5x+5)-(2x+3)(x^2-5x+5)'}{(x^2-5x+5)^2}=\text{¿}=\frac{2(x^2-5x+5)-(2x+3)(2x-5)}{(x^2-5x+5)^2}=\frac{2x^2-10x+10-4x^2+10x-6x+15}{(x^2-5x+5)^2}=\frac{-2x^2-6x+25}{(x^2-5x+5)^2}\text{¿¿}$$

13.

$$y=\frac{2}{2x-1}-\frac{1}{x} \qquad y=\frac{2x-2x+1}{(2x-1)x}=\frac{1}{2x^2-x} \qquad \{ y'=\frac{0\cdot(2x^2-x)-1\cdot(4x-1)}{(2x^2-x)^2}=\frac{-4x+1}{(2x^2-x)^2}\text{¿}$$

14.

$$y=\frac{1+\sqrt{z}}{1-\sqrt{z}} \qquad \{ y'=\frac{(1+\sqrt{z})'(1-\sqrt{z})-(1+\sqrt{z})(1-\sqrt{z})'}{(1-\sqrt{z})^2}=\frac{\frac{1}{2\sqrt{z}}(1-\sqrt{z})-(1+\sqrt{z})\left(-\frac{1}{2\sqrt{z}}\right)}{(1-\sqrt{z})^2}=\text{¿}=\frac{\frac{1}{2\sqrt{z}}-\frac{1}{2}+\frac{1}{2\sqrt{z}}+\frac{1}{2}}{(1-\sqrt{z})^2}=\frac{\frac{2}{2\sqrt{z}}}{(1-\sqrt{z})^2}=\frac{1}{\sqrt{z}(1-\sqrt{z})^2}\text{¿¿}$$

15.

$$y=5\sin x+3\cos x \qquad \{\, y'=5\cos x+3(-\sin x)=5\cos x-3\sin x \text{¿}$$

16.

$$y=tgx-ctgx \qquad \{\, y'=\frac{1}{coc^2x}-\left(-\frac{1}{\sin^2x}\right)=\frac{1}{cos^2x}+\frac{1}{\sin^2x}=\frac{\sin^2x+\cos^2x}{\sin^2x\cos^2x}=\text{¿}=\frac{1}{\frac{1}{4}\sin^2 2x}=\frac{4}{\sin^2 2x}\text{¿¿}$$

17.

$$y'=\frac{\sin x+\cos x}{\sin x-\cos x} \qquad \{\, y'=\frac{(\sin x+\cos x)'(\sin x-\cos x)-(\sin x+\cos x)(\sin x-\cos x)'}{(\sin x-\cos x)^2}=\text{¿}=\frac{(\cos x-\sin x)(\sin x-\cos x)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2}=\text{¿}=\frac{\cos x\sin x-\cos^2x-\sin^2x+\sin x\cos x-\sin x\cos x-\sin^2x-\cos^2x-\cos x\sin x}{(\sin x-\cos x)^2}=\text{¿}=\frac{-2\sin^2x-2\cos^2x}{(\sin x-\cos x)^2}=\frac{-2(\sin^2x+\cos^2x)}{(\sin x-\cos x)^2}=\frac{-2}{(\sin x-\cos x)^2}\text{¿¿}$$

18.

$$y=2t\sin t-(t^2-2)\cos t \qquad \{\, y'=2\sin t+2t\cos t-2t\cos t-(t^2-2)(-\sin t)=\text{¿}=2\sin t+2t\cos t-2t\cos t'+t^2\sin t-2\sin t=t^2\sin t\text{¿¿}$$

19.

$$y=arctgx+arcctgx \qquad \{\, y'=\frac{1}{1+x^2}-\frac{1}{1+x^2}=0\text{¿}$$

20.

$$y=x\arcsin x \qquad \{\, y'=1\cdot\arcsin x+x\cdot\frac{1}{\sqrt{1-x^2}}=\arcsin x+\frac{x}{\sqrt{1-x^2}}\text{¿}$$

21.

$$y=xctgx \qquad \{\, y'=1\cdot \text{ctgx}+x\cdot\left(-\frac{1}{\sin^2x}\right)=ctgx-\frac{x}{\sin^2x}\text{¿}$$

22.

$$y=\frac{(1+x^2)\,arctgx-x}{2} \qquad y=\frac{1}{2}\left((1+x^2)\,arctgx-x\right)=\frac{1}{2}\left[2xarctgx+(1+x^2)\cdot\frac{1}{1+x^2}-1\right]=$$

$$¿\frac{1}{2}(2xarctgx+1-1)=xractgx$$

23.

$$y=x^7e^x \qquad \{y'=7x^6e^x+x^7e^x=x^6e^x(7+x)¿$$

24.

$$y=(x-1)e^x \qquad \{y'=1\cdot e^x+(x-1)e^x=e^x+xe^x-e^x=xe^x¿$$

25.

$$y=\frac{e^x}{x^2} \qquad \{y'=\frac{e^xx^2-e^x\cdot 2x}{x^4}=\frac{xe^x(x-2)}{x^4}=\frac{e^x(x-2)}{x^3}¿$$

26.

$$y=\frac{x^5}{e^x} \qquad \{y'=\frac{5x^4e^x-x^5e^x}{(e^x)^2}=\frac{x^4e^x(5-x)}{(e^x)^2}=\frac{x^4(5-x)}{e^x}¿$$

27.

$$y=e^x\cos x \qquad \{y'=e^x\cos x+e^x(-\sin x)=e^x(\cos x-\sin x)¿$$

28.

$$y=(x^2-2x+2)e^x \qquad \{y'=(2x\text{-}2)e^x+(x^2-2x+2)e^x=2xe^x-2e^x+x^2e^x-2xe^x+2e^x=x^2e^x¿$$

29.

$$y=e^x\arcsin x \qquad \{y'=e^x\arcsin x+e^x\cdot\frac{1}{\sqrt{1-x^2}}=e^x\left(\arcsin x+\frac{1}{\sqrt{1-x^2}}\right)¿$$

30.

$$y=\frac{x^2}{\ln x} \qquad \{y'=\frac{2x\ln x-x^2\frac{1}{x}}{\ln^2 x}=\frac{2x\ln x-x}{\ln^2 x}=\frac{x(2\ln x-1)}{\ln^2 x}¿$$

31.

$$y=x^3\ln x-\frac{x^3}{3} \qquad y=x^3\ln x-\frac{1}{3}x^3 \qquad \{y'=3x^2\ln x+x^3\cdot\frac{1}{x}-\frac{1}{3}\cdot 3x^2=3x^2\ln x+x^2-x^2=3x^2\ln x¿$$

32.

$$y=\frac{1}{x}+2\ln x-\frac{\ln x}{x} \quad \{y'=\frac{0\cdot 1}{x^2}+\frac{2}{x}-\frac{\frac{1}{x}x-\ln x}{x^2}=-\frac{1}{x^2}+\frac{2}{x}-\frac{1-\ln x}{x^2}=\text{¿}=\frac{-1+2x-1+\ln x}{x^2}=\frac{-2+2x+\ln x}{x^2}\text{¿¿}$$

33.

$$y=\ln x\log x-\ln a\log_a x \quad \{y'=\frac{1}{x}\log x+\ln x\frac{1}{x\ln 10}-\ln a\frac{1}{x\ln a}=\text{¿}=\frac{\ln x}{x\ln 10}+\frac{\ln x}{x\ln 10}-\frac{1}{x}=\frac{2\ln x}{x\ln 10}-\frac{1}{x}\text{¿¿}$$

Pravilo za deriviranje složene funkcije

Ako je funkcija složena $y=f(u)$ *, gdje je* $u=u(x)$ *tada se funkcija drivira po pravilu:*

$$y'_x=y'_u\cdot u'_x$$

ZADACI

34.

$$y=(x^2-3x+3)^5 \;,\; y=u^5,\; u=x^2-3x+3 \quad \{y'=10u^4(2x-3)=10(2x-3)(x^2-3x+3)^4\text{¿}$$

35.

$$y=\sin^4 4x \;,\; y=u^4 \;,\; u=\sin v \;,\; v=4x \;;\; \{y'=4u^3\cos v\cdot 4=16\sin^3 4x\cos 4x\text{¿}$$

36.

$$y=(1+3x-5x^2)^{10} \;;\; \{y'=10(1+3x-5x^2)^9(1+3x-5x^2)'=10(1+3x-5x^2)^9(3-10x)\text{¿}$$

37.

$$y=\left(\frac{ax+b}{c}\right)^3 \;;\; \{y'=3\left(\frac{ax+b}{c}\right)^2\left(\frac{ax+b}{c}\right)'=3\left(\frac{ax+b}{c}\right)^2\left(\frac{a}{c}x+\frac{b}{c}\right)'=\text{¿}=3\left(\frac{ax+b}{c}\right)^2\cdot\frac{a}{c}=\frac{3a}{c}\left(\frac{ax+b}{c}\right)^2\text{¿¿}$$

38.

$$y=\left(\frac{ax+b}{c}\right)^3\ ;\ \{y'=3\left(\frac{ax+b}{c}\right)^2\left(\frac{ax+b}{c}\right)'=3\left(\frac{ax+b}{c}\right)^2\left(\frac{a}{c}x+\frac{b}{c}\right)'=\text{¿}=3\left(\frac{ax+b}{c}\right)^2\cdot\frac{a}{c}=\frac{3a}{c}\left(\frac{ax+b}{c}\right)^2\text{¿¿}$$

39.

$$f(y)=(2a+3by)^2\ ;\ \{f'(y)=2(2a+3by)(2a+3by)'=2(2a+3by)\cdot 3b=6b(2a+3by)=\text{¿}=12ab+18b^2y\text{¿¿}$$

40.

$$y=(3+2x^2)^4\ ;\ \{y'=4(3+2x^2)^3\cdot(3+2x^2)'=4(3+2x^2)^3\cdot 4x=16x(3+2x^2)^3\text{¿}$$

41.

$$y=\frac{3}{56(2x-1)^7}-\frac{1}{24(2x-1)^6}-\frac{1}{40(2x-1)^5}\ ,\ y=\frac{3}{56}(2x-1)^{-7}-\frac{1}{24}(2x-1)^{-6}-\frac{1}{40}(2x-1)^{-5}$$

$$y'=\frac{3}{56}(-7))(2x-1)^{-8}(2x-1)'-\frac{1}{24}(-6)(2x-1)^{-7}(2x-1)'-\frac{1}{40}(-5)(2x-1)^{-6}(2x-1)'=$$

$$\text{¿}-\frac{3}{8}(2x-1)^{-8}\cdot 2+\frac{1}{4}(2x-1)^{-7}\cdot 2+\frac{1}{8}(2x-1)^{-6}\cdot 2=-\frac{3}{4(2x-1)^8}+\frac{1}{2(2x-1)^7}+\frac{1}{4(2x-1)^6}=$$

$$\text{¿}\frac{-3+2(2x-1)+(2x-1)^2}{4(2x-1)^8}=\frac{-3+4x-2+4x^2-4x+1}{4(2x-1)^8}=\frac{-4+4x^2}{4(2x-1)^8}=\frac{4(x^2-1)}{4(2x-1)^8}=\frac{(x^2-1)}{(2x-1)^8}$$

42.

$$y=\sqrt{1-x^2}\ ;\ \{y'=\frac{1}{2\sqrt{1-x^2}}\cdot(1-x^2)'=\frac{-2x}{2\sqrt{1-x^2}}=\frac{-x}{\sqrt{1-x^2}}\text{¿}$$

43.

$$y=\sqrt[3]{a+bx^3}=(a+bx^3)^{\frac{1}{3}}\ ;\ \{y'=\frac{1}{3}(a+bx^3)^{\frac{1}{3}-1}\cdot(a+bx^3)'=\frac{1}{3}(a+bx^3)^{-\frac{2}{3}}\cdot 3bx^2=\text{¿}=\frac{bx^2}{\sqrt[3]{(a+bx^3)^2}}\text{¿¿}$$

44.

$$y=\left(a^{\frac{2}{3}}-x^{\frac{2}{3}}\right)^{\frac{3}{2}}\ ;\ \{y'=\frac{3}{2}\left(a^{\frac{2}{3}}-x^{\frac{2}{3}}\right)^{\frac{3}{2}-1}\left(a^{\frac{2}{3}}-x^{\frac{2}{3}}\right)'=\frac{3}{2}\left(a^{\frac{2}{3}}-x^{\frac{2}{3}}\right)^{\frac{1}{2}}\left(-\frac{2}{3}x^{\frac{2}{3}-1}\right)=\text{¿}=-\left(a^{\frac{2}{3}}-x^{\frac{2}{3}}\right)^{\frac{1}{2}}\cdot x^{-\frac{1}{3}}=-\left(a^{\frac{2}{3}}-x^{\frac{2}{3}}\right)^{\frac{1}{2}}\left(x^{-\frac{2}{3}}\right)^{\frac{1}{2}}=-\left(a^{\frac{2}{3}}x^{-\frac{2}{3}}-1\right)^{\frac{1}{2}}=-\sqrt{\frac{a^{\frac{2}{3}}}{x^{\frac{2}{3}}}-1}=-\sqrt{\left(\frac{a^2}{x^2}\right)^{\frac{1}{3}}-1}=\text{¿}=-\sqrt{\sqrt[3]{\frac{a^2}{x^2}}-1}\text{¿¿}$$

45.

$$y=(3-2\sin x)^5\ ;\ \{y'=5(3-2\sin x)^4(3-2\sin x)'=5(3-2\sin x)^4(-2\cos x)=\text{¿}=-10\cos x(3-2\sin x)^4\text{¿¿}$$

46.

$$y=tgx-\frac{1}{3}tg^3x+\frac{1}{5}tg^5x \quad ; \quad \{ y'=\frac{1}{\cos^2 x}-\frac{1}{3}3tg^2x\cdot(tgx)'+\frac{1}{5}5tg^4x\cdot(tgx)'=¿=\frac{1}{\cos^2 x}-tg^2x\cdot\frac{1}{\cos^2 x}+tg^4x\cdot\frac{1}{\cos^2 x}=\frac{1}{\cos^2 x}\left(1-tg^2x+tg^4x\right)=\frac{1}{\cos^2 x}\left(1-\frac{\sin^2 x}{\cos^2 x}+\frac{\sin^4 x}{\cos^4 x}\right)¿=\frac{1}{\cos^2 x}\cdot\frac{\cos^4 x-\sin^2 x\cos^2 x+\sin^4 x}{\cos^4 x}=\frac{1}{\cos^2 x}\cdot\frac{\cos^4 x-\sin^2 x\left(1-\sin^2 x\right)}{\cos^4 x}¿=\frac{1}{\cos^2 x}\cdot\frac{\cos^4 x-\sin^2 x\cos^2 x}{\cos^4 x}=\frac{\cos^2 x\left(\cos^2 x-\sin^2 x\right)}{\cos^2 x\cos^4 x}=\frac{\cos 2x}{\cos^4 x}¿¿$$

47.

$$y=\sqrt{ctgx}-\sqrt{ctg\alpha} \quad ; \quad \{ y'=\frac{1}{2\sqrt{ctgx}}\cdot(ctgx)'-0=\frac{1}{2\sqrt{ctgx}}\cdot\left(-\frac{1}{\sin^2 x}\right)=-\frac{1}{2\sin^2 x\sqrt{ctgx}}¿$$

48. $y=2x+5\cos^3 x \quad ; \quad \{ y'=2+5\cdot 3\cos^2 x\cdot(\cos x)'=2-15\sin x\cos^2 x¿$

49.

$$f(x)=-\frac{1}{6(1-3\cos x)^2} \ , \ f(x)=-\frac{1}{6}(1-3\cos x)^{-2} \quad ;$$

$$f(x)=-\frac{1}{6}\cdot(-2)(1-3\cos x)^{-3}\cdot(1-3\cos x)'=\frac{1}{3}(1-3\cos x)^{-3}(-3\cdot(-\sin x))=\frac{\sin x}{(1-3\cos x)^3}$$

50.

$$y=\frac{1}{3\cos^3 x}-\frac{1}{\cos x} \ , \ y=\frac{1}{3}\cos^{-3}x-\cos^{-1}x$$

$$y'=\frac{1}{3}(-3)\cos^{-4}x\cdot(\cos x)'-(-1)\cos^{-2}x\cdot(\cos x)'=\frac{\sin x}{\cos^4 x}-\frac{\sin x}{\cos^2 x}=$$

$$¿\sin x\left(\frac{1-\cos^2 x}{\cos^4 x}\right)=\frac{\sin^3 x}{\cos^4 x}$$

51.

$$y=\sqrt{\frac{3\sin x-2\cos x}{5}} \ , \ y=\frac{1}{\sqrt{5}}\sqrt{3\sin x-2\cos x}$$

$$y'=\frac{1}{\sqrt{5}}\cdot\frac{1}{2\sqrt{3\sin x-2\cos x}}\cdot(3\sin x-2\cos x)'=\frac{3\cos x+2\sin x}{2\sqrt{5}\sqrt{3\sin x-2\cos x}}$$

52. $y=\sqrt[3]{\sin^2 x}+\frac{1}{\cos^3 x}$, $y=\sin^{\frac{2}{3}} x-\cos^{-3} x$

$y=\frac{2}{3}\sin^{\frac{2}{3}-1} x(\sin x)'-(-3)\cos^{-4} x(\cos x)'=\frac{2}{3}\sin^{-\frac{1}{3}} x\cdot(\cos x)+\frac{3}{\cos^4 x}(-\sin x)=$

$¿\frac{2\cos x}{3\sqrt[3]{\sin x}}-\frac{3\sin x}{\cos^4 x}$

53. $y=\sqrt{1+\arcsin x}$; $\{y'=\frac{1}{2\sqrt{1+\arcsin x}}(1+\arcsin x)'=\frac{\frac{1}{\sqrt{1-x^2}}}{2\sqrt{1+\arcsin x}}=¿=\frac{1}{2\sqrt{1-x^2}\sqrt{1+\arcsin x}}¿¿$

54. $y=\sqrt{arctgx}-(\arcsin x)^3$; $\{y'=\frac{1}{2\sqrt{arctgx}}(arctgx)'-3(\arcsin x)^2\cdot(\arcsin x)'=¿=\frac{1}{2\sqrt{arctgx}}\frac{1}{1+x^2}-3(\arcsin x)^2\frac{1}{\sqrt{1-x^2}}=\frac{1}{21+x^2\sqrt{arctgx}}-\frac{3(\arcsin x)^2}{\sqrt{1-x^2}}¿¿$

55. $y=\sin 3x+\cos\frac{x}{5}+\tan\sqrt{x}$, $\{y'=\cos 3x\cdot(3x)'-\sin\frac{x}{5}\cdot\left(\frac{1}{5}x\right)'+\frac{1}{\cos^2\sqrt{x}}\cdot(\sqrt{x})'=¿=3\cos 3x-\frac{1}{5}\sin\frac{x}{5}+\frac{1}{2\sqrt{x}}\cdot\frac{1}{\cos^2\sqrt{x}}¿¿$

56. $y=\sin(x^2-5x+1)+\tan\frac{a}{x}$, $\{y'=\cos(x^2-5x+1)\cdot(x^2-5x+1)'+\frac{1}{\cos^2\frac{a}{x}}\cdot\left(\frac{a}{x}\right)'=¿=(2x-5)\cos(x^2-5x+1)+\frac{1}{\cos^2\frac{a}{x}}\cdot\frac{a'x-ax'}{x^2}=(2x-5)\cos(x^2-5x+1)+\frac{1}{\cos^2\frac{a}{x}}\cdot\frac{0-a}{x^2}¿=(2x-5)\cos(x^2-5x+1)-\frac{a}{x^2}\cdot\frac{1}{\cos^2\frac{a}{x}}=(2x-5)\cos(x^2-5x+1)-\frac{a}{x^2\cos^2\frac{a}{x}}¿¿$

57. $f(x)=\cos(\alpha x+\beta)$, $\{f'(x)=-\sin(\alpha x+\beta)\cdot(\alpha x+\beta)'=-\alpha\sin(\alpha x+\beta)¿$

58. $f(t)=\sin t\sin(t+\phi)$, $\{f'(t)=(\sin t)'\sin(t+\phi)+\sin t\cdot(\sin(t+\phi))'=¿=\cos t\sin(t+\phi)+\sin t\sin(t+\phi)\cdot(t+\phi)'=\cos t\sin(t+\phi)+\sin t\sin(t+\phi)=\sin(2t+\phi)¿¿$

59.

$$y=\frac{1+\cos 2\alpha}{1-\cos 2\alpha}\ ,\quad \{y'=\frac{(1+\cos 2\alpha)'(1-\cos 2\alpha)-(1-\cos 2\alpha)'(1+\cos 2\alpha)}{(1-\cos 2\alpha)^2}=\text{¿}\frac{-\sin 2\alpha\cdot(2\alpha)'(1-\cos 2\alpha)-\sin 2\alpha\cdot(2\alpha)'(1+\cos 2\alpha)}{(1-\cos 2\alpha)^2}=\text{¿}\frac{-2\sin 2\alpha(1-\cos 2\alpha)-2\sin 2\alpha(1+\cos 2\alpha)}{(1-\cos 2\alpha)^2}=\text{¿}\frac{-2\sin 2\alpha+2\sin 2\alpha\cos 2\alpha-2\sin 2\alpha-2\sin 2\alpha\cos 2\alpha}{(1-\cos 2\alpha)^2}=\text{¿}\frac{-4\sin 2\alpha}{(2\sin^2\alpha)^2}=\frac{-8\sin\alpha\cos\alpha}{4\sin^4\alpha}=\frac{-2\cos\alpha}{\sin^3\alpha}\text{¿¿}$$

60.

$$f(x)=a\cot\frac{x}{a}\ ,\quad \{f'(x)=a\left(-\frac{1}{\sin^2\frac{x}{a}}\right)\cdot\left(\frac{1}{a}x\right)'=-a\frac{1}{\sin^2\frac{x}{a}}\cdot\frac{1}{a}=-\frac{1}{\sin^2\frac{x}{a}}\text{¿}$$

61.

$$y=-\frac{1}{20}\cos(5x^2)-\frac{1}{4}\cos x^2\ ;\quad \{y'=-\frac{1}{20}\left(-\sin(5x^2)\right)(5x^2)'-\frac{1}{4}\left(-\sin x^2\right)(x^2)'=\text{¿}\frac{1}{20}\sin(5x^2)\cdot 10x+\frac{1}{4}\sin x^2\cdot 2x=\frac{x}{2}\sin(5x^2)+\frac{x}{2}\sin x^2=\frac{x}{2}\left[\sin(3x^2+2x^2)+\sin(3x^2-2x^2)\right]=\text{¿}\frac{x}{2}\left[\sin 3x^2\cos 2x^2+\cos 3x^2\sin 2x^2+\sin 3x^2\cos 2x^2-\cos 3x^2\sin 2x^2\right]=\frac{x}{2}2\sin 3x^2\cos 2x^2=\text{¿}x\sin 3x^2\cos 2x^2\text{¿¿}$$

62.

$$y=\arcsin 2x\ ;\quad \{y'=\frac{1}{\sqrt{1-(2x)^2}}\cdot(2x)'=\frac{2}{\sqrt{1-4x^2}}\text{¿}$$

63.

$$y=\arcsin\frac{1}{x^2}\ ;\quad \{y'=\frac{1}{\sqrt{1-\left(\frac{1}{x^2}\right)^2}}\cdot\left(\frac{1}{x^2}\right)'=\frac{1}{\sqrt{1-\frac{1}{x^4}}}\cdot\frac{1'x^2-1\cdot 2x}{x^4}=-\frac{1}{\sqrt{\frac{x^4-1}{x^4}}}\cdot\frac{2}{x^3}=\text{¿}-\frac{2}{x^3}\cdot\frac{1}{\frac{\sqrt{x^2-1}}{x^2}}=-\frac{2}{x\sqrt{x^4-1}}\text{¿¿}$$

64.

$$f(x)=\arccos\sqrt{x}\ ;\quad \{f'(x)=-\frac{1}{\sqrt{1-x^2}}\cdot(\sqrt{x})'=-\frac{1}{2\sqrt{x}}\frac{1}{\sqrt{1-x^2}}=-\frac{1}{2\sqrt{x-x^3}}\text{¿}$$

65. $y=\arctan\frac{1}{x}$; $\{y'=\frac{1}{1+\left(\frac{1}{x}\right)^2}\cdot\left(\frac{1}{x}\right)'=\frac{1}{1+\frac{1}{x^2}}\cdot\left(-\frac{1}{x^2}\right)=-\frac{1}{\frac{x^2+1}{x^2}}\cdot\frac{1}{x^2}=-\frac{1}{1+x^2}\text{¿}$

66. $y=\arctan\frac{1+x}{1-x}$; $\{y'=\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2}\cdot\left(\frac{1+x}{1-x}\right)'=\frac{1}{\frac{(1-x)^2+(1+x)^2}{(1-x)^2}}\cdot\frac{(1+x)'(1-x)-(1+x)(1-x)'}{(1-x)^2}=\text{¿}=\frac{1}{(1-x)^2+(1+x)^2}\cdot(1\cdot(1-x)-(1+x)\cdot(-1))=\frac{1-x+1+x}{1-2x+x^2+1+2x+x^2}=\frac{2}{2+2x^2}=\text{¿}=\frac{2}{2(1+x^2)}=\frac{1}{1+x^2}\text{¿¿}$

67. $y=5e^{-x^2}$; $\{y'=5e^{-x^2}\text{¿}(-x^2)'=5e^{-x^2}\text{¿}(-2x)=-10xe^{-x^2}\text{¿}$

68. $y=\frac{1}{5^{x^2}}$; $\{y'=\frac{1'\cdot 5^{x^2}-1\cdot\left(5^{x^2}\right)'}{\left(5^{x^2}\right)^2}=\frac{-5^{x^2}\text{¿}\ln 5\cdot(x^2)'}{\left(5^{x^2}\right)^2}=\frac{-2x\ln 5}{5^{x^2}}=-2x5^{-x^2}\ln 5\text{¿}$ *drugi način je da prvo funkciju napišemo u obliku* $y=5^{-x^2}$ *tada je* $\text{¿}\,y'=5^{-x^2}\ln 5\cdot(-x^2)'=-2x5^{-x^2}\ln 5\text{¿¿}$

69. $y=x^2 10^{2x}$; $\{y'=(x^2)'10^{2x}+x^2\left(10^{2x}\right)'=2x10^{2x}+x^2 10^{2x}\ln 10\cdot(2x)'=\text{¿}=2x10^{2x}+x^2 10^{2x}\ln 10\cdot 2=2x10^{2x}(1+x\ln 10)\text{¿¿}$

70. $f(t)=t\sin 2^t$; $\{f'(t)=t'\sin 2^t+t\left(\sin 2^t\right)'=\sin 2^t+t\cos 2^t\cdot\left(2^t\right)'=\sin 2^t+2^t t\cos 2^t\ln 2\text{¿}$

71. $y=\arccos e^x$; $\{y'=-\frac{1}{\sqrt{1-(e^x)^2}}\cdot(e^x)'=\frac{-e^x}{\sqrt{1-e^{2x}}}\text{¿}$

72. $y=\ln(2x+7)$; $\{y'\frac{1}{2x+7}\cdot(2x+7)'=\frac{2}{2x+7}\text{¿}$

73. $y=\log\sin x$; $\{y'=\frac{1}{\sin x\ln 10}\cdot(\sin x)'=\frac{\cos x}{\sin x\ln 10}=\cot x\log e\text{¿}$

74. $y=\ln(1-x^2)$; $\{y'=\frac{1}{1-x^2}\cdot(1-x^2)'=\frac{-2x}{1-x^2}\text{¿}$

75. $y=\ln^2 x-\ln(\ln x)$; $\{y'=2\ln x\cdot(\ln x)'-\frac{1}{\ln x}\cdot(\ln x)'=\frac{2\ln x}{x}-\frac{1}{x\ln x}\text{¿}$

76.

$$y=\ln\left(e^x+5\sin x-4\arcsin x\right)\ ;\ \ y'=\frac{1}{e^x+5\sin x-4\arcsin x}\left(e^x+5\sin x-4\arcsin x\right)'=\frac{e^x+5\cos x-\frac{4}{\sqrt{1-x^2}}}{e^x+5\sin x-4\arcsin x}=\frac{\sqrt{1-x^2}\left(e^x+5\cos x\right)-4}{\sqrt{1-x^2}\left(e^x+5\sin x-4\arcsin x\right)}$$

$$y=\arctan\left(\ln x\right)+\ln\left(\arctan x\right)\ ;\ \ y'=\frac{1}{1+\left(\ln x\right)^2}\left(\ln x\right)'+\frac{1}{\arctan x}\left(\arctan x\right)'=\frac{1}{\left(1+\ln^2 x\right)}\cdot\frac{1}{x}+\frac{1}{\arctan x}\cdot\frac{1}{1+x^2}=\frac{1}{x\left(1+\ln^2 x\right)}+\frac{1}{\left(1+x^2\right)\arctan x}$$

77.

$$y=\sqrt{\ln x+1}+\ln\left(\sqrt{x}+1\right)\ ;\ \ y'=\frac{1}{2\sqrt{\ln x+1}}\left(\ln x+1\right)'+\left(\sqrt{x}+1\right)'=\frac{1}{2\sqrt{\ln x+1}}\cdot\frac{1}{x}+\frac{1}{\sqrt{x}+1}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2x\sqrt{\ln x+1}}+\frac{1}{2\left(x+\sqrt{x}\right)}$$

78.

79.

$$y=\sin^3 5x\cos^2\frac{x}{3}\ ,\ \ y'=\left(\sin^3 5x\right)'\cos^2\frac{x}{3}+\sin^3 5x\left(\cos^2\frac{x}{3}\right)'=3\sin^2 5x\left(\sin 5x\right)'\cos^2\frac{x}{3}+\sin^3 5x\cdot 2\cos\frac{x}{3}\cdot\left(\cos\frac{x}{3}\right)'=3\sin^2 5x\cos 5x\cdot\left(5x\right)'\cos^2\frac{x}{3}+2\sin^3 5x\cos\frac{x}{3}\cdot\left(-\sin\frac{x}{3}\right)\cdot\left(\frac{x}{3}\right)'=3\sin^2 5x\cos 5x\cdot 5\cdot\cos^2\frac{x}{3}-2\sin^3 5x\cos\frac{x}{3}\cdot\sin\frac{x}{3}\cdot\frac{1}{3}=15\sin^2 5x\cos 5x\cos^2\frac{x}{3}-\frac{2}{3}\sin^3 5x\cos\frac{x}{3}\cdot\sin\frac{x}{3}$$

$$y=-\frac{11}{2\left(x-2\right)^2}-\frac{4}{x-2}\ ,\ \ y=-\frac{11}{2}\left(x-2\right)^{-2}-4\left(x-2\right)^{-1}$$

$$y'=-\frac{11}{2}\cdot\left(-2\right)\left(x-2\right)^{-3}\cdot\left(x-2\right)'-4\cdot\left(-1\right)\left(x-2\right)^{-2}\left(x-2\right)'=\frac{11}{\left(x-2\right)^3}\cdot 1+\frac{4}{\left(x-2\right)^2}\cdot 1=$$

$$=\frac{11+4x-8}{\left(x-2\right)^3}=\frac{3+4x}{\left(x-2\right)^3}$$

80.

$$y=-\frac{15}{4\left(x-3\right)^4}-\frac{10}{3\left(x-3\right)^3}-\frac{1}{2\left(x-3\right)^2}\ ,\ \ y=-\frac{15}{4}\left(x-3\right)^{-4}-\frac{10}{3}\left(x-3\right)^{-3}-\frac{1}{2}\left(x-3\right)^{-2}$$

$$y'=-\frac{15}{4}\cdot\left(-4\right)\left(x-3\right)^{-5}\left(x-3\right)'-\frac{10}{3}\cdot\left(-3\right)\left(x-3\right)^{-4}\left(x-3\right)'-\frac{1}{2}\cdot\left(-2\right)\left(x-3\right)^{-3}\left(x-3\right)'=$$

$$=15\frac{1}{\left(x-3\right)^5}\cdot 1+10\frac{1}{\left(x-3\right)^4}\cdot 1+\frac{1}{\left(x-3\right)^3}\cdot 1=\frac{15+10\left(x-3\right)+\left(x-3\right)^2}{\left(x-3\right)^5}=$$

$$=\frac{15+10x-30+x^2-6x+9}{\left(x-3\right)^5}=\frac{x^2-4x+6}{\left(x-3\right)^5}$$

81.

82.
$$y=\frac{x^8}{8\left(1-x^2\right)^4};\quad y=\frac{1}{8}\frac{x^8}{\left(1-x^2\right)^4};\quad y'=\frac{1}{8}\frac{\left(x^8\right)'\left(1-x^2\right)^4-x^8\left(\left(1-x^2\right)^4\right)'}{\left(1-x^2\right)^8}=$$
$$¿\frac{1}{8}\frac{8x^7\left(1-x^2\right)^4-x^8 4\left(1-x^2\right)^3\cdot\left(1-x^2\right)'}{\left(1-x^2\right)^8}=\frac{1}{8}\frac{8x^7\left(1-x^2\right)^4-x^8 4\left(1-x^2\right)^3\cdot\left(-2x\right)}{\left(1-x^2\right)^8}=$$
$$¿\frac{1}{8}\frac{8x^7\left(1-x^2\right)^4+8x^9\left(1-x^2\right)^3}{\left(1-x^2\right)^8}=\frac{8x^7\left(1-x^2\right)^3\left(1-x^2+x^2\right)}{8\left(1-x^2\right)^8}=\frac{x^7}{\left(1-x^2\right)^5}¿¿$$

83.
$$y=\frac{\sqrt{2x^2-2x+1}}{x};\quad y'=\frac{\left(\sqrt{2x^2-2x+1}\right)'x-\sqrt{2x^2-2x+1}\,x'}{x^2}=$$
$$¿\frac{\frac{1}{2\sqrt{2x^2-2x+1}}\left(2x^2-2x+1\right)'x-\sqrt{2x^2-2x+1}}{x^2}=¿\frac{\frac{1}{2\sqrt{2x^2-2x+1}}\left(4x-2\right)x-\sqrt{2x^2-2x+1}}{x^2}=\frac{\frac{2\left(2x-1\right)x}{2\sqrt{2x^2-2x+1}}-\sqrt{2x^2-2x+1}}{x^2}=$$
$$¿\frac{\frac{\left(2x-1\right)x}{\sqrt{2x^2-2x+1}}-\sqrt{2x^2-2x+1}}{x^2}=\frac{\frac{\left(2x^2-x\right)-\left(2x^2-2x+1\right)}{\sqrt{2x^2-2x+1}}}{x^2}=¿\frac{\frac{2x^2-x-2x^2+2x-1}{\sqrt{2x^2-2x+1}}}{x^2}=\frac{x-1}{x^2\sqrt{2x^2-2x+1}}¿¿$$

84.
$$y=\frac{x}{a^2\sqrt{a^2+x^2}}\quad,\quad y=\frac{1}{a^2}\frac{x}{\sqrt{a^2+x^2}}$$
$$y'=\frac{1}{a^2}\frac{x'\sqrt{a^2+x^2}-x\left(\sqrt{a^2+x^2}\right)'}{\left(\sqrt{a^2+x^2}\right)^2}=\frac{1}{a^2}\frac{\sqrt{a^2+x^2}-x\frac{1}{2\sqrt{a^2+x^2}}\left(a^2+x^2\right)'}{a^2+x^2}=$$
$$¿\frac{1}{a^2}\frac{\sqrt{a^2+x^2}-x\frac{1}{2\sqrt{a^2+x^2}}\cdot 2x}{a^2+x^2}=\frac{1}{a^2}\frac{\sqrt{a^2+x^2}-\frac{x^2}{\sqrt{a^2+x^2}}}{a^2+x^2}=$$
$$¿\frac{1}{a^2}\frac{\frac{a^2+x^2-x^2}{\sqrt{a^2+x^2}}}{a^2+x^2}=\frac{1}{a^2}\frac{a^2}{\left(a^2+x^2\right)\sqrt{a^2+x^2}}=\frac{1}{\left(a^2+x^2\right)\sqrt{a^2+x^2}}=\frac{1}{\sqrt{\left(a^2+x^2\right)^3}}$$

$$y=\frac{x^3}{3\sqrt{(1+x^2)^3}} \quad , \quad y=\frac{1}{3}x^3(1+x^2)^{-\frac{3}{2}}$$

$$¿\frac{1}{3}\left[(x^3)'(1+x^2)^{-\frac{3}{2}}+x^3\left((1+x^2)^{-\frac{3}{2}}\right)'\right]=\frac{1}{3}\left[3x^2(1+x^2)^{-\frac{3}{2}}+x^3\cdot\left(-\frac{3}{2}\right)(1+x^2)^{-\frac{5}{2}}(1+x^2)'\right]=$$

$$¿\frac{3}{3}\left[x^2(1+x^2)^{-\frac{3}{2}}-\frac{1}{2}x^3\cdot(1+x^2)^{-\frac{5}{2}}2x\right]=x^2(1+x^2)^{-\frac{3}{2}}-x^4\cdot(1+x^2)^{-\frac{5}{2}}=$$

$$¿(1+x^2)^{-\frac{5}{2}}\left(x^2(1+x^2)-x^4\right)=(1+x^2)^{-\frac{5}{2}}\left(x^2+x^4-x^4\right)=\frac{x^2}{\sqrt{(1+x^2)^5}}$$

85.

86.

$$y=\frac{3}{2}\sqrt[3]{x^2}+\frac{18}{7}x\sqrt[6]{x}+\frac{9}{5}x\sqrt[3]{x^2}+\frac{6}{13}x^2\cdot\sqrt[6]{x} \; , \; y=\frac{3}{2}x^{\frac{2}{3}}+\frac{18}{7}x\cdot x^{\frac{1}{6}}+\frac{9}{5}x\cdot x^{\frac{2}{3}}+\frac{6}{13}x^2\cdot x^{\frac{1}{6}}=$$

$$=\frac{3}{2}\cdot\frac{2}{3}x^{-\frac{1}{3}}+\frac{18}{7}\cdot\frac{7}{6}x^{\frac{1}{6}}+\frac{9}{5}\cdot\frac{5}{3}x^{\frac{2}{3}}+\frac{6}{13}\cdot\frac{13}{6}x^{\frac{7}{6}}=¿=x^{-\frac{1}{3}}+3x^{\frac{1}{6}}+3x^{\frac{2}{3}}+x^{\frac{7}{6}}=x^{-\frac{1}{3}}\left(1+3x^{\frac{1}{2}}+3x^1+x^{\frac{3}{2}}\right)=x^{-\frac{1}{3}}\left(1+x^{\frac{1}{2}}\right)^3=\frac{(1+\sqrt{x})^3}{\sqrt[3]{x}}¿¿$$

$$¿\frac{3}{2}x^{\frac{2}{3}}+\frac{18}{7}x^{\frac{7}{6}}+\frac{9}{5}x^{\frac{5}{3}}+\frac{6}{13}x^{\frac{13}{6}} \quad ; \quad \{y'$$

$$y=\frac{1}{8}\sqrt[3]{(1+x^3)^8}-\frac{1}{5}\sqrt[3]{(1+x^3)^5} \quad , \quad y=\frac{1}{8}(1+x^3)^{\frac{8}{3}}-\frac{1}{5}(1+x^3)^{\frac{5}{3}}$$

$$y'=\frac{1}{8}\cdot\frac{8}{3}(1+x^3)^{\frac{5}{3}}(1+x^3)'-\frac{1}{5}\cdot\frac{5}{3}(1+x^3)^{\frac{2}{3}}(1+x^3)'=\frac{1}{3}(1+x^3)^{\frac{5}{3}}\cdot 3x^2-\frac{1}{3}(1+x^3)^{\frac{2}{3}}\cdot 3x^2=$$

87. $$¿x^2(1+x^3)^{\frac{2}{3}}\left((1+x^3)^{\frac{3}{3}}-1\right)=x^2\sqrt[3]{(1+x^3)^2}(1+x^3-1)=x^5\cdot\sqrt[3]{(1+x^3)^2}$$

$$y=\frac{4}{3}\sqrt[4]{\frac{x-1}{x+2}} \; , \; y=\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} \; ; \; \{y'=\frac{4}{3}\cdot\frac{1}{4}\left(\frac{x-1}{x+2}\right)^{-\frac{3}{4}}\left(\frac{x-1}{x+2}\right)'=¿=\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{-\frac{3}{4}}\frac{(x-1)'(x+2)-(x-1)(x+2)'}{(x+2)^2}=\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{-\frac{3}{4}}\frac{x+2-x+1}{(x+2)^2}=¿=\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{-\frac{3}{4}}\frac{3}{(x+2)^2}=\left(\frac{x+2}{x-1}\right)^{\frac{3}{4}}\frac{1}{(x+2)^2}=\sqrt[4]{\frac{(x+2)^3}{(x-1)^3(x+2)^8}}=¿=\frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}¿¿$$

88.

$$y=x^4(a-2x^3)^2 \; ; \; \{y'=(x^4)'(a-2x^3)^2+x^4\left((a-2x^3)^2\right)'=¿=4x^3(a-2x^3)^2+x^4\cdot 2(a-2x^3)\cdot(a-2x^3)'=(a-2x^3)\left[4x^3(a-2x^3)+2x^4\cdot(-6x^2)\right]=¿=(a-2x^3)(4x^3a-8x^6-12x^6)=(a-2x^3)(4x^3a-20x^6)=4x^3(a-2x^3)(a-5x^3)¿¿$$

89.

$$y=\left(\frac{a+bx^n}{a-bx^n}\right)^m;\quad y'=m\left(\frac{a+bx^n}{a-bx^n}\right)^{m-1}\left(\frac{a+bx^n}{a-bx^n}\right)' \text{¿}= m\left(\frac{a+bx^n}{a-bx^n}\right)^{m-1}\frac{(a+bx^n)'(a-bx^n)-(a+bx^n)(a-bx^n)'}{(a-bx^n)^2} \text{¿}= m\left(\frac{a+bx^n}{a-bx^n}\right)^{m-1}\frac{nbx^{n-1}(a-bx^n)+nbx^{n-1}(a+bx^n)}{(a-bx^n)^2} \text{¿}= m\left(\frac{a+bx^n}{a-bx^n}\right)^{m-1}\frac{nbx^{n-1}(a-bx^n+a+bx^n)}{(a-bx^n)^2} = 2mnabx^{n-1}\frac{(a+bx^n)^{m-1}}{(a-bx^n)^{m+1}} \text{¿¿}$$

90.

$$y=\frac{9}{5(x+2)^5}-\frac{3}{(x+2)^4}+\frac{2}{(x+2)^3}-\frac{1}{2(x+2)^2}$$

$$y=\frac{9}{5}(x+2)^{-5}-3(x+2)^{-4}+2(x+2)^{-3}-\frac{1}{2}(x+2)^{-2}$$

$$y'=\frac{9}{5}(-5)(x+2)^{-6}-3(-4)(x+2)^{-5}+2(-3)(x+2)^{-4}-\frac{1}{2}(-2)(x+2)^{-3}=$$

$$\text{¿}-9(x+2)^{-6}+12(x+2)^{-5}-6(x+2)^{-4}+(x+2)^{-3}=$$

$$\text{¿}(x+2)^{-6}\left[-9+12(x+2)-6(x+2)^2+(x+2)^3\right]=$$

$$\text{¿}\frac{-9+12x+24-6x^2-24x-24+x^3+6x^2+12x+8}{(x+2)^6}=\frac{x^3-1}{(x+2)^6}$$

91.

$$y=(a+x)\sqrt{a-x};\quad y'=(a+x)'\sqrt{a-x}+(a+x)\left(\sqrt{a-x}\right)' \text{¿}=1\sqrt{a-x}+\frac{a+x}{2\sqrt{a-x}}(a-x)'=\sqrt{a-x}+\frac{a+x}{2\sqrt{a-x}}(-1)=\sqrt{a-x}-\frac{a+x}{2\sqrt{a-x}} \text{¿}=\frac{2a-2x-a-x}{2\sqrt{a-x}}=\frac{a-3x}{2\sqrt{a-x}} \text{¿¿}$$

92.

93.

$$y=\sqrt{(x+a)(x+b)(x+c)};\quad y'=\frac{1}{2\sqrt{(x+a)(x+b)(x+c)}}\left[(x+a)(x+b)(x+c)\right]' \text{¿}=\frac{1}{2\sqrt{(x+a)(x+b)(x+c)}}\left[(x+a)'(x+b)(x+c)+(x+a)(x+b)'(x+c)+(x+a)(x+b)(x+c)'\right] \text{¿}=\frac{1}{2\sqrt{(x+a)(x+b)(x+c)}}\left[(x+b)(x+c)+(x+a)(x+c)+(x+a)(x+b)\right] \text{¿}=\frac{x^2+ax+bx+bc+x^2+xc+xa+ac+x^2+xb+xa+ab}{2\sqrt{(x+a)(x+b)(x+c)}} \text{¿}=\frac{x^2+2ax+2bx+2ax+ab+ac+bc}{2\sqrt{(x+a)(x+b)(x+c)}}=\frac{x^2+2x(c+b+a)+ab+ac+bc}{2\sqrt{(x+a)(x+b)(x+c)}} \text{¿¿}$$

94.

$$z=\sqrt[3]{y+\sqrt{y}}\ ,\ z=\left(y+y^{\frac{1}{2}}\right)^{\frac{1}{3}}\quad ;\quad \{z'=\frac{1}{3}\left(y+\sqrt{y}\right)^{-\frac{2}{3}}\left(y+y^{\frac{1}{2}}\right)'=\frac{1}{3\sqrt[3]{\left(y+\sqrt{y}\right)^{2}}}\left(1+\frac{1}{2\sqrt{y}}\right)=\text{¿}=\frac{1}{3\sqrt[3]{\left(y+\sqrt{y}\right)^{2}}}\frac{2\sqrt{y}+1}{2\sqrt{y}}=\frac{2\sqrt{y}+1}{6\sqrt{y}\sqrt[3]{\left(y+\sqrt{y}\right)^{2}}}\text{¿¿}$$

$$f(t)=(2t+1)(3t+2)\sqrt[3]{3t+2}\quad ,\quad f(t)=(2t+1)(3t+2)(3t+2)^{\frac{1}{3}}=(2t+1)(3t+2)^{\frac{4}{3}}$$

$$f'(t)=(2t+1)'(3t+2)^{\frac{4}{3}}+(2t+1)\left((3t+2)^{\frac{4}{3}}\right)'=2(3t+2)^{\frac{4}{3}}+(2t+1)\frac{4}{3}(3t+2)^{\frac{1}{3}}(3t+2)'=$$

$$\text{¿}(3t+2)^{\frac{1}{3}}\left(2(3t+2)+\frac{4}{3}(2t+1)\cdot 3\right)=\sqrt[3]{3t+2}(6t+4+8t+4)=2(7t+4)\sqrt[3]{3t+2}$$

95.

$$x=\frac{1}{\sqrt{2ay-y^{2}}}\ ,\ x=\left(2ay-y^{2}\right)^{-\frac{1}{2}}\quad ;\quad \{x'=-\frac{1}{2}\left(2ay-y^{2}\right)^{-\frac{3}{2}}\left(2ay-y^{2}\right)'=\text{¿}=-\frac{1\,1}{2\sqrt{\left(2ay-y^{2}\right)^{3}}}(2a-2y)=-\frac{1\,2(a-y)}{2\sqrt{\left(2ay-y^{2}\right)^{3}}}=\frac{y-a}{\sqrt{\left(2ay-y^{2}\right)^{3}}}\text{¿¿}$$

96.

$$y=\ln\left(\sqrt{1+e^{x}}-1\right)-\ln\left(\sqrt{1+e^{x}}+1\right)\quad ;$$

$$y'=\frac{1}{\sqrt{1+e^{x}}-1}\cdot\left(\sqrt{1+e^{x}}-1\right)'-\frac{1}{\sqrt{1+e^{x}}+1}\cdot\left(\sqrt{1+e^{x}}+1\right)'=$$

$$\text{¿}\frac{1}{\sqrt{1+e^{x}}-1}\cdot\frac{1}{2\sqrt{1+e^{x}}}\cdot\left(1+e^{x}\right)'-\frac{1}{\sqrt{1+e^{x}}+1}\cdot\frac{1}{2\sqrt{1+e^{x}}}\cdot\left(1+e^{x}\right)'=$$

$$\text{¿}\frac{1}{\sqrt{1+e^{x}}-1}\cdot\frac{e^{x}}{2\sqrt{1+e^{x}}}-\frac{1}{\sqrt{1+e^{x}}+1}\cdot\frac{e^{x}}{2\sqrt{1+e^{x}}}=\frac{e^{x}}{2\sqrt{1+e^{x}}}\left(\frac{1}{\sqrt{1+e^{x}}-1}-\frac{1}{\sqrt{1+e^{x}}+1}\right)=$$

$$\text{¿}\frac{e^{x}}{2\sqrt{1+e^{x}}}\left(\frac{\left(\sqrt{1+e^{x}}+1\right)-\left(\sqrt{1+e^{x}}-1\right)}{\left(\sqrt{1+e^{x}}+1\right)\left(\sqrt{1+e^{x}}-1\right)}\right)=\frac{e^{x}}{2\sqrt{1+e^{x}}}\frac{2}{1+e^{x}-1}=\frac{e^{x}}{\sqrt{1+e^{x}}}\cdot\frac{1}{e^{x}}=\frac{1}{\sqrt{1+e^{x}}}$$

97.

$$y=\frac{1}{15}\cos^{3}x\left(3\cos^{2}x-5\right)\quad ,\quad y=\frac{1}{5}\cos^{5}x-\frac{1}{3}\cos^{3}x$$

$$y'=\frac{1}{5}\cdot 5\cos^{4}x(\cos x)'-\frac{1}{3}\cdot 3\cos^{2}x(\cos x)'=-\cos^{4}x\sin x+\cos^{2}x\sin x=$$

$$\text{¿}\cos^{2}x\sin x\left(1-\cos^{2}x\right)=\cos^{2}x\sin x\sin^{2}x=\cos^{2}x\sin^{3}x$$

98.