FUNKCIJE

1. Domena (oblast definisanosti) ili definiciono područje.
D-oznaka
skup svih vrijednosti u kojima je funkcija definisana

a) $y = f(x)$ -polinom $\Rightarrow D = \mathbb{R}$
b) $y = \frac{f(x)}{g(x)}$ uslov $g(x) \neq 0$
c) $y = \sqrt{f(x)}$ uslov $f(x) \geq 0$
d) $y = \log f(x)$ uslov $f(x) > 0$

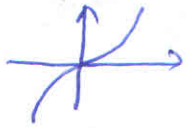
2. Parnost / neparnost

Funkcija je parna : -ako za svako $x \in D \Rightarrow \boxed{f(-x) = f(x)}$
-grafik funkcije je simetričan u odnosu na y-osu

npr. $y = x^2$

Funkcija je neparna : -ako za svako $x \in D \Rightarrow \boxed{f(-x) = -f(x)}$
-grafik je simetričan u odnosu na koordinatni početak $(0,0)$

npr.



Parnost i neparnost ispitujemo ako je D simetrično.

3. *Nule funkcije $(y=0)$ - tačke u kojima grafik siječe x-osu ~~(presjek sa~~ (izjednačimo funkciju sa nulom)

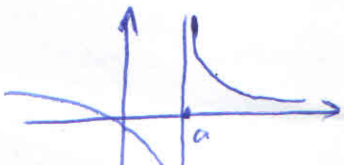
*Presjek sa y-osom $(x=0)$

4. Znak funkcije - ispitujemo intervale u kojima je funkcija pozitivna (+), odnosno negativna (–).
(crtamo tabelu za znak u koju unosimo nule i tačke iz domene)

5. Asimptote (prava kojoj se kriva približava, ali nikada ne dodiruje)

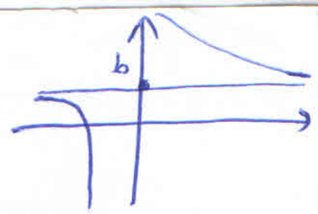
a) Vertikalna asimptota (V.A.)

$$\left.\begin{array}{l} \lim\limits_{x \to a_-} f(x) = +\infty \text{ ili } -\infty \\ \lim\limits_{x \to a_+} f(x) = +\infty \text{ ili } -\infty \end{array}\right\} \Rightarrow x = a \text{ je V.A. funkcije } f(x)$$



b) Horizontalna asimptota (H.A.)

$\lim_{x \to \pm\infty} f(x) = b$, $b \in R$ $\Rightarrow$ $y = b$ je H.A.



c) Kosa asimptota (K.A.)

$y = kx + n$ $\quad k = \lim_{x \to \pm\infty} \frac{f(x)}{x}$, $n = \lim_{x \to \pm\infty} [f(x) - k \cdot x]$

* Ako imamo H.A. $\Rightarrow$ nema K.A.
Ako nema H.A. $\Rightarrow$ ispitujemo K.A.

6. Prvi izvod i primjena (rast, opadanje, stacionarne tačke)
- funkcija opada (↘) na intervalu (a,b) ako za svako $x \in (a,b)$ $\quad f'(x) < 0$
- funkcija raste (↗) na intervalu (a,b) ako za svako $x \in (a,b)$ $\quad f'(x) > 0$

Rješenje jednačine $f'(x) = 0$ su stacionarne tačke koje su kandidati za ekstreme funkcije.

* Ekstremi (minimum i maksimum)
- funkcija ima max ako ↗ pa ↘ tj. $+$↗ max ↘$-$ (I izvod mijenja znak sa $+$ na $-$)
- funkcija ima min ako ↘ pa ↗ tj. $-$↘ min ↗$+$

II način za ekstreme: x_1 - stacionarna tačka
- ako je $f''(x_1) < 0 \Rightarrow$ ∩ max u tački x_1
- ako je $f''(x_1) > 0 \Rightarrow$ ∪ min

7. Drugi izvod i primjena (konveksnost, konkavnost, prevojne tačke)
- ako $\forall x \in (a,b)$ $\quad f''(x) < 0 \Rightarrow$ konkavna ∩ (max)
- ako $\forall x \in (a,b)$ $\quad f''(x) > 0 \Rightarrow$ konveksna ∪ (min)

Ako je $f''(x_0) = 0$ za svako $x \in (a,b)$, ako funkcija prelazi iz konveksne u konkavnu ili obrnuto (∩∪, ∪∩)
$\Rightarrow$ x_0 je PREVOJNA TAČKA (P.T.)

8. Na osnovu 1, 2, 3, 4, 5, 6, 7 crtamo grafik funkcije.

IZVODI

1. $c' = 0$, $c = const.$
2. $(x^n)' = n \cdot x^{n-1}$
3. $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$
4. $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$
5. $(a^x)' = a^x \ln a$ $(a = const)$
6. $(e^x)' = e^x$
7. $(\log_a x)' = \frac{1}{x \ln a}$, $(0 < a \neq 1)$
8. $(\ln x)' = \frac{1}{x}$
9. $(\sin x)' = \cos x$
10. $(\cos x)' = -\sin x$
11. $(tg\, x)' = \frac{1}{\cos^2 x}$
12. $(ctg\, x)' = -\frac{1}{\sin^2 x}$
13. $(arc \sin x)' = \frac{1}{\sqrt{1-x^2}}$
14. $(arc \cos x)' = -\frac{1}{\sqrt{1-x^2}}$
15. $(arc\, tg\, x)' = \frac{1}{1+x^2}$
16. $(arc\, ctg\, x)' = -\frac{1}{1+x^2}$
17. $(c \cdot u)' = c \cdot u'$ $(c = const)$
18. $(u \pm v)' = u' \pm v'$
19. $(u \cdot v)' = u'v + u \cdot v'$
20. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ ✓
21. $(u \cdot v \cdot w)' = u' \cdot v \cdot w + uv'w + uvw'$
22. $x' = 1$

IZVOD SLOŽENE FUNKCIJE

$y = f[u(x)]$ – složena funkcija

$y'(x) = y'(u) \cdot u'(x)$

1. $\left\{[f(x)]^n\right\}' = n \cdot [f(x)]^{n-1} \cdot f'(x)$
2. $\left[e^{f(x)}\right]' = e^{f(x)} \cdot f'(x)$
3. $[\ln f(x)]' = \frac{1}{f(x)} \cdot f'(x)$
4. $\left[\sqrt{f(x)}\right]' = \frac{f'(x)}{2\sqrt{f(x)}}$
5. $[\sin f(x)]' = \cos f(x) \cdot f'(x)$
6. $[\cos f(x)]' = -\sin f(x) \cdot f'(x)$
7. $[tg\, f(x)]' = \frac{1}{\cos^2 f(x)} \cdot f'(x)$
8. $[ctg\, f(x)]' = -\frac{1}{\sin^2 f(x)} \cdot f'(x)$
9. $[arc \sin f(x)]' = \frac{1}{\sqrt{1-[f(x)]^2}} \cdot f'(x)$
10. $[arc \cos f(x)]' = -\frac{1}{\sqrt{1-[f(x)]^2}} \cdot f'(x)$
11. $[arc\, tg\, f(x)]' = \frac{1}{1+[f(x)]^2} \cdot f'(x)$
12. $[arc\, ctg\, f(x)]' = -\frac{1}{1+[f(x)]^2} \cdot f'(x)$
13. $\left[a^{f(x)}\right]' = a^{f(x)} \cdot \ln a \cdot f'(x)$
14. $[\log_a f(x)]' = \frac{1}{f(x) \ln a} \cdot f'(x)$

DATUM:

Granična vrijednost niza

A - granična vrijednost

limes - granica

X_n - niz

$\boxed{\lim_{n\to\infty} X_n = A}$ - granica niza X_n je broj A kad $n\to\infty$

Neodređeni izrazi: $\frac{0}{0}$, $\infty-\infty$, $0\cdot\infty$, $\frac{\infty}{\infty}$, $\frac{\infty}{0}$

Određeni -II- : $\infty\cdot\infty=\infty$, $\infty+\infty=\infty$, $\frac{0}{\infty}=0$

Vrijedi: $\frac{c}{0}=\infty$ $\frac{c}{\infty}=0$ c-const.

Operacije sa limesima

Pr. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \to 0$ kada $n\to\infty$

$\boxed{\lim_{n\to\infty}\frac{1}{n}=0}$

a) $\lim_{n\to\infty}(a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$

b) $\lim_{n\to\infty}(a_n \cdot b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$

c) $\lim_{n\to\infty}\frac{a_n}{b_n} = \frac{\lim a_n}{\lim_{n\to\infty} b_n}$

d) $\lim_{n\to\infty}\sqrt[k]{a_n} = \sqrt[k]{\lim_{n\to\infty} a_n}$

e) $\lim_{n\to\infty} b^{a_n} = b^{\lim_{n\to\infty} a_n}$

f) $\lim_{n\to\infty} \log_b a_n = \log_b \lim_{n\to\infty} a_n$ $(b>1)$

DATUM:

1. $\lim\limits_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$ $\quad \left(\lim\limits_{n\to\infty} \frac{c}{n} = 0 \right)$

$\lim\limits_{n\to\infty} 7 = 7$ $\quad \lim\limits_{n\to\infty} n^2 = \infty^2 = \infty$

2. $\lim\limits_{n\to\infty} \frac{3n-1}{n+5} = \frac{\infty}{\infty}$ neodređeni izraz → potrebno je transformirati niz $\frac{3n-1}{n+5}$

(podijeliti i br. i naz. sa n da bi iskoristili 1. $\lim\limits_{n\to\infty} \frac{1}{n} = 0$)

$\lim\limits_{n\to\infty} \frac{3n-1}{n+5} \begin{matrix} /:n \\ /:n \end{matrix} = \lim\limits_{n\to\infty} \frac{\frac{3n}{n} - \frac{1}{n}}{\frac{n}{n} + \frac{5}{n}} = 3$

3. $\lim\limits_{n\to\infty} \frac{n^2+2n+3}{2n^2+n-5} \begin{matrix} /:n^2 \\ /:n^2 \end{matrix} \ldots = \frac{1}{2}$

(dijelimo sa najvećom ~~oda~~ stepenom od n)

4. $\lim\limits_{n\to\infty} \frac{2n^3+n-1}{2n^4+1} \begin{matrix} /:n^4 \\ /:n^4 \end{matrix} = \frac{0}{2} = 0$ $\quad \lim\limits_{n\to\infty} 3$

5. $\lim\limits_{n\to\infty} \frac{n+(-1)^n}{3n-(-1)^n} \begin{matrix} /:n \\ /:n \end{matrix} = \lim\limits_{n\to\infty} \frac{1 + \frac{(-1)^n}{n}}{3 - \frac{(-1)^n}{n}} = \frac{1}{3}$

6. $\lim\limits_{n\to\infty} \frac{2^n+3^n}{n^2+3^n} = \lim\limits_{n\to\infty} \frac{\frac{2^n}{3^n}+1}{\frac{n^2}{3^n}+1} = \lim\limits_{n\to\infty} \frac{\left(\frac{2}{3}\right)^n+1}{\frac{n^2}{3^n}+1} = \frac{0+1}{0+1} = 1$

(sa 3^n jer najbrže raste kad $n\to\infty$)

DATUM: ____________

7. $\lim_{n\to\infty} 3^{\frac{6n+2}{3n-2}} = 3^{\lim_{n\to\infty}\frac{6n+2/:n}{3n-2/:n}} = 3^{2} = 9$

8. $\lim_{n\to\infty} \log_5 \frac{2n^2}{3n^2+1} = \log_5 \lim_{n\to\infty} \frac{2n^2}{3n^2+1} = \log_5 \frac{2}{3}$

9. $\lim_{n\to\infty} \left(\sqrt{2n+3} - \sqrt{n-1}\right) = \lim_{n\to\infty} \left(\sqrt{2n+3} - \sqrt{n-1}\right) \frac{\sqrt{2n+3} + \sqrt{n-1}}{\sqrt{2n+3} + \sqrt{n-1}}$

$= \lim_{n\to\infty} \frac{2n+3-n+1}{\sqrt{2n+3}+\sqrt{n-1}} = \lim_{n\to\infty} \frac{n+4 \;/:n}{\sqrt{2n+3}+\sqrt{n-1} \;/:n} =$

$= \lim_{n\to\infty} \frac{1+\frac{4}{n}}{\sqrt{\frac{2}{n}+\frac{3}{n^2}} + \sqrt{\frac{1}{n}-\frac{1}{n^2}}} = \frac{1}{0} = \infty$

10. $\lim_{n\to\infty} \left(\sqrt{n^2-5n+6} - n\right) = \dots = -\frac{5}{2} = -2{,}5$

11. $\lim_{n\to\infty} \left(\frac{1+3+5+\dots+(2n-1)}{n+1} - \frac{2n+1}{2}\right) = \dots = -\frac{3}{2}$

12. $\lim_{n\to\infty} \left(\frac{1+2+3+\dots+n}{n+2} - \frac{n}{2}\right) = \dots = -\frac{1}{2}$

13. $\lim_{n\to\infty} \left(\frac{4n^3-4n^2+7}{n^3+90n^2-6}\right)^2 =$

DATUM:

$(a+b)(a^2-ab+b^2)=a^3+b^3$

14. $\lim_{n\to\infty}\left(\sqrt[3]{1+n}+\sqrt[3]{1-n}\right)=\ldots=0$

15. ~~$\lim_{n\to\infty}\left(1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\ldots+\right.$~~

16. $\lim_{n\to\infty}\frac{n!}{(n+1)!-n!}=\ldots=0$

DATUM: ________________

Granična vrijednost funkcije

Funkcija $f(x) \to A$ kada $x \to \varsigma$ (A, ς - brojevi)
ili $\lim_{x \to a} f(x) = A$ ako $\forall$ broj $\varepsilon > 0$
postoji $\delta > 0$ tako da je $|f(x) - A| < \varepsilon$ ~~za~~
$0 < |x - \varsigma| < \delta$.

1. $\lim_{x \to 2} \frac{x^2-4}{x^2-3x+2} = \frac{0}{0}$ → neodređeni izraz

$\Rightarrow \lim_{x \to 2} \underbrace{\frac{x^2-4}{x^2-3x+2}}_{\text{rastavimo na faktore}} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x-1)} = \lim_{x \to 2} \frac{x+2}{x-1} = 4$

2. $\lim_{x \to 3} \frac{3x^2-10x+3}{2x^2-7x+3} =$ †trebamo rastaviti na faktore i brojnik i nazivnik

1) † $ax^2+bx+c = a(x-x_1)\cdot(x-x_2)$ → rastavljena kvadratni trinom na faktore.

$x_{1,2} = \frac{-b \pm \sqrt{D}}{2 \cdot a}$ $\quad D = b^2 - 4 \cdot a \cdot c$

ili:

2) Rastavimo srednji član pa grupišemo!

$3x^2-10x+3 =$

$D = 10^2 - 4 \cdot 3 \cdot 3 = 64$ $\quad x_1 = \frac{10+8}{6} = 3$ $\quad x_2 = \frac{10-8}{6} = \frac{1}{3}$

$\Rightarrow 3x^2-10x+3 = 3(x-3)(x-\frac{1}{3}) = (x-3)(3x-1)$

$$2x^2-7x+3 = 2x^2-6x-x+3 = 2x(x-3)-(x-3) =$$
$$= (x-3)(2x-1)$$

$$\Rightarrow \lim_{x\to 3} \frac{3x^2-10x+3}{2x^2-7x+3} = \lim_{x\to 3} \frac{\cancel{(x-3)}(3x-1)}{\cancel{(x-3)}(2x-1)} = \frac{9-1}{6-1} = \frac{8}{5}$$

3\. $$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} = \left(\frac{0}{0}\right) = \left| \begin{array}{l} \text{smjena } 1+x=t^6 \\ \text{kada } x\to 0 \Rightarrow t\to 1 \end{array} \right|$$

$$= \lim_{t\to 1} \frac{\sqrt{t^6}-1}{\sqrt[3]{t^6}-1} = \lim_{x\to 1} \frac{t^3-1}{t^2-1} = \lim_{t\to 1} \frac{\cancel{(t-1)}(t^2+t+1)}{\cancel{(t-1)}(t+1)}$$

$$= \frac{1^2+1+1}{1+1} = \frac{3}{2}$$

Osnovni limesi

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$

$$\boxed{\lim_{x\to \infty} \left(1+\frac{1}{x}\right)^x = e}$$

$$\lim_{x\to \infty} \left(1+\frac{k}{x}\right)^x = e^k$$

4\. $$\lim_{x\to 0} \frac{\sin 5x}{2x} = \lim_{x\to 0} \frac{\frac{\sin 5x}{5x}}{\frac{2x}{5x}} = \lim_{x\to 0} \frac{\frac{\sin 5x}{5x}}{\frac{2}{5}} =$$

$$= \frac{\overbrace{\lim_{x\to 0} \frac{\sin 5x}{5x}}^{=1}}{\frac{2}{5}} = \frac{1}{\frac{2}{5}} = \frac{5}{2}$$

DATUM:

(5) $\lim_{x\to 0} \frac{\sin 4x}{\sin 3x} = \lim_{x\to 0} \frac{\frac{\sin 4x}{4x}\cdot 4\cancel{x}}{\frac{\sin 3x}{3x}\cdot 3\cancel{x}} =$

$= \lim_{x\to 0} \frac{\frac{\sin 4x}{4x}\cdot 4}{\frac{\sin 3x}{3x}\cdot 3} = \frac{4}{3} \lim_{x\to 0} \frac{\overbrace{\frac{\sin 4x}{4x}}^{=1}}{\underbrace{\frac{\sin 3x}{3x}}_{=1}}$

$= \frac{4}{3}$

(6.) $\lim_{n\to\infty} \left(1+\frac{5}{n}\right)^n = e^5$

(7) $\lim_{n\to\infty} \left(1+\frac{1}{4n}\right)^{n+1} = \lim_{n\to\infty} \left(1+\frac{1}{4n}\right)^n \cdot \left(1+\frac{1}{4n}\right)^1$

$= \underbrace{\lim_{n\to\infty} \left(1+\frac{1}{4n}\right)^{4n\cdot\frac{1}{4}}}_{e} \cdot \lim_{n\to\infty} \left(1+\frac{1}{4n}\right)^{1} =$

$= e^{\frac{1}{4}} \cdot 1 = e^{\frac{1}{4}} = \sqrt[4]{e}$

(8) $\lim_{x\to\infty} \left(\frac{x-3}{x+1}\right)^x = \lim_{x\to\infty} \left(\frac{x+1-4}{x+1}\right)^x = \lim_{x\to\infty} \left(\frac{x+1}{x+1} - \frac{4}{x+1}\right)^x =$

$= \lim_{x\to\infty} \left(1-\frac{4}{x+1}\right)^x = \lim_{x\to\infty} \left(1+\frac{-4}{x+1}\right)^x = \lim_{x\to\infty} \left(1+\frac{1}{\frac{x+1}{-4}}\right)^x =$

$= \lim_{x\to\infty} \left(1+\frac{1}{\frac{x+1}{-4}}\right)^{\frac{x+1}{-4}\cdot(-4)-1} = \left(\lim_{x\to\infty} \left(1+\frac{1}{\frac{x+1}{-4}}\right)^{\frac{x+1}{-4}}\right)^{-4} \cdot$

$\cdot \lim_{x\to\infty} \left(1+\frac{1}{\frac{x+1}{-4}}\right)^{-1} = e^{-4} \cdot \lim_{x\to\infty} \left(1+\frac{-4}{x+1}\right)^{-1} = e^{-4}\cdot 1 = e^{-4}$

DATUM:

5

IZVODI

(*dodatak - pravila za izvode)

y' - prvi izvod

y'' - drugi izvod itd.

Neka su u i v funkcije ($u = u(x)$, $v = v(x)$).

Vrijedi: $(u \pm v)'(x) = u'(x) \pm v'(x)$

$$(u \cdot v)' = u' \cdot v + u \cdot v'$$

$$(c \cdot u)' = c \cdot u' \qquad c - \text{const.}$$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2} \qquad v \neq 0$$

1. $5' = 0$ (pravilo 1.)

2. $(3x)' = 3$ (jer $x' = 1 \Rightarrow 3 \cdot 1 = 3$)

3. $y = x^3$

$$y' = (x^3)' = 3 \cdot x^2$$

4. $y = x^5 - 4 \cdot x^3 + 2x - 3$

$$y' = 5x^4 - 12x^2 + 2$$

5. $y = \dfrac{2x+5}{x^2 - 5x + 9}$

$$y' = \frac{(2x+5)' \cdot (x^2 - 5x + 9) - (2x+5)(x^2 - 5x + 9)'}{(x^2 - 5x + 9)^2} =$$

DATUM:

$$= \frac{2(x^2-5x+9)-(2x+5)(2x-5)}{(x^2-5x+9)^2} = \cancel{2x^2-10x+18-4x^2+10x+10}$$

$$= \frac{2x^2-10x+18-4x^2+\cancel{10x}-\cancel{10x}+25}{(x^2-5x+9)^2} =$$

$$= \frac{-2x^2-10x+43}{(x^2-5x+9)^2}$$

6. $y = \underbrace{2x}_{u}\cdot\underbrace{\sin x}_{v} - \underbrace{(x^2-2)}_{u}\cdot\underbrace{\cos x}_{v}$

$y' = (2x)'\sin x + (2x)(\sin x)' - [(x^2-2)'\cos x + (x^2-2)\cdot(\cos x)']$

$= 2\sin x + 2x\cos x - 2x\cos x - (x^2-2)(-\sin x)$

$= 2\sin x + \cancel{2x\cos x} - \cancel{2x\cos x} + (x^2-2)\sin x$

$= \cancel{2\sin x} + x^2\sin x - \cancel{2\sin x} = x^2\sin x$

7. $y = \dfrac{x^5\cdot\ln x}{e^x}$

$$y' = \frac{(x^5\ln x)'\cdot e^x - x^5\ln x\cdot(e^x)'}{(e^x)^2} =$$

$$= \frac{\left(5x^4\ln x + x^5\cdot\frac{1}{x}\right)e^x - x^5\ln x\cdot e^x}{(e^x)^2} =$$

$$= \frac{(5x^4\ln x + x^4)e^x - x^5\ln x\, e^x}{(e^x)^2} = \frac{e^x(5x^4\ln x + x^4 - x^5\ln x)}{(e^x)^2}$$

$$= \frac{5x^4\ln x + x^4 - x^5\ln x}{e^x}$$

8) $y = \operatorname{arc\,ctg} x - \frac{x}{1+x^2}$

$$y' = -\frac{1}{1+x^2} - \frac{1+x^2 - x\cdot 2x}{(1+x^2)^2} =$$

$$= -\frac{1}{1+x^2} - \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{-(1+x^2)-1-x^2+2x^2}{(1+x^2)^2} =$$

$$= \frac{-1-\cancel{x^2}-1+\cancel{x^2}}{(1+x^2)^2} = \frac{-2}{(1+x^2)^2}$$

Izvod složenih funkcija

1) $y = (\underbrace{x^2-3x+2}_{u})^2$ $\quad \boxed{y = u^2 \Rightarrow y' = 2u\cdot u'}$

$$y' = 2(x^2-3x+2)\cdot(x^2-3x+2)' =$$

$$= 2(x^2-3x+2)\cdot(2x-3)$$

2) $y = \ln\frac{1-x^2}{x^2-3}$ $\quad \boxed{y = \ln u \Rightarrow y' = \frac{1}{u}\cdot u'}$

$$y' = \frac{1}{\frac{1-x^2}{x^2-3}}\cdot\left(\frac{1-x^2}{x^2-3}\right)' =$$

$$= \frac{\cancel{x^2-3}}{1-x^2}\cdot\frac{-2x(x^2-3)-(1-x^2)\cdot 2x}{(x^2-3)^{\cancel{2}}} =$$

$$= \frac{-\cancel{2x^3}+6x-2x+\cancel{2x^3}}{(1-x^2)(x^2-3)} = \frac{4x}{(1-x^2)(x^2-3)}$$

DATUM:

(3) $y = e^{3x^5+6x-1}$ $\qquad y' = e^u \cdot u'$

$y' = e^{3x^5+6x-1} \cdot (3x^5+6x-1)' =$

$= e^{3x^5+6x-1} \cdot (15x^4+6)$

(4)* $y = \operatorname{arc\,tg} \ln x$ $\qquad$ *$y = \operatorname{arc\,tg} u$

$y' = \frac{1}{1+\ln^2 x} \cdot (\ln x)'$ $\qquad y' = \frac{1}{1+u^2} \cdot u'$

$= \frac{1}{1+\ln^2 x} \cdot \frac{1}{x} = \frac{1}{x(1+\ln^2 x)}$

(5) ~~$y =$~~

$y = \ln \operatorname{tg} 3x$

$y' = \frac{1}{\operatorname{tg} 3x} \cdot (\operatorname{tg} 3x)' = \frac{1}{\operatorname{tg} 3x} \cdot \frac{1}{\cos^2 3x} \cdot (3x)' =$

$= \frac{3}{\operatorname{tg} 3x \cdot \cos^2 3x}$

V $y = \operatorname{arc\,tg} \ln(x-1)$

DATUM:

Izvodi višeg reda

y' - 1. izvod

$y'' = (y')'$ - drugi izvod

$y''' = (y'')'$

$\vdots$

(1.) $y = x \cdot e^x \qquad y''' = ?$

$y' = e^x + x \cdot e^x$

$y'' = e^x + e^x + xe^x = 2e^x + xe^x = e^x(2+x)$

$y''' = e^x(2+x) + e^x = e^x(2+x+1) = e^x(x+3)$

(2) $y = \ln \frac{x^2+3}{x^2+1} \qquad y'' = ?$

$y' = \frac{1}{\frac{x^2+3}{x^2+1}} \cdot \left(\frac{x^2+3}{x^2+1}\right)' =$

$= \frac{\cancel{x^2+1}}{x^2+3} \cdot \frac{2x(x^2+1) - (x^2+3) \cdot 2x}{(x^2+1)^{\cancel{2}}} =$

$= \frac{1}{x^2+3} \cdot \frac{\cancel{2x^3} + 2x - \cancel{2x^3} - 6x}{x^2+1} = \frac{-4x}{(x^2+3)(x^2+1)} =$

$= \frac{-4x}{x^4+x^2+3x^2+3} = \frac{-4x}{x^4+4x^2+3}$

DATUM:

$$y'' = \left(\frac{-4x}{x^4+4x^2+3} \right)'$$

$$y'' = \frac{-4(x^4+4x^2+3)+4x(4x^3+8x)}{(x^4+4x^2+3)^2}$$

$$= \frac{-4x^4-16x^2-12+16x^4+32x^2}{(x^4+4x^2+3)^2} = \frac{12x^4+16x^2-12}{(x^4+4x^2+3)^2}$$

③ $y = \ln(\sin\sqrt{x^2+5})$

$y'' = ?$ $\quad y' = \frac{1}{\sin\sqrt{x^2+5}} \cos\sqrt{x^2+5} \; \frac{1}{2\sqrt{x^2+5}} \cdot 2x$

$$= \frac{x\cos\sqrt{x^2+5}}{\sin\sqrt{x^2+5}\cdot\sqrt{x^2+5}}$$

$y' =$

③' $y = \ln(\sin\sqrt{x^2+5})$

④

$y = \frac{2x^2}{x-5} \qquad y'' = \dots = \frac{100}{(x-5)^3}$

DATUM:

L'HOSPITAL-ovo pravilo (L.P.)

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \left(\frac{0}{0} \text{ ili } \frac{\infty}{\infty}\right) = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

1) $$\lim_{x\to 0} \frac{\ln x}{\operatorname{ctg} x} = \frac{\infty}{\infty} \overset{L.P.}{=} \lim_{x\to 0} \frac{\frac{1}{x}}{-\frac{1}{\sin^2 x}} =$$

$$= \lim_{x\to 0} \frac{-\sin^2 x}{x} = \left(\frac{0}{0}\right) = -\underbrace{\lim_{x\to 0} \frac{\sin x}{x}}_{=1} \cdot \sin x =$$

$$= -1 \cdot \lim_{x\to 0} \sin x = -1 \cdot 0 = 0$$

2) $$\lim_{x\to\infty} \frac{x^5}{e^x} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{5x^4}{e^x} = \left(\frac{\infty}{\infty}\right) =$$

$$= \lim_{x\to 0} \frac{20x^3}{e^x} = \left(\frac{\infty}{\infty}\right) = \lim_{x\to 0} \frac{60x^2}{e^x} = \lim_{x\to 0} \frac{120x}{e^x} =$$

$$= \lim_{x\to\infty} \frac{120}{e^x} = \frac{120}{\infty} = 0$$

3) $$\lim_{x\to\infty} \frac{\ln x}{x^4} = \left(\frac{\infty}{\infty}\right) = \lim_{x\to\infty} \frac{\frac{1}{x}}{4x^3} =$$

$$= \lim_{x\to\infty} \frac{1}{4x^4} = \frac{1}{\infty} = 0$$

Funkcije

1. Ispitati i nacrtati funkciju $y = \frac{3x^2}{x-1}$

1) Domena: $x-1 \neq 0 \quad x \neq 1$

$x \in (-\infty, 1) \cup (1, +\infty)$

2) Parnost: $f(-x) = \frac{3\cdot(-x)^2}{-x-1} = \frac{3x^2}{-(x+1)}$ ni parna
ni neparna

3) Nule: $\frac{3x^2}{x-1} = 0 \Rightarrow 3x^2 = 0 \;/:3$

$x^2 = 0$

$\boxed{x = 0}$

presjek sa y-osom: $x = 0 \Rightarrow \underline{y = 0}$

4) Znak:

x	$-\infty$	0		1	$+\infty$
$3x^2$	+	0	+		+
$x-1$	−		−	0	+
y	−		−		+

5) Asimptote:

H.A. $\lim\limits_{x\to\pm\infty} \frac{3x^2 \;/:x^2}{x-1 \;/:x^2} = \lim\limits_{x\to\pm\infty} \frac{3}{\frac{1}{x}-\frac{1}{x^2}} = \infty$ nema H.A.

V.A. Kandidat iz domene: $x = 1$

$$\left.\begin{aligned} \lim_{x\to 1^-} \frac{3x^2}{x-1} &= \frac{3}{0_-} = -\infty \\ \lim_{x\to 1^+} \frac{3x^2}{x-1} &= \frac{3}{0_+} = +\infty \end{aligned}\right\} \Rightarrow \text{V.A. } \underline{x = 1}$$

K.A. $y = k \cdot x + n$

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\frac{3x^2}{x-1}}{x} = \lim_{x\to\infty} \frac{3x^2 \;/ : x^2}{x^2 - x \;/ : x^2} =$$

$$= 3$$

$$n = \lim_{x\to\infty} [f(x) - k\cdot x] = \lim_{x\to\infty} \left[\frac{3x^2}{x-1} - 3x\right] =$$

$$= \lim_{x\to\infty} \frac{3x^2 - 3x(x-1)}{x-1} = \lim_{x\to\infty} \frac{\cancel{3x^2} - \cancel{3x^2} + 3x \;/ : x}{x - 1 \;/ : x} = 3$$

K.A. $\boxed{y = 3x + 3}$

x	-1	0	1
$y = 3x+3$	0	3	6

6) $$y' = \left(\frac{3x^2}{x-1}\right)' = \frac{6x(x-1) - 3x^2 \cdot 1}{(x-1)^2} = \frac{6x^2 - 6x - 3x^2}{(x-1)^2} =$$

$$= \frac{3x^2 - 6x}{(x-1)^2}$$ → uvijek pozitivno pa ne utiče na znak

$y' = 0 \Rightarrow 3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ → $\boxed{x = 0}$

$x - 2 = 0$ → $\boxed{x = 2}$

stacionarne tačke

	$-\infty$ … 0	0 … 2	2 … $+\infty$
$3x$	$-$	0 $+$	$+$
$x-2$	$-$	$-$	0 $+$
y'	$+$	$-$	$+$
y	$\nearrow$	$\searrow$	$\nearrow$

max $\in D$ (at $x=0$) min $\in D$ (at $x=2$)

DATUM: ____________

$x_{max} = 0$ $\qquad\qquad$ $x_{min} = 2$

$y_{max} = \frac{3\cdot 0^2}{0-1} = \frac{0}{-1} = 0$ $\qquad\qquad$ $y_{min} = \frac{3\cdot 2^2}{2-1} = \frac{12}{1} = 12$

$T_{max}(0,0)$ $\qquad\qquad$ $T_{min}(2,12)$

7) $y'' = \left(\frac{3x^2-6x}{(x-1)^2}\right)' = \frac{(6x-6)(x-1)^2-(3x^2-6x)2(x-1)}{(x-1)^4}$

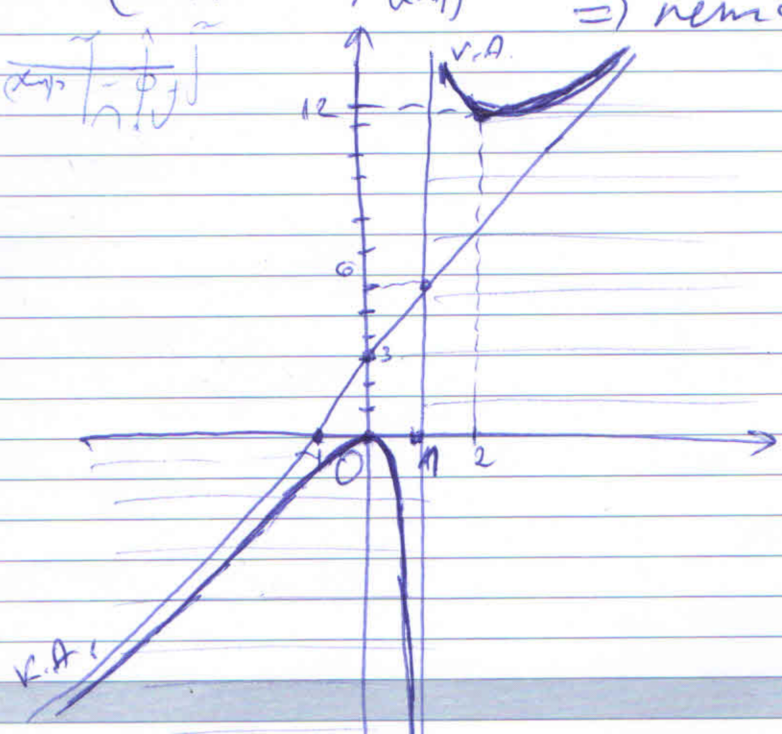
$= \frac{\cancel{(x-1)}[(6x-6)(x-1)-2(3x^2-6x)]}{(x-1)^{\cancel{4}}} =$

$= \frac{\cancel{6x^2}-\cancel{6x}-\cancel{6x}+6-\cancel{6x^2}+\cancel{12x}}{(x-1)^3} = \frac{6}{(x-1)^3}$

$y''=0 \Rightarrow \frac{6}{(x-1)^3}=0 \,/\cdot(x-1)^3 \Rightarrow 6=0$ nemoguće

$\Rightarrow$ nema prevojnih tačaka

8) crtež



DATUM: ______________

~~2.~~ $\cancel{y = \frac{x^2+5x}{x^2+2x+1}}$

2\. $y = \dfrac{x^2+5x}{x^2+2x+1}$

3\. $y = \dfrac{x^2}{x^2-4}$