Mechatronics Engineering Students Society – College of Engineering

JACOBIAN MATRIX CALCULATION OF SCARA MANIPULATOR

THIS IS THE FORMULA USED IN CALCULATING THE JACOBIAN MATRIX.

	PRISMATIC	REVOLUTE
LINEAR	$R^0_{i-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R^0_{i-1}\begin{bmatrix}0\\0\\1\end{bmatrix} \times (d^0_n - d^0_{n-1})$
ROTATIONAL	$\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R^0_{f-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$

WHEREIN:
R = ROTATIONAL MATRIX
d = POSITION VECTOR
f = WHICH JOINT IS BEING SOLVED
n = NO. OF JOINTS

USING THE CROSS-PRODUCT METHOD

$$\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix} \times \begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix} = \begin{bmatrix}a_2b_3 - a_3b_2\\a_3b_1 - a_1b_3\\a_1b_2 - a_2b_1\end{bmatrix}$$

CROSS-PRODUCT METHOD

BY APPLYING THE CROSS-PRODUCT METHOD WE CAN NOW MULTIPLY OUR 3x1 MATRIX AND DOING IT WOULD GIVE US THIS 3x1 MATRIX.

SUBSTITUTING THE FORMULAS FOR LINEAR AND ROTATIONAL PART

$$\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{z}\\\omega_x\\\omega_y\\\omega_z\end{bmatrix} = \begin{bmatrix} R^0_0\begin{bmatrix}0\\0\\1\end{bmatrix}(d^0_3 - d^0_0) & R^0_0\begin{bmatrix}0\\0\\1\end{bmatrix}(d^0_3 - d^0_1) & R^0_0\begin{bmatrix}0\\0\\1\end{bmatrix} \\ R^0_0\begin{bmatrix}0\\0\\1\end{bmatrix} & R^0_1\begin{bmatrix}0\\0\\1\end{bmatrix} & \begin{bmatrix}0\\0\\1\end{bmatrix} \end{bmatrix} \begin{bmatrix}\dot{\theta}_1\\\dot{\theta}_2\\d_3\end{bmatrix}$$

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SOLVING FOR THE COLUMN 2 OF JACOBIAN MATRIX

$$\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix} \times \left(\begin{bmatrix} a_2\cos\theta_1 + a_4\cos(\theta_1+\theta_2) \\ a_2\sin\theta_1 + a_4\sin(\theta_1+\theta_2) \\ a_1 + a_3 - a_5 - d_3 \end{bmatrix} - \begin{bmatrix} a_2\cos\theta_1 \\ a_2\sin\theta_1 \\ a_1 \end{bmatrix} \right)$$

$$\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix} a_4\cos(\theta_1+\theta_2) \\ a_4\sin(\theta_1+\theta_2) \\ a_3 - a_5 - d_3 \end{bmatrix}$$

$$\begin{bmatrix}0\\0\\1\end{bmatrix}$$

SOLVING USING THE CROSS-PRODUCT METHOD

$$\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix} \times \begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix} = \begin{bmatrix} 0 - a_4\sin(\theta_1+\theta_2) \\ a_4\sin(\theta_1+\theta_2) - 0 \\ 0 - 0 \end{bmatrix}$$

$$\begin{bmatrix} -a_4\sin(\theta_1+\theta_2) \\ a_4\sin(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix}0\\0\\1\end{bmatrix}$$

SOLVING FOR THE COLUMN 3 OF JACOBIAN MATRIX

$$\begin{bmatrix} \cos\theta_1\cos\theta_2 + \sin\theta_1\cos\theta_2 & -\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1 & 0 \\ \cos\theta_1\cos\theta_2 + \sin\theta_1\cos\theta_2 & -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\begin{bmatrix}0\\0\\-1\end{bmatrix} \qquad \begin{bmatrix}0\\0\\0\end{bmatrix}$$

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SUBSTITUTING THE MATRICES OF COLUMN 1, 2, AND 3 TO THE JACOBIAN MATRIX

$$\begin{vmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{vmatrix} = \begin{vmatrix} -a_2 \sin\theta_1 + a_4 \sin(\theta_1 + \theta_2) & -a_4 \sin(\theta_1 + \theta_2) & 0 \\ a_2 \cos\theta_1 + a_4 \cos(\theta_1 + \theta_2) & a_4 \cos(\theta_1 + \theta_2) & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ d_3 \end{bmatrix}$$

ASSIGNING VALUES TO THE LINK LENGTHS AND JOINT VARIABLES

$a_1 = 40$ $\quad$ $a_5 = 30$

$a_2 = 30$ $\quad$ $\theta_1 = 0$

$a_3 = 70$ $\quad$ $\theta_2 = 0$

$a_4 = 50$ $\quad$ $d_3 = 100$

SUBSTITUTING THE ASSIGNED VALUES TO JACOBIAN VARIABLES

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -30 \sin(0) + 50 \sin(0+0) & -50 \sin(0+0) & 0 \\ 30 \cos(0) + 50 \cos(0+0) & 50 \cos(0+0) & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ d_3 \end{bmatrix}$$

JACOBIAN MATRIX

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 80 & 50 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ d_3 \end{bmatrix}$$

END EFFECTOR VELOCITY VECTOR

$$\dot{x} = 0$$
$$\dot{y} = 80\theta_1 + 50\theta_2$$
$$\dot{z} = -d_3$$
$$\omega_x = 0$$
$$\omega_y = 0$$
$$\omega_z = \theta_1 + \theta_2$$

SINGULARITY CALCULATION OF SCARA MANIPULATOR

JACOBIAN MATRIX

$$\begin{bmatrix} 0 & 0 & 0 \\ 80 & 50 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

DETERMINANT OF SINGULARITY
SINGULARITY = D(J)
IF D(J) = 0, SINGULARITY
IF D(J) ≠ 0, NOT SINGULARITY

$$\mathrm{Det}(J) = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 80 & 50 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{matrix} 0 & 0 \\ 80 & 50 \\ 0 & 0 \end{matrix}$$

$D(J) = (0+0+0) - (0+0+0)$
$D(J) = 0$
$D(J) = 0$