Week 1 Tutorial Attempt

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1 Question 1

If $f(x) = 2x^3 - x$, find $f(-1), f(0), f(x^2), f(\sqrt{x}), f(\frac{1}{x})$

Answer:

Just substitute the numbers accordingly

$$\begin{aligned} f(-1) &= 2(-1)^3 - (-1) \\ &= 2(-1) + 1 \\ &= -2 + 1 \\ &= -1 \quad \blacksquare \end{aligned}$$

$$\begin{aligned} f(0) &= 2(0)^3 - (0) \\ &= 2(0) \\ &= 0 \quad \blacksquare \end{aligned}$$

$$\begin{aligned}
f(x^2) &= 2(x^2)^3 - (x^2) \\
&= 2(x^6) - x^2 \\
&= 2x^6 - x^2 \\
&= x^2(2x^4 - 1) \quad \blacksquare
\end{aligned}$$

$$\begin{aligned}
f(\sqrt{x}) &= 2(\sqrt{x})^3 - (\sqrt{x}) \\
&= 2(x^{\frac{3}{2}}) - \sqrt{x} \\
&= 2x^{\frac{3}{2}} - x^{\frac{1}{2}} \\
&= x^{\frac{1}{2}}(2x - 1) \quad \blacksquare
\end{aligned}$$

$$\begin{aligned}
f\left(\frac{1}{x}\right) &= 2\left(\frac{1}{x^3}\right) - \left(\frac{1}{x}\right) \\
&= 2\left(\frac{1}{x^3}\right) - \frac{1}{x} \\
&= \frac{2}{x^3} - \frac{1}{x} \\
&= \frac{1}{x}\left(\frac{2}{x^2} - 1\right) \quad \blacksquare
\end{aligned}$$

2 Question 2

If $f(x) = \begin{cases} x^2 + 1, & \textbf{if } x \leq 0 \\ \sqrt{x}, & \textbf{if } x > 0 \end{cases}$**, find** $f(-2), f(0)$ **and** $f(1)$

Answer:

Same as Q1, proper substitution needs to be performed here.

$$\begin{aligned} f(-2) &= 2(-2)^2 + 1 \quad x < 0 \textit{ hence take 1st option} \\ &= 2(4) + 1 \\ &= 7 \ \blacksquare \end{aligned}$$

$$\begin{aligned} f(0) &= 2(0)^2 + 1 \quad x = 0 \textit{ hence take 1st option} \\ &= 1 \ \blacksquare \end{aligned}$$

$$\begin{aligned} f(1) &= 2(1)^2 + 1 \quad x > 0 \textit{ hence take 2nd option} \\ &= 2(1) + 1 \\ &= 3 \ \blacksquare \end{aligned}$$

3 Question 3

Find domain of the following function:

$f(x) = \sqrt{x-2} + \sqrt{4-x}$

$f(x) = \frac{\sqrt{x+2}+\sqrt{2-x}}{x^3-x}$

Answer

Honestly, I have a problem with this question. There is no indication of what the field is. Is it $\mathbb{R}$ or $\mathbb{C}$ field? Or is it only for $\mathbb{N}$? But most likely it will be either real or complex field. As such I'll write both fields for this question. Going forward, the domain is just what numbers can fit into this function to produce an output within the field.

$$f(x) = \sqrt{x-2} + \sqrt{4-x}$$

$$\mathbb{C} \text{ field for domain:} (-\infty, \infty)$$

$$\mathbb{R} \text{ field for domain:} [2, 4]$$

$\blacksquare$

So in the complex field, we don't care. Honestly, the square roots means nothing. You might as well eat them. But once we reached the real field, $\sqrt{x}$ matters more to us. Looking at the first part of $\sqrt{x-2}$, $x - 2 \leq 0$ for the real answersw hence $x >= 2$. Now onto the second part, $\sqrt{4-x}$, $4 - x \geq 0$ hence $x > 4$. This only leaves us with the domain between 2 and 4 or $[2, 4]$

$$
\begin{aligned}
f(x) &= \frac{\sqrt{x+2} + \sqrt{2-x}}{x^3 - x} \\
&= \frac{\sqrt{x+2} + \sqrt{2-x}}{x(x^2 - 1)} \\
&= \frac{\sqrt{x+2} + \sqrt{2-x}}{x(x+1)(x-1)}
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{C} \text{ field for domain: } \mathbb{R} &\neq -1, 0, 1 \\
\mathbb{R} \text{ field for domain: } [-2, 2] &\neq -1, 0,
\end{aligned}
$$

$\blacksquare$

This function is a little bit more ugly. Fractions with variables as the denominator are always going to have some problems. So even in the complex field, we need to take note of the denominator. I factorised $x^3 - x$ to $x(x+1)(x-1)$ such that any of these values cannot hit 0. As such, we can have any values except from -1 to 1. Since we cannot have -1 or 1, we need to use open intervals (). And I just remembered, the values between -1 to 1 are valid except for 0, which means more more unions. But that's stupid. Instead, I'll write out the three invalid numbers within here.

For the real field, it is easier. Just need the values inside the square roots to not be negative. So $x + 2 \geq 0 => x \geq -2$
$2 - x \leq 0 => 2 \leq x$. This gives us the closed range of $[-2 \leq x \leq 2]$. So all we have to do is exclude the same values as before and we're done. So tiring, isn't it?