Week 1 Tutorial Attempt

XK

October 22, 2024

Contents

1 Question 1

If $f(x) = 2x^3 - x$, find $f(-1), f(0), f(x^2), f(\sqrt{x}), f(\frac{1}{x})$

Answer:

Just substitute the numbers accordingly

$$\begin{aligned} f(-1) &= 2(-1)^3 - (-1) \\ &= 2(-1) + 1 \\ &= -2 + 1 \\ &= -1 \quad \blacksquare \end{aligned}$$

$$\begin{aligned} f(0) &= 2(0)^3 - (0) \\ &= 2(0) \\ &= 0 \quad \blacksquare \end{aligned}$$

$$\begin{aligned} f(x^2) &= 2(x^2)^3 - (x^2) \\ &= 2(x^6) - x^2 \\ &= 2x^6 - x^2 \\ &= x^2(2x^4 - 1) \quad \blacksquare \end{aligned}$$

$$\begin{aligned} f(\sqrt{x}) &= 2(\sqrt{x})^3 - (\sqrt{x}) \\ &= 2(x^{\frac{3}{2}}) - \sqrt{x} \\ &= 2x^{\frac{3}{2}} - x^{\frac{1}{2}} \\ &= x^{\frac{1}{2}}(2x - 1) \quad \blacksquare \end{aligned}$$

$$\begin{aligned} f\left(\frac{1}{x}\right) &= 2\left(\frac{1}{x^3}\right) - \left(\frac{1}{x}\right) \\ &= 2\left(\frac{1}{x^3}\right) - \frac{1}{x} \\ &= \frac{2}{x^3} - \frac{1}{x} \\ &= \frac{1}{x}\left(\frac{2}{x^2} - 1\right) \quad \blacksquare \end{aligned}$$

2 Question 2

If $f(x) = \begin{cases} x^2 + 1, & \textbf{if } x \leq 0 \\ \sqrt{x}, & \textbf{if } x > 0 \end{cases}$**, find** $f(-2), f(0)$ **and** $f(1)$

Answer:

Same as Q1, proper substitution needs to be performed here.

$$\begin{aligned} f(-2) &= 2(-2)^2 + 1 \quad x < 0 \textit{ hence take 1st option} \\ &= 2(4) + 1 \\ &= 7 \ \blacksquare \end{aligned}$$

$$\begin{aligned} f(0) &= 2(0)^2 + 1 \quad x = 0 \textit{ hence take 1st option} \\ &= 1 \ \blacksquare \end{aligned}$$

$$\begin{aligned} f(1) &= 2(1)^2 + 1 \quad x > 0 \textit{ hence take 2nd option} \\ &= 2(1) + 1 \\ &= 3 \ \blacksquare \end{aligned}$$

3 Question 3

Find domain of the following function:

$f(x) = \sqrt{x-2} + \sqrt{4-x}$

$f(x) = \frac{\sqrt{x+2}+\sqrt{2-x}}{x^3-x}$

Answer

Honestly, I have a problem with this question. Specifically the first function. There no indication of what the field is. Is it $\mathbb{R}$ or $\mathbb{C}$ field, as such I'll write both fields for both the first and second functions. Going forward, the domain is just what numbers can fit into this function to produce an output.

$$f(x) = \sqrt{x-2} + \sqrt{4-x}$$
$$\mathbb{C} \text{ field for domain:}(-\infty, \infty)$$
$$\mathbb{R} \text{ field for domain:}[2, 4]$$

So in the complex field, we don't care. Honestly, the square roots means nothing. You might as well eat them. But once we reached the real field, $\sqrt{x}$ matters more to us. Looking at the first part of

$\sqrt{x-2},\ x-2>0$ for the real answersw hence $x>2$. Now onto the second part, $\sqrt{4-x},\ 4-x>0$ hence $x>4$. This only leaves us with the domain between 2 and 4 or $[2,4]$

$$\begin{aligned} f(x) &= \frac{\sqrt{x+2}+\sqrt{2-x}}{x^3-x} \\ &= \frac{\sqrt{x+2}+\sqrt{2-x}}{x(x^2-1)} \\ &= \frac{\sqrt{x+2}+\sqrt{2-x}}{x(x+1)(x-1)} \end{aligned}$$

$$\mathbb{C} \text{ field for domain:} (-\infty,-1)\cup(1,\infty)$$

$$\mathbb{R} \text{ field for domain:} (-\infty,-2]\cup[2,\infty)$$