Supplementary Notes

I think this text assumes an intimate familiarity with linear algebra, which is particularly evidenced in exercise 2.1. I am familiar with the notation that ket's, $|\Psi\rangle$, are column-vectors, and bra's, $\langle\Psi|$, are row-vectors. They are related via $|\Psi\rangle^{\dagger} = \langle\Psi|$. This allows us to calculate quantities such as $\langle\Psi|\Psi\rangle$ and $|\Psi\rangle\langle\Psi|$ via traditional matrix operations (see Lidar Ch. 2, A). In Mathematica, this means an extra set of curly braces.

```
In[68]:= list = {a, b};
array = {{a}, {b}};
list // MatrixForm
array // MatrixForm
Dimensions[list]
Dimensions[array]
```

Out[70]//MatrixForm=

$$\begin{pmatrix} a \\ b \end{pmatrix}$$

Out[71]//MatrixForm=

$$\begin{pmatrix} a \\ b \end{pmatrix}$$

Out[72]=

```
{2}
```

Out[73]=

```
{2, 1}
```

The list *does* default to being a column-vector, but look how dot products are handled:

```
In[61]:= list.ConjugateTranspose[list] // MatrixForm
array.ConjugateTranspose[array] // MatrixForm
```

Out[61]//MatrixForm=

```
a Conjugate[a] + b Conjugate[b]
```

Out[62]//MatrixForm=

$$\begin{pmatrix} \text{a Conjugate[a]} & \text{a Conjugate[b]} \\ \text{b Conjugate[a]} & \text{b Conjugate[b]} \end{pmatrix}$$

The lists return a standard dot product, but the arrays return a matrix multiplication! We want the behavior of the second case, as $|\Psi\rangle\langle\Psi|$ should indeed be a 2x2 matrix. In Schmied, this same result is achieved via the KroneckerProduct function (Kronecker Products are also covered in Lidar Ch. 2). This is why I say the text assumes an intimate familiarity with linear algebra - you need to know when to use the dot product and when to use the Kronecker product, versus just standard matrix multiplication.

In[74]:=
```
KroneckerProduct[list, ConjugateTranspose[list]] // MatrixForm
```

Out[74]//MatrixForm=

$$\begin{pmatrix} a\,\text{Conjugate}[a] & a\,\text{Conjugate}[b] \\ b\,\text{Conjugate}[a] & b\,\text{Conjugate}[b] \end{pmatrix}$$

Exercises

Q2.1

In[49]:=
```
$Assumptions = θ ∈ Reals && ϕ ∈ Reals;
up[θ_, ϕ_] := {{Cos[θ/2]}, {e^(I*ϕ) Sin[θ/2]}};
dn[θ_, ϕ_] := {{-e^(-I*ϕ) Sin[θ/2]}, {Cos[θ/2]}};
```

We can convert the bra to a ket via the Conjugate command.

In[46]:=
```
up[θ, ϕ] // MatrixForm
ConjugateTranspose[up[θ, ϕ]] // MatrixForm // Simplify
```

Out[46]//MatrixForm=

$$\begin{pmatrix} \text{Cos}\left[\frac{\theta}{2}\right] \\ e^{i\phi}\,\text{Sin}\left[\frac{\theta}{2}\right] \end{pmatrix}$$

Out[47]//MatrixForm=

$$\begin{pmatrix} \text{Cos}\left[\frac{\theta}{2}\right] \\ e^{-i\phi}\,\text{Sin}\left[\frac{\theta}{2}\right] \end{pmatrix}$$

1.

To show that the two states are orthonormal:

orthogonal - dot product is zero

normal - magnitude (dot product with self) is one.

In[39]:=
```
ConjugateTranspose[up[θ, ϕ]].dn[θ, ϕ] // Simplify
ConjugateTranspose[up[θ, ϕ]].up[θ, ϕ] // Simplify
ConjugateTranspose[dn[θ, ϕ]].dn[θ, ϕ] // Simplify
ConjugateTranspose[dn[θ, ϕ]].up[θ, ϕ] // Simplify
```

Out[39]=
{0}

Out[40]=
1

Out[41]=
{{1}}

Out[42]=
{0}

If I want to manually check any math, I can remove the simplify command. It is a little frustrating that there isn't an easy way to simplify only the Conjugate part. It appears to be either all or nothing.

In[ ]:=
```
ConjugateTranspose[up[θ, ϕ]].dn[θ, ϕ]
```

Out[ ]=

$$-e^{-i\phi}\,\mathrm{Cos}\left[\frac{\mathrm{Conjugate}[\theta]}{2}\right]\,\mathrm{Sin}\left[\frac{\theta}{2}\right]+e^{-i\,\mathrm{Conjugate}[\phi]}\,\mathrm{Cos}\left[\frac{\theta}{2}\right]\,\mathrm{Sin}\left[\frac{\mathrm{Conjugate}[\theta]}{2}\right]$$

2.

An important point of order here is that $\mathbb{1}$ in the text is shorthand for the identity matrix:

In[ ]:=
```
MatrixForm[IdentityMatrix[{2, 2}]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[ ]:=
```
MatrixForm[KroneckerProduct[{a, b}, {c, d}]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} a\,c & a\,d \\ b\,c & b\,d \end{pmatrix}$$

In[ ]:=
```
MatrixForm[
  KroneckerProduct[up[θ, ϕ], Conjugate[up[θ, ϕ]]]
   + KroneckerProduct[dn[θ, ϕ], Conjugate[dn[θ, ϕ]]]
 ] // FullSimplify
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[54]:=
```
up[θ, ϕ].ConjugateTranspose[up[θ, ϕ]] // Simplify // MatrixForm
```

Out[54]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}\left[\frac{\theta}{2}\right]^2 & \frac{1}{2}\,e^{-i\phi}\,\mathrm{Sin}[\theta] \\ \frac{1}{2}\,e^{i\phi}\,\mathrm{Sin}[\theta] & \mathrm{Sin}\left[\frac{\theta}{2}\right]^2 \end{pmatrix}$$