Supplementary Notes

I think this text assumes an intimate familiarity with linear algebra, which is particularly evidenced in exercise 2.1.

Exercises

Q2.1

In[1]:=
```
$Assumptions = θ ∈ Reals && ϕ ∈ Reals;
```
$$\text{up}[\theta\_, \phi\_] := \left\{\left\{\text{Cos}\left[\frac{\theta}{2}\right]\right\}, \left\{e^{I*\phi}\,\text{Sin}\left[\frac{\theta}{2}\right]\right\}\right\};$$
$$\text{dn}[\theta\_, \phi\_] := \left\{\left\{-e^{-I*\phi}\,\text{Sin}\left[\frac{\theta}{2}\right]\right\}, \left\{\text{Cos}\left[\frac{\theta}{2}\right]\right\}\right\};$$

We can convert the bra to a ket via the Conjugate command.

In[8]:=
```
up[θ, ϕ] // MatrixForm
ConjugateTranspose[up[θ, ϕ]] // MatrixForm // Simplify
```

Out[8]//MatrixForm=
$$\begin{pmatrix} \text{Cos}\left[\frac{\theta}{2}\right] \\ e^{i\,\phi}\,\text{Sin}\left[\frac{\theta}{2}\right] \end{pmatrix}$$

Out[9]//MatrixForm=
$$\begin{pmatrix} \text{Cos}\left[\frac{\theta}{2}\right] & e^{-i\,\phi}\,\text{Sin}\left[\frac{\theta}{2}\right] \end{pmatrix}$$

1.

To show that the two states are orthonormal:

- orthogonal - dot product is zero
- normal - magnitude (dot product with self) is one.

```
ConjugateTranspose[up[θ, ϕ]].dn[θ, ϕ] // Simplify
ConjugateTranspose[up[θ, ϕ]].up[θ, ϕ] // Simplify
ConjugateTranspose[dn[θ, ϕ]].dn[θ, ϕ] // Simplify
ConjugateTranspose[dn[θ, ϕ]].up[θ, ϕ] // Simplify
```

Out[∘]=

0

Out[∘]=

1

Out[∘]=

1

Out[∘]=

0

If I want to manually check any math, I can remove the simplify command. It is a little frustrating that there isn't an easy way to simplify only the Conjugate part. It appears to be either all or nothing.

In[∘]:=
```
ConjugateTranspose[up[θ, ϕ]].dn[θ, ϕ]
```

Out[∘]=

$$-e^{-i\,\phi}\,\mathrm{Cos}\left[\frac{\mathrm{Conjugate}[\theta]}{2}\right]\,\mathrm{Sin}\left[\frac{\theta}{2}\right]+e^{-i\,\mathrm{Conjugate}[\phi]}\,\mathrm{Cos}\left[\frac{\theta}{2}\right]\,\mathrm{Sin}\left[\frac{\mathrm{Conjugate}[\theta]}{2}\right]$$

2.

An important point of order here is that 𝟙 in the text is shorthand for the identity matrix:

In[∘]:=
```
MatrixForm[IdentityMatrix[{2, 2}]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[∘]:=
```
MatrixForm[KroneckerProduct[{a, b}, {c, d}]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} a\,c & a\,d \\ b\,c & b\,d \end{pmatrix}$$

In[∘]:=
```
MatrixForm[
  KroneckerProduct[up[θ, ϕ], Conjugate[up[θ, ϕ]]]
   + KroneckerProduct[dn[θ, ϕ], Conjugate[dn[θ, ϕ]]]
 ] // FullSimplify
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$