Homework 4 林樸誠 E24105038 資訊114甲

1)

i	$depth_T(k_i)$	$depth_T(k_i) \cdot p_i$
1	2	0.2
2	1	0.2
3	0	0
4	1	0.05
5	2	0.1
6	2	0.2
7	2	0.4
		1.15

$\therefore E[\text{search cost}] = 1 + 1.15$
$\boxed{= 2.15}$ #

2)

		1	2	3	4	5
	0	0	0	0	0	0
2	0	0↑	1↖	1←	1←	1←
2	0	0↑	1↖	1↑	1↑	1↑
6	0	0↑	1↑	1↑	1↑	1↑
4	0	0↑	1↑	1↑	2↖	2←
5	0	0↑	1↑	1↑	2↑	3↖

$\boxed{\therefore LCS = \langle 2, 4, 5 \rangle}$ #

3) $\boxed{\text{0-1 knapsack problem}}$ cannot be solved using the greedy strategy.

$\Rightarrow W = 50$ (at most $W = 50$)

i	1	2	3
V_i	60	100	120
W_i	10	20	30
$\frac{V_i}{W_i}$	6	5	4

↳ Using greedy solution,

Step ①: choose $i = 1$ $\Rightarrow$ current $W = 10$, $V = 60$

②: choose $i = 2$ $\Rightarrow$ current $W = 30$, $V = 160$

③: cannot choose $i = 3$ already because it has already exceeded ($W = 50$)

$\therefore$ Using greedy solution, $\boxed{V = 160}$ # only and $\boxed{W = 30}$ # waste the remaining space(20).

$\Rightarrow$ Optimal solution: Take items 2 and 3

↳ $\boxed{V = 220, \ W = 50}$ #

$\Rightarrow$ no leftover capacity.

$\therefore$ 0-1 knapsack problem cannot be solved using the greedy strategy #

4) ① cast the optimization problem as one in which we make a choice and are left with one subproblem to solve.

② Prove that there's always an optimal solution that make the greedy choice to ensure that the greedy choice is always safe.

③ Show that the greedy choice and optimal solution to the subproblem is the optimal solution to the problem.

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5)

	A	B	C	D	E
price/weight (kg)	$\$\frac{1}{3}$	\$1	\$1	\$2.5	\$2

$\frac{P_D}{W_D} > \frac{P_E}{W_E} > \boxed{\frac{P_B}{W_B} = \frac{P_C}{W_C}} > \frac{P_A}{W_A}$

∴ Selected order : $D \to E \to B \to C \to A$ $\Rightarrow$ Ans : $D + E + B + C + \frac{7}{12} A$

$\Rightarrow \boxed{D, E, B, C, A}$ #

∴ $\boxed{w = 15}$, $V = 2 + 10 + 1 + 2 + \frac{7}{12}(4)$

$\boxed{= (\$) 17\frac{1}{3}}$

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6) ⇒ Final result of selection $= B + C + D + E$ $\boxed{\Rightarrow B, C, D, E}$ #

$= 2 + 1 + 10 + 2$

$p = \{4, 2, 1, 10, 2\}$

$W = \{12, 2, 1, 4, 1\}$

$\boxed{= (\$) 15}$, $\boxed{W = 8}$

ω

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4
2	0	0	2	2	2	2	2	2	2	2	2	2	4	4	6	6
3	0	1	2	3	3	3	3	3	3	3	3	3	4	5	6	7
4	0	1	2	3	10	11	12	13	13	13	13	13	13	13	13	13
5	0	2	3	4	10	12	13	14	15	15	15	15	15	15	15	15

15-2=13

2-2=0 3-1=2 13-10=3

7) (2, 4, 6, 8, 10, 12, 14)

$\Rightarrow$ their associated probabilities (0.05, 0.1, 0.15, 0.1, 0.2, 0.3, 0.1)

$\Rightarrow$ (2, 4, 6, 8, 10, 12, 14)

e	0	1	2	3	4	5	6	7
1	0	0.05	0.2	0.5	0.7	1.2	1.9	2.2
2		0	0.1	0.35	0.55	1.05	1.7	2
3			0	0.15	0.35	0.8	1.4	1.7
4				0	0.1	0.4	1	1.2
5					0	0.2	0.7	0.9
6						0	0.3	0.5
7							0	0.1
8								0

root	1	2	3	4	5	6	7
1	1	2	2	3	3	5	5
2		2	3	3	3	5	5
3			3	3	4	5	5
4				4	5	5	6
5					5	6	6
6						6	6
7							7

$$e[1,2] = \min \begin{cases} k=1, e[1,0] + e[2,2] + \sum_{i=1}^{2} p_i \\ k=2, \underbrace{e[1,1]}_{(k-1)} + \underbrace{e[3,2]}_{(k+1)} + \sum_{i=1}^{2} p_i \end{cases} = \min\{0+0.1+(0.05+0.1), 0.05+0+(0.05+0.1)\}$$

$$= \min\{0.25, 0.2\}$$

$$e[2,3] = \min\{0+0.15+0.25, 0.1+0+0.25\} \boxed{= 0.2}$$

$$= \min\{0.4, 0.35\} \boxed{= 0.35}$$

$$e[4,5] = \min\{0.5, 0.4\} \boxed{= 0.4}$$

$$e[5,6] = \min\{0.8, 0.7\} \boxed{= 0.7}$$

$$e[6,7] = \min\{0.5, 0.7\} \boxed{= 0.5}$$

$$e[3,4] = \min\{0+0.1+0.25, 0.15+0+0.25\}$$

$$= \min\{0.35, 0.4\} \boxed{= 0.35}$$

$$e[1,3] = \min\{0.65, 0.5, 0.5\} \boxed{= 0.5}$$

$$e[2,4] = \min\{0.7, 0.55, 0.7\} \boxed{= 0.55}$$

$$e[3,5] = \min\{0.85, 0.8, 0.8\} \boxed{= 0.8}$$

$$e[4,6] = \min\{1.3, 1, 1\} \boxed{= 1}$$

$$e[5,7] = \min\{1.1, 0.9, 1.3\} \boxed{= 0.9}$$

$$e[1,4] = \min\{0.95, 0.8, 0.7, 0.9\} \boxed{= 0.7}$$

$$e[2,5] = \min\{1.35, 1.05, 1.1, 1.1\} \boxed{= 1.05}$$

$$e[3,6] = \min\{1.75, 1.6, 1.4, 1.55\} \boxed{= 1.4}$$

$$e[4,7] = \min\{1.6, 1.3, 1.2, 1.7\} \boxed{= 1.2}$$

$$e[1,5] = \min\{1.65, 1.45, 1.2, 1.3, 1.3\} \boxed{= 1.2}$$

$$e[2,6] = \min\{2.25, 1.95, 1.9, 1.7, 1.9\} \boxed{= 1.7}$$

$$e[3,7] = \min\{2.05, 1.9, 1.7, 1.75, 2.25\} \boxed{= 1.7}$$

$$e[1,6] = \min\{2.6, 2.35, 2.1, 2.1, 1.9, 2.1\} \boxed{= 1.9}$$

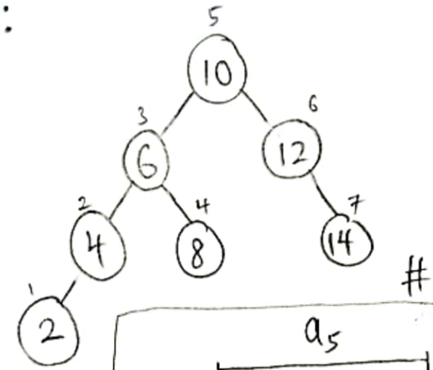
$$e[2,7] = \min\{2.65, 2.25, 2.2, 2, 2.1, 2.65\} \boxed{= 2}$$

$$e[1,7] = \min\{3, 2.75, 2.4, 2.4, 2.2, 2.3, 2.9\}$$

$$\boxed{= 2.2}$$

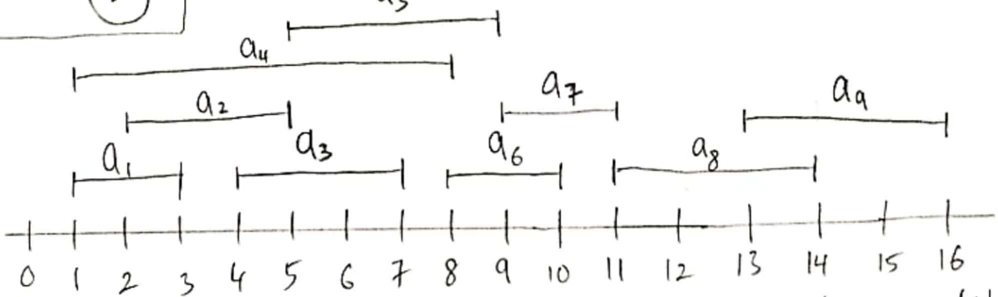
W	0	1	2	3	4	5	6	7
1	0	0.05	0.15	0.3	0.4	0.6	0.9	1
2		0	0.1	0.25	0.35	0.55	0.85	0.95
3			0	0.15	0.25	0.45	0.75	0.85
4				0	0.1	0.3	0.6	0.7
5					0	0.2	0.5	0.6
6						0	0.3	0.4
7							0	0.1
8								0

=> Ans:



#

8) *



∴ largest possible set of nonoverlapping (mutually compatible):

↳ $\boxed{[a_1, a_3, a_6, a_8]}$ #

- Assume activities already sorted by monotonically increasing finish time. If not, then sort in $O(n \lg n)$ time.

=>
```
Greedy-Activity Selector (s, f, n)
    A ← {a1}
    i ← 1
    for m ← 2 to n
        do if sm ≥ fi
            then A ← A ∪ {am}
                 i ← m
    return A
```

$\boxed{\therefore \text{Time complexity} = \Theta(n)}$ #

9) (1) (B) dynamic programming

(2) (B) 36

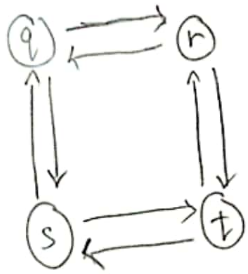
(3) (E) None of above → $\boxed{(A_1(A_2(A_3(A_4A_5))))}$

m	1	2	3	4	5
1	0	24	36	56	36
2		0	24	64	28
3			0	30	16
4				0	10
5					0

$A_1 : 2\times4$
$A_2 : 4\times3$
$A_3 : 3\times2$
$A_4 : 2\times5$
$A_5 : 5\times1$

s	1	2	3	4	5
1		1	2	3	1
2			2	3	2
3				3	3
4					4
5					

10)



⇒ Consider $q \to r \to t$ = longest simple path of $q \rightsquigarrow t$

↳ its subpath is not a longest simple path ($q \rightsquigarrow r$, $r \rightsquigarrow t$)

⇒ subpath $q \rightsquigarrow r$

↳ its longest simple path is $q \to s \to t \to r$

⇒ subpath $r \rightsquigarrow t$

↳ its longest simple path is $r \to q \to s \to t$

∴ Combining the longest simple path of both subpaths

⇒ $q \to s \to t \to r \to q \to s \to t$

↳ It is not a simple path

∴ The unweighted longest simple path problem does not exhibits optimal substructure.

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