林業誠 E24105038 資訊114甲

Homework 1

1)
① m
② m+1
③ i
④ j
⑤ k+p

2) a) true

↳ $0 \le 2^{n+1} \le c2^n$, when $c=2$, $n_0=1$, for all $n \ge n_0$

b) false

↳ Because there's no c, n_0 meet the conditions.

3) $\frac{n^2}{2}+8=\Theta(n^2)$

$\Rightarrow \quad c_1 n^2 \le \frac{n^2}{2}+8 \le c_2 n^2$

$\Leftrightarrow n^2 \le \frac{n^2}{2}+8$

$\Leftrightarrow c_1 \le \frac{1}{2}+\frac{8}{n^2}$

$\therefore c_1=\frac{1}{2} \quad \therefore n_0=1$

$\Leftrightarrow \frac{n^2}{2}+8 \le c_2 n^2$

$\Leftrightarrow \frac{1}{2}+\frac{8}{n^2} \le c_2$

$\therefore n_0=1 \quad \therefore c_2=\frac{17}{2}$

$\boxed{\text{Ans: } c_1=\frac{1}{2},\ c_2=\frac{17}{2},\ n_0=1}$

4) $9n-6=\Theta(3n)$

$\Rightarrow \quad c_1(3n) \le 9n-6 \le c_2(3n)$

$\Leftrightarrow 9n-6 \le c_2(3n)$

$\Leftrightarrow 9-\frac{6}{n} \le 3c_2$

$\Leftrightarrow 3-\frac{2}{n} \le c_2$

$\boxed{\therefore c_2=3 \quad \therefore n_0=1}$

$\Leftrightarrow c_1(3n) \le 9n-6$

$c_1 \le 3-\frac{2}{n}$

$\boxed{\therefore n_0=1 \quad \therefore c_1=1}$

$\boxed{\text{Ans: } c_1=1,\ c_2=3,\ n_0=1}$

5) Best situation : A [2, 4, 5, 6, 8]

Worst case : A $[\underbrace{8, 2, 5}_{L}, \underbrace{6, 4}_{R}]$

⇒ left and right sub-arrays contain alternate numbers, so every element for both arrays needs to be compared at least one to get merged in sorted manner

---

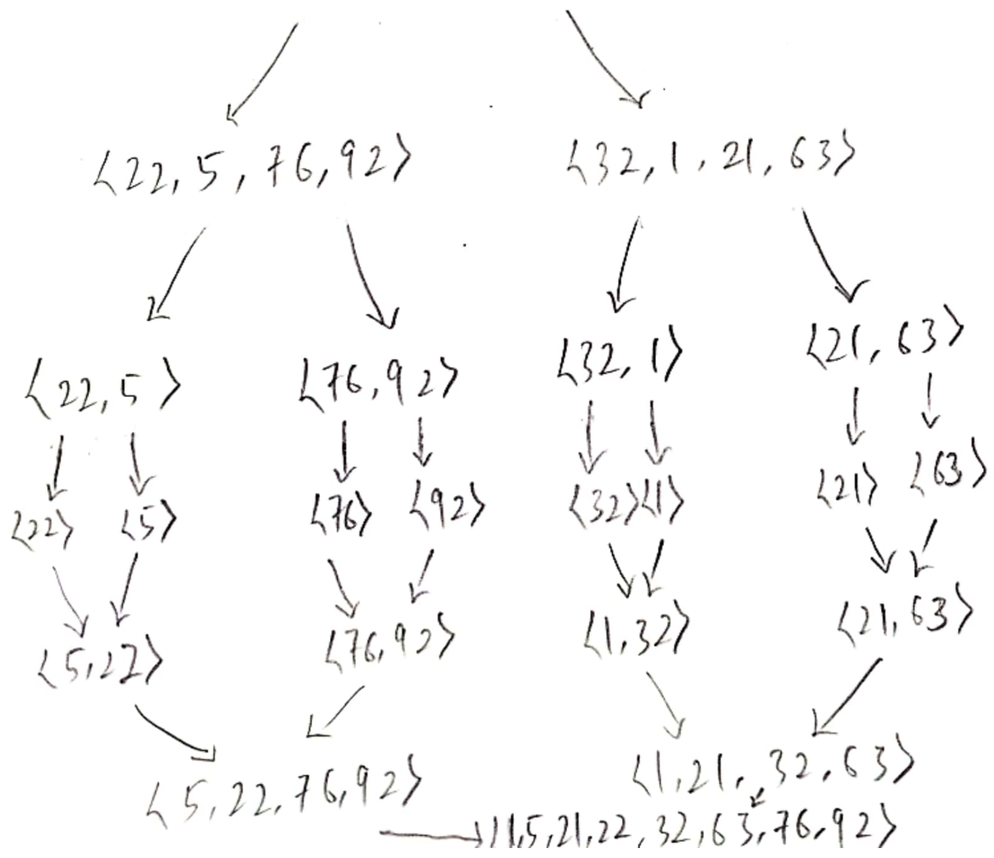
6) Best case : A [2, 4, 5, 6, 8]

Worst case : A [8, 6, 5, 4, 2]

⇒ In the worst case for insertion sort (when the input array is reverse-sorted), insertion sort performs just as many comparisons as selection sort.

---

7) A = ⟨22, 5, 76, 92, 32, 1, 21, 63⟩



8) - As the functions are asymptotically non-negative, we can assume that for some $n_0 > 0$, $f(n) \geq 0$ and $g(n) \geq 0$

$\Rightarrow n \geq n_0$,

$\hookrightarrow f(n) + g(n) \geq \max(f(n), g(n))$ — ①

$\hookrightarrow f(n) \leq \max(f(n), g(n))$ and $g(n) \leq \max(f(n), g(n))$

$\therefore f(n) + g(n) \leq 2\max(f(n), g(n))$

$\therefore \frac{1}{2}(f(n) + g(n)) \leq \max(f(n), g(n))$ — ②

$\Rightarrow$ By ① and ② : $0 \leq \frac{1}{2}(f(n) + g(n)) \leq \max(f(n), g(n)) \leq f(n) + g(n)$ for $n \geq n_0$

$\therefore \max(f(n), g(n)) = \Theta(f(n) + g(n))$ because there exists $c_1 = \frac{1}{2}$, $c_2 = 1$.

---

9) $\log n < \log^2 n < \sqrt{n} < n < n \log n < n^{1+\varepsilon}$

$< n^2 = n^2 + \log n < n^3 < 7n^5 - n^3 + n$

$n \cdot n^{\varepsilon}$ $(0 < \varepsilon < 1)$

$\Rightarrow n^{\varepsilon} > \log n$

$< 2^{n-1} = 2^n < e^n < n!$

---

10)

```
fib(n) {
    if (n = 0)      return 0;
    else if (n = 1)  return 1;
    else {
        x ← 0;
        y ← 1;
        while (n-2 ≥ 0) {
            ans ← x + y;
            x ← y;
            y ← ans;
            n--;
        }
    return ans;
}
```