Homework 1 林業誠 E24105038 資訊114

Section 1.1

9) $-m_1x_1 + x_2 = b_1$ — ① , where m_1, m_2, b_1, b_2 are constants

$-m_2x_1 + x_2 = b_2$ — ②

a) $\Rightarrow$ if $\boxed{m_1 \neq m_2}$

$\Rightarrow$ ① $\times m_2$ − ② $\times m_1$: $-m_1m_2x_1 + m_2x_2 + m_1m_2x_1 - m_1x_2 = m_2b_1 - m_1b_2$

$$m_2x_2 - m_1x_2 = m_2b_1 - m_1b_2$$

$$(m_2-m_1)x_2 = m_2b_1 - m_1b_2$$

$\therefore \Rightarrow \begin{cases} -m_1x_1 + x_2 = b_1 \\ (m_2-m_1)x_2 = m_2b_1 - m_1b_2 \end{cases}$ — ③ (strict triangular form)

↳ since $m_1 \neq m_2$, b_1, b_2 are constants, from ③

$\Rightarrow \boxed{m_2 - m_1 \neq 0}$ # $\therefore \boxed{x_2 \text{ has an unique answer}}$

$\therefore$ There is an unique solution if $m_1 \neq m_2$

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b) $\Rightarrow$ if $m_1 = m_2$

$\Rightarrow$ From (a) question : $(m_2-m_1)x_2 = m_2b_1 - m_1b_2$

$\Rightarrow \begin{cases} -m_1x_1 + x_2 = b_1 & \text{— ①} \\ (m_2-m_1)x_2 = m_2b_1 - m_1b_2 & \text{— ②} \end{cases}$

$\Rightarrow$ Since $m_1 = m_2$, $\therefore$ ② : $m_2b_1 - m_1b_2 = 0$

$\Rightarrow m_1b_1 - m_1b_2 = 0$ (since $m_1 = m_2$)

$m_1(b_1 - b_2) = 0$

↳ Since m_1 is constant, so the only way that this system be consistent is if $\boxed{b_1 - b_2 = 0}$

$\Rightarrow b_1 - b_2 = 0$ $\boxed{\therefore b_1 = b_2}$ #

$\therefore$ If $m_1 = m_2$, the system will be consistent only if $b_1 = b_2$

Section 1.2

6) a) $\begin{cases} 2x+y=1 \\ 7x+6y=1 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 2 & 1 & 1 \\ 7 & 6 & 1 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cc|c} 2 & 1 & 1 \\ 7 & 6 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{6}{7} & \frac{1}{7} \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{5}{14} & \frac{5}{14} \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -1 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right]$ $\boxed{\therefore (x,y)=(1,-1)}$ #

b) $\begin{cases} x_1+x_2-x_3+x_4=6 \\ 2x_1-x_2+x_3-x_4=-3 \\ 3x_1+x_2-2x_3+x_4=9 \end{cases} \Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 6 \\ 2 & -1 & 1 & -1 & -3 \\ 3 & 1 & -2 & 1 & 9 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 6 \\ 0 & 3 & -3 & 3 & 15 \\ 0 & 2 & -1 & 2 & 9 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 6 \\ 0 & 1 & -1 & 1 & 5 \\ 0 & 0 & -1 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 6 \\ 0 & 1 & -1 & 1 & 5 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right]$ $\therefore x_1, x_2, x_3$: leading variables, x_4 : free variable, let $x_4=\alpha$

$\boxed{\therefore (x_1,x_2,x_3,x_4)=(1,4-\alpha,-1,\alpha)}$ #

c) $\begin{cases} x_1-10x_2+5x_3=-4 \\ x_1+x_2+x_3=1 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & -10 & 5 & -4 \\ 1 & 1 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -10 & 5 & -4 \\ 0 & -11 & 4 & -5 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & -10 & 5 & -4 \\ 0 & 1 & -\frac{4}{11} & \frac{5}{11} \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & \frac{15}{11} & \frac{6}{11} \\ 0 & 1 & -\frac{4}{11} & \frac{5}{11} \end{array}\right]$ $\therefore x_1, x_2$: leading variables, x_3 : free variable

let $x_3=\alpha$ $\boxed{\therefore (x_1,x_2,x_3) = \left(\frac{6}{11}-\frac{15}{11}\alpha, \frac{5}{11}+\frac{4}{11}\alpha, \alpha\right)}$ #

d) $\begin{cases} x_1-2x_2+3x_3+x_4=4 \\ 2x_1+x_2-x_3+x_4=1 \\ x_1+3x_2+x_3+x_4=3 \end{cases} \Rightarrow \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 4 \\ 2 & 1 & -1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 3 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 4 \\ 0 & -5 & 7 & 1 & 7 \\ 0 & -5 & 2 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 4 \\ 0 & 1 & -\frac{7}{5} & -\frac{1}{5} & -\frac{7}{5} \\ 0 & 0 & -5 & -1 & -6 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & -2 & 0 & \frac{2}{5} & \frac{2}{5} \\ 0 & 1 & 0 & \frac{2}{25} & \frac{7}{25} \\ 0 & 0 & 1 & \frac{1}{5} & \frac{6}{5} \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{14}{25} & \frac{24}{25} \\ 0 & 1 & 0 & \frac{2}{25} & \frac{7}{25} \\ 0 & 0 & 1 & \frac{1}{5} & \frac{6}{5} \end{array}\right]$ $\therefore x_1, x_2, x_3$: leading variables, x_4 : free variable

$\Rightarrow$ let $x_4=\alpha$ $\boxed{\therefore (x_1,x_2,x_3) = \left(\frac{24}{25}-\frac{14}{25}\alpha, \frac{7}{25}-\frac{2}{25}\alpha, \frac{6}{5}-\frac{1}{5}\alpha, \alpha\right)}$ #

15) $\begin{cases} x_1 + 380 = x_2 + 430 \\ 420 + 450 = x_1 + x_4 \\ x_2 + 540 = x_3 + 420 \\ x_3 + 470 = 420 + 400 \end{cases} \Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 1 & 0 & 0 & 1 & 870 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \\ 1 & 0 & 0 & 1 & 870 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \\ 0 & 1 & 0 & 1 & 820 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \\ 0 & 0 & 1 & 1 & 940 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \\ 0 & 0 & 0 & 1 & 590 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 280 \\ 0 & 1 & 0 & 0 & 230 \\ 0 & 0 & 1 & 0 & 350 \\ 0 & 0 & 0 & 1 & 590 \end{array}\right]$ $\boxed{\therefore (x_1, x_2, x_3, x_4) = (280, 230, 350, 590)}$ #

Section 1.3

13) a) $\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

a) $A = \begin{bmatrix} 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 1 & 2 & 4 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$

$\therefore Ax = \begin{bmatrix} x_1 + 2x_2 + 3x_4 + x_5 \\ x_3 + 2x_4 + 4x_5 \end{bmatrix}$

$\Rightarrow x_1, x_3$: leading variables, x_2, x_4, x_5 : free variables

$\Rightarrow$ let $x_2 = \alpha$, $x_4 = \beta$, $x_5 = \gamma$

$\Rightarrow \boxed{(x_1, x_2, x_3, x_4, x_5) = (-2 - 2\alpha - 3\beta - \gamma, \alpha, 5 - 2\beta - 4\gamma, \beta, \gamma)}$ #

5)

$\Rightarrow \begin{cases} x_1 = -2 - 2\alpha - 3\beta - \gamma \\ x_2 = \alpha \\ x_3 = 5 - 2\beta - 4\gamma \\ x_4 = \beta \\ x_5 = \gamma \end{cases}$ $\Rightarrow a_1 = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 4 \end{pmatrix}$, $a_3 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 3 \end{pmatrix}$

$\Rightarrow$ As x_2, x_4, x_5 are free variables, so we choose $x_2 = x_4 = x_5 = 0$

$\Rightarrow \therefore x_1 = -2$, $x_2 = 0$, $x_3 = 5$, $x_4 = 0$, $x_5 = 0$

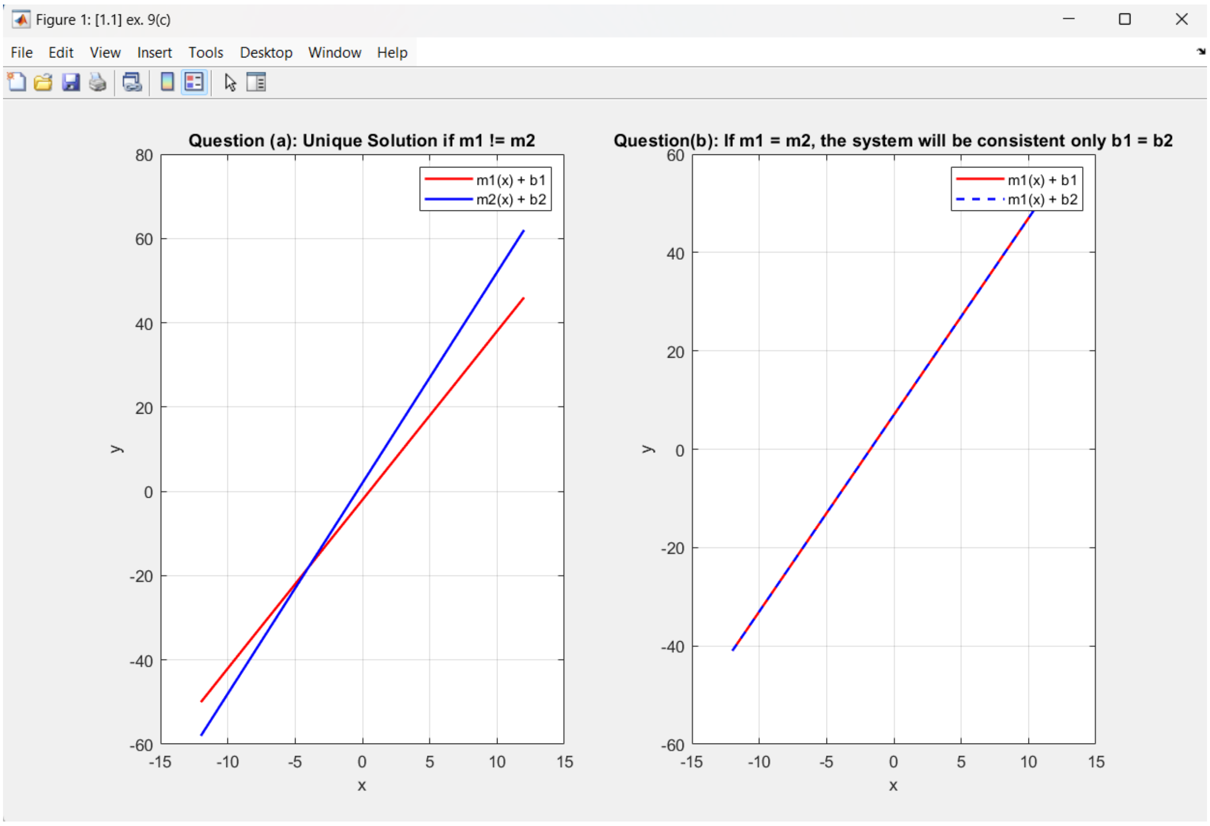
$\therefore b = -2a_1 + 5a_3$

$= -2\begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix} + 5\begin{bmatrix} 2 \\ -1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ -1 \\ 7 \end{bmatrix}$

$\boxed{\therefore b = \begin{bmatrix} 8 \\ -7 \\ -1 \\ 7 \end{bmatrix}}$ #

Section 1.1

9(c):



- a 小題的 b1， b2 使用 random 的方式生成 constants， 以證明不影響當 m1 != m2 會擁有 unique solution 的事實， 但是因爲作圖的範圍我有限制， 因此 b1， b2 的 random value， 我也有設置一個範圍， 不然最終交匯點會超出我的圖的範圍

[Matlab ex]

ex. 1(a), (b):

```
>> HW1_1a
   100    69    87    74   119
   161    93   163   160   145
   162    45   163   192   212
   236    87   237   198   243
   234    73   242   222   244

   201   159   226   190   191
   123   122   136   142   169
   170   168   156   167   166
   180   207   210   231   250
    91   105    88   102    88

   201   159   226   190   191
   123   122   136   142   169
   170   168   156   167   166
   180   207   210   231   250
    91   105    88   102    88

   100    69    87    74   119
   161    93   163   160   145
   162    45   163   192   212
   236    87   237   198   243
   234    73   242   222   244
```

```
>> HW1_1b
    71   168   193   155   222
    78   141   145   144   197
    57   127   124    85   125
    54   121   113    78   121
    50   147   147    76   131

    61   129   135    70    83
    90   166   155    96    86
   132   202   170   173    78
    64   118   100    55    54
   111   195   174   148    93

    71   168   193   155   222
    78   141   145   144   197
    57   127   124    85   125
    54   121   113    78   121
    50   147   147    76   131

    61   129   135    70    83
    90   166   155    96    86
   132   202   170   173    78
    64   118   100    55    54
   111   195   174   148    93
```

- From the result 1a above, we can conclude that A1 = A4 and A2 = A3
- From the result 1b above, we can conclude that A1 = A3 and A2 = A4