Homework 3 林業誠 E24105038 資訊114

Section 2.1

11) Let A, B be 2×2 matrices

$\Rightarrow A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

a) $\Rightarrow \det(A+B) = (a+e)(d+h) - (b+f)(c+g) = ad + ah + ed + eh$
$- bc - bg - cf - fg$

$\Rightarrow \det(A) + \det(B) = (ad - bc) + (eh - fg)$

$\boxed{\therefore \det(A+B) \neq \det(A) + \det(B)}$ #

b) $\Rightarrow AB = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$

$\Rightarrow \det(AB) = (ae+bg)(cf+dh) - (ce+dg)(af+bh)$
$= acef + adeh + bcfg + bdgh - acef - bceh - adfg$
$- bdhg$
$= adeh - adfg + bcfg - bceh$

$\Rightarrow \det(A)\det(B) = (ad-bc)(eh-fg)$
$= adeh - adfg - bceh + bcfg$

$\boxed{\therefore \det(AB) = \det(A)\det(B)}$ #

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c) $\Rightarrow \det(AB) = adeh - adfg + bcfg - bceh$ (Using the answer from (b))

$$\Rightarrow BA = \begin{bmatrix} ae+cf & be+df \\ ag+ch & bg+dh \end{bmatrix}$$

$$\Rightarrow \det(BA) = (ae+cf)(bg+dh) - (be+df)(ag+ch)$$
$$= abeg + adeh + bcfg + cdfh - abeg - adfg - bceh - cdfh$$
$$= adeh - adfg + bcfg - bceh$$

$$\boxed{\therefore \det(AB) = \det(BA)} \quad \#$$

Section 2.2

2) a)
$$A = \begin{bmatrix} 2 & 1 & 2 & 2 \\ -3 & 3 & 1 & 3 \\ 2 & 1 & -1 & -4 \\ 1 & -3 & 2 & 0 \end{bmatrix} \xrightarrow{\times\frac{3}{2},\ \times(-1),\ \times(-\frac{1}{2})} \begin{bmatrix} 2 & 1 & 2 & 2 \\ 0 & \frac{9}{2} & 4 & 6 \\ 0 & 0 & -3 & -6 \\ 0 & -\frac{7}{2} & 1 & -1 \end{bmatrix} \xrightarrow{\times\frac{7}{9}} \begin{bmatrix} 2 & 1 & 2 & 2 \\ 0 & \frac{9}{2} & 4 & 6 \\ 0 & 0 & -3 & -6 \\ 0 & 0 & \frac{37}{9} & \frac{33}{9} \end{bmatrix}$$

$$\xrightarrow{\times\frac{37}{27}} \begin{bmatrix} 2 & 1 & 2 & 2 \\ 0 & \frac{9}{2} & 4 & 6 \\ 0 & 0 & -3 & -6 \\ 0 & 0 & 0 & -\frac{41}{9} \end{bmatrix}$$

$$\therefore \det(A) = 2 \cdot \frac{9}{2} \cdot (-3) \cdot \left(-\frac{41}{9}\right)$$
$$= (-27) \cdot \left(-\frac{41}{9}\right)$$
$$\boxed{\therefore \det(A) = 123} \quad \#$$

b)
$$\begin{vmatrix} 2 & 1 & 2 & 2 \\ 2 & 1 & -1 & -4 \\ 1 & -3 & 2 & 0 \\ -3 & 3 & 1 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 1 & 2 & 2 \\ -3 & 3 & 1 & 3 \\ -1 & 4 & 0 & -1 \\ -2 & 0 & 3 & 3 \end{vmatrix}$$

$$= (-1)\cdot(-1) \begin{vmatrix} 2 & 1 & 2 & 2 \\ -3 & 3 & 1 & 3 \\ 2 & 1 & -1 & -4 \\ 1 & -3 & 2 & 0 \end{vmatrix} \xrightarrow{+,\ \times(-1),\ \times(-1)} \begin{vmatrix} 2 & 1 & 2 & 2 \\ -3 & 3 & 1 & 3 \\ 2 & 1 & -1 & -4 \\ 1 & -3 & 2 & 0 \end{vmatrix}$$

(2 times of interchanging the row)

(2 times of row operation III $\Rightarrow$ will not affect the result of det.

$$= 123 + 123 \ \boxed{= 246} \quad \#$$

18) ⇒ A : $k \times k$ matrix , B : $(n-k) \times (n-k)$ matrix

$$\Rightarrow E = \begin{bmatrix} I_k & O \\ O & B \end{bmatrix}, \quad F = \begin{bmatrix} A & O \\ O & I_{n-k} \end{bmatrix}, \quad C = \begin{bmatrix} A & O \\ O & B \end{bmatrix}$$

a) By the rule of when the matrix is triangular, its determinant is the product of the diagonal elements

$\Rightarrow \det(E) = \det(I_k) \times \det(B)$

$\because$ determinat of an identity matrix always $= 1$

$\boxed{\therefore \det(E) = (1)\det(B) = \det(B)}$ (proved) #

b) By the rule of when the matrix is triangular, its determinant is the product of the diagonal elements

$\Rightarrow \det(F) = \det(A) \times \det(I_{n-k})$

$= \det(A) \times 1$ (determinant of an identity matrix is always 1)

$\boxed{\therefore \det(F) = \det(A)}$ (proved) #

c) By the rule of when the matrix is triangular, its determinant is the product of the diagonal elements

$$\Rightarrow \boxed{\begin{aligned} \det(C) &= \det(A) \times \det(B) \\ &= \det(A)\det(B) \end{aligned}}$$ (proved) #

Section 2.3

2) a) $2x_1 + 3x_2 = 2$

$8x_1 - 6x_2 = 2$

$\Rightarrow A = \begin{bmatrix} 2 & 3 \\ 8 & -6 \end{bmatrix}$

$\Rightarrow \det(A) = \begin{vmatrix} 2 & 3 \\ 8 & -6 \end{vmatrix} = -12 - 24 = -36$

$\Rightarrow \det(A_1) = \begin{vmatrix} 2 & 3 \\ 2 & -6 \end{vmatrix} = -12 - 6 = -18$

$\Rightarrow \det(A_2) = \begin{vmatrix} 2 & 2 \\ 8 & 2 \end{vmatrix} = 4 - 16 = -12$

$\therefore x_1 = \dfrac{\det(A_1)}{\det(A)} = \dfrac{-18}{-36} = \dfrac{1}{2}$

$\therefore x_2 = \dfrac{\det(A_2)}{\det(A)} = \dfrac{-12}{-36} = \dfrac{1}{3}$ #

b) $6x_1 + 3x_2 = 5$

$5x_1 + 7x_2 = 12$

$\Rightarrow A = \begin{bmatrix} 6 & 3 \\ 5 & 7 \end{bmatrix}$

$\Rightarrow \det(A) = \begin{vmatrix} 6 & 3 \\ 5 & 7 \end{vmatrix} = 42 - 15 = 27$

$\Rightarrow \det(A_1) = \begin{vmatrix} 5 & 3 \\ 12 & 7 \end{vmatrix} = 35 - 36 = -1$

$\Rightarrow \det(A_2) = \begin{vmatrix} 6 & 5 \\ 5 & 12 \end{vmatrix} = 72 - 25 = 47$

$\therefore x_1 = \dfrac{\det(A_1)}{\det(A)} = \dfrac{-1}{27} = -\dfrac{1}{27}$

$\therefore x_2 = \dfrac{\det(A_2)}{\det(A)} = \dfrac{47}{27} = \dfrac{47}{27}$ #

c) $4x_1 \quad -2x_3 = 26$

$7x_1 + 7x_2 + 2x_3 = -4$

$3x_1 - 2x_2 + 1x_3 = -3$

$\Rightarrow A = \begin{bmatrix} 4 & 0 & -2 \\ 7 & 7 & 2 \\ 3 & -2 & 1 \end{bmatrix}$

$\Rightarrow \det(A) = \begin{vmatrix} 4 & 0 & -2 \\ 7 & 7 & 2 \\ 3 & -2 & 1 \end{vmatrix}$ (choose first row) $= 4(7+4) - 2(-14-21) = 44 + 70 \boxed{=114}$

$\Rightarrow \det(A_1) = \begin{vmatrix} 26 & 0 & -2 \\ -4 & 7 & 2 \\ -3 & -2 & 1 \end{vmatrix}$ (choose first row) $= 26(11) - 2(8+21) = 286 - 58 \boxed{=228}$

$\Rightarrow \det(A_2) = \begin{vmatrix} 4 & 26 & -2 \\ 7 & -4 & 2 \\ 3 & -3 & 1 \end{vmatrix}$ (first column) $= 4(-4+6) - 7(26-6) + 3(52-8) = 8 - 140 + 132 \boxed{=0}$

$\Rightarrow \det(A_3) = \begin{vmatrix} 4 & 0 & 26 \\ 7 & 7 & -4 \\ 3 & -2 & -3 \end{vmatrix}$ (first row) $= 4(-21-8) + 26(-14-21) = -116 - 910 \boxed{=-1026}$

$\therefore x_1 = \dfrac{\det(A_1)}{\det(A)} = \dfrac{228}{114} = 2$

$\therefore x_2 = \dfrac{\det(A_2)}{\det(A)} = \dfrac{0}{114} = 0$

$\therefore x_3 = \dfrac{\det(A_3)}{\det(A)} = \dfrac{-1026}{114} = -9$ #

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d) $2x_1 - 2x_2 + 3x_3 = 0$

$x_1 + 2x_2 + 3x_3 = 8$

$-2x_1 + 4x_2 + x_3 = 6$

$\Rightarrow A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 2 & 3 \\ -2 & 4 & 1 \end{bmatrix}$ $\Rightarrow \det(A) = \begin{vmatrix} 2 & -2 & 3 \\ 1 & 2 & 3 \\ -2 & 4 & 1 \end{vmatrix}$

(choose first column)

$= 2(2-12) - (-2-12) - 2(-6-6)$

$= -20 + 14 + 24 \; \boxed{= 18}$

$\Rightarrow \det(A_1) = \begin{vmatrix} 0 & -2 & 3 \\ 8 & 2 & 3 \\ 6 & 4 & 1 \end{vmatrix} \overset{\text{(first row)}}{=} 2(8-18) + 3(32-12) = -20 + 60 \; \boxed{= 40}$

$\Rightarrow \det(A_2) = \begin{vmatrix} 2 & 0 & 3 \\ 1 & 8 & 3 \\ -2 & 6 & 1 \end{vmatrix} \overset{\text{(first row)}}{=} 2(8-18) + 3(6+16) = -20 + 66 \; \boxed{= 46}$

$\Rightarrow \det(A_3) = \begin{vmatrix} 2 & -2 & 0 \\ 1 & 2 & 8 \\ -2 & 4 & 6 \end{vmatrix} \overset{\text{(first row)}}{=} 2(12-32) + 2(6+16) = -40 + 44 \; \boxed{= 4}$

$$\boxed{\therefore x_1 = \frac{\det(A_1)}{\det(A)} = \frac{40}{18} = \frac{20}{9}, \quad x_2 = \frac{\det(A_2)}{\det(A)} = \frac{46}{18} = \frac{23}{9} \quad x_3 = \frac{\det(A_3)}{\det(A)} = \frac{4}{18} = \frac{2}{9}}$$ #

e) $-x_1 + 2x_2 + x_3 + x_4 = 9$

$2x_1 + x_2 - 2x_3 + x_4 = 3$

$3x_1 + 3x_2 + x_3 = 6$

$2x_2 + x_3 - x_4 = 2$

$$\Rightarrow A = \begin{bmatrix} -1 & 2 & 1 & 1 \\ 2 & 1 & -2 & 1 \\ 3 & 3 & 1 & 0 \\ 0 & 2 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow \det(A) = \begin{vmatrix} -1 & 2 & 1 & 1 \\ 2 & 1 & -2 & 1 \\ 3 & 3 & 1 & 0 \\ 0 & 2 & 1 & -1 \end{vmatrix} \overset{\text{first column}}{=} - \begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & -1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & -1 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$\overset{\text{second row}}{=} -(-3(1) + (-3)) \overset{\text{second row}}{-} 2(-3(-2) + (-4))$$

$$\overset{\text{first row}}{+} 3(2(1) - (-3) + (5))$$

$$= -(-6) - 2(2) + 3(10) = 6 - 4 + 30 = 32$$

$$\Rightarrow \det(A_1) = \begin{vmatrix} 9 & 2 & 1 & 1 \\ 3 & 1 & -2 & 1 \\ 6 & 3 & 1 & 0 \\ 2 & 2 & 1 & -1 \end{vmatrix} \overset{\text{last column}}{=} - \begin{vmatrix} 3 & 1 & -2 \\ 6 & 3 & 1 \\ 2 & 2 & 1 \end{vmatrix} + \begin{vmatrix} 9 & 2 & 1 \\ 6 & 3 & 1 \\ 2 & 2 & 1 \end{vmatrix} - \begin{vmatrix} 9 & 2 & 1 \\ 3 & 1 & -2 \\ 6 & 3 & 1 \end{vmatrix}$$

$$\overset{\text{first row}}{=} -(3(1) - (4) - 2(6)) \overset{\text{first row}}{+} (9(1) - 2(4) + (6))$$

$$\overset{\text{first row}}{-} (9(7) - 2(15) + (3))$$

$$= 13 + 7 - 36 = -16$$

$$\Rightarrow \det(A_2) = \begin{vmatrix} -1 & 9 & 1 & 1 \\ 2 & 3 & -2 & 1 \\ 3 & 6 & 1 & 0 \\ 0 & 2 & 1 & -1 \end{vmatrix} \overset{\text{last column}}{=} - \begin{vmatrix} 2 & 3 & -2 \\ 3 & 6 & 1 \\ 0 & 2 & 1 \end{vmatrix} + \begin{vmatrix} -1 & 9 & 1 \\ 3 & 6 & 1 \\ 0 & 2 & 1 \end{vmatrix} - \begin{vmatrix} -1 & 9 & 1 \\ 2 & 3 & -2 \\ 3 & 6 & 1 \end{vmatrix}$$

$$\overset{\text{last row}}{=} -(-2(8) + (3)) \overset{\text{last row}}{+} (-2(-4) + (-33)) \overset{\text{first row}}{-} (-(15) - 9(8) + (3))$$

$$= -(-13) + (-25) - (-84) = 72$$

$$\Rightarrow \det(A_3) = \begin{vmatrix} -1 & 2 & 9 & 1 \\ 2 & 1 & 3 & 1 \\ 3 & 3 & 6 & 0 \\ 0 & 2 & 2 & -1 \end{vmatrix} \overset{\text{first column}}{=} - \begin{vmatrix} 1 & 3 & 1 \\ 3 & 6 & 0 \\ 2 & 2 & -1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 9 & 1 \\ 3 & 6 & 0 \\ 2 & 2 & -1 \end{vmatrix} + 3 \begin{vmatrix} 2 & 9 & 1 \\ 1 & 3 & 1 \\ 2 & 2 & -1 \end{vmatrix}$$

$$\overset{\text{second row}}{=} -\left(-3(-5)+6(-3)\right) \overset{\text{second row}}{-2}\left(-3(-11)+6(-4)\right)$$

$$\overset{\text{first row}}{+3}\left(2(-5)-9(-3)+(-4)\right)$$

$$= -(-3) - 2(9) + 3(13) = 24$$

$$\Rightarrow \det(A_4) = \begin{vmatrix} -1 & 2 & 1 & 9 \\ 2 & 1 & -2 & 3 \\ 3 & 3 & 1 & 6 \\ 0 & 2 & 1 & 2 \end{vmatrix} \overset{\text{first column}}{=} - \begin{vmatrix} 1 & -2 & 3 \\ 3 & 1 & 6 \\ 2 & 1 & 2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 & 9 \\ 3 & 1 & 6 \\ 2 & 1 & 2 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 & 9 \\ 1 & -2 & 3 \\ 2 & 1 & 2 \end{vmatrix}$$

$$\overset{\text{first row}}{=} -\left((-4)+2(-6)+3(1)\right) \overset{\text{first row}}{-2}\left(2(-4)-(-6)+9(1)\right)$$

$$\overset{\text{first row}}{+3}\left(2(-7)-(-4)+9(5)\right)$$

$$= (-13) - 2(7) + 3(35) = 104$$

$$\boxed{\begin{aligned} \therefore x_1 &= \frac{\det(A_1)}{\det(A)} = \frac{-16}{32} = -\frac{1}{2} \\ \therefore x_2 &= \frac{\det(A_2)}{\det(A)} = \frac{72}{32} = \frac{9}{4} \\ \therefore x_3 &= \frac{\det(A_3)}{\det(A)} = \frac{24}{32} = \frac{3}{4} \\ \therefore x_4 &= \frac{\det(A_4)}{\det(A)} = \frac{104}{32} = \frac{13}{4} \end{aligned}}$$

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4)

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 4 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow \det(A) = \begin{vmatrix} 2 & 1 & 2 \\ 0 & 3 & 4 \\ 1 & 1 & 2 \end{vmatrix} \overset{\text{second row}}{=} 3(4-2) - 4(2-1) = 2$$

$$\Rightarrow A_1 = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix},\ A_2 = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix},\ A_3 = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \det(A_1) \overset{\text{first column}}{=} -(2-2) = 0$$

$$\Rightarrow \det(A_2) \overset{\text{second column}}{=} 1(4-2) = 2$$

$$\Rightarrow \det(A_3) \overset{\text{third column}}{=} -(2-1) = -1$$

$$\therefore x_1 = \frac{\det(A_1)}{\det(A)} = \frac{0}{2} = 0$$

$$\therefore x_2 = \frac{\det(A_2)}{\det(A)} = \frac{2}{2} = 1$$

$$\therefore x_3 = \frac{\det(A_3)}{\det(A)} = \frac{-1}{2} = -\frac{1}{2}$$

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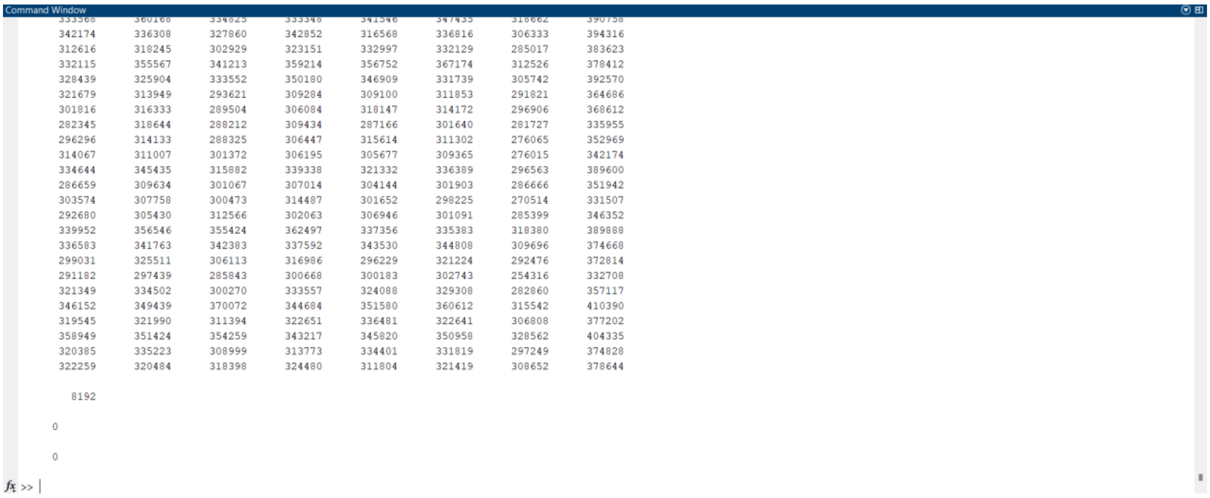
$\therefore$ The second column of A^{-1} is $\left(0, 1, \frac{-1}{2}\right)^T$ \#

1.3

4)



Command Window

342174	336308	327860	342852	316568	336816	306333	394316
312616	318245	302929	323151	332997	332129	285017	383623
332115	355567	341213	359214	356752	367174	312526	378412
328439	325904	333552	350180	346909	331739	305742	392570
321679	313949	293621	309284	309100	311853	291821	364686
301816	316333	289504	306084	318147	314172	296906	368612
282345	318644	288212	309434	287166	301640	281727	335955
296296	314133	288325	306447	315614	311302	276065	352969
314067	311007	301372	306195	305677	309365	276015	342174
334644	345435	315882	339338	321332	336389	296563	389600
286659	309634	301067	307014	304144	301903	286666	351942
303574	307758	300473	314487	301652	298225	270514	331507
292680	305430	312566	302063	306946	301091	285399	346352
339952	356546	355424	362497	337356	335383	318380	389888
336583	341763	342383	337592	343530	344808	309696	374668
299031	325511	306113	316986	296229	321224	292476	372814
291182	297439	285843	300668	300183	302743	254316	332708
321349	334502	300270	333557	324088	329308	282860	357117
346152	349439	370072	344684	351580	360612	315542	410390
319545	321990	311394	322651	336481	322641	306808	377202
358949	351424	354259	343217	345820	350958	328562	404335
320385	335223	308999	313773	334401	331819	297249	374828
322259	320484	318398	324480	311804	321419	308652	378644

8192

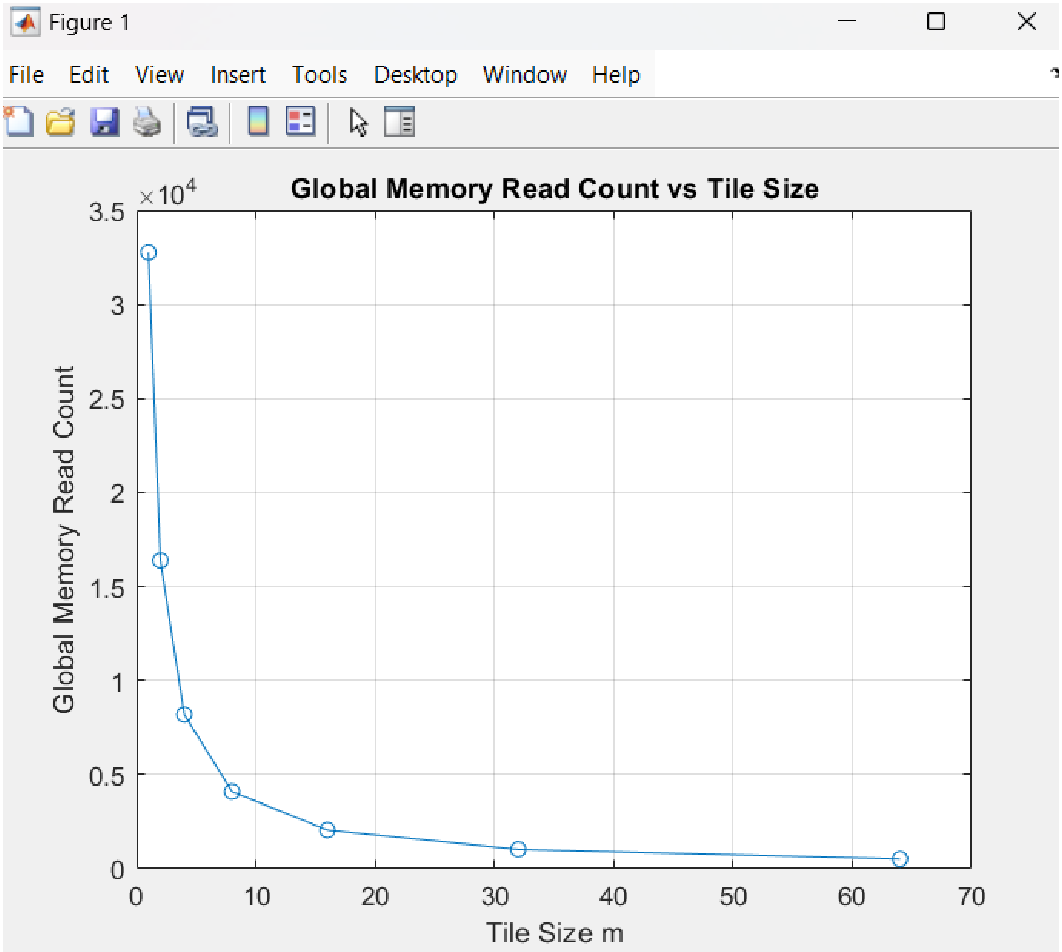
0

0

fx >>

- **As you can see the result of errors of both C's are zero, so they(C_naive & C_tiled) are accurate.**

5)



- **This plot has shown an the decrease in memory reads as tile size increases, illustrating the efficiency gained by reading blocks of data. However, if the tile size becomes too large (approaching the size of the matrix), you might see diminishing returns or even increased reads due to inefficiencies in handling very large tiles.**