Homework 5 林業誠 E24105038 資訊甲 114

5.14) a) $b(4;4,0.9) = \sum_{x=0}^{4} b(x;4,0.9) - \sum_{x=0}^{3} b(x;4,0.9)$

$= 1 - 0.3439$

$\boxed{= 0.6561}$ #

b) $b^*(4;4,0.9) + b^*(5;4,0.9) + b^*(6;4,0.9) + b^*(7;4,0.9)$

$= 0.9\Big[\Big(\sum_{x=0}^{3} b(x;3,0.9) - \sum_{x=0}^{2} b(x;2,0.9)\Big) + \Big(\sum_{x=0}^{3} b(x;4,0.9) - \sum_{x=0}^{2} b(x;4,0.9)\Big)$

$+ \Big(\sum_{x=0}^{3} b(x;5,0.9) - \sum_{x=0}^{2} b(x;5,0.9)\Big) + \Big(\sum_{x=0}^{3} b(x;6,0.9) - \sum_{x=0}^{2}(x;6,0.9)\Big)\Big]$

$= 0.9\,(1 - 0.2710 + 0.3439 - 0.0523 + 0.0815 - 0.0086 + 0.0159 - 0.0013)$

$= 0.9(1.1081)$ $\boxed{= 0.99729}$ #

c) The probability that the Bulls win is always $\boxed{0.9}$ #

5.26)

a) $b(6;8,\frac{6}{10}) = \binom{8}{6}(0.6)^6(0.4)^2$

$= \frac{8!}{2!6!}(0.6)^6(0.4)^2$

$\boxed{= 0.20901888}$ #

b) $b(6;8,\frac{6}{10})$

$= \sum_{x=0}^{6} b(x;8,0.6) - \sum_{x=0}^{5} b(x;8,0.6)$

$= 0.8936 - 0.6846$ $\boxed{= 0.209}$ #

5.50)

a) $b^*(7;3,0.5) = 0.5\Big[\sum_{x=0}^{2} b(x;6,0.5) - \sum_{x=0}^{1} b(x;6,0.5)\Big]$

$= 0.5\,(0.3438 - 0.1094)$

$\boxed{= 0.1172}$ #

b) $g(4;0.5) = b^*(4;1,0.5) = 0.5\Big[\sum_{x=0}^{0} b(x;3,0.5)\Big]$

$= 0.5(0.125)$

$\boxed{= 0.0625}$ #

CamScanner

5.56)

a) $\mu = 3$

$\therefore p(5;3) = \sum_{x=0}^{5} p(x;3) - \sum_{x=0}^{4} p(x;3)$

$= 0.9161 - 0.8153 \boxed{= 0.1008}$ #

b) $P(X<3) = \sum_{x=0}^{2} p(x;3)$

$\boxed{= 0.4232}$ #

c) $P(X \geq 2) = 1 - \sum_{x=0}^{1} p(x;3)$

$= 1 - 0.1991$

$\boxed{= 0.8009}$ #

5.80) $\mu = 2.7$

a) $P(X \leq 4) = \sum_{x=0}^{4} p(x;2.7)$

$= \sum_{x=0}^{4} \frac{e^{-2.7}(2.7)^x}{x!}$

$= e^{-2.7} + (e^{-2.7})(2.7) + \frac{e^{-2.7}(2.7)^2}{2} + \frac{e^{-2.7}(2.7)^3}{2\cdot 3} + \frac{e^{-2.7}(2.7)^4}{2\cdot 3\cdot 4}$

$\simeq \boxed{0.8629}$ #

b) $P(X<2) = \sum_{x=0}^{1} p(x;2.7)$

$= e^{-2.7} + (e^{-2.7})(2.7)$

$\boxed{\simeq 0.2487}$ #

c) * in a 5-minute period $\therefore \mu = 2.7 \times 5 = 13.5$

$\Rightarrow P(X>10) = 1 - \sum_{x=0}^{10} p(x;13.5)$

$= 1 - \Big[e^{-13.5} + (e^{-13.5})(13.5) + \frac{e^{-13.5}(13.5)^2}{2!} + \frac{e^{-13.5}(13.5)^3}{3!} + \frac{e^{-13.5}(13.5)^4}{4!}$

$+ \frac{e^{-13.5}(13.5)^5}{5!} + \frac{e^{-13.5}(13.5)^6}{6!} + \frac{e^{-13.5}(13.5)^7}{7!} + \frac{e^{-13.5}(13.5)^8}{8!} + \frac{e^{-13.5}(13.5)^9}{9!}$

$+ \frac{e^{-13.5}(13.15)^{10}}{10!} \Big]$ $\boxed{\simeq 0.7888}$ #

HW5 Matlab assignment

1:

(c)

Command Window

n	r	p = 0.1	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0	0.9	0.8	0.75	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	1	1	1	1	1	1	1	1	1	1	1
2	0	0.81	0.64	0.5625	0.49	0.36	0.25	0.16	0.09	0.04	0.01
2	1	0.99	0.96	0.9375	0.91	0.84	0.75	0.64	0.51	0.36	0.19
2	2	1	1	1	1	1	1	1	1	1	1
3	0	0.729	0.512	0.4219	0.343	0.216	0.125	0.064	0.027	0.008	0.001
3	1	0.972	0.896	0.8438	0.784	0.648	0.5	0.352	0.216	0.104	0.028
3	2	0.999	0.992	0.9844	0.973	0.936	0.875	0.784	0.657	0.488	0.271
3	3	1	1	1	1	1	1	1	1	1	1
4	0	0.6561	0.4096	0.3164	0.2401	0.1296	0.0625	0.0256	0.0081	0.0016	0.0001
4	1	0.9477	0.8192	0.7383	0.6517	0.4752	0.3125	0.1792	0.0837	0.0272	0.0037
4	2	0.9963	0.9728	0.9492	0.9163	0.8208	0.6875	0.5248	0.3483	0.1808	0.0523
4	3	0.9999	0.9984	0.9961	0.9919	0.9744	0.9375	0.8704	0.7599	0.5904	0.3439
4	4	1	1	1	1	1	1	1	1	1	1
5	0	0.5905	0.3277	0.2373	0.1681	0.0778	0.0313	0.0102	0.0024	0.0003	0
5	1	0.9185	0.7373	0.6328	0.5282	0.337	0.1875	0.087	0.0308	0.0067	0.0005
5	2	0.9914	0.9421	0.8965	0.8369	0.6826	0.5	0.3174	0.1631	0.0579	0.0086
5	3	0.9995	0.9933	0.9844	0.9692	0.913	0.8125	0.663	0.4718	0.2627	0.0815
5	4	1	0.9997	0.999	0.9976	0.9898	0.9688	0.9222	0.8319	0.6723	0.4095
5	5	1	1	1	1	1	1	1	1	1	1
6	0	0.5314	0.2621	0.178	0.1176	0.0467	0.0156	0.0041	0.0007	0.0001	0
6	1	0.8857	0.6554	0.5339	0.4202	0.2333	0.1094	0.041	0.0109	0.0016	0.0001
6	2	0.9842	0.9011	0.8306	0.7443	0.5443	0.3438	0.1792	0.0705	0.017	0.0013
6	3	0.9987	0.983	0.9624	0.9295	0.8208	0.6563	0.4557	0.2557	0.0989	0.0158
6	4	0.9999	0.9984	0.9954	0.9891	0.959	0.8906	0.7667	0.5798	0.3446	0.1143
6	5	1	0.9999	0.9998	0.9993	0.9959	0.9844	0.9533	0.8824	0.7379	0.4686
6	6	1	1	1	1	1	1	1	1	1	1
7	0	0.4783	0.2097	0.1335	0.0824	0.028	0.0078	0.0016	0.0002	0	0
7	1	0.8503	0.5767	0.4449	0.3294	0.1586	0.0625	0.0188	0.0038	0.0004	0
7	2	0.9743	0.852	0.7564	0.6471	0.4199	0.2266	0.0963	0.0288	0.0047	0.0002
7	3	0.9973	0.9667	0.9294	0.874	0.7102	0.5	0.2898	0.126	0.0333	0.0027
7	4	0.9998	0.9953	0.9871	0.9712	0.9037	0.7734	0.5801	0.3529	0.148	0.0257
7	5	1	0.9996	0.9987	0.9962	0.9812	0.9375	0.8414	0.6706	0.4233	0.1497
7	6	1	1	0.9999	0.9998	0.9984	0.9922	0.972	0.9176	0.7903	0.5217
7	7	1	1	1	1	1	1	1	1	1	1

fx >>

(d)

```
>> HW5_1d
Poisson Probability Sums Σ(x=0~r)[p(x;μ)]
```

r	μ = 5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5
0	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001
1	0.0266	0.0174	0.0113	0.0073	0.0047	0.003	0.0019	0.0012	0.0008
2	0.0884	0.062	0.043	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042
3	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149
4	0.3575	0.2851	0.2237	0.173	0.1321	0.0996	0.0744	0.055	0.0403
5	0.5289	0.4457	0.369	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885
6	0.686	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649
7	0.8095	0.744	0.6728	0.5987	0.5246	0.453	0.3856	0.3239	0.2687
8	0.8944	0.8472	0.7916	0.7291	0.662	0.5925	0.5231	0.4557	0.3918
9	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.653	0.5874	0.5218
10	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.706	0.6453
11	0.989	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.803	0.752
12	0.9955	0.9912	0.984	0.973	0.9573	0.9362	0.9091	0.8758	0.8364
13	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981
14	0.9994	0.9986	0.997	0.9943	0.9897	0.9827	0.9726	0.9585	0.94
15	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.978	0.9665
16	0.9999	0.9998	0.9996	0.999	0.998	0.9963	0.9934	0.9889	0.9823
17	1	0.9999	0.9998	0.9996	0.9992	0.9984	0.997	0.9947	0.9911
18	1	1	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957
19	1	1	1	1	0.9999	0.9997	0.9995	0.9989	0.998
20	1	1	1	1	1	0.9999	0.9998	0.9996	0.9991
21	1	1	1	1	1	1	0.9999	0.9998	0.9996
22	1	1	1	1	1	1	1	0.9999	0.9999
23	1	1	1	1	1	1	1	1	0.9999
24	1	1	1	1	1	1	1	1	1

```
fx >>
```

(e)

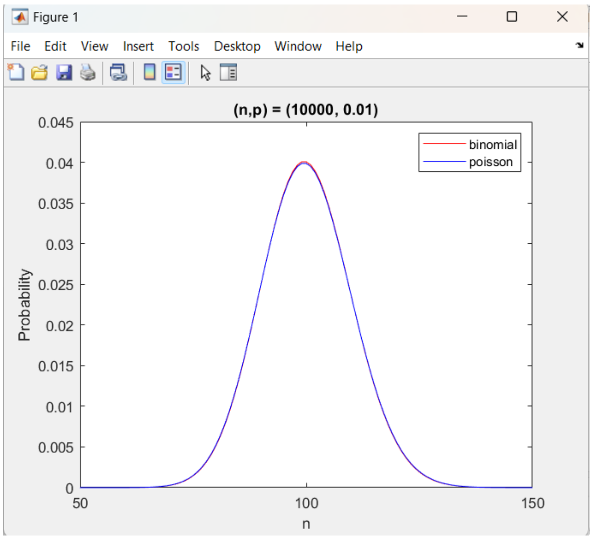


Figure 1
File Edit View Insert Tools Desktop Window Help
(n,p) = (10000, 0.01)
binomial
poisson
Probability
n
0.045 0.04 0.035 0.03 0.025 0.02 0.015 0.01 0.005 0
50 100 150

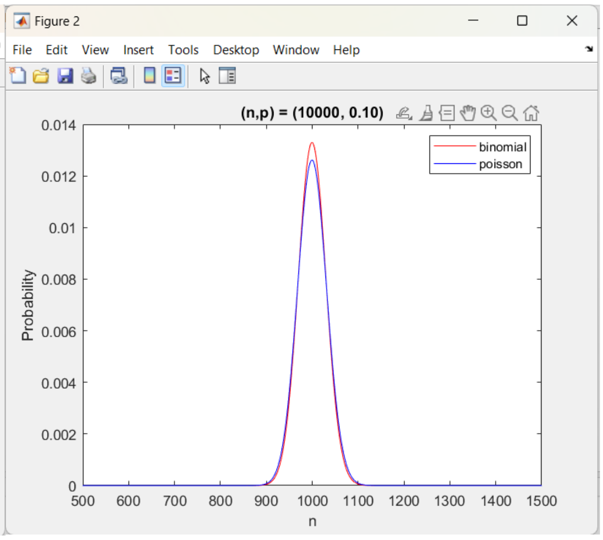


Figure 2
File Edit View Insert Tools Desktop Window Help
(n,p) = (10000, 0.10)
binomial
poisson
Probability
n
0.014 0.012 0.01 0.008 0.006 0.004 0.002 0
500 600 700 800 900 1000 1100 1200 1300 1400 1500

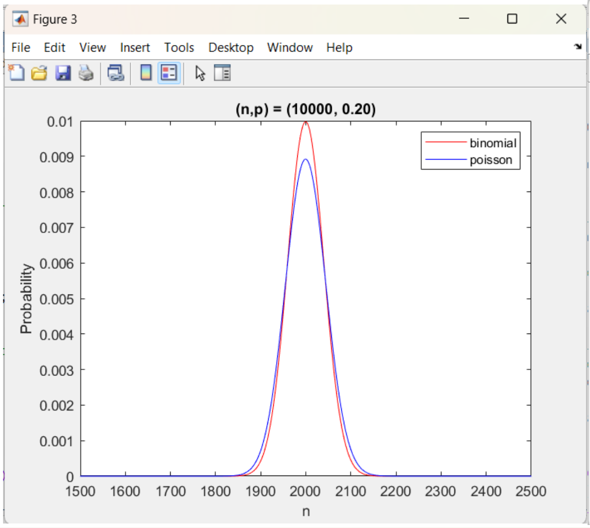


Figure 3
File Edit View Insert Tools Desktop Window Help
(n,p) = (10000, 0.20)
binomial
poisson
Probability
n
0.01 0.009 0.008 0.007 0.006 0.005 0.004 0.003 0.002 0.001 0
1500 1600 1700 1800 1900 2000 2100 2200 2300 2400 2500

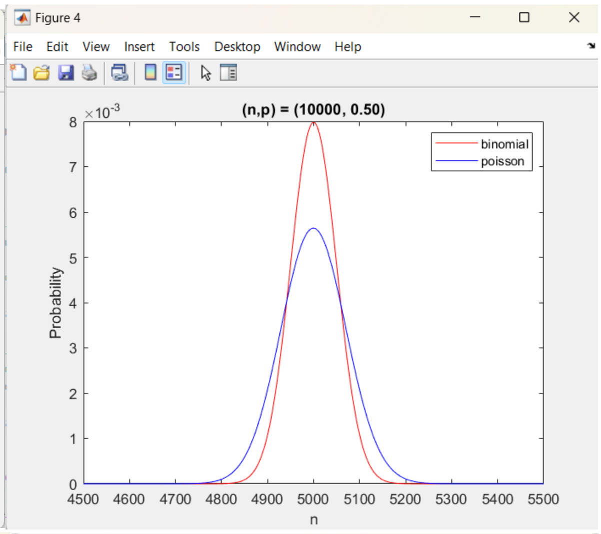


Figure 4
File Edit View Insert Tools Desktop Window Help
(n,p) = (10000, 0.50)
binomial
poisson
Probability
n
×10-3
8 7 6 5 4 3 2 1 0
4500 4600 4700 4800 4900 5000 5100 5200 5300 5400 5500

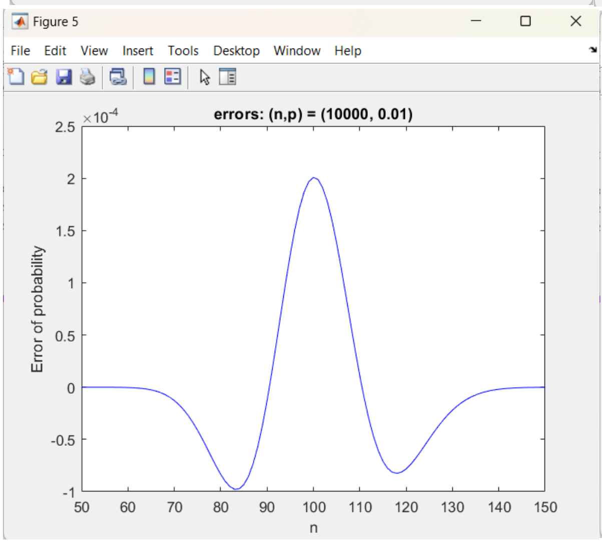


Figure 5
File Edit View Insert Tools Desktop Window Help
errors: (n,p) = (10000, 0.01)
Error of probability
n
×10-4
2.5 2 1.5 1 0.5 0 -0.5 -1
50 60 70 80 90 100 110 120 130 140 150

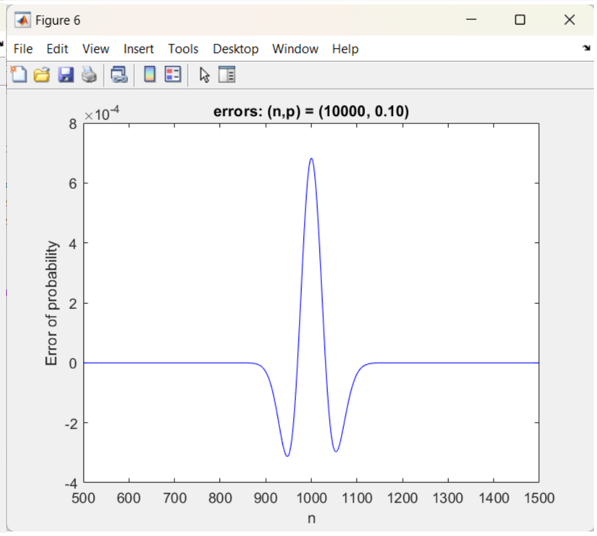
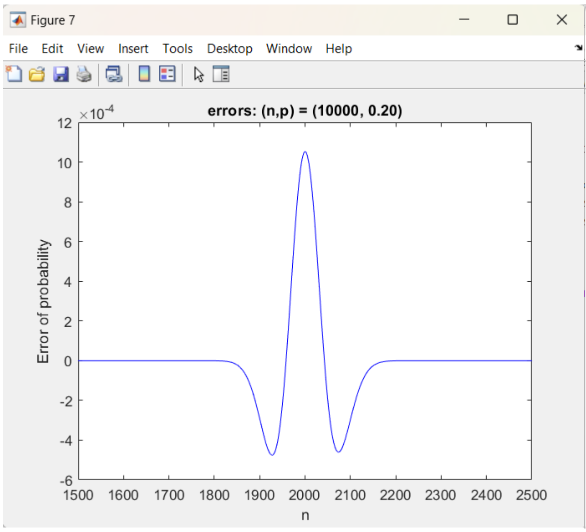
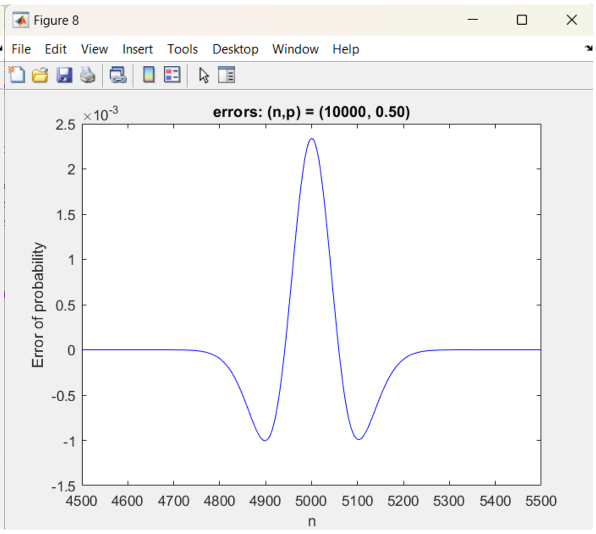


Figure 6
File Edit View Insert Tools Desktop Window Help
errors: (n,p) = (10000, 0.10)
Error of probability
n
×10-4
8 6 4 2 0 -2 -4
500 600 700 800 900 1000 1100 1200 1300 1400 1500





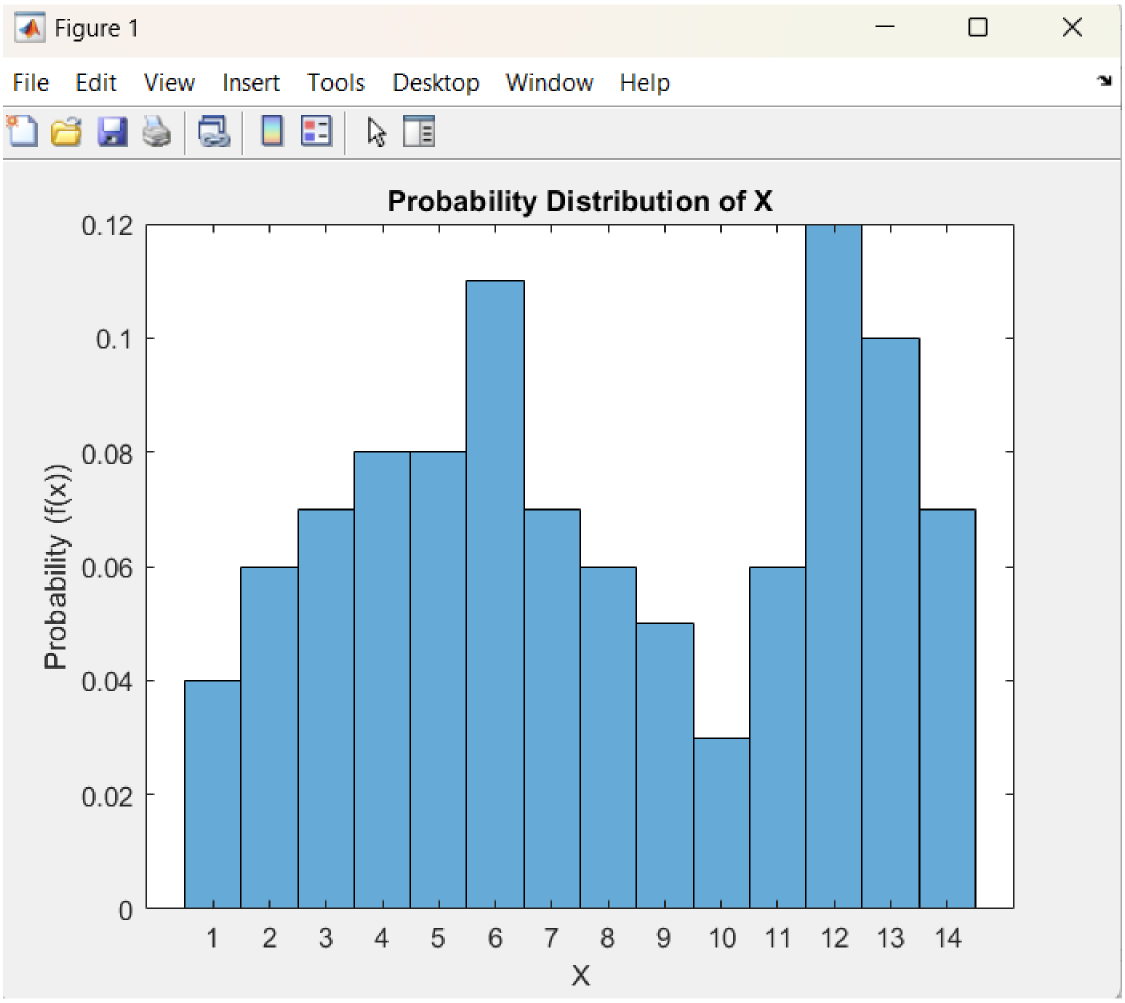
Referring to these graphs, we can conclude that:

1. **(n, p) = (10^4, 0.01):**
   - **The accuracy of approximation is the highest among them, as you can see the graph of Binomial and Poisson distribution are about to overlapping each other completely and the errors are the lowest among them.**
2. **(n, p) = (10^4, 0.1):**
   - **The accuracy of approximation is lower than when p = 0.01 as the errors are higher than when p = 0.01.**
3. **(n, p) = (10^4, 0.2):**
   - **The accuracy of approximation is lower than when p = 0.01 and 0.1, as the errors are higher than when p = 0.01 and 0.1.**
4. **(n, p) = (10^4, 0.5):**
   - **The accuracy of approximation is the lowest among them as the errors are the highest among them.**

As the (success) probability (p) increases, the accuracy of the approximation tends to diminish. On the other hand, when p is small and n is large, the Poisson approximation is generally accurate.

2:

(a)



(b) The relative frequency plot generated in part 2(b) using the random samples look alike to the plot in 2(a) but not exactly alike, which represents the true probability distribution. The similarity between the two plots depends on the number of samples generated and the randomness involved in the sampling process.

As the number of samples increases, the relative frequency plot should be more alike to the true probability distribution represented by the plot in 2(a). By generating a large number of samples (10^4 number of samples are considered as enough to let the plot seems alike to the true probability distribution) and comparing the relative frequency plot to the true distribution plot,

we can assess the overall agreement between the two and evaluate the effectiveness of the sampling approach.