Homework 4 林業誠 E24105038 資訊114甲

4.24)

$$\Rightarrow f(x,y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}, \quad 1 \le x+y \le 4, \quad x=0,1,2,3, \quad y=0,1,2$$

(a)

$$E(X^2Y-2XY) = \sum_{x=0}^{3}\sum_{y=0}^{2}(x^2y-2xy)f(x,y)$$

$$= (1-2)\left[\frac{\binom{3}{1}\binom{2}{1}\binom{3}{2}}{\binom{8}{4}}\right] + (2-4)\left[\frac{\binom{3}{1}\binom{2}{2}\binom{3}{1}}{\binom{8}{4}}\right] + \underbrace{(4-4)}_{0}\left[\frac{\binom{3}{2}\binom{2}{1}\binom{3}{1}}{\binom{8}{4}}\right]$$

$$+ \underbrace{(8-8)}_{0}\left[\frac{\binom{3}{2}\binom{2}{2}}{\binom{8}{4}}\right] + (9-6)\left[\frac{\binom{3}{3}\binom{2}{1}}{\binom{8}{4}}\right]$$

$$= -\frac{9}{35} - \frac{9}{35} + \frac{3}{35} \boxed{= -\frac{3}{7}} \#$$

(b)

x	0	1	2	3
$g(x)$	$\frac{1}{14}$	$\frac{3}{7}$	$\frac{3}{7}$	$\frac{1}{14}$

y	0	1	2
$h(y)$	$\frac{3}{14}$	$\frac{4}{7}$	$\frac{3}{14}$

$$\hookrightarrow \mu_X = E(X) = (0)\left(\frac{1}{14}\right) + (1)\left(\frac{3}{7}\right) + (2)\left(\frac{3}{7}\right) + (3)\left(\frac{1}{14}\right) \boxed{= \frac{3}{2}}$$

$$\therefore \mu_X - \mu_Y = \frac{3}{2} - 1 \boxed{= \frac{1}{2}} \#$$

$$\hookrightarrow \mu_Y = E(Y) = (0)\left(\frac{3}{14}\right) + (1)\left(\frac{4}{7}\right) + (2)\left(\frac{3}{14}\right) \boxed{= 1}$$

4.44)

$$E(XY) = \sum_{x=0}^{3}\sum_{y=0}^{2} xy f(x,y) = (1)(1)\left(\frac{9}{35}\right) + (1)(2)\left(\frac{9}{70}\right) + (2)(1)\left(\frac{9}{35}\right) + (2)(2)\left(\frac{3}{70}\right)$$

$$+ (3)(1)\left(\frac{1}{35}\right)$$

$$\boxed{= \frac{9}{7}} \#$$

$\because \mu_X = \frac{3}{2}, \mu_Y = 1$ (from 4.24)

$$\therefore \sigma_{XY} = E(XY) - \mu_X\mu_Y$$

$$= \frac{9}{7} - \left(\frac{3}{2}\right)(1)$$

$$\boxed{= -\frac{3}{14}} \#$$

4.60)

$Y \backslash x$	2	4	$h(y)$
1	0.15	0.10	0.25
3	0.25	0.25	0.50
5	0.15	0.10	0.25
$g(x)$	0.55	0.45	1

(a) $E(2X-3Y) = 2E(X) - 3E(Y)$

$\Rightarrow E(X) = 2(0.55) + 4(0.45) = 2.9$

$\Rightarrow E(Y) = 1(0.25) + 3(0.50) + 5(0.25)$

$= 3$

$\therefore E(2X-3Y) = 2(2.9) - 3(3) \boxed{= -3.2}$ #

(b) $E(XY) = E(X)E(Y)$

$= (2.9)(3) \boxed{= 8.7}$

4.78) $\mu = E(X) = 30\int_0^1 x^3(1-x)^2 dx = 30\int_0^1 x^3 - 2x^4 + x^5 dx$

$= 30\left[\frac{x^4}{4} - \frac{2x^5}{5} + \frac{x^6}{6}\right]\Big|_0^1$

$= 30\left(\frac{1}{4} - \frac{2}{5} + \frac{1}{6}\right)$

$= \frac{1}{2}$

$E(X^2) = 30\int_0^1 x^4(1-x)^2 dx$

$= 30\int_0^1 x^4 - 2x^5 + x^6 dx$

$= 30\left[\frac{x^5}{5} - \frac{x^6}{3} + \frac{x^7}{7}\right]\Big|_0^1$

$= 30\left(\frac{1}{5} - \frac{1}{3} + \frac{1}{7}\right)$

≈ 0.2857

$\Rightarrow \sigma^2 = 0.2857 - (0.5)^2$

$= 0.0357$

$\therefore \sigma \approx 0.1889$

$\therefore P(\mu - 2\sigma < X < \mu + 2\sigma)$

$= P(0.5 - 2(0.1889) < X < 0.5 + 2(0.1889))$

$= P(0.1222 < X < 0.8778)$

$= 30\int_{0.1222}^{0.8778} x^2(1-x^2) dx$

$= 30\int_{0.1222}^{0.8778} x^2 - 2x^3 + x^4 dx$

$= 30\left[\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\right]\Big|_{0.1222}^{0.8778}$

$\boxed{\simeq 0.9682 \geq 1 - \frac{1}{k^2} = \frac{3}{4}}$ #

↳ The answer is 0.9682, so it matches with the result of at least 0.75 given by Chebyshev's theorem.

4.98)

	y		
x	0	1	2
0	0.12	0.04	0.04
1	0.08	0.19	0.05
2	0.06	0.12	0.30

(a) ↳

x	0	1	2
$g(x)$	0.2	0.32	0.48

#

↳

Y	0	1	2
$h(y)$	0.26	0.35	0.39

#

↳

x	0	1	2
$f(x\|2)$	$\frac{4}{39}$	$\frac{5}{39}$	$\frac{10}{13}$

#

(b) ↳ $E(X) = 0(0.2) + 1(0.32) + 2(0.48)$

$\boxed{= 1.28}$ #

$\Rightarrow E(X^2) = 0^2(0.2) + 1^2(0.32) + 2^2(0.48)$

$= 2.24$

↳ $\sigma^2 = E(X^2) - \mu^2$

$= 2.24 - 1.28^2$

$\boxed{= 0.6016}$ #

(c) ↳ $E(X|Y=2)$

$= 0(\frac{4}{39}) + 1(\frac{5}{39}) + 2(\frac{10}{13})$

$\boxed{= \frac{5}{3}}$ #

$\Rightarrow E(X^2|Y=2) = 0^2(\frac{4}{39}) + 1^2(\frac{5}{39}) + 2^2(\frac{10}{13})$

$= \frac{125}{39}$

↳ $\sigma^2 = E(X^2|Y=2) - \mu^2_{X|Y=2}$

$= \frac{125}{39} - (\frac{5}{3})^2$

$\boxed{= \frac{50}{117}}$ #