HW3

Tuesday, 1 April 2025 12:28 PM

HW3 林業誠 E24105038 資訊114

3.3) Find the Laplace $X(s)$ given that $x(t)$ is

(a) $2t\,u(t-4)$ (b) $5\cos t\,\delta(t-2)$ (c) $e^{-t}u(t-\tau)$ (d) $\sin 2t\,u(t-\tau)$

(a) $X(s) = \mathcal{L}\{2t\,u(t-4)\}$

$\Rightarrow 2t\,u(t-4) = [2(t-4)+8]\,u(t-4)$

$\therefore \mathcal{L}\{[2(t-4)+8]\,u(t-4)\}$

$= \left(\frac{2}{s^2}+\frac{8}{s}\right)e^{-4s}$ #

(b) $X(s) = \int_0^\infty 5\cos t\,\delta(t-2)\cdot e^{-st}\,dt$

$= 5\cos t\cdot e^{-st}\Big|_{t=2}$

$= 5\cos 2\cdot e^{-2s}$ #

(c) $\Rightarrow e^{-t}u(t-\tau) = e^{-(t-\tau)}\,\overset{\text{const.}}{e^{-\tau}}\,u(t-\tau)$

$\therefore X(s) = e^{-\tau}\cdot\frac{1}{s+1}\cdot e^{-\tau s}$

$= \frac{e^{-\tau(s+1)}}{s+1}$ #

(d) $\Rightarrow \sin 2t\,u(t-\tau) = \sin[2(t-\tau)+2\tau]\,u(t-\tau)$

$\therefore x(t) = [\sin 2(t-\tau)\cos 2\tau + \cos 2(t-\tau)\sin 2\tau]\,u(t-\tau)$

$\mathcal{L}\{x(t)\} = \left(\frac{2}{s^2+4}\cos 2\tau + \frac{s}{s^2+4}\sin 2\tau\right)e^{-\tau s} = X(s)$ #

3.13) Find the inverse Laplace transform for

(a) $X(s) = \frac{3s+1}{s^2+2s+5}$ (b) $Y(s) = \frac{3s+7}{s^2+3s+2}$

(c) $Z(s) = \frac{s^2-8}{s^2-4}$ (d) $H(s) = \frac{12}{(s+2)^2}$

(a) $X(s) = \frac{3s+1}{s^2+2s+5}$

$= \frac{3(s+1)}{(s+1)^2+4} - \frac{2}{(s+1)^2+4}$

$\therefore \mathcal{L}^{-1}\{X(s)\} = x(t) = [3\cos 2t\cdot e^{-t} - \sin 2t\cdot e^{-t}]\,u(t)$

$= u(t)\cdot e^{-t}(3\cos 2t - \sin 2t)$ #

(b) $Y(s) = \frac{3s+7}{s^2+3s+2} = \frac{3s+7}{(s+2)(s+1)}$

$= \frac{-1}{s+2} + \frac{4}{s+1}$

$\therefore \mathcal{L}\{Y(s)\} = y(t) = (-e^{-2t} + 4e^{-t})\,u(t)$ #

(c) $Z(s) = \frac{s^2-8}{s^2-4} = 1 - \frac{4}{s^2-4}$

$= 1 - \frac{4}{(s+2)(s-2)}$

$= 1 + \frac{1}{s+2} - \frac{1}{s-2}$

$$\begin{array}{r} 1 \\ s^2+0-4\,\big)\,\overline{s^2+0-8} \\ -)\;s^2+0-4 \\ \hline -4 \end{array}$$

$\therefore \mathcal{L}^{-1}\{Z(s)\} = z(t) = \delta(t) + (e^{-2t} - e^{2t})\,u(t)$ #

(d) $H(s) = \frac{12}{(s+2)^2}$

$\Rightarrow \mathcal{L}^{-1}\{H(s)\} = h(t) = (e^{-2t}\cdot 12t\cdot\frac{1}{1!})\,u(t)$

$= 12te^{-2t}u(t)$ #

3.14) Find inverse Laplace transform

(a) $F(s) = \frac{20(s+2)}{s(s^2+6s+25)}$ (b) $\frac{6s^2+36s+20}{(s+1)(s+2)(s+3)} = P(s)$

(a) $F(s) = \frac{20(s+2)}{s(s^2+6s+25)}$

$\Rightarrow \frac{20(s+2)}{s(s^2+6s+25)} = \frac{A}{s} + \frac{Bs+C}{s^2+6s+25}$

$A = \frac{40}{25} = \frac{8}{5}$

↳ By Comparing coef.

$s:\ 20 = \frac{48}{5} + C \quad \therefore C = \frac{52}{5}$

$s^2:\ 0 = \frac{8}{5} + B \quad \therefore B = -\frac{8}{5}$

$\therefore F(s) = \frac{\frac{8}{5}}{s} + \frac{-\frac{8}{5}s + \frac{52}{5}}{s^2+6s+25}$

$= \frac{\frac{8}{5}}{s} + \frac{-\frac{8}{5}(s+3)}{(s+3)^2+16} + \frac{\frac{76}{5}}{(s+3)^2+16}$

$\therefore \mathcal{L}^{-1}\{F(s)\} = \left(\frac{8}{5} - \frac{8}{5}\cos 4t\cdot e^{-3t} + \frac{19}{5}\sin 4t\cdot e^{-3t}\right)u(t) = f(t)$ #

(b) $P(s) = \frac{6s^2+36s+20}{(s+1)(s+2)(s+3)}$

$= \frac{-5}{s+1} + \frac{28}{s+2} - \frac{17}{s+3}$

$\therefore p(t) = (-5e^{-t} + 28e^{-2t} - 17e^{-3t})\,u(t)$ #

$\Rightarrow \frac{6s^2+36s+20}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$

$\Rightarrow A = \frac{6-36+20}{(-1+2)(-1+3)} = -5$

$\Rightarrow B = \frac{24-72+20}{(-2+1)(-2+3)} = 28$

$\Rightarrow C = \frac{54-108+20}{(-3+1)(-3+2)} = -17$

3.18) Let $F(s) = \frac{5(s+1)}{(s+2)(s+3)}$

(a) Use initial and final value thms. to find $f(0)$ and $f(\infty)$

(b) Verify answer in part (a) by finding $f(t)$ using partial fractions.

(a) $f(0) = \lim_{s\to\infty} sF(s)$

$= \lim_{s\to\infty}\frac{5s(s+1)}{(s+2)(s+3)}$

$= 5$ #

$f(\infty) = \lim_{s\to 0} sF(s)$

$= \lim_{s\to 0}\frac{5s(s+1)}{(s+2)(s+3)}$

$= 0$ #

(b) $F(s) = \frac{5(s+1)}{(s+2)(s+3)}$

$= \frac{A}{s+2} + \frac{B}{s+3}$

$= \frac{-5}{s+2} + \frac{10}{s+3}$

$\Rightarrow A = \frac{-5}{1} = -5,\ B = \frac{5(-2)}{-1} = 10$

$\therefore f(t) = (-5e^{-2t} + 10e^{-3t})\,u(t)$

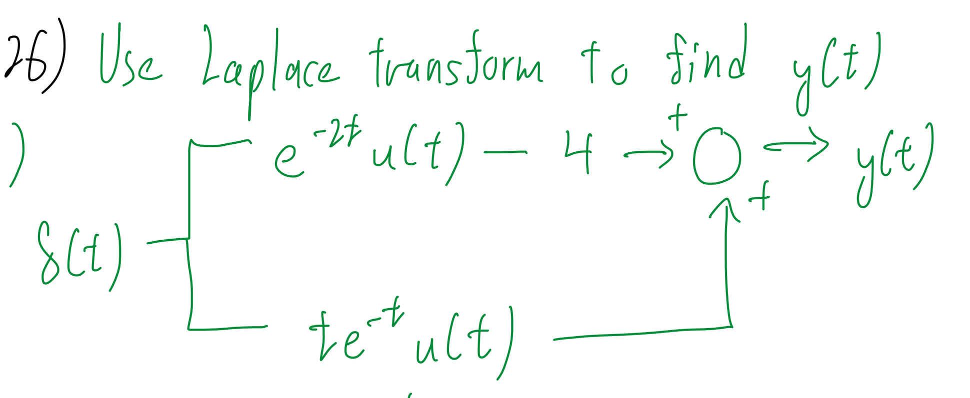
$\therefore f(0) = -5 + 10 = 5$

$\therefore f(\infty) = 0 + 0 = 0$

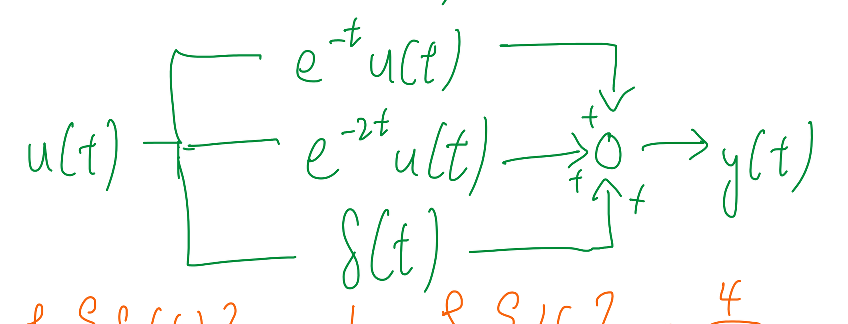
} (verified) #

3.26) Use Laplace transform to find $y(t)$

(a)



(b)



(a) $\mathcal{L}\{\delta(t)\} = 1,\ \mathcal{L}\{4\} = \frac{4}{s}$

$\mathcal{L}\{e^{-2t}u(t)\} = \frac{1}{s+2} \quad \Rightarrow \cancel{e^{-2t}u(t) * 4} = \frac{4}{s(s+2)}$

$\mathcal{L}\{te^{-t}u(t)\} = \frac{1}{s^2}\Big|_{s\to s+1} = \frac{1}{(s+1)^2}$

$\therefore Y(s) = 1\left[\frac{4}{s(s+2)} + \frac{1}{(s+1)^2}\right] = \frac{2}{s} - \frac{2}{s+2} + \frac{1}{(s+1)^2}$

$\therefore y(t) = (2 - 2e^{-2t} + te^{-t})\,u(t)$ #

(b) $\mathcal{L}\{u(t)\} = \frac{1}{s},\ \mathcal{L}\{e^{-t}u(t)\} = \frac{1}{s+1}$

$\mathcal{L}\{e^{-2t}u(t)\} = \frac{1}{s+2},\ \mathcal{L}\{\delta(t)\} = 1$

$\therefore Y(s) = \frac{1}{s}\left[\frac{1}{s+1} + \frac{1}{s+2} + 1\right]$

$= \frac{1}{s(s+1)} + \frac{1}{s(s+2)} + \frac{1}{s}$

$= \left[\frac{1}{s} - \frac{1}{s+1}\right] + \left[\frac{\frac{1}{2}}{s} - \frac{\frac{1}{2}}{s+2}\right] + \frac{1}{s}$

$\therefore y(t) = u(t) - e^{-t}u(t) + \frac{1}{2}u(t) - \frac{1}{2}e^{-2t}u(t) + u(t)$

$= \left[\frac{5}{2} - e^{-t} - \frac{1}{2}e^{-2t}\right]u(t)$ #

3.43) An LTI system is described by

$\Rightarrow \frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} - 3x(t)$

(a) Determine transfer function

(b) Obtain impulse response

(a) $[s^2Y(s) - sy(0) - y'(0)] + [2sY(s) - y(0)] + 2Y(s) = sX(s) - 3X(s)$

$s^2Y(s) + 2sY(s) + 2Y(s) = sX(s) - 3X(s)$

$(s^2+2s+2)Y(s) = (s-3)X(s)$

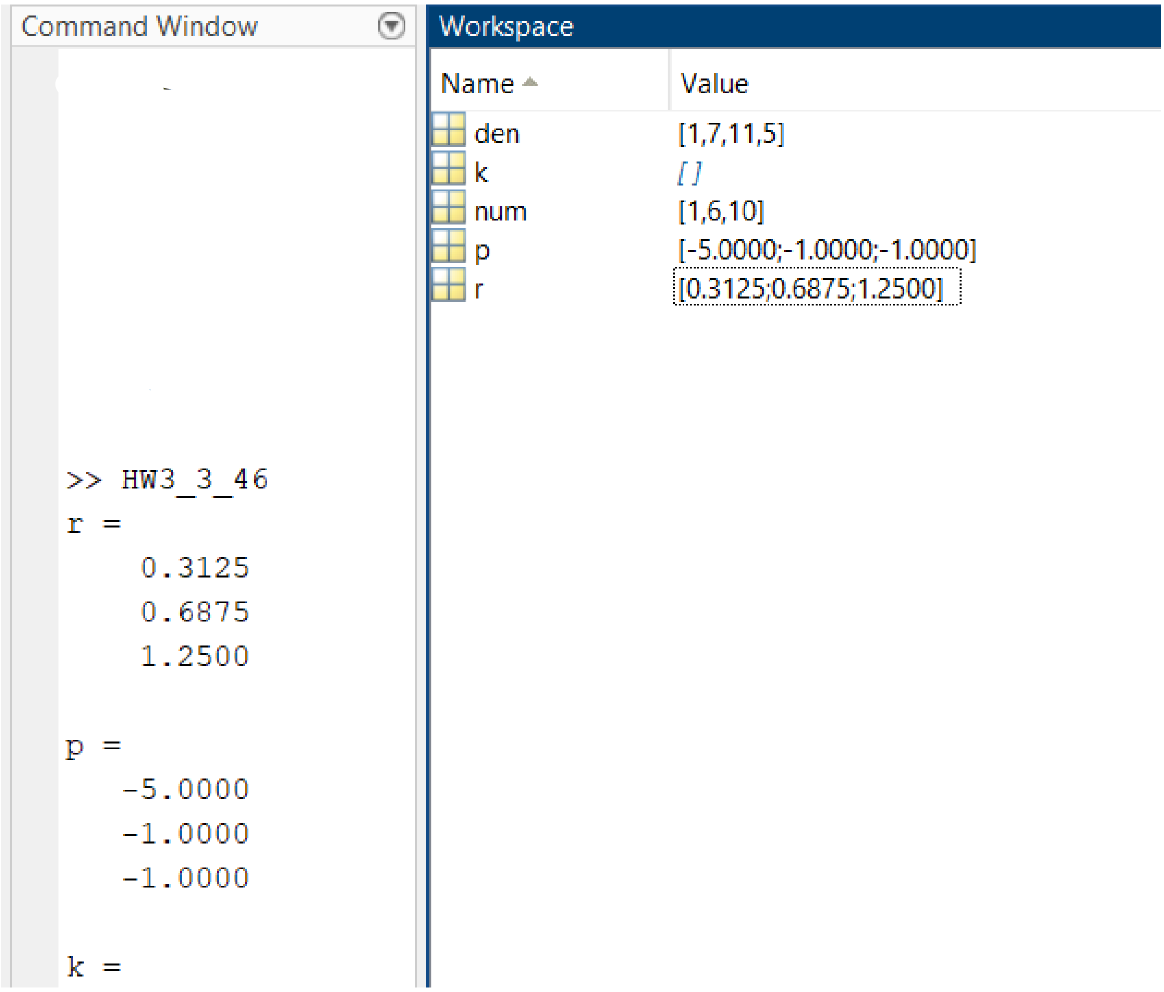
$\therefore H(s) = \frac{Y(s)}{X(s)} = \frac{s-3}{s^2+2s+2}$ #

(b) $H(s) = \frac{s-3}{s^2+2s+2} = \frac{s+1}{(s+1)^2+1} - \frac{4}{(s+1)^2+1}$

$\therefore h(t) = e^{-t}(\cos t - 4\sin t)\,u(t)$ #

<u>**MATLAB Simulation**</u>

3.46)

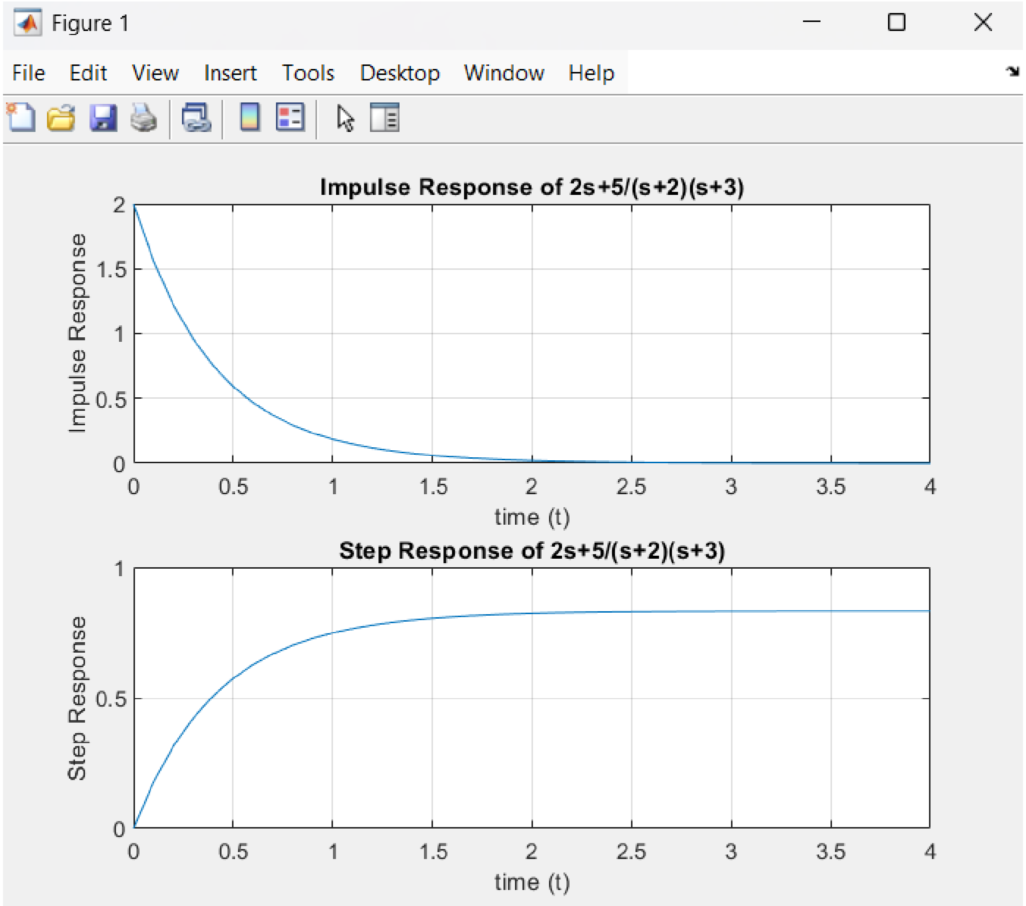


$$H(s) = \frac{0.3125}{s-(-5)} + \frac{0.6875}{s-(-1)} + \frac{1.25}{[s-(-1)]^2}$$

$$= \frac{0.3125}{s+5} + \frac{0.6875}{s+1} + \frac{1.25}{(s+1)^2}$$

$$\therefore h(t) = \left(0.3125e^{-5t} + 0.6875e^{-t} + 1.25te^{-t}\right)u(t) \quad \#$$

3.49)



3.51)

```
>> HW3_3_51
Question (a):
zeros =
     2

poles =
  -1.0000 + 3.0000i
  -1.0000 - 3.0000i

Question (b):
zeros =
  -1.0000 + 2.0000i
  -1.0000 - 2.0000i

poles =
   0.0000 + 0.0000i
  -2.0000 + 3.0000i
  -2.0000 - 3.0000i

Question (c):
zeros =
   -9.4721
   -0.5279

poles =
  -1.5956 + 2.2075i
  -1.5956 - 2.2075i
  -0.8087 + 0.0000i
```

3.53)

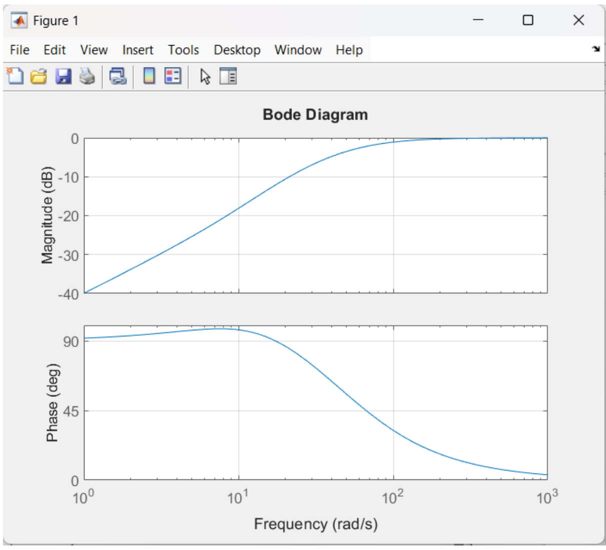


Figure 1
File Edit View Insert Tools Desktop Window Help
Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
Phase (deg)
90
45
0
10^0
10^1
10^2
10^3
Frequency (rad/s)

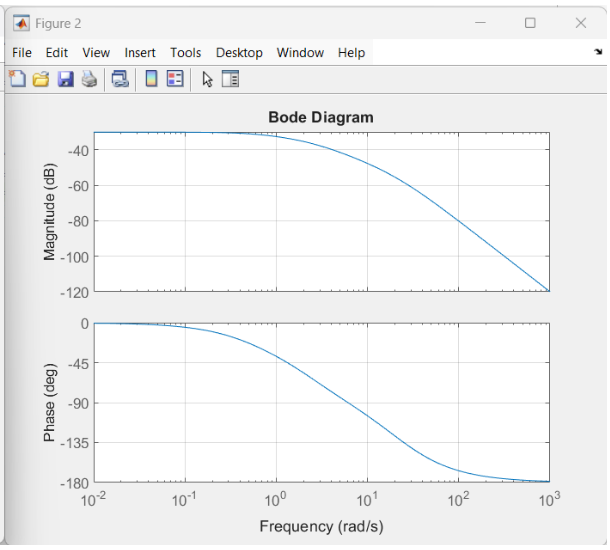


Figure 2
File Edit View Insert Tools Desktop Window Help
Bode Diagram
Magnitude (dB)
-40
-60
-80
-100
-120
Phase (deg)
0
-45
-90
-135
-180
10^-2
10^-1
10^0
10^1
10^2
10^3
Frequency (rad/s)