HW2

Tuesday, 25 March 2025 4:37 PM

HW2 林業誌 E24105038 資訊14

2.4) $x(t)=2\delta(t)$, $y(t)=4u(t)$, $z(t)=e^{-2t}u(t)$

(a) $x(t)*y(t)$ (b) $x(t)*z(t)$ (c) $y(t)*z(t)$ (d) $y(t)*[y(t)+z(t)]$

(a) $x(t)*y(t)$

$=\int_{-\infty}^{\infty}x(\tau)y(t-\tau)\,d\tau=\int_{-\infty}^{\infty}2\delta(\tau)4u(t-\tau)\,d\tau$ ⇒ $\delta(t)=\begin{cases}1, & t\geq 0\\ 0, & t\neq 0\end{cases}$

$=8u(t)$ # $u(t-\tau)=\begin{cases}1, & t-\tau\geq 0\\ 0, & t-\tau<0\end{cases}$

(b) $x(t)*z(t)=\int_{-\infty}^{\infty}2\delta(\tau)e^{-2(t-\tau)}(t-\tau)\,d\tau$

$=2e^{-2t}u(t)$ #

(c) $y(t)*z(t)=\int_{-\infty}^{\infty}4u(\tau)e^{-2(t-\tau)}u(t-\tau)\,d\tau$ ⇒ $u(\tau)=\begin{cases}1, & \tau\geq 0\\ 0, & \tau<0\end{cases}$

$=4\int_0^t e^{-2t}\cdot e^{2\tau}\,d\tau$ $u(t-\tau)=\begin{cases}1, & t-\tau\geq 0\\ 0, & t-\tau<0\end{cases}$ $t\geq\tau$

$=4e^{-2t}\int_0^t e^{2\tau}\,d\tau$ ∴ $t\geq\tau\geq 0$

$=4e^{-2t}\left[\frac{1}{2}e^{2\tau}\Big|_0^t\right]$

$=4e^{-2t}\left(\frac{1}{2}e^{2t}-\frac{1}{2}\right)$

$=2(1-e^{-2t})$ #

(d) $y(t)*[y(t)+z(t)]=4u(t)*[4u(t)+e^{-2t}u(t)]$

$=\int_{-\infty}^{\infty}4u(\tau)\left[4+e^{-2(t-\tau)}\right]u(t-\tau)\,d\tau$

$=4\int_0^t(4+e^{-2t}\cdot e^{2\tau})\,d\tau$

$=4\left(4t+\frac{1}{2}-\frac{1}{2}e^{-2t}\right)$

$=16t-2e^{-2t}+2$ #

2.14) Impulse response of a low-pass filter: $h(t)=e^{-t}u(t)$.

Determine the step response (the output when the input is a unit step)

∴ $x(t)=u(t)$, $h(t)=e^{-t}u(t)$

∴ $y(t)=x(t)*h(t)$

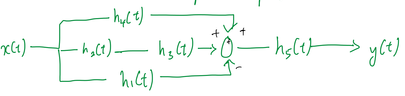
$=\int_{-\infty}^{\infty}u(\tau)e^{-(t-\tau)}u(t-\tau)\,d\tau$ ⇒ $u(\tau)=\begin{cases}1, & \tau\geq 0\\ 0, & \tau<0\end{cases}$

$=e^{-t}\int_0^t e^{\tau}\,d\tau$ $u(t-\tau)=\begin{cases}1, & t-\tau\geq 0\\ 0, & t-\tau<0\end{cases}$

$=e^{-t}(e^{t}-1)u(t)$ ∴ $t\geq\tau\geq 0$

$=(1-e^{-t})u(t)$ #

2.24) Determine the overall impulse response

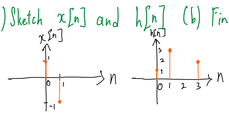


⇒ $h(t)=h_5(t)*[h_4(t)+(h_2(t)*h_3(t))-h_1(t)]$ #

2.30) $x[n]=\begin{cases}1, & n=0\\ -1, & n=1\\ 0, & \text{otherwise}\end{cases}$, $h[n]=\begin{cases}1, & n=0\\ 3, & n=1\\ 2, & n=3\\ 0, & \text{otherwise}\end{cases}$

(a) Sketch $x[n]$ and $h[n]$ (b) Find $x[n]*h[n]$

a)



b) ∵ $x[n]=\delta[n]-\delta[n-1]$ ※ $x[n]*\delta[n-k]=x[n-k]$

∵ $h[n]=\delta[n]+3\delta[n-1]+2\delta[n-3]$

∴ $x[n]*h[n]=x[n]*\{\delta[n]+3\delta[n-1]+2\delta[n-3]\}$

$=x[n]+3x[n-1]+2x[n-3]$

$=\delta[n]-\delta[n-1]+3\delta[n-1]-3\delta[n-2]+2\delta[n-3]-2\delta[n-4]$

$=\delta[n]+2\delta[n-1]-3\delta[n-2]+2\delta[n-3]-2\delta[n-4]$ #

2.33) $h_1[n]=(0.4)^n u[n]$, $h_2[n]=\delta[n]+0.5\delta[n-1]$

Determine the response to the input $x[n]=0.4^n u[n]$ if

(a) Connected in parallel (b) Connected in cascade

a) $h=h_1+h_2=0.4^n u[n]+\delta[n]+0.5\delta[n-1]$

$y[n]=x[n]*h[n]$

$=0.4^n u[n]*\{0.4^n u[n]+\delta[n]+0.5\delta[n-1]\}$

$=(n+1)(0.4)^n u[n]+0.4^n u[n]+0.5(0.4)^{n-1}u[n-1]$

$=(n+2)(0.4)^n u[n]+0.5(0.4)^{n-1}u[n-1]$ #

b) $h=h_1*h_2=0.4^n u[n]*\{\delta[n]+0.5\delta[n-1]\}$

$=0.4^n u[n]+0.5(0.4)^{n-1}u[n-1]$

$y[n]=x[n]*h[n]=0.4^n u[n]*\{0.4^n u[n]+0.5(0.4)^{n-1}u[n-1]\}$

$=(n+1)0.4^n u[n]+0.5(n-1+1)(0.4)^{n-1}u[n-1]$

$=(n+1)0.4^n u[n]+0.5n(0.4)^{n-1}u[n-1]$ #

2.36) Input $x[n]=[1\ \ -1]$, output $y[n]=[4\ \ 2\ \ 5\ \ 1]$

⇒ Determine the impulse response

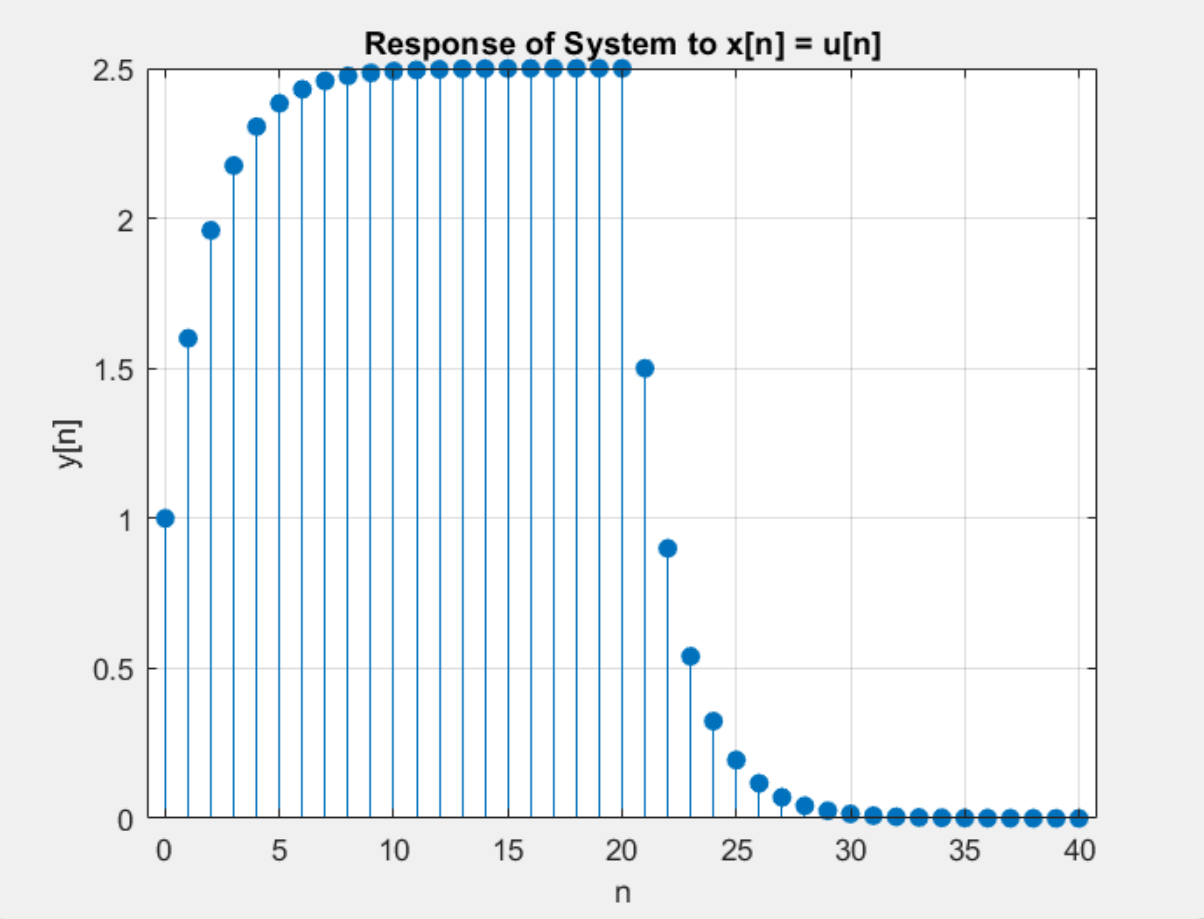
⇒ $x(z)=z-1$, $y(z)=4z^3+2z^2+5z+1$

$$
\begin{array}{r}
4z^2+6z+11\\
z-1\ \overline{)\ 4z^3+2z^2+5z+1}\\
-)\ 4z^3-4z^2\\
\hline
6z^2+5z\\
-)\ 6z^2-6z\\
\hline
11z+1\\
-)\ 11z-11\\
\hline
12
\end{array}
$$

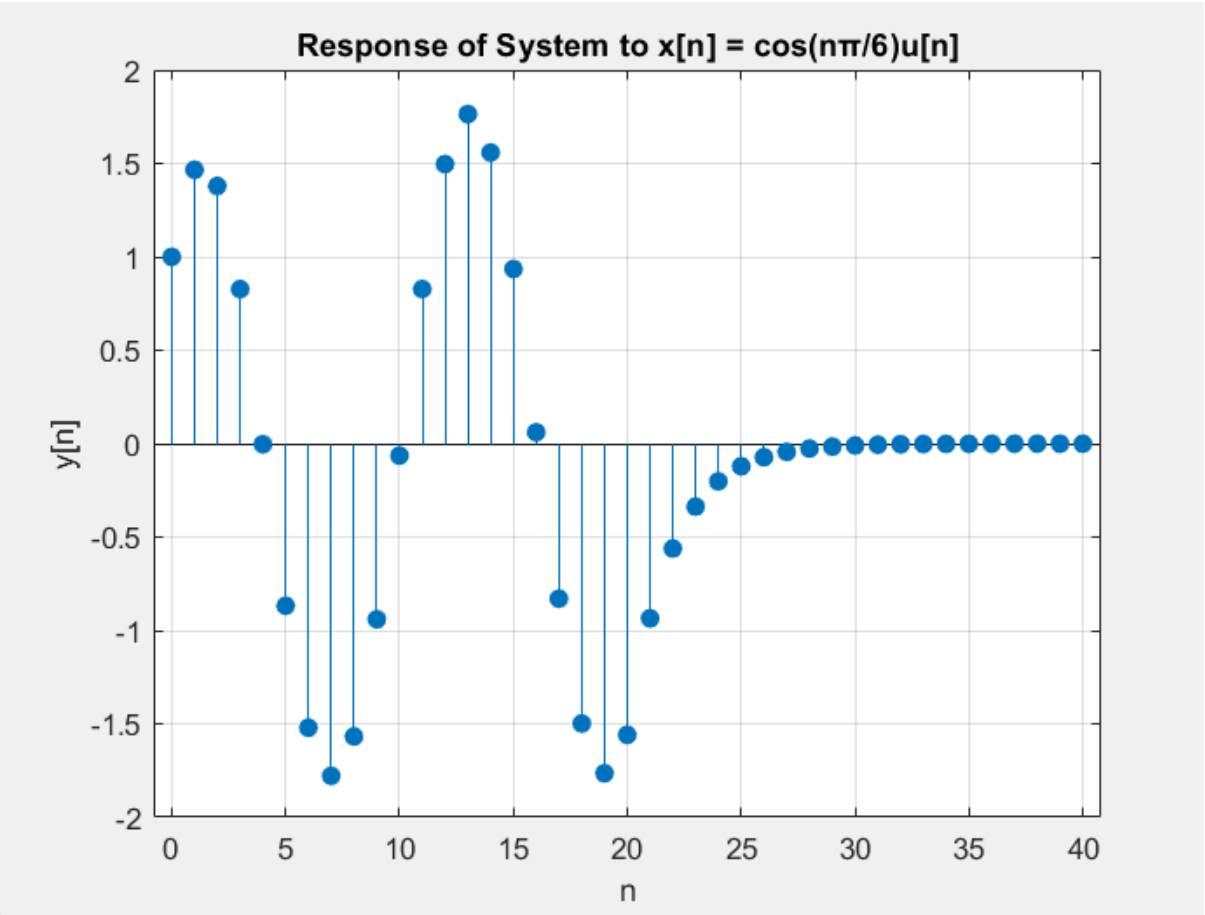
∴ 有余數 ∴ 找不到題目要求的 impulse response #

MATLAB Simulation

2.39)



2.40)



2.41)

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>> HW2_2_41
Impulse response h[n] (Recursive Method):
     2     8     8   -16

Impulse response h[n] (Deconvolution Method):
     2     8     8   -16
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2.41) $x[n] = [1\ -1\ 2\ 4]$, $y[n] = [2\ 6\ 4\ 0\ 8\ 5\ 12]$

$$
\begin{array}{r|l}
z^2 - z^2 + 2z + 4 & 2z^3 + 8z^2 + 8z - 16 \\
\hline
 & 2z^6 + 6z^5 + 4z^4 + 0 + 8z^2 + 5z + 12 \\
-) & 2z^6 - 2z^5 + 4z^4 + 8z^3 \\
\hline
 & 8z^5 + 0 - 8z^3 + 8z^2 \\
-) & 8z^5 - 8z^4 + 16z^3 + 32z^2 \\
\hline
 & 8z^4 - 24z^3 - 24z^2 + 5z \\
-) & 8z^4 - 8z^3 + 16z^2 + 32z \\
\hline
 & -16z^3 - 40z^2 - 27z + 12 \\
-) & -16z^3 + 16z^2 - 32z - 64 \\
\hline
 & -56z^2 + 5z + 76
\end{array}
$$

∵有余數 ∴沒有找到符合的 Impulse Response #