PH4419/PAP723: Computational Physics

Assignment 0

INSTRUCTIONS

- *Submission:* Assignment is due: **12th February 2019**.

- *Submission format:* All submission are to be done through NTULearn. There are 3 (three) questions in this assignment. Answer each question in a separate **python .py** file. Label your files using the following convention:

  Matric Number_Assignment 0_Question X.py

  For example: U1234567A_Assignment 0_Question 0.py

  Each file should begin with the lines

  ```
  from scipy import *
  import matplotlib.pyplot as plt
  ```

  Any other Python libraries used must be listed directly below these two lines.

- *Grading:* For full credit, code must follow good programming style. Key code blocks must be clearly commented. Function names must be properly named for ease of understanding your codes. The program structure should be modular; avoid unnecessary code duplication, and group numerical constant definitions neatly together. The output of your programs should be clear. Each generated plot should be well labeled, and if multiple curves are shown, they should be well-labeled.

- *PAP723:* Parts of questions marked with a ($*$) are for students taking the course PAP723, they must be attempted to receive credit. Undergraduate students who can produce a working code with correct output for these parts will receive 1 (one) bonus mark per question.

- *Documentation:* Whenever the assignment mentions a Scipy function you might need, consult the Scipy/Numpy online documentation to learn what the function does and how to use it.

- *Techniques:* The core techniques you will need to apply in this problem set are: (i) Numerical Differentiation (ii) Numerical Integration, (iii) Phase plot/Vector plot and (iv) Plots as a means of visualisation.

00. Magnetic Fields of Coils [10 marks]

Consider a circular current loop. The current is I, and the loop has radius R; the wire is infinitesimally thin. For convenience, take the coordinate origin to be at the center of the loop. According to the Biot-Savart law, in cylindrical coordinates, the magnetic field at $\vec{r} = [x, y, z]$ is

$$\vec{B} = \frac{\mu_0 I R}{4\pi} \int_0^{2\pi} \frac{z \sin(\varphi)\, \hat{\rho} + [R - \rho \sin(\varphi)]\, \hat{z}}{[R^2 + \rho^2 + z^2 - 2\rho R \sin(\varphi)]^{3/2}} \, d\varphi$$

where μ_0 is the permeability of free space. We take $\mu_0 = 1$ in this problem.

(a) **Single Coil Magnetic Field.** Consider a circular current loop as shown in Figure 1(a). The loop is lying in the xy plane.

(i) **[4 marks]** Write a function that calculates the magnetic field produced by a coil at a point $\vec{r}$ relative to the center of the loop.

def Coil_Bfield(r, current, radius):	
Input	
r	A 1D array $[x, y, z]$, specifying a position $\vec{r}$ in Cartesian coordinates relative to the center of the loop.
current	A positive real. The current I
radius	A positive real. The loop radius R
Output	
B	The magnetic field at position $\vec{r}$, in Cartesian coordinates.

(ii) **[1 marks]** Hence, write a function that plots how B_z (the z component of the magnetic field) varies with z.

(b) **Helmholtz Coil. [5 marks]** A Helmholtz coil consists of two parallel circular loops of radius R, separated by distance L, with each loop carrying equal current I, flowing in the same direction. Assume the origin is exactly between the two loops, as shown in Figure 1(b). Write the following function:

def Helmholtz_Bfield(z, current, radius, distance):	
Input	
z	The z coordinate to plot the magnetic field.
current	A positive real number. The current I
radius	A positive real number. The loop radius R
distance	A positive real number. The coil separation L

There are no return values for this function, but the function should generate a plot that shows the relationship between B_z (the z component of the magnetic field) versus ρ (the radial cylindrical coordinate). Discuss in your code comments the behaviour of the magnetic field at $z = 0$ for $R = L$, $R < L$ and $R > L$ as well as the behaviour of the magnetic field for $z \neq 0$.

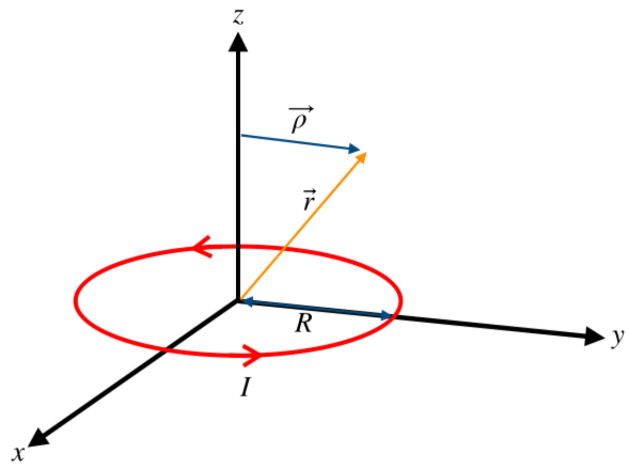

Figure 1(a): Coil with current in spherical coordinate representation. With origin at center of coil lying parallel to the xy plane.

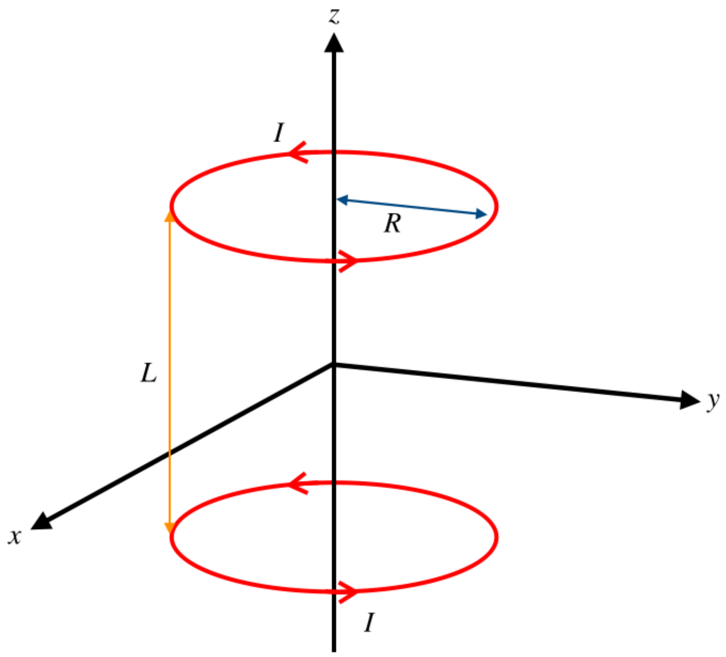

Figure 1(b): Helmholtz coil of radius R separated over a distance L.

01. Electric Fields of Point Charges [6 marks]

In classical electrostatics, electric potential is a scalar quantity denoted by V, it is a property of the electric field itself. The potential and electric field of point charge, q is given by:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{1}$$

$$E = -\nabla V \tag{2}$$

where, r is the displacement measured from the point charge. We will take $\epsilon_0 = 1$ and $Q = 1$ C. Consider the charge configuration in Figure 2.

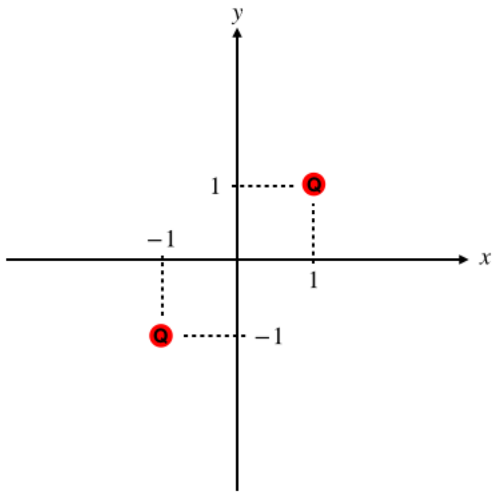

Figure 2: Charge configuration of 2 positive point charges.

(a) **[4 marks]** Write a function to plot the equipotential lines and electric field vectors of the above charge configuration. You are free to decide the domain of your plot. Please ensure that the main details and properties of both the electric potential and electric field can be clearly seen.

(b) **[2 marks]** Write a function to plot the potential V of the above charge configuration. You are free to decide the domain of your plot. Please ensure that the main details and properties of your plot are clearly seen.

02. Volume of Torus [4+5 marks]

A torus is the product of 2 circles, generated by revolving a circle around an axis which forms another circle

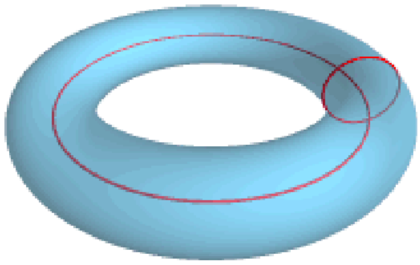

Figure 2: A torus.

(a) **[3 marks]** Write a code to calculate the volume of a torus where the major radius $R = 1$ and the minor radius $r = 0.5$. You are to use the Monte Carlo Integration method to determine the volume. The equation of a torus geometrically symmetric about the z axis is given by

$$\left(\sqrt{x^2 + y^2} - R\right)^2 + z^2 = r^2 \tag{3}$$

def torus_estimate(r, R, N):	
Input	
r	The minor radius of the torus, defined as the radius of the vertical circle in the polodial direction.
R	The major radius of the torus, defined as the radius of the greater horizontal circle in the torodial direction
N	The number of randomly chosen points in the domain $[-1.5, 1.5] \times [-1.5, 1.5] \times [-1.0, 1.0]$
Output	
est_vol	The estimated volume using Monte Carlo Integration
points	An array of size $3 \times N$ containing the (x, y, z) points that lies in and on the torus.

(b) **[1 mark]** Using the data that you obtain from the Monte Carlo integration, make a 3D scatter plot of the results to visualise the torus.

(c) **[5 marks]** (*) The actual volume of a torus is given by

$$V_0 = (\pi r^2)(2\pi R) \tag{4}$$

Plot a *log-log* graph that shows how error obtain by your code in part (a) varies with N, the number of sample points. We define the error as $\epsilon = |V - V_0|$, where V_0 is the actual volume of the torus and V is the estimated volume obtained by the Monte Carlo Integration. Take range of values of $N = 10$ to $N = 10^6$. Comment on the rate of convergence of your code in (a).