Homework 1 Due: 1002

1.

1\. (1)

$$\frac{d}{dt}a(t) = k_0 - k_1 a(t) b(t)$$

$$\frac{d}{dt}d(t) = k_1 a(t) b(t) - k_2 d(t)$$

$$\frac{d}{dt}b(t) = -k_1 a(t) b(t) + k_2 d(t)$$

$$\frac{d}{dt}e(t) = k_3 c(t) - k_4 e(t)$$

$$\frac{d}{dt}c(t) = k_1 a(t) b(t) - k_3 c(t)$$

$$\frac{d}{dt}f(t) = k_3 c(t) - k_5 f(t)$$

(2)

state variables : $a(t)$, $b(t)$, $c(t)$, $d(t)$, $e(t)$

parameters : k_1, k_2, k_3, k_4, k_5

2.

(a)

The three molecules are viral RNA, viral antigen and antibody

(b)

`Inactive whole virus`: When injected, the immune system recognizes the viral antigens and produces antibodies to fight the virus. These antibodies will be ready to respond to future exposures to the live virus.

`Antigen proteins`: The immune system identifies these proteins as foreign antigens and generates an immune response, including producing antibodies that recognize and neutralize the virus if encountered later.

`mRNA(genetic instructions)`: The mRNA vaccines contain a small piece of the virus's genetic material that provides instructions for cells to make the spike protein found on the surface of SARS-CoV-2. Once cells produce this spike protein, the immune system detects it and generates a response, including the production of antibodies and memory cells to recognize the virus in the future.

`DNA (genetic instructions)`: After being delivered into the body, cells take up the DNA, produce the spike protein, and display it on their surface. The immune system responds by producing antibodies and T cells, which prepare the body to fight the actual virus if it is encountered later.

3.

(a)

Solution implemented in with Python in `plot.py`

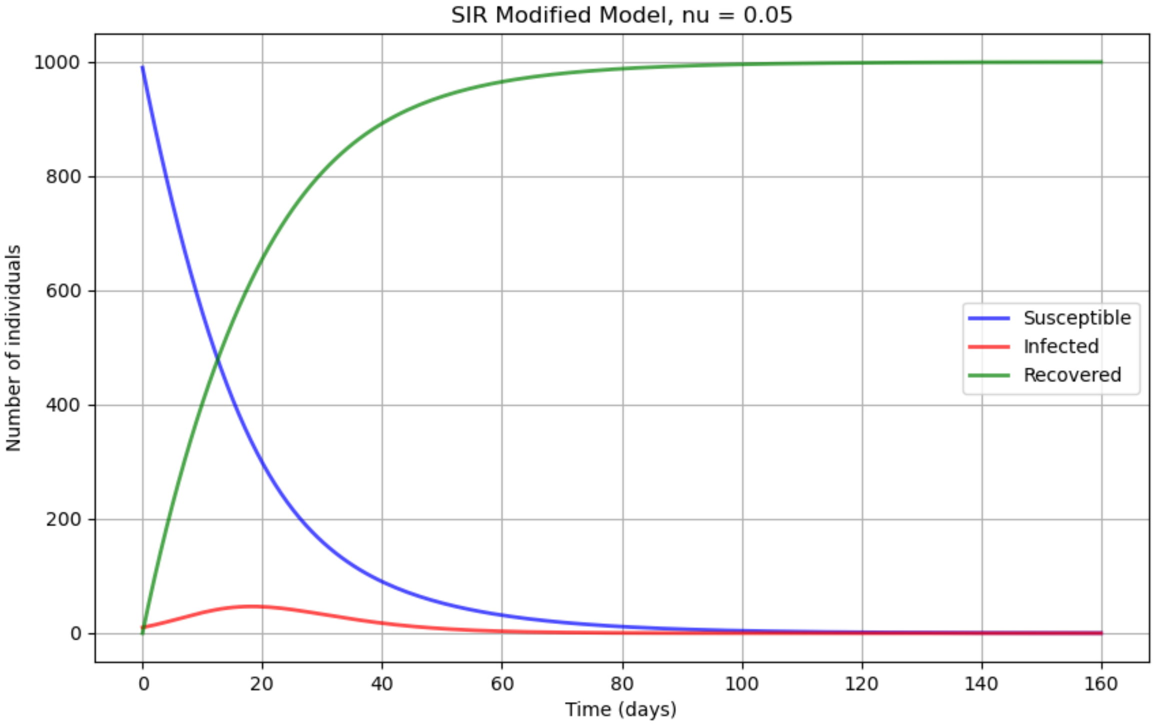
```python
# SIR model differential equations
Tabnine: Edit | Test | Explain | Document | Ask
def sir_model(y, t, beta, gamma, v, N):
    S, I, R = y
    dSdt = -beta * S * I / N  - v * S
    dIdt = beta * S * I / N - gamma * I
    dRdt = gamma * I + v * S
    return [dSdt, dIdt, dRdt]

# Set parameters
N = 1000  # Total population
I0, R0 = 10, 0  # Initial number of infected and recovered individuals
S0 = N - I0 - R0  # Initial number of susceptible individuals
beta = 0.3  # Contact rate
gamma = 0.1  # Recovery rate
vs = [0.05, 0.00, 0.10, 0.30]     # Vaccination rate
t = np.linspace(0, 160, 160)  # Time grid


for v in vs:
    # Initial conditions vector
    y = [S0, I0, R0]

    # Solve the differential equations
    solution = odeint(sir_model, y, t, args=(beta, gamma, v, N))
    S, I, R = solution.T
```

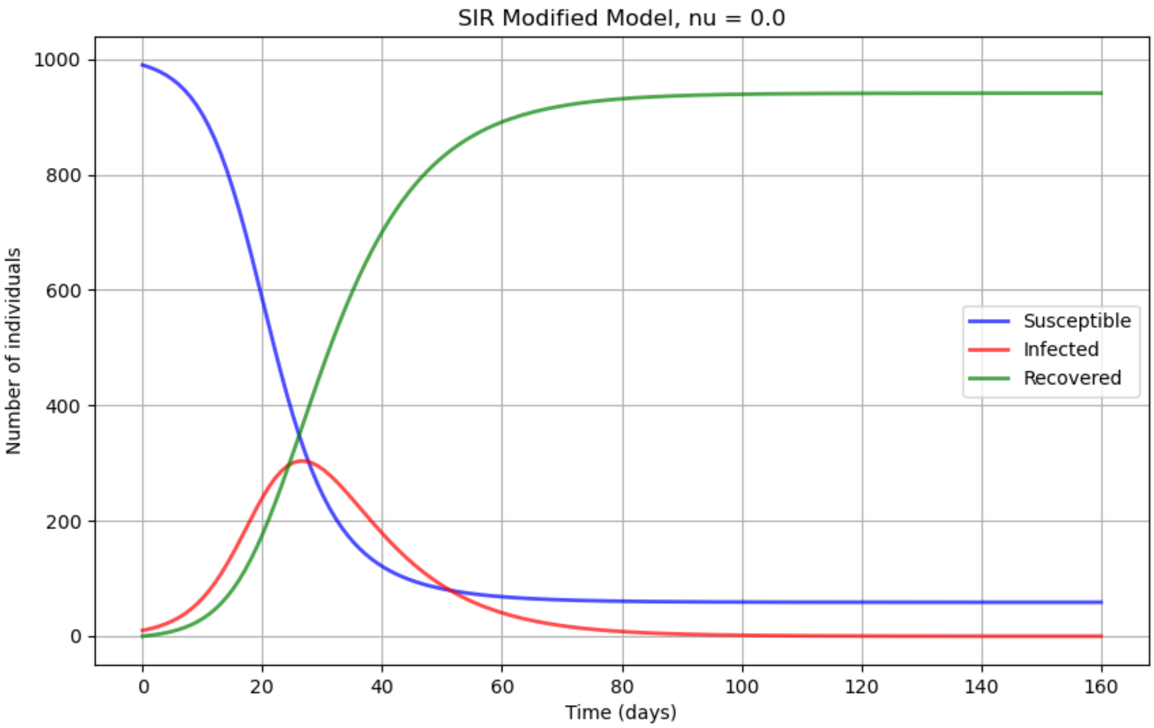
Result when $\nu = 0.05$



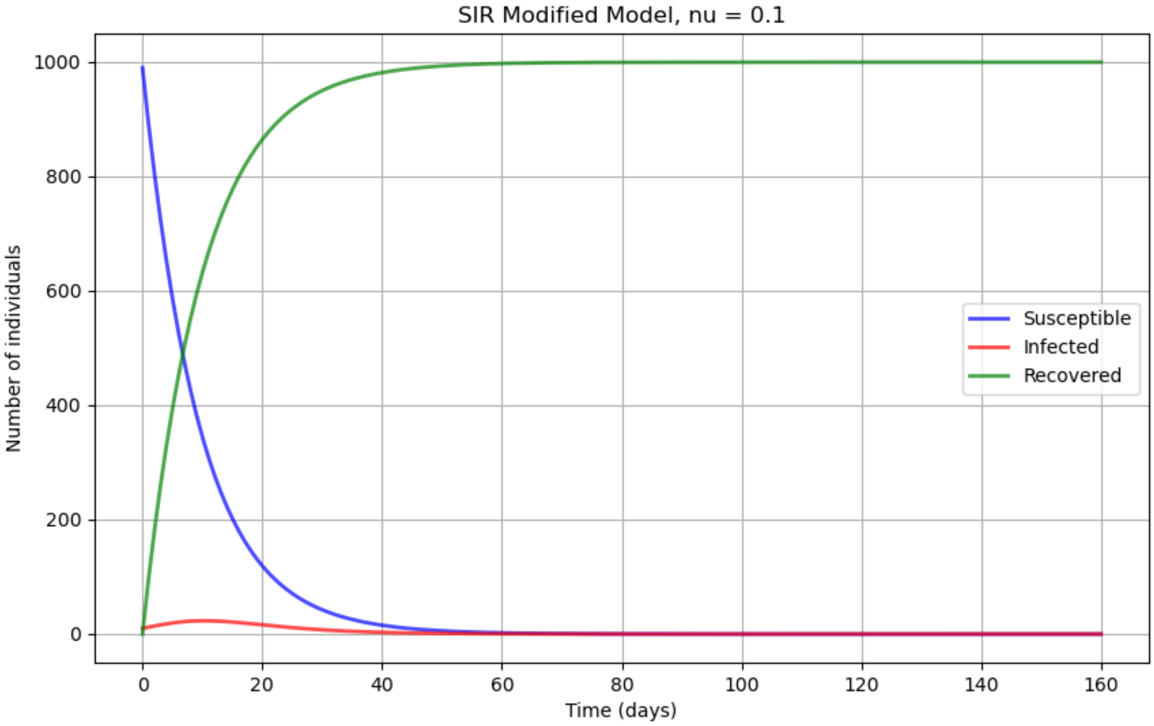
(b)

Compare result of different ν

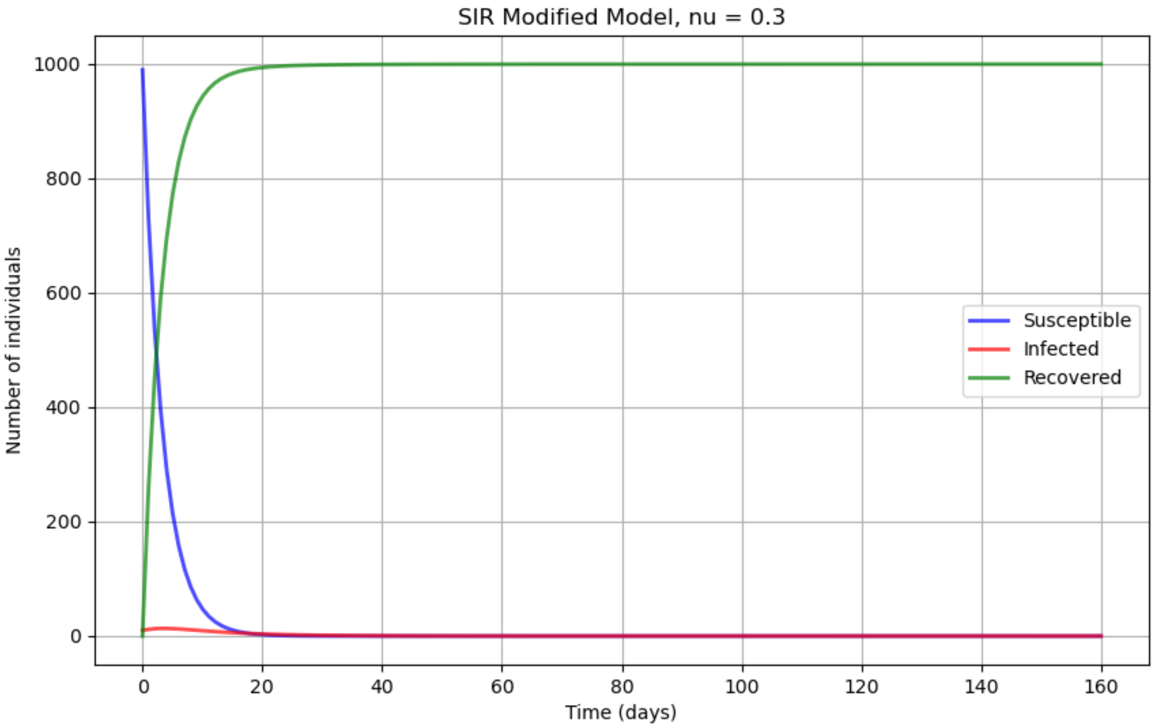
$\nu = 0.0$



$\nu = 0.10$



$\nu = 0.30$



By comparing the results of different ν, we can find that when ν increases, the number of susceptible individuals decreases faster, and the number of recovered individuals increases faster. This is because the vaccination rate ν reduces the number of susceptible individuals by vaccinating them, and also reduces the number of infected individuals by reducing the contact rate.

4. Souce Code

github