Homework: 2024/11/27

1. Information Matrix of Binary Choice Models

a. $L_n(\beta) = \sum_{i=1}^{n} \left(Y_i \ln G(X_i'\beta) + (1-Y_i) \ln(1 - G(X_i'\beta)) \right) = \sum_{i=1}^{n} \left(Y_i \ln G(X_i'\beta) + (1-Y_i) \ln(-G(X_i'\beta)) \right)$

score function $S_n(\beta) = \nabla_\beta L_n(\beta) = \sum_{i=1}^{n} \frac{\nabla_\beta G(X_i'\beta)}{G(X_i'\beta)} = \sum_{i=1}^{n} X_i h(X_i'\beta)$, for $h(x) = \frac{\partial}{\partial x} \ln G(x)$

Hessian matrix $H_n(\beta) = \nabla_\beta^2 L_n(\beta) = \sum_{i=1}^{n} X_i X_i' h'(X_i'\beta) = \sum_{i=1}^{n} X_i X_i' H(X_i'\beta)$, for $H(x) = \frac{\partial^2}{\partial x^2} \ln G(x)$

$\therefore$ Information matrix $B_n(\beta) = \nabla_\beta L_n(\beta) \nabla_{\beta'} L_n(\beta) = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j' h(X_i'\beta) h(X_j'\beta)$, for $h(x) = \frac{\partial}{\partial x} \ln G(x)$

b. To prove the information matrix equality, we must have $E[H_n(\beta)] + E[B_n(\beta)] = 0$

$E[H_n(\beta)] = \sum_{i=1}^{n} X_i X_i' E[H(X_i'\beta)] = \sum_{i=1}^{n} X_i X_i' E\left[\frac{G''(X_i'\beta)}{G(X_i'\beta)} - \left(\frac{G'(X_i'\beta)}{G(X_i'\beta)} \right)^2 \right]$

$E[B_n(\beta)] = \sum_{i=1}^{n} X_i X_i' E[h(X_i'\beta)^2] + \sum_{i=1}^{n} \sum_{j \neq i}^{n} X_i X_j' E[h(X_i'\beta) h(X_j'\beta)]$, and since $h(X_i'\beta)$ and $h(X_j'\beta)$ are independent
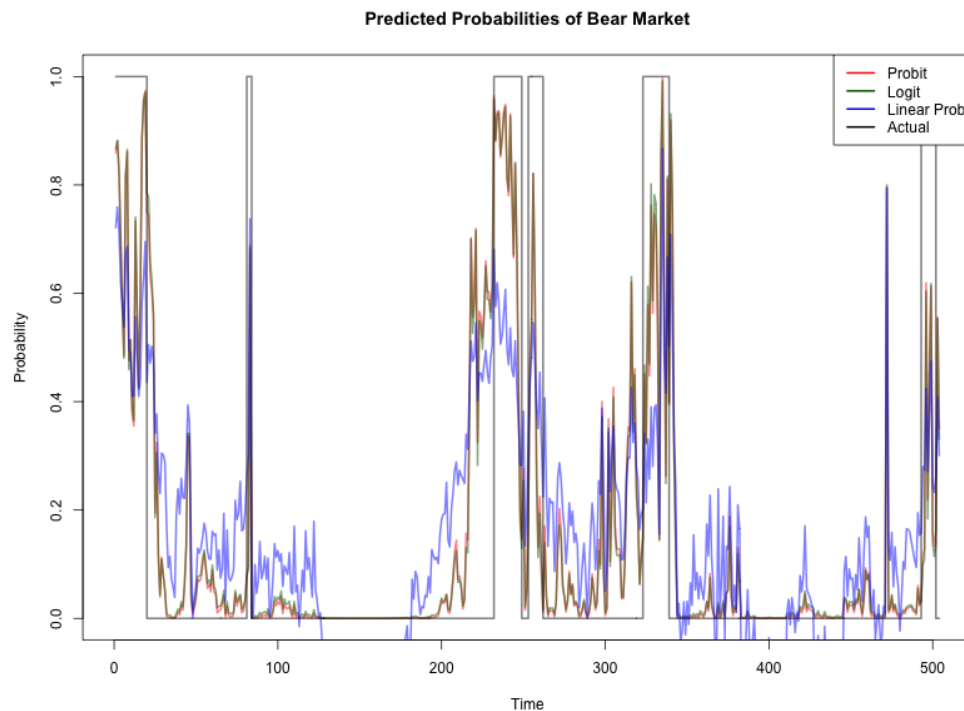
$= \sum_{i=1}^{n} X_i X_i' E\left[\left(\frac{G'(X_i'\beta)}{G(X_i'\beta)} \right)^2 \right]$

$\therefore E[H_n(\beta)] + E[B_n(\beta)] = \sum_{i=1}^{n} X_i X_i' E\left[\frac{G''(X_i'\beta)}{G(X_i'\beta)} \right] \neq 0$, the information matrix equality

doesn't hold.

2.

market-cycle-index sequence & its predictions of different models

Predicted Probabilities of Bear Market

Score values of probit and logit models

```
Probit Model Score Values:
                  dfy            infl           svar             tms
-0.0120446060  0.0010997689  0.0016579720 -0.0027711492  0.0047622655
         tbl            dfr             dp            ltr             bmr
 0.0026251451 -0.0043932080  0.0137075978  0.0011586185 -0.0061308072
        ntis
 0.0001298595                                                    2 / 2

Logit Model Score Values:
                  dfy            infl           svar             tms
-0.0018310312  0.0009013303  0.0006512284 -0.0002166276 -0.0003751593
         tbl            dfr             dp            ltr             bmr
-0.0002529876  0.0002769116 -0.0034284784  0.0001790371 -0.0053869289
        ntis
-0.0005339635
```

By observing the score values, we can see that the score values of probit and logit models are close to zero, which indicates that the numerical optimization is successfully converged.

3. Source Code

Source Code