Homework: 2024/11/27

1. Information Matrix of Binary Choice Models

a. $L_n(\beta) = \sum_{i=1}^{n} \left(Y_i \ln G(X_i'\beta) + (1-Y_i) \ln (1 - G(X_i'\beta)) \right) = \sum_{i=1}^{n} \left(Y_i \ln G(X_i'\beta) + (1-Y_i) \ln (G(-X_i'\beta)) \right)$

$= \sum_{i=1}^{n} \ln G(Z_i'\beta)$, By the definition

$Z_i = \begin{cases} X_i , & Y_i = 1 \\ -X_i , & Y_i = 0 \end{cases}$

score function $S_n(\beta) = \nabla_\beta L_n(\beta) = \sum_{i=1}^{n} \frac{\nabla_\beta G(Z_i'\beta)}{G(Z_i'\beta)} = \sum_{i=1}^{n} Z_i h(Z_i'\beta)$, for $h(x) = \frac{\partial}{\partial x} \ln G(x)$

Hessian matrix $H_n(\beta) = \nabla_\beta^2 L_n(\beta) = \sum_{i=1}^{n} Z_i Z_i' h'(Z_i'\beta) = \sum_{i=1}^{n} Z_i Z_i' H(Z_i'\beta)$, for $H(x) = \frac{\partial^2}{\partial x^2} \ln G(x)$

$\therefore$ Information matrix $B_n(\beta) = \nabla_\beta L_n(\beta) \nabla_{\beta'} L_n(\beta) = \sum_{i=1}^{n}\sum_{j=1}^{n} Z_i Z_j' h(Z_i'\beta) h(Z_j'\beta)$, for $h(x) = \frac{\partial}{\partial x} \ln G(x)$

b. To prove the information matrix equality, we must have $E[H_n(\beta)] + E[B_n(\beta)] = 0$

$E[H_n(\beta)] = \sum_{i=1}^{n} Z_i Z_i' E[H(Z_i'\beta)] = \sum_{i=1}^{n} Z_i Z_i' E\left[\frac{G''(Z_i'\beta)}{G(Z_i'\beta)} - \left(\frac{G'(Z_i'\beta)}{G(Z_i'\beta)}\right)^2 \right]$

$E[B_n(\beta)] = \sum_{i=1}^{n} Z_i Z_i' E[h(Z_i'\beta)^2] + \sum_{i=1}^{n}\sum_{j\neq i}^{n} Z_i Z_j' E[h(Z_i'\beta) h(Z_j'\beta)]$. and since $h(Z_i'\beta)$ and $h(Z_j'\beta)$ are independent

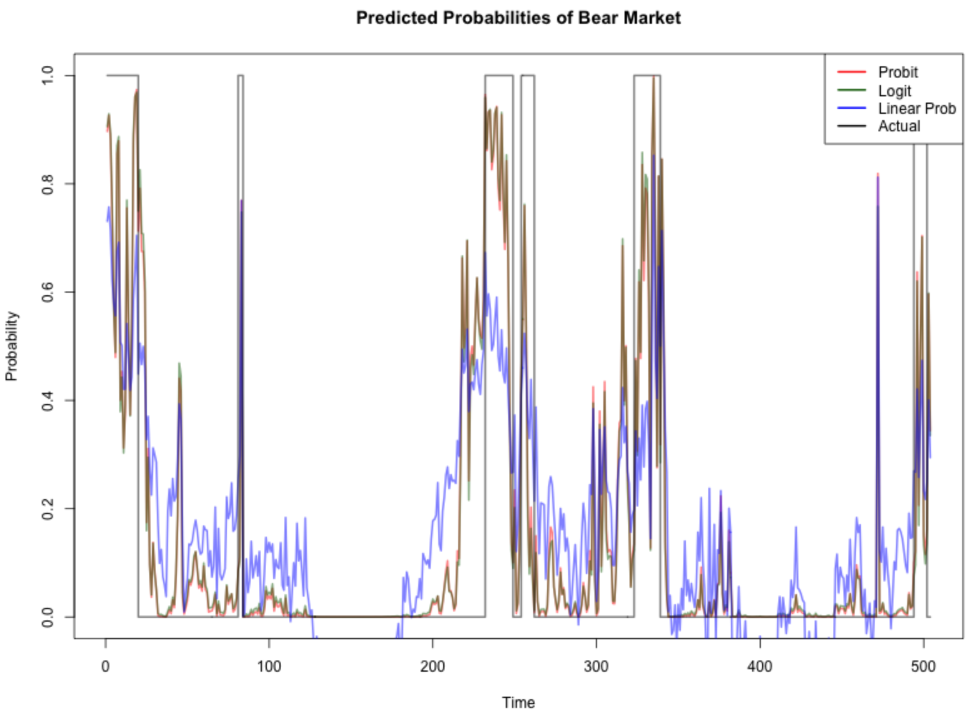
$= \sum_{i=1}^{n} Z_i Z_i' E\left[\left(\frac{G'(Z_i'\beta)}{G(Z_i'\beta)}\right)^2\right]$

$E[H_n(\beta)] + E[B_n(\beta)] = \sum_{i=1}^{n} Z_i Z_i' E\left[\frac{G''(Z_i'\beta)}{G(Z_i'\beta)}\right]$, since G is symmetric, we have $E\left[\frac{G''(Z_i'\beta)}{G(Z_i'\beta)}\right] = 0$

$\therefore E[H_n(\beta)] + E[B_n(\beta)] = 0$, information matrix equality holds.

2.

market-cycle-index sequence & its predictions of different models



Score values of probit and logit models

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Probit Model Score Values:
                        dfy          infl          svar           tms
-3.153491e-02  5.566595e-03  1.069826e-03 -1.183443e-02  9.980074e-05
          tbl           dfr            dp           ltr            ep
 3.347931e-02  1.836287e-02  1.131310e-01  9.999583e-03  1.080522e-01
          bmr          ntis
 1.721287e-02  3.724040e-03

Logit Model Score Values:
                        dfy          infl          svar           tms
 9.167080e-03  1.201280e-03  3.991171e-04  1.976480e-04 -6.553894e-04
          tbl           dfr            dp           ltr            ep
-1.381212e-04 -2.052281e-03 -3.783134e-02  5.836199e-05 -2.658905e-02
          bmr          ntis
-1.890811e-03 -2.159018e-03
```

By observing the score values, we can see that the score values of probit and logit models are close to zero, which indicates that the numerical optimization is successfully converged.

3. Source Code

Source Code