Homework: 2024/12/04

1. Prove the asymptotic distribution of $T\hat{\rho}^2(k)$

The numerator of $\hat{\rho}(k)$ can be written as $\frac{1}{T-k}\sum_{t=k+1}^{T}(Y_t-\bar{Y})(Y_{t-k}-\bar{Y})$

As $T\to\infty$, $\frac{1}{T-k}\sum_{t=k+1}^{T}(Y_t-\bar{Y})(Y_{t-k}-\bar{Y}) \xrightarrow{d} \frac{1}{T-k}\sum_{t=k+1}^{T}(Y_t-\mu)(Y_{t-k}-\mu)$

And since $\{Y_t\}_{t=1}^{T}$ is IID, $E[(Y_t-\mu)(Y_{t-k}-\mu)]=0$, and $Var[\frac{1}{T-k}\sum_{t=k+1}^{T}(Y_t-\mu)(Y_{t-k}-\mu)]=\frac{\sigma^4}{T}$

$\sqrt{T}(\frac{1}{T-k}\sum_{t=k+1}^{T}(Y_t-\mu)(Y_{t-k}-\mu)) \xrightarrow{d} N(0,\sigma^4)$

so given $\hat{\rho}(k)=\frac{1}{T-k}\sum_{t=k+1}^{T}\frac{(Y_t-\bar{Y})}{\hat{\sigma}}\cdot\frac{(Y_{t-k}-\bar{Y})}{\hat{\sigma}} \xrightarrow{d} \frac{1}{T-k}\sum_{t=k+1}^{T}\frac{(Y_t-\mu)}{\sigma}\cdot\frac{(Y_{t-k}-\mu)}{\sigma}$

$\therefore \sqrt{T}\hat{\rho}(k) \xrightarrow{d} N(0,1)$, which implies $T\hat{\rho}(k)=(\sqrt{T}\hat{\rho}(k))^2 \xrightarrow{d} \chi^2(1)$, QED.

2. Prove the asymptotic distribution of Box-Pierce statistic

By #1, $Q(m)=T\cdot\sum_{k=1}^{m}\hat{\rho}(k) \xrightarrow{d} \sum_{k=1}^{m}\chi_k^2(1)$

since $\{Y_t\}_{t=1}^{T}$ is IID, which implies for different lagging k, $\chi_k^2(1)$ are independent

$\therefore Q(m) \xrightarrow{d} \sum_{k=1}^{m}\chi_k^2(1) \xrightarrow{d} \chi^2(m)$ as $T\to\infty$, QED.

3. Plot the sample autocorrelation function and test the null of IIDness for each of the 12 time series

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Plot of sample autocorrelation function
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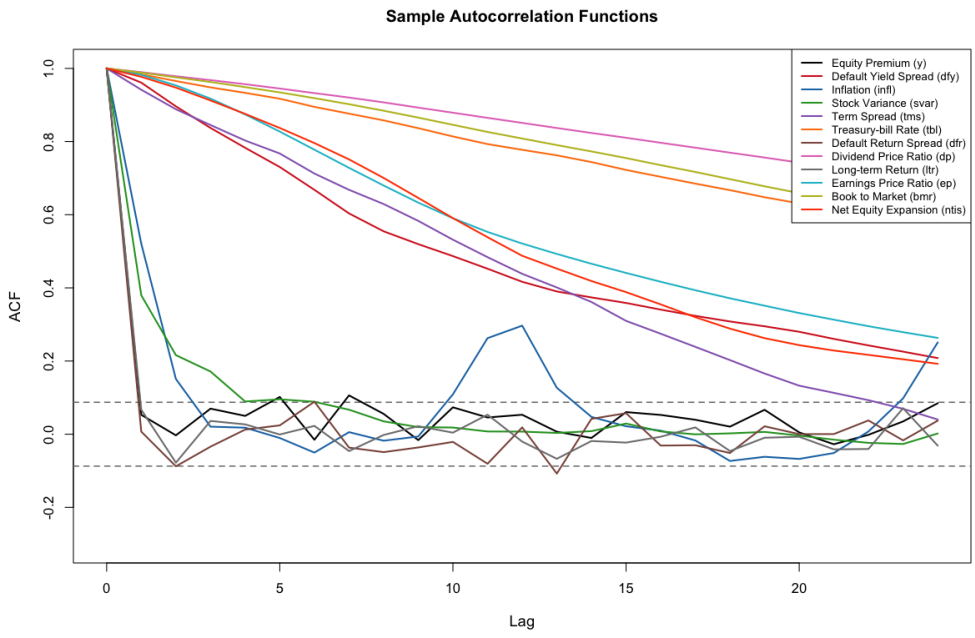

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Test the null of IIDness for each of the 12 time series
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Box-Pierce test results for Equity Premium (y) :
m=12: Q = 23.03 , p-value = 0.0275
m=24: Q = 34.2 , p-value = 0.0812

Box-Pierce test results for Default Yield Spread (dfy) :
m=12: Q = 2810.91 , p-value = 0
m=24: Q = 3376.78 , p-value = 0

Box-Pierce test results for Inflation (infl) :
m=12: Q = 235.12 , p-value = 0
m=24: Q = 289.65 , p-value = 0

Box-Pierce test results for Stock Variance (svar) :
m=12: Q = 126.84 , p-value = 0
m=24: Q = 128.13 , p-value = 0

Box-Pierce test results for Term Spread (tms) :
m=12: Q = 3035.22 , p-value = 0
m=24: Q = 3354.73 , p-value = 0

Box-Pierce test results for Treasury-bill Rate (tbl) :
m=12: Q = 4742.99 , p-value = 0
m=24: Q = 7443.11 , p-value = 0

Box-Pierce test results for Default Return Spread (dfr) :
m=12: Q = 15.08 , p-value = 0.2371
m=24: Q = 27.53 , p-value = 0.2804

Box-Pierce test results for Dividend Price Ratio (dp) :
m=12: Q = 5172.4 , p-value = 0
m=24: Q = 8723.79 , p-value = 0

Box-Pierce test results for Long-term Return (ltr) :
m=12: Q = 9.56 , p-value = 0.6543
m=24: Q = 18.36 , p-value = 0.7851

Box-Pierce test results for Earnings Price Ratio (ep) :
m=12: Q = 3572.37 , p-value = 0
m=24: Q = 4422.18 , p-value = 0

Box-Pierce test results for Book to Market (bmr) :
m=12: Q = 4973.9 , p-value = 0
m=24: Q = 7891.06 , p-value = 0

Box-Pierce test results for Net Equity Expansion (ntis) :
m=12: Q = 3595.62 , p-value = 0
m=24: Q = 4174.72 , p-value = 0
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We can see that the Box-Pierce statistic implies that the null of IIDness is rejected for most of the time series. For dfr & ltr, the null of IIDness is not rejected at 5% level, which implies that these two time series are IID. And for y, the null of IIDness is rejected when m=12, but not when m=24.

4. Source Code

Source Code