Homework: 2024/11/27

1. Information Matrix of Binary Choice Models

a. $L_n(\beta) = \sum_{i=1}^{n}\left(Y_i \ln G(X_i'\beta) + (1-Y_i)\ln(1-G(X_i'\beta))\right) = \sum_{i=1}^{n}\left(Y_i \ln G(X_i'\beta) + (1-Y_i)\ln(G(-X_i'\beta))\right)$

$= \sum_{i=1}^{n} \ln G(z_i'\beta)$, By the definition

$$z_i = \begin{cases} X_i, & Y_i = 1 \\ -X_i, & Y_i = 0 \end{cases}$$

score function $S_n(\beta) = \nabla_\beta L_n(\beta) = \sum_{i=1}^{n} \frac{\nabla_\beta G(z_i'\beta)}{G(z_i'\beta)} = \sum_{i=1}^{n} z_i h(z_i'\beta)$, for $h(x) = \frac{\partial}{\partial x}\ln G(x)$

Hessian matrix $H_n(\beta) = \nabla_\beta^2 L_n(\beta) = \sum_{i=1}^{n} z_i z_i' h'(z_i'\beta) = \sum_{i=1}^{n} z_i z_i' H(z_i'\beta)$, for $H(x) = \frac{\partial^2}{\partial x^2}\ln G(x)$

∴ Information matrix $B_n(\beta) = \nabla_\beta L_n(\beta)\,\nabla_\beta' L_n(\beta) = \sum_{i=1}^{n}\sum_{j=1}^{n} z_i z_j' h(z_i'\beta) h(z_j'\beta)$, for $h(x) = \frac{\partial}{\partial x}\ln G(x)$

b. To prove the information matrix equality, we must have $E[H_n(\beta)] + E[B_n(\beta)] = 0$

$E[H_n(\beta)] = \sum_{i=1}^{n} z_i z_i' E[H(z_i'\beta)] = \sum_{i=1}^{n} z_i z_i' E\left[\frac{G''(z_i'\beta)}{G(z_i'\beta)} - \left(\frac{G'(z_i'\beta)}{G(z_i'\beta)}\right)^2\right]$

$E[B_n(\beta)] = \sum_{i=1}^{n} z_i z_i' E[h(z_i'\beta)^2] + \sum_{i=1}^{n}\sum_{j\neq i}^{n} z_i z_j' E[h(z_i'\beta)h(z_j'\beta)]$, and since $h(z_i'\beta)$ and $h(z_j'\beta)$ are independent

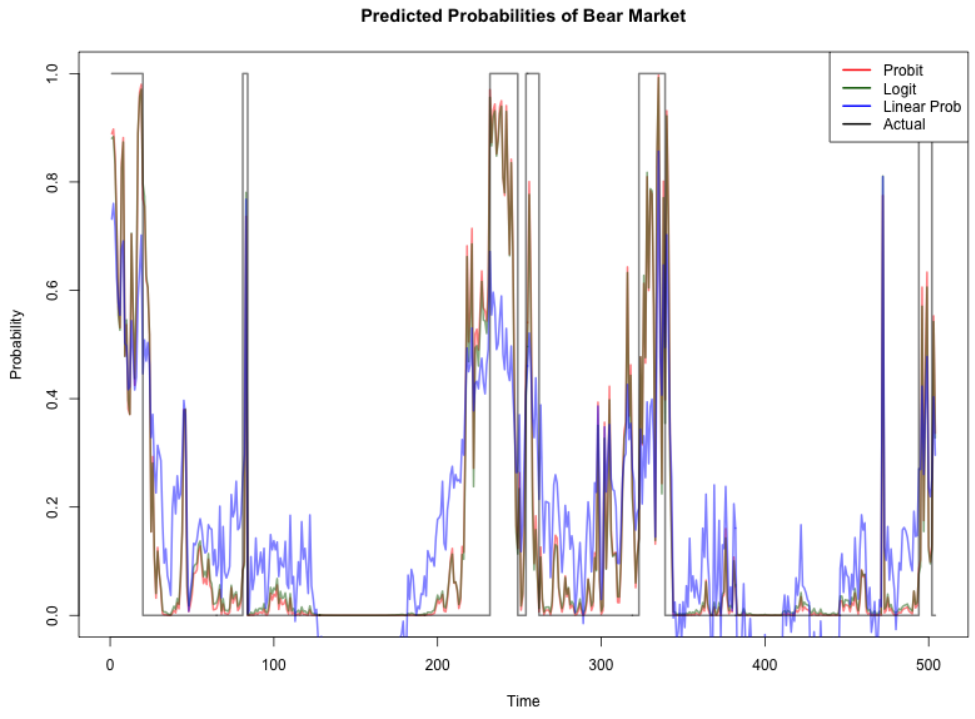
$= \sum_{i=1}^{n} z_i z_i' E\left[\left(\frac{G'(z_i'\beta)}{G(z_i'\beta)}\right)^2\right]$

$E[H_n(\beta)] + E[B_n(\beta)] = \sum_{i=1}^{n} z_i z_i' E\left[\frac{G''(z_i'\beta)}{G(z_i'\beta)}\right]$, since G is symmetric, we have $E\left[\frac{G''(z_i'\beta)}{G(z_i'\beta)}\right] = 0$

∴ $E[H_n(\beta)] + E[B_n(\beta)] = 0$, information matrix equality holds.

2.

market-cycle-index sequence & its predictions of different models



Score values of probit and logit models

```
Probit Model Score Values:
                   dfy          infl          svar          tms
-0.0128984350  0.0007682938  0.0005893370  0.0009055447 -0.0014594192
          tbl           dfr            dp           ltr           bmr
-0.0010994555 -0.0021968756  0.0268646348  0.0055537665 -0.0147808426
         ntis
 0.0009461258

Logit Model Score Values:
                  dfy         infl         svar          tms          tbl
-0.019470939  0.005170674  0.004002578 -0.002400097 -0.002289225 -0.001404856
         dfr           dp          ltr          bmr         ntis
 0.001943741  0.034258791 -0.005860236 -0.001116910 -0.003785653
```

By observing the score values, we can see that the score values of probit and logit models are close to zero, which indicates that the numerical optimization is successfully converged.

3. Source Code

Source Code