

Seguridad Informatica - Fall 2024

Module IV: Physical security
Lecture 4

Non-interference

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Module IV

- Lecture 1 – Introduction
- Lecture 2 – Cache-based side channel attacks
- Lecture 3 – Speculative execution attacks
- Lecture 4 – Non-interference
- Lecture 5 – Automated detection of speculative leaks

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Recommended reading

Language-based information-flow basics

by Aslan Askarov

Available at [https://github.com/aslanix/SmallStepNI/
blob/master/infowflow-basics.pdf](https://github.com/aslanix/SmallStepNI/blob/master/infowflow-basics.pdf)

Introduction

- ***Problem:*** Seemingly secure programs can leak sensitive information

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- The ***question*** that we will answer:
 - When is a program ***secure?***

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 - When is a program ***secure***? ← Security as ***confidentiality!***

Introduction

- **Problem:** Seemingly secure programs can leak sensitive information
- The **question** that we will answer:
 - When is a program **secure**? ← Security as **confidentiality!**
 - How can we **reason** about a program security?
 - **Non-interference** provides theoretical foundations for talking about **program security**



What is the
meaning of a
program?

Programs

Programs map *memories* to *memories*

$$\langle\langle Prg \rangle\rangle : Mems \rightarrow Mems$$

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Program

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$$\langle\langle Prg \rangle\rangle : Mems \rightarrow Mems$$

Program

Program memories: map
variables to values

Programs

Programs map *memories* to *memories*

Examples

$$\langle\langle x \leftarrow x + 1 \rangle\rangle([x \mapsto 0]) = [x \mapsto 1]$$

$$\langle\langle x \leftarrow x + 1 \rangle\rangle([x \mapsto 1, y \mapsto 2]) = [x \mapsto 2, y \mapsto 2]$$

Program

Program memories: map
variables to values

Formalizing programs - *Syntax*

$x \in Vars$

$n \in \mathbb{N}$

$\Theta \in \{ \neg, \dots \}$

$\oplus \in \{ +, -, *, \dots \}$

$expr = n \mid x \mid \Theta\ expr \mid expr_1 \oplus\ expr_2$

$stmt = \text{stop} \mid \text{skip} \mid x \leftarrow expr \mid stmt_1; stmt_2$
 $\mid \text{if}(expr)\{stmt_1\}\text{else}\{stmt_2\}$

Formalizing programs - *Syntax*

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Formalizing programs - *Semantics*

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- Program memories

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- Our goal: Programs transform memories

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- There are many ways of specifying program semantics
 - ***Small-step semantics***

Formalizing programs - *Semantics*

- Program memories

$$\sigma \in Mems = Vars \rightarrow \mathbb{N}$$

- Our goal: Programs transform memories

$$\langle\langle Prg \rangle\rangle : Mems \rightarrow Mems$$

- There are many ways of specifying program semantics
 - ***Small-step semantics*** Define meaning of ***each instruction***! Derive program semantics ***compositionally***!

Small-step semantics - 1

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Evaluation of ***expressions***: $\llbracket e \rrbracket : Mems \rightarrow \mathbb{N}$

Small-step semantics - 1

Evaluation of ***expressions***: $\llbracket e \rrbracket : Mems \rightarrow \mathbb{N}$

$$\llbracket n \rrbracket(m) = n$$

$$\llbracket x \rrbracket(m) = m(x)$$

$$\llbracket \Theta e \rrbracket(m) = \Theta \llbracket e \rrbracket(m)$$

$$\llbracket e_1 \oplus e_2 \rrbracket(m) = \llbracket e_1 \rrbracket(m) \oplus \llbracket e_2 \rrbracket(m)$$

Small-step semantics - 1

Evaluation of **expressions**:

$$[e] : Mems \rightarrow \mathbb{N}$$

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- The meaning of ***each instruction*** is formalized using one or more ***inference rules*** over program configurations

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$$\langle p, m \rangle \in Prg \times Mem$$

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- ***Program configurations***

Program models the state of ***computation***

$$\langle p, m \rangle \in Prg \times Mem$$

Small-step semantics - 2

- The meaning of ***each instruction*** is formalized using one or more ***inference rules*** over program configurations

- ***Program configurations***

$$\langle p, m \rangle \in Prg \times Mem$$

Program models the state of ***computation***

Memory models the state of the ***data***

Small-step semantics - 2

- The meaning of ***each instruction*** is formalized using one or more ***inference rules*** over program configurations

- ***Program configurations***

$$\langle p, m \rangle \in Prg \times Mem$$

Program models the state of ***computation***

Memory models the state of the ***data***

- One application of the rules models a “***small step***” of the computation

Small-step semantics - 3

Evaluation of $\textcolor{violet}{statements} : \rightarrow \subseteq \textit{Conf} \times \textit{Conf}$

Small-step semantics - $\text{Conf} = \text{Prg} \times \text{Mem}$

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Small-step semantics -

$$\text{Conf} = \text{Prg} \times \text{Mem}$$

Evaluation of $\text{statements} : \rightarrow \subseteq \text{Conf} \times \text{Conf}$

Stop

Small-step semantics -

$$\text{Conf} = \text{Prg} \times \text{Mem}$$

Evaluation of $\text{statements} : \rightarrow \subseteq \text{Conf} \times \text{Conf}$

Stop

$$\overline{\langle \text{stop}, m \rangle \rightarrow \langle \text{stop}, m \rangle}$$

Small-step semantics -

$$\text{Conf} = \text{Prg} \times \text{Mem}$$

Evaluation of $\text{statements} : \rightarrow \subseteq \text{Conf} \times \text{Conf}$

Skip

Small-step semantics -

$\text{Conf} = \text{Prg} \times \text{Mem}$

Evaluation of $\text{statements} : \rightarrow \subseteq \text{Conf} \times \text{Conf}$

Skip

$$\langle \text{skip}, m \rangle \rightarrow \langle \text{stop}, m \rangle$$

Small-step semantics - $\text{Conf} = \text{Prg} \times \text{Mem}$

Evaluation of $\text{statements} : \rightarrow \subseteq \text{Conf} \times \text{Conf}$

Assignment

Small-step semantics - $\text{Conf} = \text{Prg} \times \text{Mem}$

Evaluation of $\text{statements} : \rightarrow \subseteq \text{Conf} \times \text{Conf}$

Assignment

$$\frac{\llbracket e \rrbracket(m) = v}{\langle x \leftarrow e, m \rangle \rightarrow \langle \text{stop}, m[x \mapsto v] \rangle}$$

Small-step semantics - $\text{Conf} = \text{Prg} \times \text{Mem}$

Evaluation of $\text{statements} : \rightarrow \subseteq \text{Conf} \times \text{Conf}$

Sequential composition

Small-step semantics - $\text{Conf} = \text{Prg} \times \text{Mem}$

Evaluation of $\text{statements} : \rightarrow \subseteq \text{Conf} \times \text{Conf}$

Sequential composition

$$\frac{\langle \text{stmt}_1, m \rangle \rightarrow \langle \text{stop}, m' \rangle}{\langle \text{stmt}_1; \text{stmt}_2, m \rangle \rightarrow \langle \text{stmt}_2, m' \rangle}$$

Small-step semantics - $\text{Conf} = \text{Prg} \times \text{Mem}$

Evaluation of $\text{statements} : \rightarrow \subseteq \text{Conf} \times \text{Conf}$

Sequential composition

$$\frac{\langle \text{stmt}_1, m \rangle \rightarrow \langle \text{stop}, m' \rangle}{\langle \text{stmt}_1; \text{stmt}_2, m \rangle \rightarrow \langle \text{stmt}_2, m' \rangle}$$

$$\frac{\langle \text{stmt}_1, m \rangle \rightarrow \langle \text{stmt}'_1, m' \rangle \quad \text{stmt}'_1 \neq \text{stop}}{\langle \text{stmt}_1; \text{stmt}_2, m \rangle \rightarrow \langle \text{stmt}'_1; \text{stmt}_2, m' \rangle}$$

Small-step semantics - $\text{Conf} = \text{Prg} \times \text{Mem}$

Evaluation of $\text{statements} : \rightarrow \subseteq \text{Conf} \times \text{Conf}$

If statements



Small-step semantics - $\text{Conf} = \text{Prg} \times \text{Mem}$

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Small-step semantics - $\text{Conf} = \text{Prg} \times \text{Mem}$

Evaluation of $\text{statements} : \rightarrow \subseteq \text{Conf} \times \text{Conf}$

If statements

$$\frac{\llbracket e \rrbracket \neq 0}{\langle \text{if}(e)\{stmt_1\}\text{else}\{stmt_2\}, m \rangle \rightarrow \langle stmt_1, m \rangle}$$

Small-step semantics - $\text{Conf} = \text{Prg} \times \text{Mem}$

Evaluation of $\text{statements} : \rightarrow \subseteq \text{Conf} \times \text{Conf}$

If statements

$$\frac{\llbracket e \rrbracket \neq 0}{\langle \text{if}(e)\{stmt_1\}\text{else}\{stmt_2\}, m \rangle \rightarrow \langle stmt_1, m \rangle}$$

$$\frac{\llbracket e \rrbracket = 0}{\langle \text{if}(e)\{stmt_1\}\text{else}\{stmt_2\}, m \rangle \rightarrow \langle stmt_2, m \rangle}$$

Small-step semantics - 4

Program semantics:

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Program semantics: $\langle\!\langle Prg \rangle\!\rangle : Mems \rightarrow Mems$

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Program semantics: $\langle\!\langle Prg \rangle\!\rangle : Mems \rightarrow Mems$

$$\langle\!\langle Prg \rangle\!\rangle(m) := \begin{cases} m' & \text{if } \langle Prg, m \rangle \xrightarrow{*} \langle \text{stop}, m' \rangle \\ \perp & \text{otherwise} \end{cases}$$

Small-step semantics - 4

Program semantics: $\langle\!\langle Prg \rangle\!\rangle : Mems \rightarrow Mems$

$$\langle\!\langle Prg \rangle\!\rangle(m) := \begin{cases} m' & \text{if } \langle Prg, m \rangle \xrightarrow{*} \langle \text{stop}, m' \rangle \\ \perp & \text{otherwise} \end{cases}$$

Partial function!

Small-step semantics - 4

Program semantics: $\langle\!\langle Prg \rangle\!\rangle : Mems \rightarrow Mems$

$$\langle\!\langle Prg \rangle\!\rangle(m) := \begin{cases} m' & \text{if } \langle Prg, m \rangle \xrightarrow{*} \langle \text{stop}, m' \rangle \\ \perp & \text{otherwise} \end{cases}$$

Transitive closure of \rightarrow

Partial function!



Programs have
a precise
meaning





What is a
secure
program?

Attacker model

- Split variables into:
 - **Public** variables x_{public}
 - **Secret** variables x_{secret}



Attacker can **observe** final value of
public variables

Non-interference (informally)



Non-interference (informally)

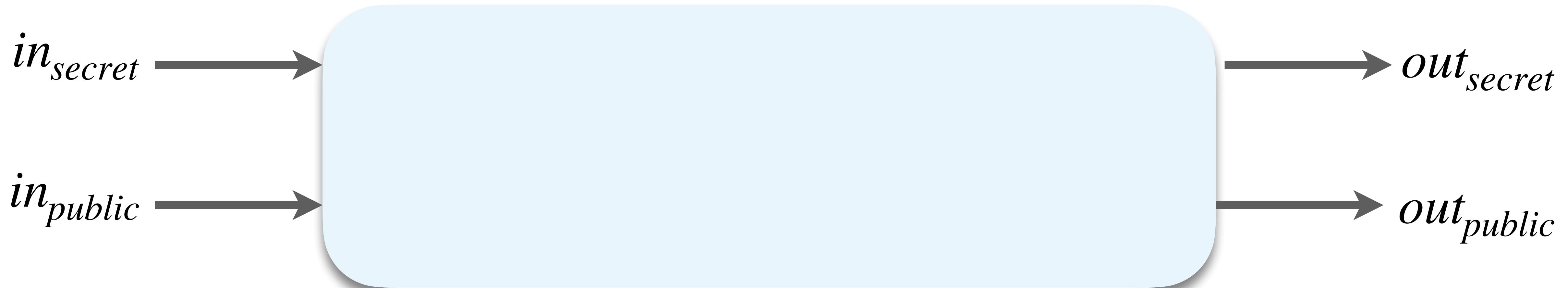


Security (confidentiality)

Attacker cannot derive secret values!



Non-interference (informally)

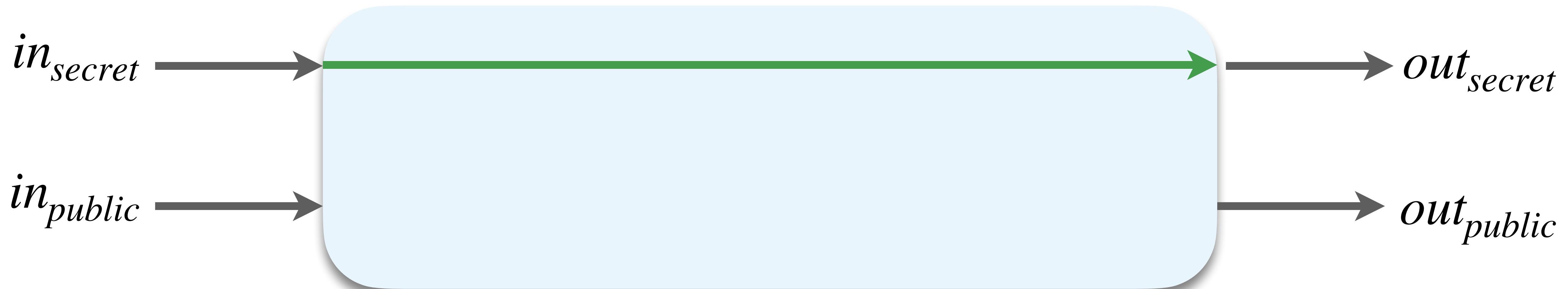


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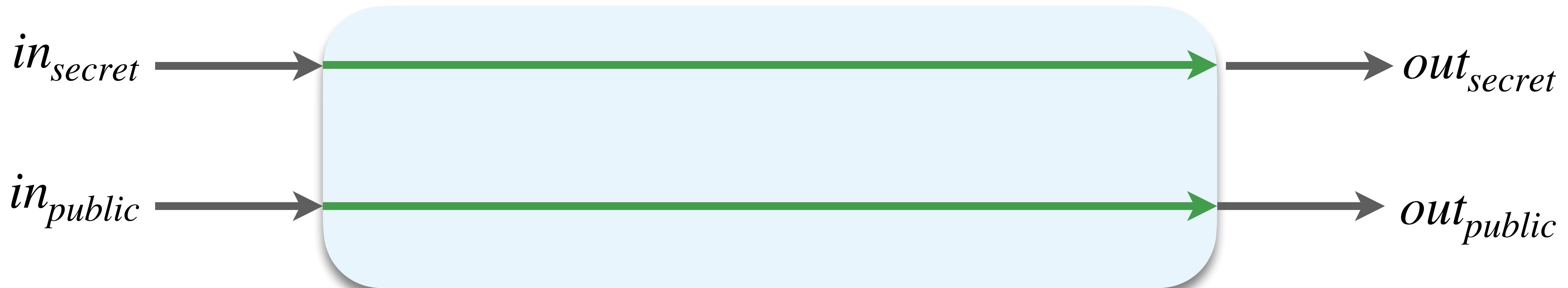


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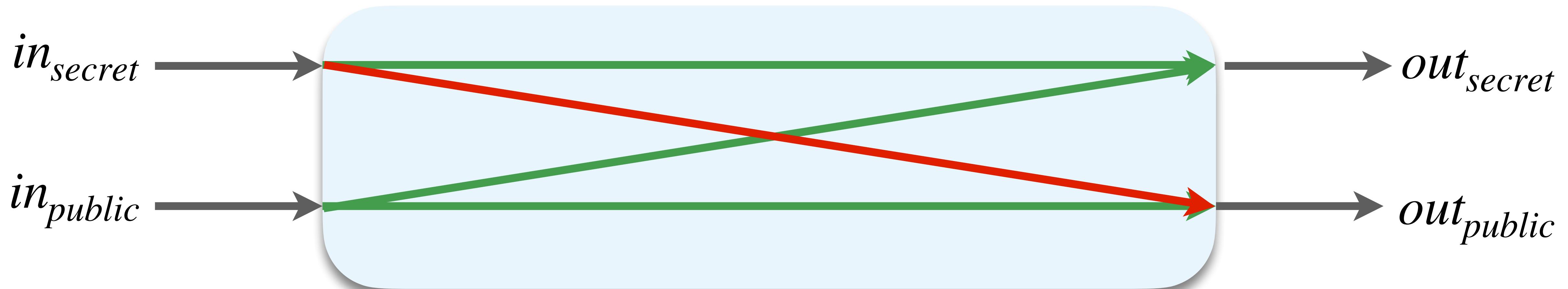


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Non-interference (informally)



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Non-interference (formally)

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Two memories \mathbf{m} and \mathbf{m}' are **public-equivalent** $\mathbf{m} \sim \mathbf{m}'$ if

For all public variables $x_{public} \in Vars$

$$\mathbf{m}(x_{public}) = \mathbf{m}'(x_{public})$$

Non-interference (formally)

Two memories m and m' are **public-equivalent** $m \sim m'$ if

For all public variables $x_{public} \in Vars$

$$m(x_{public}) = m'(x_{public})$$

Program **Prg** is **non-interferent** if

For all memories m and m' :

$$m \sim m' \Rightarrow \langle\langle Prg \rangle\rangle(m) \sim \langle\langle Prg \rangle\rangle(m')$$

Non-interference (formally)



Program **Prg** is *non-interferent* if

For all memories m and m' :

$$m \sim m' \Rightarrow \langle\langle Prg \rangle\rangle(m) \sim \langle\langle Prg \rangle\rangle(m')$$

Non-interference – examples

```
program f1(xpublic, xsecret) {  
    xpublic = xsecret + 1  
}
```

Non-interference – examples

```
program f1(xpublic, xsecret) {  
    xpublic = xsecret + 1  
}
```



Non-interference – examples

```
program f2(xpublic, xsecret) {  
    xsecret = 0  
    xpublic = xsecret + 1  
}
```

Non-interference – examples

```
program f2(xpublic, xsecret) {  
    xsecret = 0  
    xpublic = xsecret + 1  
}
```



Non-interference – examples

```
program f3(xpublic, xsecret) {  
    xsecret = xpublic + 1  
}
```

Non-interference – examples

```
program f3(xpublic, xsecret) {  
    xsecret = xpublic + 1  
}
```



Non-interference – examples

```
program f4(xpublic, xsecret) {  
    xsecret = 1  
}
```

Non-interference – examples

```
program f4(xpublic, xsecret) {  
    xsecret = 1  
}
```



Non-interference – examples

```
program f5 (xpublic, xsecret) {  
    if (xsecret==0)  
        xpublic = 1  
    else  
        xpublic = 0  
}
```

Non-interference – examples

```
program f5 (xpublic, xsecret) {  
    if (xsecret==0)  
        xpublic = 1  
    else  
        xpublic = 0  
}
```



We have a
notion of
security for
programs





What can we
do with non-
interference?

What can we do with NI?

- *Non-interference* provides theoretical foundations for talking about *program security*

What can we do with NI?

- ***Non-interference*** provides theoretical foundations for talking about ***program security***
- We can ensure that ***programs are secure***:

What can we do with NI?

- ***Non-interference*** provides theoretical foundations for talking about ***program security***
- We can ensure that ***programs are secure***:
 - ***Static*** enforcement

What can we do with NI?

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- We can ensure that ***program***
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Check statically if
programs are NI

What can we do with NI?

- ***Non-interference*** provides theoretical foundations for talking about ***program security***
- We can ensure that ***program***
 - ***Static*** enforcement
 - ***Dynamic*** enforcement

Check statically if
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What can we do with NI?

- ***Non-interference*** provides theoretical foundations for talking about ***program security***
- We can ensure that ***program***
 - ***Static*** enforcement
 - ***Dynamic*** enforcement

Check statically if programs are NI

Modify program ***behavior*** to enforce NI at ***runtime***

Static enforcement of NI

- Use ***program analysis*** techniques to check if programs are non-interferent:
 - Type systems
 - Symbolic execution
 - Abstract interpretation
 - ...

Static enforcement of NI

- Use ***program analysis*** techniques to check if programs are non-interferent:
 - Type systems
 - Symbolic execution
 - Abstract interpretation
 - ...



See lecture 5!



We can make
programs
secure



Backup

A type system for NI

A type system for NI

- Context:

A type system for NI

- Context:
 - $\Gamma(x_{\text{public}}) = \text{public}$
 - $\Gamma(x_{\text{secret}}) = \text{secret}$

A type system for NI

- Context:
 $\Gamma(x_{\text{public}}) = \text{public}$
 $\Gamma(x_{\text{secret}}) = \text{secret}$
- Typing rules:

Stop & Skip

 $pc \vdash \text{stop}$

 $pc \vdash \text{skip}$

A type system for NI

- Context:
 $\Gamma(x_{\text{public}}) = \text{public}$
 $\Gamma(x_{\text{secret}}) = \text{secret}$
- Typing rules:

Stop & Skip

$\frac{}{pc \vdash \text{stop}}$

$\frac{}{pc \vdash \text{skip}}$

$pc \in \{\text{public}, \text{secret}\}$

A type system for NI

- Context:
 $\Gamma(x_{\text{public}}) = \text{public}$
 $\Gamma(x_{\text{secret}}) = \text{secret}$
- Typing rules:

Assignment

$$\frac{\Gamma(e) \sqcup pc \subseteq \Gamma(x)}{pc \vdash x \leftarrow e}$$

A type system for NI

- Context:
 $\Gamma(x_{\text{public}}) = \text{public}$
 $\Gamma(x_{\text{secret}}) = \text{secret}$
- Typing rules:

Assignment

$$\frac{\Gamma(e) \sqcup pc \subseteq \Gamma(x)}{pc \vdash x \leftarrow e}$$

public \sqcup public = public
secret \sqcup public = secret

public \sqcup secret = secret
secret \sqcup secret = secret

A type system for NI

- Context:
- Typing rules:

$$\Gamma(x_{\text{public}}) = \text{public}$$
$$\Gamma(x_{\text{secret}}) = \text{secret}$$

public \sqsubseteq secret
secret $\not\sqsubseteq$ public

Assignment

$$\frac{\Gamma(e) \sqcup pc \sqsubseteq \Gamma(x)}{pc \vdash x \leftarrow e}$$

public \sqcup public = public
secret \sqcup public = secret

public \sqcup secret = secret
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A type system for NI

- Context:
 $\Gamma(x_{\text{public}}) = \text{public}$
 $\Gamma(x_{\text{secret}}) = \text{secret}$
- Typing rules:

Sequential composition

$$\frac{pc \vdash stmt_1 \quad pc \vdash stmt_2}{pc \vdash stmt_1; stmt_2}$$

A type system for NI

- Context:
 $\Gamma(x_{\text{public}}) = \text{public}$
 $\Gamma(x_{\text{secret}}) = \text{secret}$
- Typing rules:

If statements

$$\frac{pc \sqcup \Gamma(e) \vdash stmt_1 \quad pc \sqcup \Gamma(e) \vdash stmt_2}{pc \vdash \text{if } (e) \{stmt_1\} \text{ else } \{stmt_2\}}$$

A type system for NI

- Context:
 $\Gamma(x_{\text{public}}) = \text{public}$
 $\Gamma(x_{\text{secret}}) = \text{secret}$
- Typing rules:

For all **programs** Prg , if $\text{public} \vdash Prg$ then
 Prg satisfies non-interference

Dynamic enforcement of NI

Dynamic enforcement of NI

- ***Modify*** program ***semantics*** to enforce non-interference at runtime

Dynamic enforcement of NI

- **Modify** program *semantics* to enforce non-interference at runtime
- Context $\Delta : Vars \cup \{pc\} \rightarrow \{\text{public}, \text{secret}\}$

Assignment

$$\frac{[\![e]\!](m) = v \quad \Delta(pc) \sqcup \Delta(e) \sqsubseteq \Delta(x)}{\langle x \leftarrow e, m, \Delta \rangle \Rightarrow \langle \text{stop}, m[x \mapsto v], \Delta[x \mapsto \Delta(e) \sqcup \Delta(pc)] \rangle}$$

Dynamic enforcement of NI

- **Modify** program *semantics* to enforce non-interference at runtime
- Context $\Delta : Vars \cup \{pc\} \rightarrow \{\text{public}, \text{secret}\}$

If statements

$$[\![e]\!] \neq 0$$

$$\langle \text{if}(e)\{\text{stmt}_1\}\text{else}\{\text{stmt}_2\}, m, \Delta \rangle \Rightarrow \langle \text{stmt}_1, m, \Delta[pc \mapsto \Delta(pc) \sqcup \Delta(e)] \rangle$$

Dynamic enforcement of NI

- **Modify** program ***semantics*** to enforce non-interference at runtime
- Context $\Delta : Vars \cup \{pc\} \rightarrow \{\text{public}, \text{secret}\}$

For all ***programs Prg***, executing *Prg* under the semantics
 \Rightarrow satisfies non-interference

$$\langle \text{if}(e)\{stmt_1\}\text{else}\{stmt_2\}, m, \Delta \rangle \Rightarrow \langle stmt_1, m, \Delta[pc \mapsto \Delta(pc) \sqcup \Delta(e)] \rangle$$