CO221 - Compilers

6th January 2020

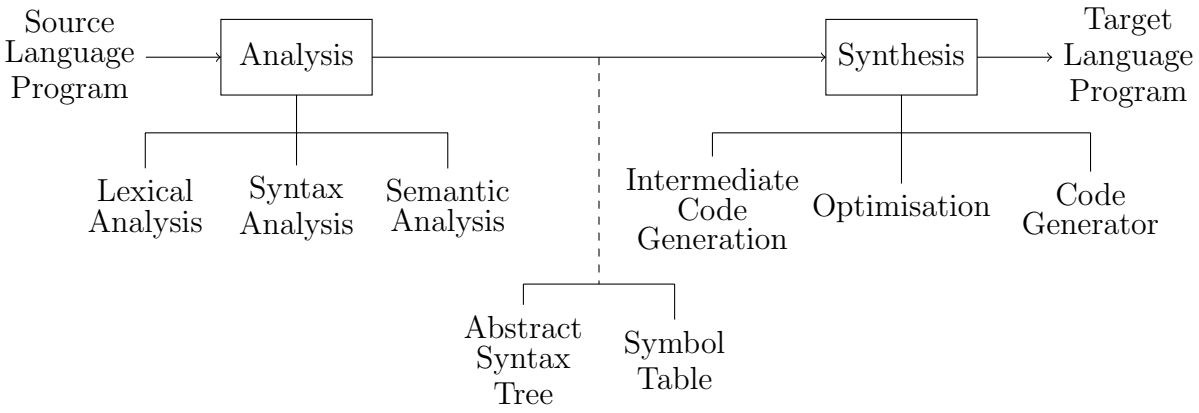
A compiler is a program which processes programs, including translating a program written in one language (usually higher level) to another programming language (usually in a lower level). In our course, the focus is to generate assembly code from the high level language. This translation goes between high level human concepts, and the data manipulation the machine performs.

Structure

The general structure of a compiler is as follows;

1. **input** takes in an input program in some language
2. **analysis** constructs an internal representation of the source structure
3. **synthesis** walks the representation to generate the output code
4. **output** creates an output in the target language

In more detail, it can be represented as follows;



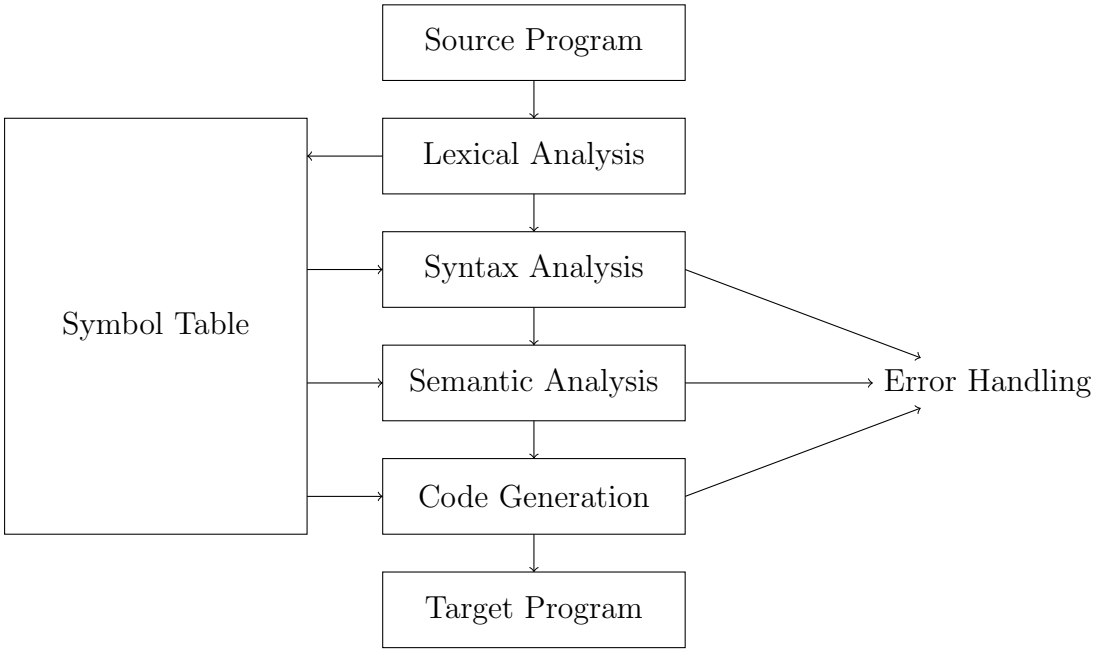
- **lexical analysis** looks at characters of input program, analyses which are keywords (such as converting `if`, and `while` to corresponding tokens), which are user defined words, and which are punctuation, etc.
- **syntax analysis** discovers structure of input
- **semantic analysis** checks that variables are declared before they are used, and that they are used consistently with their types etc.

Simple compilers go straight to code generation, but optimising compilers do several passes of intermediate code generation and optimisation.

The symbol table holds data on variables, such as types. Sometimes we need to know the type of the variable, in order to generate code for the variable, for example if we were to print a variable, it would need to generate different code for strings than it would need to do for integers. Scope rules are also needed.

Phases

Whether all of these phases are done in the order shown is a design choice. For example, lexical analysis and syntax analysis are often interleaved. This can be done when the syntax analysis stage needs the next symbol, and therefore the lexical analysis stage can be used.



Syntax Analysis

This is also known as parsing. Languages have a grammatical structure specified by grammatical rules in a **context-free grammar** such as BNF (**Backus-Naur Form**). The output of the analyser is a data structure which represents the program structure; an **abstract syntax tree**. The writer of the compiler must design the AST carefully such that it is easy to build, as well as easy to use by the code generator.

A language specification consists of the following;

- **syntax** grammatical structure

  in order to determine that a program is syntactically correct, one must determine how the rules were used to construct it
- **semantics** meaning

For example, we can encode the rules for a statement as follows (anything in quotes is a terminal), in BNF;

$$\text{stat} \rightarrow \text{'if' '(' exp ')' stat 'else' stat}$$

Each BNF production is a valid way for a non-terminal (LHS) to be expanded (RHS) into a combination of terminals and non-terminals. Only terminals can appear in the final results (they are lexical tokens).

To prove the following is a valid example of stat, we'd need to show that a can be derived from exp, and that both b and c can be derived from stat.

$$\text{if (a) b else c}$$

Context-Free Grammars

Formally, a context-free grammar consists of the following four components;

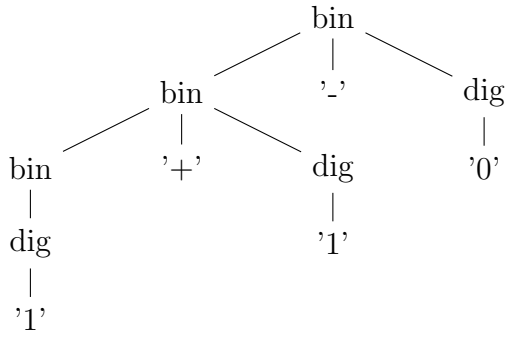
- S a non-terminal start symbol
- P a set of productions
- t a set of tokens (terminals)
- nt a set of non-terminals

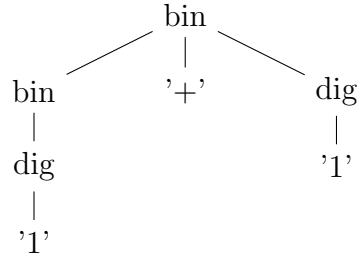
For example, consider the following BNF, and their associated components;

$$\begin{aligned}
&\text{bin} \rightarrow \text{bin '+' dig} \mid \text{bin '-' dig} \mid \text{dig} \\
&\text{dig} \rightarrow \text{'0'} \mid \text{'1'} \\
&t = \{\text{'+'}, \text{'-'}, \text{'0'}, \text{'1'}\} \\
&nt = \{\text{bin}, \text{dig}\} \\
&S = \text{bin}
\end{aligned}$$

A string of only terminals (**sentential form**) can be derived using the grammar by beginning with the start symbol, and repeatedly replacing each non-terminal with the RHS from a corresponding production. We refer to the set of all sentential forms derived from the start symbol as the **language** of a grammar.

We can prove that some string is in the language of a grammar by constructing a **parse tree**. For example, to prove that "1+1-0" $\in L(G)$, and "1+1" $\in L(G)$ we can use the following trees;

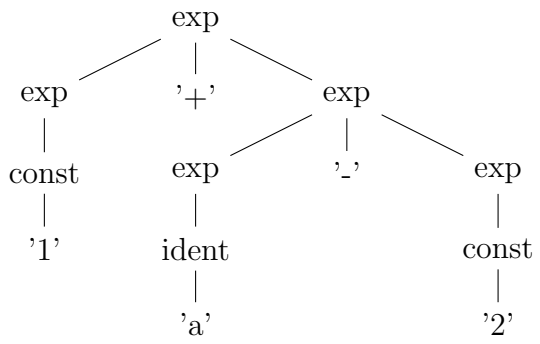


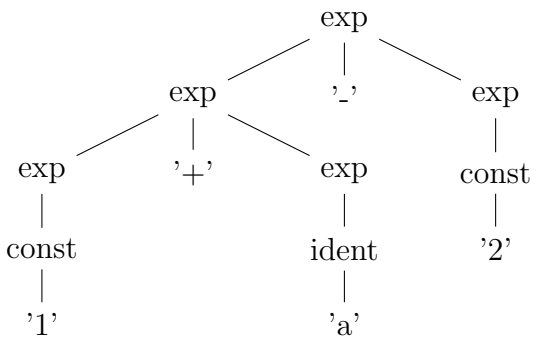


Ambiguity

A grammar is referred to as **ambiguous** if its language contains strings which can be generated in two different ways. Essentially, there exists some string in $L(G)$ which has two different parse trees. Consider string "1 + a - 3" in the following grammar, and the parse tree(s) associated;

$$\text{exp} \rightarrow \text{exp '+' exp} \mid \text{exp '-' exp} \mid \text{const} \mid \text{ident}$$





While the string is still valid, and in the language, our issue is with the ambiguity, as we want to generate a program uniquely. The reason our grammar is broken is due to the recursive use of the non-terminal exp on both sides, which means we're given a choice of which side to expand when generating.

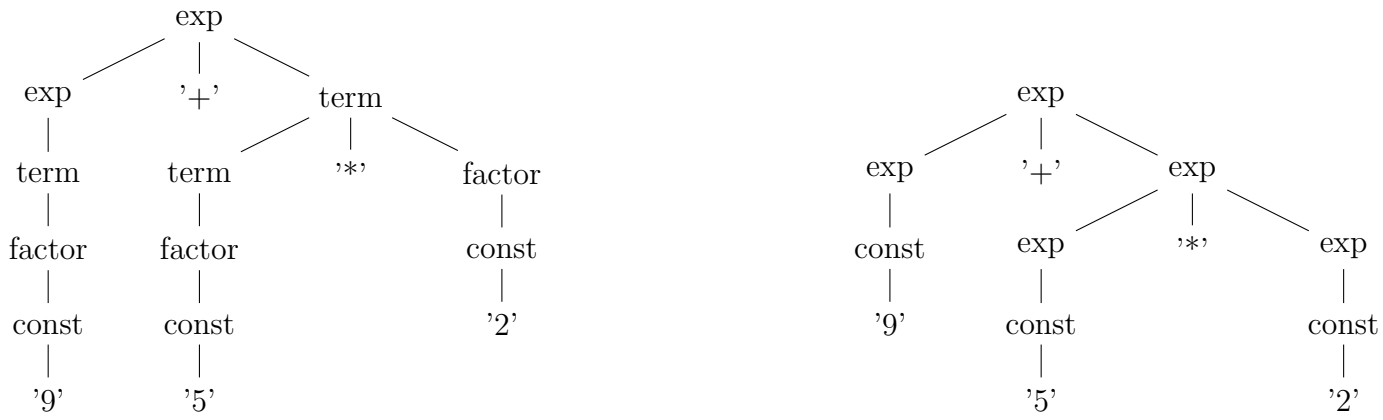
Associativity and Precedence

For our example language, we're using all left-associative operators. We also want to maintain that '*' and '/' have higher precedence than '+' and '-'. One way of doing this is to split the grammar

into layers, by having separate non-terminals for precedence levels. This method can be done with the following unambiguous grammar for arithmetic expressions;

$$\begin{aligned} \text{exp} &\rightarrow \text{exp } \text{'+'} \text{ term} \mid \text{exp } \text{'-'} \text{ term} \mid \text{term} \\ \text{term} &\rightarrow \text{term } \text{'*'} \text{ factor} \mid \text{term } \text{'/'} \text{ factor} \mid \text{factor} \\ \text{factor} &\rightarrow \text{const} \mid \text{ident} \end{aligned}$$

Now, we can unambiguously generate the parse tree (and thus the unique abstract syntax tree) for "9+5*2";



It's important to note that the **abstract** syntax tree doesn't need this in contrast, as only the parse tree needs it.

Parsers

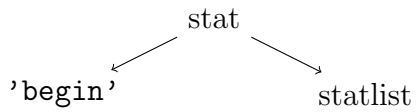
The parser checks that the input is grammatically correct, and builds an AST representing the structure. In general, there are two classes of parsing algorithms;

- **top-down / predictive** — we are using recursive descent
- **bottom-up** — also known as shift-reduce

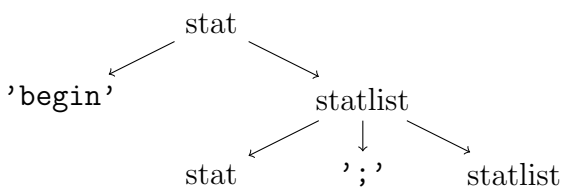
For this, we will use the input "`begin S; S; end`", with the following grammar;

$$\begin{aligned} \text{stat} &\rightarrow \texttt{'begin'} \text{ statlist} \mid \texttt{'S'} \\ \text{statlist} &\rightarrow \texttt{'end'} \mid \text{stat } \texttt{';'} \text{ statlist} \end{aligned}$$

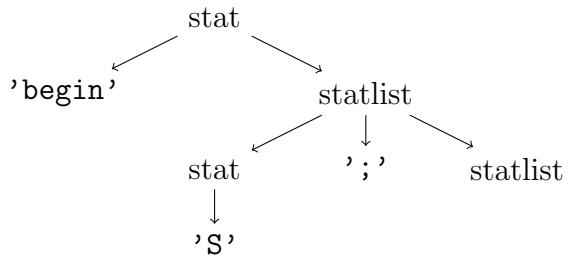
When we start top-down parsing, we start with the non-terminal stat. The first token we identify is the `'begin'`, thus our tree becomes the following;



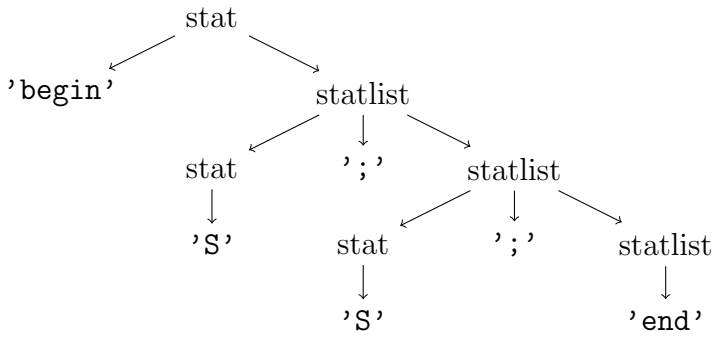
However, as the next symbol isn't the terminal `'end'`, we have to use an alternative. As we only have one alternative, we can predict it, and thus the tree becomes;



As the next symbols are the terminal `'S'`, and the terminal `';'`, we can tick them off, thus the tree becomes;



This process continues, until we reach the final tree;



On the other hand, bottom-up parsing tries to use all the RHSs (whereas top-down tries to match a non-terminal by trying each of the RHSs), and replaces it with a non-terminal, by using the production in reverse. Bottom-up succeeds when the whole input is replaced by the start symbol.

In general, we push the current symbol onto the stack (or the reduction if we can reduce it). For example, once we encounter `'S'`, we can reduce it to stat, and similarly once we encounter `'end'`, we can reduce it to statlist.

stack	current symbol	remaining tokens	S/R	note
	'begin'	'S' ';' 'S' ';' 'end'	S	nothing to do yet
'begin'	'S'	';' 'S' ';' 'end'	R	terminal for stat
'begin'	stat	';' 'S' ';' 'end'	S	no more work
'begin' stat	';'	'S' ';' 'end'	S	nothing to do yet
'begin' stat ';'	'S'	';' 'end'	R	terminal for stat
'begin' stat ';'	stat	';' 'end'	S	no more work
'begin' stat ';' stat	';'	'end'	S	nothing to do yet
'begin' stat ';' stat ';'	'end'		R	terminal for statlist
'begin' stat ';' stat ';'	statlist			
'begin' stat ';' stat ';'	statlist		R	match for statlist
'begin' stat ';'	statlist			
'begin' stat ';'	statlist		R	match for statlist
'begin'	statlist			
'begin'	statlist		R	match for stat
	stat			complete

Simple Compiler in Haskell

If the input to the parser is a simple string of characters, representing an arithmetic expression, following the BNF defined below;

expr → fact '+' expr | fact
fact → number | identifier

The string `a + b + 1` would become the following sequence of tokens, after lexical analysis;

```
[IDENT "a", PLUS, IDENT "b", PLUS, NUM 1]
```

It's important to note that this is a right-recursive grammar, as a recursive descent method **will not** work with left-recursive grammars.

```
1  data Token
2    = IDENT [Char] | NUM Int | PLUS
3  data Ast
4    = Ident [Char] | Num Int | Plus Ast Ast
5    deriving (Show)
6  data Instr
7    = PushVar [Char] | PushConst Int | Add
8    deriving (Show)
9  parse :: [Token] -> Ast
10 parse ts =
11   let (tree, ts') = parseExpr ts
12   in case ts' of
13     [] -> tree
14     _  -> error "Excess tokens"
15 parseExpr :: [Token] -> (Ast, [Token])
16 parseExpr ts
17   = let (factTree, ts') = parseFact ts
18     in case ts' of
19       (PLUS : ts') ->
20         let (sExpTree, ts'') = parseExpr ts'
21         in (Plus factTree sExpTree, ts'')
22       other -> (factTree, other)
23 parseFact :: [Token] -> (Ast, [Token])
24 parseFact (t:ts)
25   = case t of
26       NUM n -> (Num n, ts)
27       IDENT x -> (Ident x, ts)
28       _ -> error "Expected a number or identifier"
29 translate :: Ast -> [Instr]
30 translate ast
31   = case ast of
32       Num n -> PushConst n
33       Ident x -> PushVar x
34       Plus e1 e2 -> translate e1 ++ translate e2 ++ [Add]
```

Comments on the code;

- Note that the structures for `Token` and `Ast`, defined on lines 2 and 4 respectively, are very similar - however, the latter represents a tree structure
- We require a parsing function for each non-terminal in the code, hence we have `parseExpr` and `parseFact`
- It's easier to start with the non-recursive cases, which are the factors
- From here, you can see that the recursion structure of the code closely follows the recursion structure of the grammar, as we have the recursion in line 20 (on expressions, after the factor is parsed)
- Each function returns the part of the AST it has generated and the **remaining** tokens after consuming input
- The final translation function generates instructions for a very simple stack machine

13th January 2020

Bootstrapping

Imagine the scenario where there is a new language, and only one machine. The process of writing a compiler for this language, with this language, is to first manually write a compiler in the assembly language for the machine for a small subset of the new language. Using the subset of this new language (that can be compiled), we can write a compiler to compile more of the language, and this process continues until we can compile the entire language.

Lexical Analysis

The lexical analyser (sometimes called a scanner) converts characters into tokens. This is because the compiler shouldn't have to deal with strings directly. Normally, this removes whitespace, as it isn't needed in code generation (other than for string / character literals). In this course, the regular expressions that the scanners use will be converted into finite automata.

Identifiers are usually classified into the following;

- **keywords**

  These are defined by the language, and are reserved. For example, words such as "return", "for", "class", and so on are represented as their own tokens (`RETURN`, `FOR`, and `CLASS` respectively), since there is a (relatively) small finite set to work with. The scanner needs to be able to quickly verify if something is a keyword, and therefore something such as a "perfect" hash function is used.

- **user-defined**

  These are defined by the programmer. Since there can be (theoretically) an infinite amount of them, it's not possible to generate a unique token for each one, and therefore it usually falls under a general identifier token with a string parameter (such that "xyz" becomes `IDENT("xyz")`, or similar).

In the case of literals, some special consideration may be needed for cases where the language we are compiling can support more than the language we are writing the compiler in. For example, if the language we are writing a compiler for can support arbitrarily large integers, special consideration will be required if the language the compiler is written in cannot support such values. Some examples of literals are as follows (more can exist, such as booleans, characters, and so on);

- **integers**

  An integer would likely be represented by a general integer token, such that the string "123" would become `INTEGER(123)`, or similar.

- **strings**

  Similar to integers, but the token constructor will now take a string parameter instead of an integer, such that ""foo"" would become `STRING("foo")`.

There are other tokens, not just the two cases above, such as (but not limited to);

- **operators / symbols**

  Normally operators / symbols such as "+", "¡=", "(" are represented as their own unique token, such as `PLUS`, `LTE`, `LPAREN`, respectively.

- **whitespace / comments**

  Whitespace characters are normally removed (unless they are in the case where they exist within a literal), but are needed to separate adjacent identifiers. Comments are also usually removed.

Regular Expressions (Regex)

This allows us to formally define the acceptable tokens of the language.

regex	matches
`a`	a literal **symbol** of the language's alphabet (that isn't a regex meta-character)
ϵ	the empty string **epsilon**
`R1 R2`	**concatenation** of regex `R1` followed by `R2` (medium precedence)
`R1|R2`	**alternation** of regex `R1` or `R2` (lowest precedence)
`R*`	**repetition** of regex `R` (0 or more times) (highest precedence)
`(R)`	**grouping** `R` by itself, used to override precedence
`\a`	"escaping", used to have a literal of a meta-character
shortcut	(can be made by the rules above)
`R?`	0 or 1 occurrences of regex `R`
`R+`	1 or more occurrences of regex `R`
`[aeiou123]`	any character from the given set
`[a-zA-Z0-9]`	any alphanumeric character
`[^a-zA-Z]`	any character **except** the ones in the set
`.`	any character except a newline

15th January 2020

Regular Expression Rules

We write rules or productions in the form $\alpha \rightarrow$ `X`, where α is a non-terminal (the name of the rule), and `X` is some regular expressions constructed by any combination of terminals (symbols) and non-terminals (names of **other** rules - recursion is not allowed, therefore all non-terminals must be defined before being used in another rule). For example, we have the following regular expressions for a simple grammar (note that it looks very similar to WACC).

```
      Digit → [0-9]
        Int → Digit+
SignedDigit → (+ | -)?  Int
    Keyword → if | while | do
 Identifier → Letter (Letter | Digit)*
```

However, we can run into the issue of ambiguity, when a character sequence can match to more than one regex. For example, the input string `dough` matches to the identifier `dough`, as well as partially to the keyword `do`. Two strategies are either to match the longest character sequence (causing the former), or to have textual precedence, where the first regex takes precedence (causing the latter).

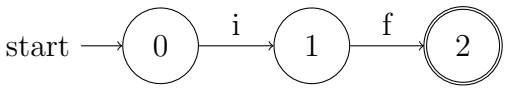
Finite Automata

When we draw a finite automata, its important to note the following symbols we use;

- **states** are circles
  - the **start** state has an unlabelled arrow going into it
  - the **accepting** (end) state is a double circle
  - all non-accepting states have arrows leading to an error state, but this is often omitted
- **transitions** are arrows between states, with the matched **symbol** being the labels of the arrows

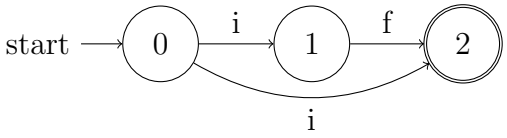
The types of finite automata we look at are the following;

- **deterministic finite automata (DFA)**



The example above is deterministic as there are no two transitions from the same state with the same symbol.

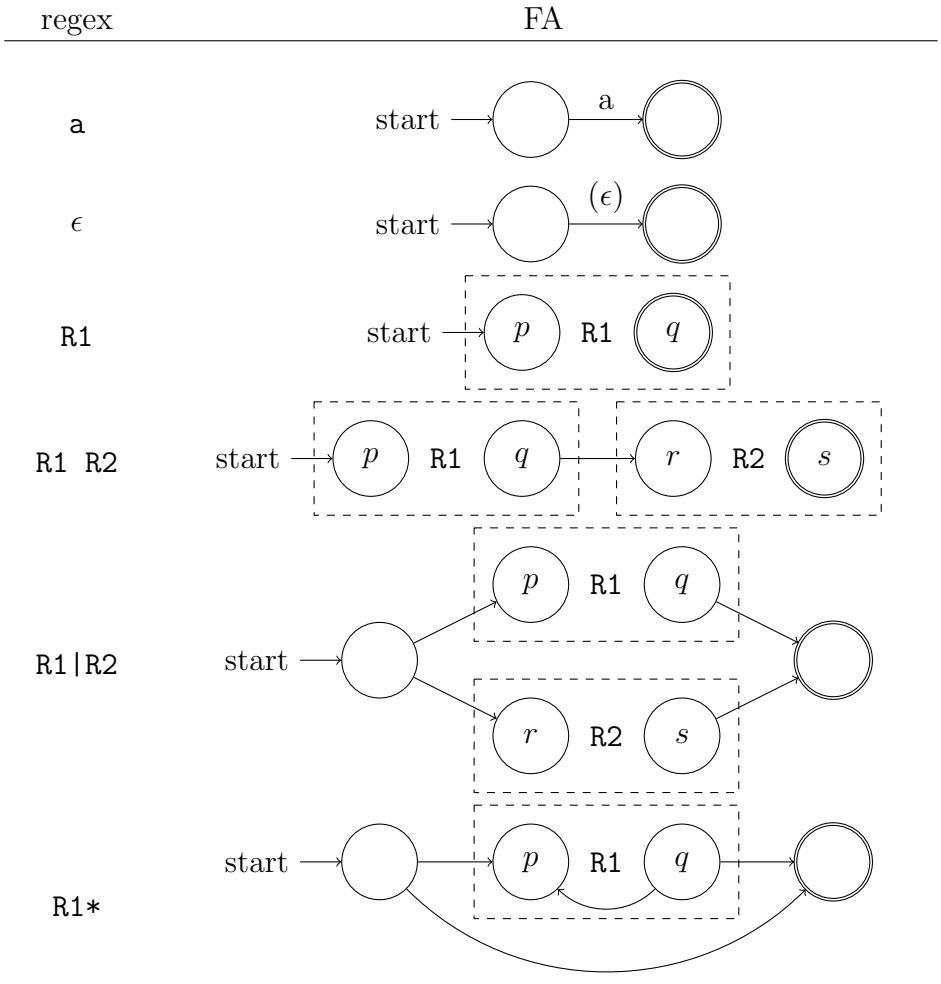
- **non-deterministic finite automata (NFA)**



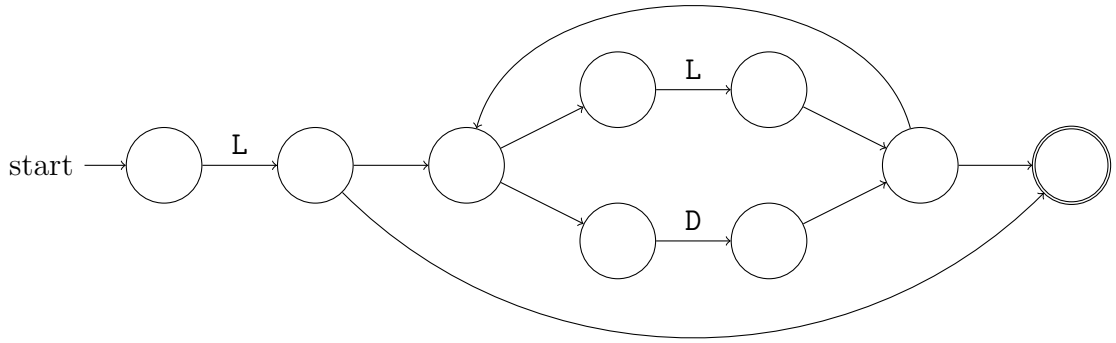
The example above is non-deterministic as there is more than one transition from state 0 with the same symbol. It allows for choice and more compact solutions, but requires backtracking.

Conversion of Regex to (Non-Deterministic) Finite Automata

Thompson's construction uses ϵ-transitions to "glue" together automata. While this is usually represented with an ϵ over the transition, it can be omitted for brevity, and therefore any transitions without labels will be assumed to be ϵ-transitions. In lieu of repeatedly drawing the same thing, let the regular expression `R1` have an initial state of p and an accepting / end state of q, and let `R2` have an initial state r and accepting state s. Also note that within the dotted lines can exist an arbitrarily complex automata.



For example, consider the regular expressions for an identifier, following the form `L(L | D)*`, which represents a letter followed by any combination of letters and digits. Note that we are allowed to abbreviate the use of `Letter` to `L`, for brevity, and similar for `Digit` to `D`.



Conversion from NFA to DFA

It's important to note the worst case complexities for NFA and DFA are as follows, where n is the length of the input string, and r is the length of the regular expression;

type	space complexity	time complexity
NFA	$O(r)$	$O(nr)$
DFA	$O(2^r)$	$O(n)$

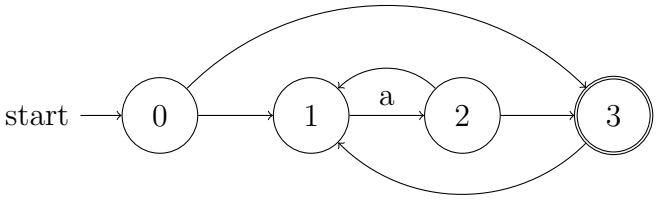
As you can see, DFAs are much faster, but can be exponentially bigger than NFAs. However, the worst case space complexity for a DFA is rarely reached for lexer analyser generators.

In order to convert from NFA to DFA, we use ϵ-closures. To avoid repetition, I will be denoting the ϵ closure of s as $\epsilon_c(s)$.

$$\epsilon_c(s) = \text{set of states reachable by zero or more } \epsilon\text{-transitions from } s$$

$$\epsilon_c(\{s_1, \ldots, s_n\}) = \bigcup_{i=1}^{n} s_i$$

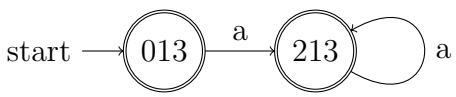
For example, take the following NFA;



Which has the following closures;

$\epsilon_c(0) = \{0, 1, 3\}$
$\epsilon_c(1) = \{1\}$
$\epsilon_c(2) = \{1, 2, 3\}$
$\epsilon_c(3) = \{1, 3\}$

From our start state, using subset construction, we then create a node consisting of its ϵ-closure. We look at this new node, find the ones that have a non-ϵ-transition, and handle those cases. For each new case, we take the state that it goes to, and create a node consisting of its ϵ-closure, and repeat the process. Finally, we mark each of the states that contain an accepting state as an accepting state. The example above becomes the following;



17th January 2020

At the start of this lecture, he goes over the grammar for an example language that has statements, consisting of assignment and iteration.

Code Generation for a Stack Machine

He then continues with a representation of a stack machine, it's a lot of reading, and most of it is on the slides. The general premise of the stack machine is that it fulfils a promise where it leaves the result of a computation at the top of the stack, and doesn't modify anything that was below it. As such, we end up with the following code (note that `label1` and `label2` are unique labels that haven't been used before - generating this in Haskell isn't as trivial as in other languages);

```
type Name = [Char]
type Label = [Char]

data Stat = Assign Name Exp | Seq Stat Stat | ForLoop Name Exp Exp Stat
data Exp = Binop Op Exp Exp | Unop Op Exp | Ident Name | Const Int
data Op = Plus | Minus | Times | Divide
data Instruction = Add | Sub | Mul | Div | Negate
| PushImm Int | PushAbs Name | Pop Name
| CompEq | JTrue Label | JFalse Label
| Define Label -- not executed

-- naive code generator
transStat :: Stat -> [Instruction]
transStat s
= case s of
    Assign id exp -> transExp exp ++ [Pop id]
    Seq s1 s2 -> transStat s1 ++ transStat s2
    ForLoop id e1 e2 body ->
        transExp e1 ++ [Pop id] ++ -- initialisation
        [Define label1] ++
        transExp e2 ++ [PushAbs id] ++ [CompGt] ++ [JTrue label2] ++ -- test
        transStat body ++ -- loop body
        [PushAbs id] ++ [PushImm 1] ++ [Add] ++ [Pop id] ++ -- increment
        [Jump label1] ++ -- go back to test
        [Define label2] -- define end of loop

transExp :: Exp -> [Instruction]
transExp e
= case e of
    Ident id -> [PushAbs id]
    Const v -> [PushImm v]
    Binop op e1 e2 -> transExp e1 ++ transExp e2 ++ transOp op
    Unop op e -> transExp e ++ transUnop op

transOp :: Op -> [Instruction]
transOp Plus = [Add]
transOp Minus = [Sub]
transOp Times = [Mul]
transOp Divide = [Div]

transUnop :: Op -> [Instruction]
```

```
42 transUnop Minus = [Negate]
```

We need to remember that the compiler does **not** execute code, or evaluate the instructions, since it cannot know the value of variables which are determined at runtime. Looking at line 16, in the code above, the instruction **list** `transExp exp`, after execution, leaves the result of evaluating `exp` at the top of the stack, ready for `Pop id` to store. It's also important to remember that `Define` isn't actually executed, but used for the assembler to figure out addresses for jumps.

Code Generation for a Machine with Registers

While a stack machine isn't unrealistic, it will be much slower compared to one that has efficient use of registers. For this part, we want to concentrate on the effective use of registers for arithmetic operations. In the previous code snippet given in Haskell, we modify the instructions to use registers, as such (replacing instances where sensible, otherwise preserving them);

```
1  type Reg = Int
2
3  data Instruction = Add Reg Reg | -- and so on
4    | Load Reg Name | LoadImm Reg Int | Store Reg Name | Push Reg | Pop Reg
5    | CompEq Reg Reg | JTrue Reg Label | JFalse Reg Label
6
7  transExp :: Exp -> Reg -> [Instruction]
8  transExp e r
9    = case e of
10       Ident id -> Load r id
11       Const v -> LoadImm r v
12       Binop op e1 e2 ->
13         transExp e1 r ++
14         transExp e2 (r + 1) ++
15         [binop r (r + 1)]
16         where
17           binop = case op of
18             Plus -> Add
19 -- and so on
```

Notice the additional parameter given into `transExp`. This specifies the register in which the result of the operation should go into. The allocation of registers in this translator mirrors the "slot" that the expression would be stored in on the stack. As we mirror the stack in this sense, specifying something goes into register i allows the program to modify anything $\geq i$, but nothing below it. This is why we specify the second expression must be evaluated into $r + 1$, as we don't want to modify the result of the first.

However, one caveat of this, if we were to evaluate `(x * 4) + 3`, is that we would have to use a total of 2 registers. This is because all our operations currently only work between registers, and therefore immediate values have to be loaded in. However, many assembly languages allow for immediate operations on registers, meaning that the entire execution can be achieved with only a single register. The general idea of this is that we are able to take advantage of pattern matching, given that the language the compiler is written in supports it, to look for these obvious patterns, such as doing arithmetic with a constant term. This is called **instruction selection**.

Combination of Register and Stack

When there aren't enough registers for us to use, in the case of a complex arithmetic operation for example, we want to start utilising the stack. This gives us the performance benefits of using registers, but also allows us to perform arbitrarily complex computations.

To do this, we consider the example of an accumulator machine. This only has one register and also uses the stack. Generating code for this allows us to generate code for the case where the registers are all full, except for one, which we then treat as the accumulator. Therefore the general strategy is to consider it as a register machine until all but one register is used, and then treat it as an accumulator machine.

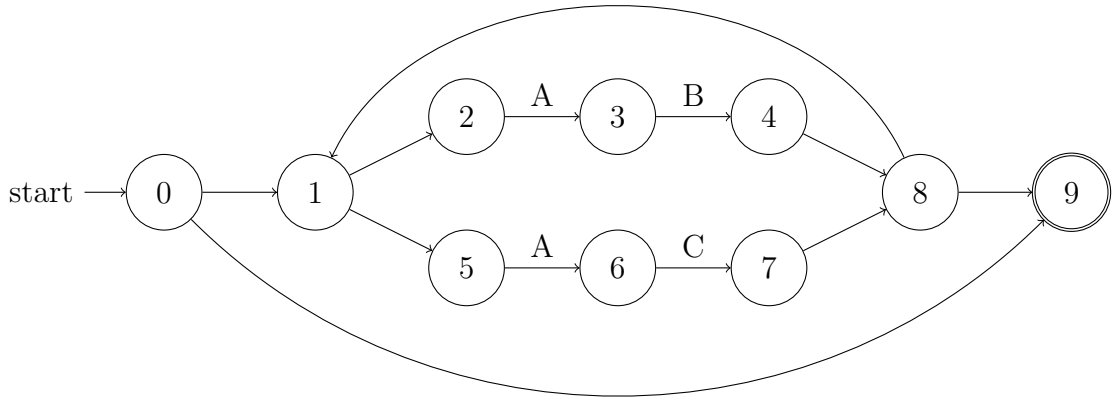
Note that when we do work on an accumulator machine, the binary operation $e_1 \bullet e_2$ has the following order; evaluate e_2 into the accumulator, push it to the stack, and then evaluate e_1 into the accumulator, then perform an add instruction that uses both the register and the stack.

```
transExp :: Exp -> Reg -> [Instruction]
transExp e r
  = case e of
      -- etc
      Binop op e1 e2 ->
        if (r == MAXREG) then
          transExp e2 r ++
          [Push r] ++
          transExp e1 r ++
          transBinopStack op r -- one operand is register
        else
          transExp e1 r ++
          transExp e2 (r + 1) ++
          transBinop op r (r + 1) -- both operands are registers
      -- etc
```

20th January 2020

Subset Construction of Regex

This lecture starts by working through the subset construction of the regex `(AB|AC)*`. We start by constructing the NFA for the corresponding regular expression;

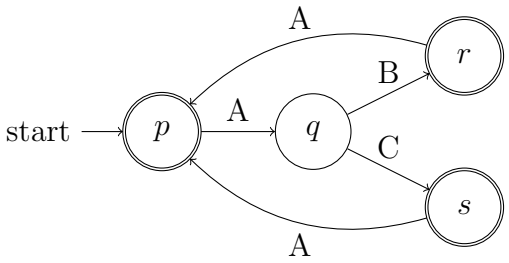


The DFA is then generated as follows (note that in lieu of writing out the entire closure, I use a single character, however it's best to write out the entire thing);

1. create a state for 0 with its closure (01259), let it be p
2. looking at the states within p, we see that the ones with non-ϵ-transitions are 2 and 5, both with transitions with label 'A', to 3 and 6 respectively
3. create a state for 3 and 6, with its closure (36), let it be q

4. looking at the states within q, we see that the ones with non-ϵ-transitions are both 3 and 6, with transitions 'B' to 4, and 'C' to 7 respectively
5. create a state for 4, with its closure (124589), let it be r
6. looking at the states within r, we see that the ones with non-ϵ-transitions are 2 and 5 - however we have already seen this, it goes to the state q
7. create a state for 7, with its closure (125789), let it be s
8. s also goes to q for the same reasoning as above
9. all the states that contain 9 are now marked as accepting states, which are p, r, s

This gives us the following DFA



LR (Bottom-up) Parsing

The rules in the context free grammar exist as `R` $\rightarrow$ `(R|t)*`, where `R` is a rule, and `t` is a token. Therefore rules are represented as some arbitrary combination of rules and tokens (thus supporting recursion, unlike regular expressions). The goal of a parser is to convert a sequence of tokens into an AST (or parse tree). In the case of the AST we can discard tokens that are simply syntactic sugar, as the structure of the program is represented in the tree.

Note that we need to look at subsets of CFGs, as they can be cubic ($O(n^3)$) to parse in the worst case, and we want to consider the largest subsets which take $O(n)$. The types we consider are as follows (node that LL(n) $\subseteq$ LR(n), therefore LR parsers are more powerful);

- LL(k) **l**eft to right scanning, with k token look-ahead, and **l**eft-most derivation top-down, as it builds the AST from the root nodes to the leaf nodes
- LR(k) **l**eft to right scanning, with k token look-ahead, and **r**ight-most derivation bottom-up, as it builds the AST from the leaf nodes to the root node

We will be first looking at LR(0), then LR(1), and then finally LALR(1).

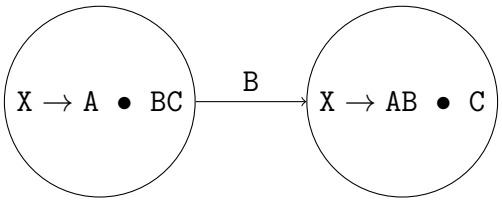
Starting with LR(0), it doesn't need the token for a reduce. An LR(0) item is a "rule" with a dot ($\bullet$) at some position in the right hand side. An item indicates how much of a rule we've seen. For example, the item `E` $\rightarrow$ `E + ` $\bullet$ `int` indicates that we've seen an expression, followed by a plus, and we are hoping to see an integer to complete the item. This means that LR(0) items represent the steps to recognise the RHS of a given production.

For example, we can look at the rules for `X` $\rightarrow$ `ABC`;

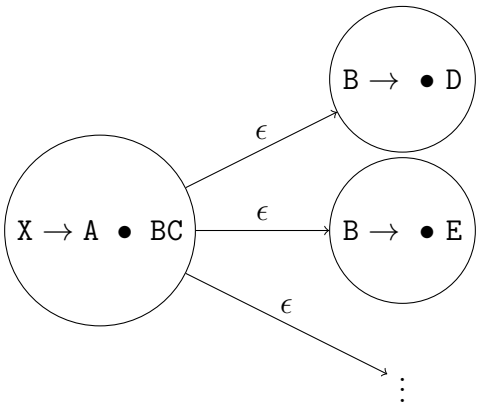
`X` $\rightarrow$ $\bullet$ `ABC`	initial item
`X` $\rightarrow$ `A` $\bullet$ `BC`	
`X` $\rightarrow$ `AB` $\bullet$ `C`	
`X` $\rightarrow$ `ABC` $\bullet$	complete item

$$\texttt{X} \rightarrow \underbrace{\texttt{AB}}_{\text{seen}} \bullet \overbrace{\texttt{C}}^{\text{hope to see}}$$

These items are used as states of a finite automaton to maintain information about the progress of a shift-reduce parser. We can build an NFA from these items as follows (and then build a DFA with subset construction). Working with the single rule $\texttt{X} \rightarrow \texttt{A} \bullet \texttt{BC}$, we add the following transition;



Additionally, if `B` is non-terminal, such that $\texttt{B} \rightarrow \texttt{D} \mid \texttt{E} \mid \cdots$, add an ϵ-transition for each rule;



We also need to add a start rule, with symbol to indicate the end of input `$` (it is implied if omitted). For example, given the grammar

$$\texttt{E} \rightarrow \texttt{E '+' int} \mid \texttt{int}$$

We'd add a rule $\texttt{E'} \rightarrow \texttt{E \$}$. This therefore has a total of 8 items;

$\texttt{E'} \rightarrow \bullet\ \texttt{E}$	initial item
$\texttt{E'} \rightarrow \texttt{E} \bullet$	complete / reduce item
$\texttt{E} \rightarrow \bullet\ \texttt{E + int}$	initial item
$\texttt{E} \rightarrow \texttt{E} \bullet \texttt{+ int}$	
$\texttt{E} \rightarrow \texttt{E +} \bullet \texttt{int}$	
$\texttt{E} \rightarrow \texttt{E + int} \bullet$	complete item
$\texttt{E} \rightarrow \bullet\ \texttt{int}$	initial item
$\texttt{E} \rightarrow \texttt{int} \bullet$	complete item

Chomsky Hierarchy

In the following, `R` is a non-terminal (name of a rule), `t` is a sequence of terminals, and α, β, φ are sequences of terminals and non-terminals.

type 3: $\texttt{R} \rightarrow \texttt{t}$	regular grammars (DFA)
type 2: $\texttt{R} \rightarrow \alpha$	context free grammars (pushdown automata)
type 1: $\alpha\texttt{R}\beta \rightarrow \alpha\varphi\beta$	context sensitive grammars (linear bounded automata)
type 0: $\alpha \rightarrow \beta$	unrestricted grammars

22nd January 2020

DFA to LR(0) Parsing Table

From the LR(0) items, it's fairly straightforward to determine the ϵ-closures, therefore the DFA can be constructed directly (however it can also be constructed with subset construction from a NFA). Continuing on with the simple expression example from last lecture, we can construct the DFA with the following rules in each state;

state 0:

`E'` $\rightarrow$ • `E`

`E` $\rightarrow$ • `E + int`

`E` $\rightarrow$ • `int`

state 1:

`E'` $\rightarrow$ `E` •

`E` $\rightarrow$ `E` • `+ int`

state 2:

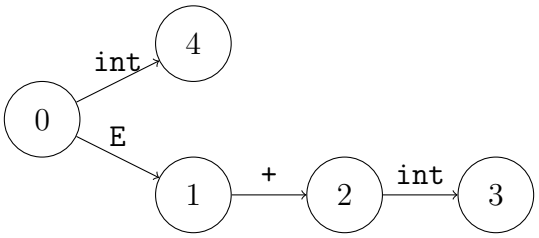
`E` $\rightarrow$ `E +` • `int`

state 3:

`E` $\rightarrow$ `E + int` •

state 4:

`E` $\rightarrow$ `int` •



Converting from this to a parsing table, we follow these rules;

- for each terminal transition `X` $\xrightarrow{\texttt{T}}$ `Y` add `P[X, T] = sY` (shift `Y`)

  "move from state `X` to state `Y`"
- for each non-terminal transition `X` $\xrightarrow{\texttt{N}}$ `Y` add `P[X, N] = gY` (goto `Y`)
- for each state `X` containing `R'` $\rightarrow \cdots$ • add `P[X, $] = a` (accept)

  this corresponds to the auxiliary rule, when we are done parsing
- for each state `X` which contains a `R` $\rightarrow \cdots$ • add `P[X, T] = rn` (reduce) for every terminal `T`, where `n` is `R`'s rule number

  the number does not correspond to a state, it instead corresponds to a rule number, by looking at the corresponding rule, we know how many items need to be popped off the stack and reduced

For the DFA above, we can represent it as the following LR(0) parsing table; Anything left blank is considered an error.

state	action			goto
	`int`	`+`	`$`	`E`
0	`s4`			`g1`
1		`s2`	`a`	
2	`s3`			
4	`r1`	`r1`	`r1`	
3	`r2`	`r2`	`r2`	

Model of an LR Parser

Assuming all the states are integers, we can approach the LR parser as follows;

1. push state 0 onto the stack
2. repeatedly decide what to do next depending on the top of the stack and the current character (`switch P[S[top], curr]`)

   - shift `n` — push the state `n` onto the stack, and go to the next token
   - reduce `n` — (complex)
     - remove `K` elements off the stack, where `K` is the length of the RHS of **rule** `n`
     - push `P[S[top], L]` where `L` is the LHS of **rule** `n`; this essentially looks at the state it was in before, and performs the "goto"
       in our example if we reduced to an `E`, state 0 is most likely on the top of the stack, and therefore it goes to state 1
     - generate an AST node with this data for the rule
   - accept
   - error — report an error
   - goto `n` — not directly selected, looked up in reduce case

FIRST and FOLLOW Sets

While it is possible to formally write out the algorithm for deriving the sets, it's easier to do it by intuition.

- FIRST set

  The FIRST set of a terminal is itself. However, the FIRST set for a rule is anything that can begin the derivation for such that particular rule. This set is constructed recursively, by looking through the FIRST sets of the first item of the LHS of each rule. However, note that because we are generating a set, we can stop when we encounter something we've already checked, as it wouldn't be added anyways. Let the rule be `R`, let `X` be an arbitrary rule, and let `t` be a terminal;

  - $\texttt{R} \to \texttt{t} \cdots$ — starts with a terminal
    add `t` to the FIRST set of `R`
  - $\texttt{R} \to \texttt{X} \cdots$ — starts with a non-terminal
    add the FIRST set of `X`, this can lead to recursion (which can be dealt with)

- FOLLOW set

  The FOLLOW set of a rule is something that can come after it. The intuition in this is to look at how it is used in the RHS of productions. On the RHS, there are three cases (or more, I could be wrong), let the rule be `R`, let `X`, `Y` be arbitrary rules, and let `t` be a terminal;

  - $\texttt{X} \to \cdots \texttt{R}$ — at the end of a rule
    add the FOLLOW set of `X`, as anything following `X` could follow `R`
  - $\texttt{X} \to \cdots \texttt{R t} \cdots$ — followed by a terminal
    add `t` to the FOLLOW set of `R`
  - $\texttt{X} \to \cdots \texttt{R Y} \cdots$ — followed by a non-terminal / rule
    add the FIRST set of `Y`, as anything that could start a derivation of `Y` could follow `X`

  Note that the same intuition on recursion applies. For example, in the first case, if $\texttt{R} = \texttt{X}$, then the FOLLOW set of `R` needs the FOLLOW set of `R`, but there's no point in doing this.

It's also important to consider the case where it can be ϵ.

24th January 2020

Weights

In the grammar, we've defined expressions to be right associative, hence it is a right growing tree. This requires more registers if we evaluate the left side first, as we need to retain this result for the next calculation. This can be shown in the example below, for the evaluation of $1+2+3$, with the left tree begin left associative $(1+2)+3$, and the right tree being right associative $1+(2+3)$ - note that the former uses 2 registers, whereas the latter uses 3.

The general idea for this is to first evaluate the subexpression that requires more registers. For example, let there be a binary operator with operands e_1 and e_2, needing L registers, and R registers respectively. If we choose to evaluate e_1 first, then we require L registers, and $R+1$ registers - this is because in the evaluation of e_2, we need to use R registers, as well as maintaining the result of e_1. Therefore, the cost (in registers) for evaluating e_1 first is $c_1 = \max\{L, R+1\}$. Under similar reasoning, the cost of evaluating e_2 first is $c_2 = \max\{L+1, R\}$. Therefore, assuming we always choose the optimal subtree in terms of registers, we have the overall cost of evaluating the binary operation is $\min\{c_1, c_2\}$. We can represent the weights as follows;

```
1  weight :: Exp -> Int
2  weight (Const _) = 1
3  weight (Ident _) = 1
4  weight (Binop _ e1 e2) = min c1 c2
5    where
6      c1 = max (weight e1) (weight e2) + 1
7      c2 = max (weight e1) + 1 (weight e2)
```

However, when we specify the target registers (naively) we have one of two issues;

- it stores the result of `e2` into `r`, and `e1` into `(r + 1)`, when `e2` is evaluated first, this leads to issues with operators that aren't commutative
- trying to fix the above, and storing `e2` into `(r + 1)`, then evaluating `e1` into `r` - the evaluation of `e1` may modify the contents of `(r + 1)`, hence overwriting the result of `e2`

The fix is to give the expression translation function a list of registers it's able to use. The result should be stored into the first register in that list.

```
1  transExp :: Exp -> [Register] -> [Instruction]
2  transExp (Const n) (dst:_) = [LoadImm dst n]
3  transExp (Ident x) (dst:_) = [LoadAbs dst x]
4  transExp (Binop op e1 e2) (dst:nxt:rest)
5    | weight e1 > weight e2 =
6        transExp e1 (dst:nxt:rest) ++
7        transExp e2 (nxt:rest) ++
8        transBinop op dst nxt
9    | otherwise =
10       transExp e1 (dst:nxt:rest) ++
11       transExp e2 (nxt:rest) ++
12       transBinop op dst nxt
```

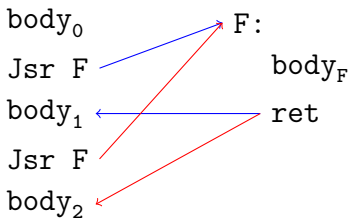
However, this can be further optimised by using more advanced instructions such as doing immediate addressing whenever possible. The translator can use pattern-matching to detect these cases.

The worst case for this is a perfectly balanced tree. This is because if it is unbalanced, then we can always choose a more optimal side. When there are k operators, and $k-1$ intermediate values, the number of registers is the depth of the tree ($\lceil \log_2(k) \rceil$). This means that with N registers, we can support expressions with up to 2^N operators.

Note that the algorithm above can lead to issues when working with side effecting expressions, as the order of execution is important. For example, `(x)+(x++)` is different from `(x++)+(x)`.

Function Calls

At the function call, registers may already be in use, and therefore those will need to be saved. Note that the address of the next instruction also has to be saved onto the stack, for the function to return to the correct place (notice how the control paths are coloured). Consider the example below, where $\texttt{body}_0$ requires the set of registers M, $\texttt{body}_1$ requires set N, and $\texttt{body}_\texttt{F}$ requires set P.



There are two conventions (and a hybrid of the two) for saving registers with regards to calls;

- **caller-saved**

  Before the instruction `Jsr F` is executed (after $\texttt{body}_0$), it saves the intersection of registers currently in use, and registers that the function will use ($M \cap P$). After the function returns, it restores ($M \cap P$).

- **callee-saved**

  Since the callee doesn't know which jump it's coming from, it has to save everything that **might** be in use. Before $\texttt{body}_\texttt{F}$ is executed, it saves the set of registers $(M \cup N) \cap P$, and then restores it before returning.

Optimising Compilers

While the Sethi-Ullman algorithm we followed is fast, and very easy to test, it's essentially a tree walk. Therefore it doesn't take into context what it's translating (other than the small optimisation we used to determine weights), Currently, in our implementation, we don't use registers to carry values between states, and only use them to store intermediate values for computation. An optimising compiler can have named variables in the register, with no reference to the main memory, therefore being much faster. Note that Sethi-Ullman is optimal in trees that have no shared subtrees.

Graph Colouring

An obvious optimisation of the code below;

```
1 a1 := b1 + s * k
2 a2 := b2 + s * k
```

would be compute the value of `s * k` into a temporary variable (stored in a register) and use it for both expressions;

```
1 t  := s * k
2 a1 := b1 + t
3 a2 := b2 + t
```

We need to consider all variables on equal terms (not just ones defined by the programmer) - including intermediate values during computation.

This can be achieved by doing the following;

(1) perform a simple tree walk to generate intermediate code where temporary values are saved in named locations (three address code - looks similar to assembly but with infinite registers)

(2) construct an interference graph, where the nodes are temporary locations, and an arc between two nodes represents an overlap in live ranges (if they must be stored simultaneously)

(3) colour the graph, with each register being its own colour - no connected nodes can have the same register

This is very straightforward in the case of straight line code, as we simply consider the live-in to be the first use of the value (when it is first declared), and the live-out to be the last use. Note that the LHS and RHS of an assignment does not count as a simultaneous use.

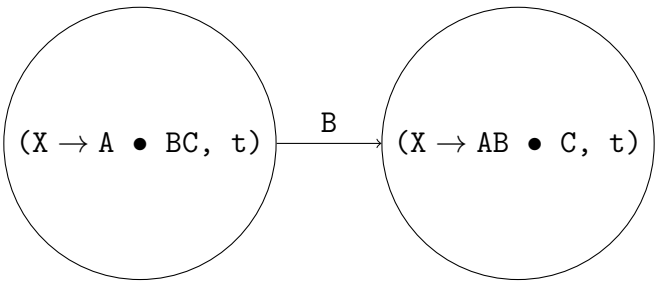
```
1 A := e1
2 B := e2
3 ...
4 ... B ...
5 C := A + B
6 ...
7 D := A * 5
8 ... D ...
9 ... C ...
```

This gives the following interference graph, and the result of the colouring;

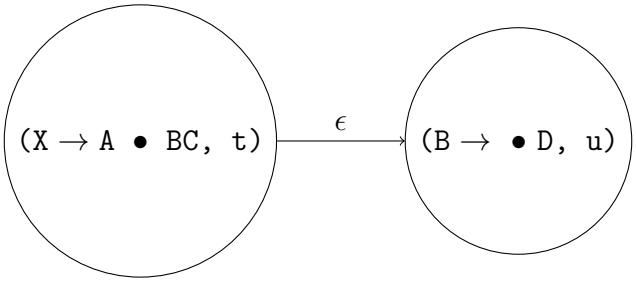
27th January 2020

LR(1) Items

An LR(1) item is a pair consisting of an LR(0) item, and a lookahead token `t`. The process for a terminal doesn't change much;



However, if `B` is a rule, then for each rule in the form `B` → `D`, we add an ϵ-transition for every token `u` in FIRST(`Ct`), we add a transition if and only if the string following `B` is derivable from `Ct`.



The auxiliary item `(X' → • X, $)` is also added to start the construction of the parsing table. The reduction is only done if the current token matches `t`. Consider the following grammar (not written with the vertical separation), and the states of the DFA;

`S' → S $`	
`S → id`	`r1`
`S → V = E`	`r2`
`V → id`	`r3`
`E → V`	`r4`
`E → int`	`r5`

state 0:

`S' → • S, $`	starting state [1]
`S → • id, $`	derived from non-terminal in (1) [2]
`S → • V = E, $`	derived from non-terminal in (1) [3]
`V → • id, =`	derived from non-terminal in (3) [4]

state 1:

`S' → S • , $`	shift from (1) [5]

state 2:

`S → id • , $`	shift from (2) [6]
`V → id • , =`	shift from (4) [7]

state 3:

`S → V • = E, $`	shift from (3) [8]

state 4:

`S → V = • E, $`	shift from (8) [9]
`E → • int, $`	derived from non-terminal in (9) [10]
`E → • V, $`	derived from non-terminal in (9) [11]
`V → • id, $`	derived from non-terminal in (11) [12]

state 5:

`S → V = E • , $`	shift from (8) [13]

state 6:

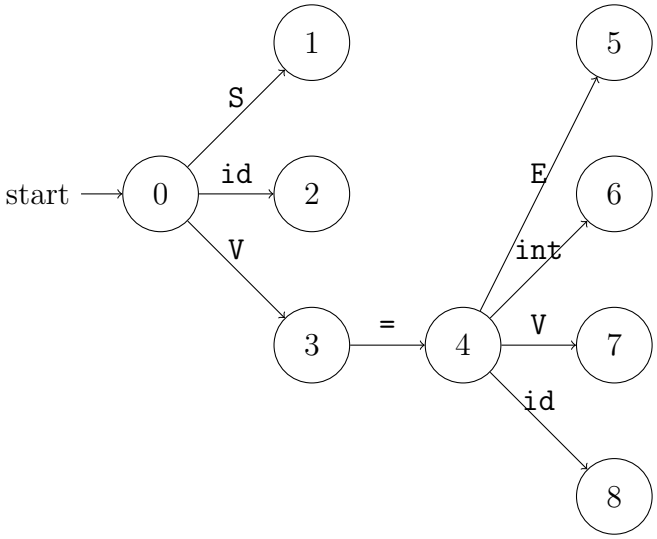
`E → int • , $`	shift from (10) [14]

state 7:

`E → V • , $`	shift from (11) [15]

state 8:

`V → id • , $`	shift from (12) [16]



This is represented in the following parsing table - note that the reduce state now only happens when it matches `t`;

state	action				goto		
	id	int	=	$	E	V	S
0	s2					g3	g1
1				a			
2			r3	r1			
3			s4				
4	s8	s6			g5	g7	
5				r2			
6				r5			
7				r4			
8				r3			

LALR(1)

In reality, LR(1) parsers don't really exist. We optimise LR(1) by merging LR(1) items that have the same LR(0) item, but different lookaheads into a single LALR(1) item that has the LR(0) item and the combined set of lookaheads. For example, we can combine these states;

[LR(1)] state 2:

X → E • T, =
V → id • , +

[LR(1)] state 3:

X → E • T, $
V → id • , /

[LALR(1)] state 23:

X → E • T, {=, $}
V → id • , {+, /}

Ambiguity and Conflicts

Ambiguity can be removed by rewriting the grammar, or adding additional rules. For example, we can get a shift-reduce conflict when we don't know whether to reduce an item or to continue, or we can get a reduce-reduce conflict, when it could reduce to either rule;

shift-reduce conflict

`A` $\rightarrow$ `id`	can be reduced
`A` $\rightarrow$ `id '[' Expr ']'`	can be shifted

reduce-reduce conflict

`Expr` $\rightarrow$ `id`
`Var` $\rightarrow$ `id`

Similar to lexing, we'd normally continue shifting in the first case, and in the second case we give precedence to the first rule declared.

Parse Trees

When we perform a shift, we are recognising an input and it builds a leaf node for the parse tree. On the other hand, when we perform a reduce, we take the leaf nodes, and put them into a parent node for the parse tree. However, this tree can have a lot of junk in it (thus being inconvenient to work with), and is either pruned in a separate pass to generate an abstract syntax tree (AST), or generated with instructions augmented onto the existing rules for the grammar to generate nodes as it is being parsed.

29th January 2020

LL Parsing

LL parsing is a natural way of hand writing a compiler, as we don't work with DFAs, but instead work with functions for each production. This also can easily produce AST nodes at the same time. A grammar is considered LL(k) if a k-token lookahead is sufficient to determine the next step (alternative to a rule).

Extended BNF

Due to the frequency of repeated and optional constructs in language syntax, many specifications have additional shortcuts for context free grammars;

- $\{\alpha\}$ zero or more occurrences of α
- $[\alpha]$ zero or one occurrences of α
- (α) grouping, useful for alternatives like $(\alpha \mid \beta \mid \gamma)$

Grammar to Functions

We assume there is some global variable `token` which holds the current token. It's also important to note that a rule `A` goes to a parsing function for that rule `A()`, and a token `T` goes to a match function `match(T)`, however in the code below, we will be using the function for rules (substitute `A()` for `match(A)` if we have a token instead);

```
1 "A B" => A(); B();
2 "A|B" => if   token in FIRST(A): A();
3          elif token in FIRST(B): B();
```

```
          # FIRST(A) and FIRST(B) must be disjoint
"{A}" => while token in FIRST(A): A();
          # FIRST(A) must be disjoint with what follows [A]
"[A]" => if token in FIRST(A): A();
          # FIRST(A) must be disjoint with what follows [A]
```

For example, when we work with the following grammar, we can create the functions as follows;

$$\begin{aligned}
\texttt{Statement} &\rightarrow \texttt{IfStatement} \mid \texttt{BeginStatement} \mid \texttt{PrintStatement} \\
\texttt{IfStatement} &\rightarrow \texttt{'if' Expr 'then' Statement ['else' Statement] 'fi'} \\
\texttt{BeginStatement} &\rightarrow \texttt{'begin' Statement \{';' Statement\} 'end'} \\
\texttt{PrintStatement} &\rightarrow \texttt{'print' Expr}
\end{aligned}$$

With the following code listing, for the structure of the AST, you can see it has a direct parallel with the grammar;

```
class StatementAST extends AST:
  ...
class IfAST extends StatementAST:
  ExprAST e
  StatementAST t, f
class BeginAST extends StatementAST:
  StatementAST[] statlist
class PrintAST extends StatementAST:
  ExprAST e
```

From here, we can also directly generate the AST.

```
def Statement():
  if token == IF:      return IfStatement()
  elif token == BEGIN: return BeginStatement()
  elif token == PRINT: return PrintStatement()
  else: error()
def IfStatement():
  match(IF)
  e = Expr()
  match(THEN)
  t = Statement()
  if token == ELSE:
    match(ELSE)
    f = Statement()
  else:
    f = None
  match(FI)
  return IfAST(e, t, f)
def BeginStatement():
  match(BEGIN)
  statlist = []
  statlist.append(Statement())
  while token == SEMICOLON:
    match(SEMICOLON)
    statlist.append(Statement())
  match(END)
  return BeginAST(statlist)
def PrintStatement():
```

```
28    match(PRINT)
29    return PrintAST(Expr())
```

However, it's obvious from these examples that it cannot deal with left recursion. If we had an additional rule that was in the form `Expr` → `Expr '+' Expr`, we'd end up recursing infinitely. Translation from a non-LL(1) grammar to a LL(1) grammar cannot be done fully automatically, and requires some ingenuity - it needs to ensure the transformed grammar has the same semantics. These three transformations are commonly used;

(1) left factorisation — usually carried out after the others

if two or more alternatives share a prefix, we can factor that out, such that (in EBNF) `A` → `B C` | `B D` can be factored out as `A` → `B (C | D)`, and `A` → `B | B C` would become `A` → `B [C]` (these would need auxiliary rules in BNF)

(2) substitution

if there is a non-terminal `B` on the right hand side of a rule, it may make sense to replace `B` with a group consisting of the alternatives for `B` - this can make the grammar clearer, and also easier to do left factorisation

(3) left recursion removal

issues with left recursion have already been discussed, the idea is to write a left recursive rule with right repetition, hence `A` → `X | A Y` should become `A` → `X { Y }`, or more generally;

$$\texttt{A} \rightarrow \texttt{X}_1 \mid \cdots \mid \texttt{X}_\texttt{m} \mid \texttt{A}\ \texttt{Y}_1 \mid \cdots \mid \texttt{A}\ \texttt{Y}_\texttt{n} \quad \Rightarrow$$
$$\texttt{A} \rightarrow (\texttt{X}_1 \mid \cdots \mid \texttt{X}_\texttt{m}) \mid \texttt{A}\ (\texttt{Y}_1 \mid \cdots \mid \texttt{Y}_\texttt{n}) \quad \Rightarrow$$
$$\texttt{A} \rightarrow (\texttt{X}_1 \mid \cdots \mid \texttt{X}_\texttt{m})\ \{\texttt{Y}_1 \mid \cdots \mid \texttt{Y}_\texttt{n}\}$$

This is harder to deal with when the left recursion isn't direct.

Error Recovery

Ideally, we want to be able to pick up as many (meaningful) errors as possible, and report them to the user - ensuring that the errors are accurate. To pick up multiple errors, we need a way of recovering from an error, and attempting to continue parsing.

One method of error recovery is to use the concept of a "syncset". If we don't encounter the token we were expecting, we can skip to a point where we do encounter it, or skip to the syncset of the higher level call. Generally, the syncsets grow as you go deeper into the AST, as there are more points it can attempt to "recover" to.

31st January 2020

When Colouring Fails

This continues on the last lecture in this part of the course, where we ended on graph colouring. In complex programs, we are likely to run into the case where colouring fails - when we simply don't have enough registers to use for a complex graph that has too many overlapping live ranges.

One method of tackling this is to split a live range, by spilling (saving) registers that aren't used onto the stack, and then restoring them when they are needed. Ideally, this makes the graph colourable, and doesn't hurt the performance of our compiled program too much. Consider these three examples;

(1) Even though `A` is used after the loop, there isn't any point in keeping it in a register, especially since the loop can be intensive, and require heavy use of registers.

```
A = ... # 'A' declared
for (...) {
    # high register pressure
    # 'A' not used
}
... A ... # 'A' used
```

(2) This highlights the idea that while `A` isn't used in an area of high register pressure, it still makes sense to keep it live at points - hence we don't simply choose a variable to live on the stack, but instead pick points where it is saved when not needed, and recovered before it is.

```
A = ... # 'A' declared
for (...) {
    ... A ... # 'A' used
}
for (...) {
    # high register pressure
    # 'A' not used
}
for (...) {
    ... A ... # 'A' used
}
```

(3) While the loop uses `A` within it, it isn't always needed as it is conditionally used, whereas there are other registers that would be needed unconditionally, therefore it doesn't make sense for `A` to be stored for the entire body of the loop.

```
A = ... # 'A' declared
for (...) {
    # high register pressure
    # 'A' not used
    if (...) {
        ... A ... # 'A' used
    }
}
```

Optimisations

It's important to note that a good optimisation needs a combination of high and low level principles. High level principles, such as inlining, are useful for picking up optimisations that are architecture-independent. Inlining replaces function calls with the body of the function, which removes the overhead required by performing the calls and returning, as well as opening up possibilities for more optimisations. For example, if we inline a function into a loop, and find loop invariant code, then that could be taken out (whereas it wouldn't have been possible with calls, as the function doesn't know where it's being called from).

On the other hand, low level principles are much more architecture dependent, such as instruction selection, and require a deeper understanding of the internals of the machine.

Peephole optimisation works at an instruction level, and is able to generate "smarter" code by removing pointless actions, for example;

```
peep :: [Instruction] -> [Instruction]
peep (Store r1 dst : Load r2 src : rest)
  | src == dst = Store r1 dst : (peep (Load r2 r1 : rest))
  | otherwise  = Store r1 dst : (peep (Load r2 src : rest))
-- etc
```

However, this also raises the question; what order should optimisations be done in? While all optimisations (hopefully) improve the code, it can also make it more complex - leading to difficulties when trying to apply another optimisation over it.

There exists an entire spectrum of optimisations;

- instruction level (peephole)
- expression level (Sethi-Ullman weights)
- local (basic blocks - instruction sequence with single entry and exit point, no jumps or labels)
  generally runs quickly, easy to validate
- global (looks at whole procedure)
  worse than linear complexity
- interprocedural (entire program)

Loop Optimisations

- **loop-invariant code motion** (an instruction is loop-invariant if its operands can only arrive from outside the loop)
- detection of induction variables (variables that increase or decrease by a loop-invariant constant on each iteration)
- strength reduction (calculate induction variables by addition instead of multiplying other induction variables)
- control variable selection (replace an unused loop control variable with a induction variable that's actually used)

Note that in the example below, we can see some of these rules;

```
1  int P(int N, int M) {
2      int i, u, v, w, x, y;
3      int z = 0;
4      for (i = 0; i < N; i++) {
5          w = w + 10;
6          x = w * 10;
7          y = z * (w - x);
8          u = w + x + y + N + M;
9          v = v + u
10     }
11     return v;
12 }
```

(1) `y` is constant (product of `0 * ???`) (line 7)

(2) `w - x` is dead code (multiplied by constant 0) (line 7)

(3) `y + N + M` is loop invariant (`y` is constant, `N`, `M` come from outside) (line 8)

(4) `i`, `w`. `x` are induction variables (`w + x` also is)

(5) `x` increases by 100 every iteration (strength reduction) (line 6)

(6) `i` is only used for loop control, another induction variable can be exit condition

Intermediate Code

As previously mentioned, we should generate intermediate code on the traversal of the AST, instead of going directly to assembly. The art of optimising compiler design lies in designing an IR satisfying the following;

- represent all primitive instructions for execution
- easy to analyse and manipulate
- independent of target architecture

Appel argues that the initial IR should be a tree representation before performing instruction selection, and a control flow graph afterwards. The IR should assume infinite registers, and these will be mapped to physical registers after optimisation and colouring.

Data Flow Analysis

The goal of data flow analysis is to deduce the program properties from the IR. It first analyses every instruction for its meaning, and then combines this information with the meaning of other instructions in order to derive a meaning for the program.

Control Flow Graph

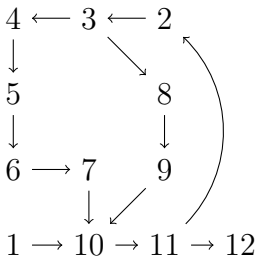
Consider the following source code, and its generated IR (with the node numbers on the left);

```
1  while (b < 10) {
2      if (b < a) {
3          a = a * 7;
4          b = a + 1;
5      } else {
6          a = b - 1;
7      }
8  }
```

```
1  [ 1]   bra L2
2       L1:
3  [ 2]   cmp b a
4  [ 3]   bge L3
5  [ 4]   mul #7 a
6  [ 5]   mov a b
7  [ 6]   add #1 b
8  [ 7]   bra L4
9       L3:
10 [ 8]   mov b a
11 [ 9]   sub #1 a
12      L4:
13      L2:
14 [10]   cmp b #10
15 [11]   blt L1
16 [12]
```

This leads to the following table and control flow graph;

id	uses (reads)	defs (updates)	successors
1			10
2	a, b		3
3			4, 8
4	a	a	5
5	a	b	6
6	b	b	7
7			10
8	b	a	9
9	a	a	10
10	b		11
11			12, 2



We have the following terms in live variable analysis;

- a **point** is any location between adjacent nodes
- a **path** is a sequence of points, where p_{i+1} is the immediate successor of p_i in the CFG
- saying x is live at p is to say that the value of x could be **used** along some path starting at p

For example, if we consider `b` after node 4, you will see that in node 5, the only successor to node 4, it is replaced with the value of `a`, and therefore "killed" - thus it will not reach any point after that.

This can be solved with a depth first search (for each variable, at every program point) - which is very expensive. It will need to be modified to detect cycles and stop, as well as detect when the value is updated, and halt the search there.

Let us define the following functions, where the difference between the two sets is **caused** by the instruction;

- LiveIn(n) is the set of temporaries live immediately before node n
  - it is live before n if it is live after node n (some later instruction reads it), unless n overwrites it
  - or it is used by node n (the instruction reads it)

$$\text{LiveIn}(n) = \text{uses}(n) \cup (\text{LiveOut}(n) - \text{defs}(n))$$

- LiveOut(n) is the set of temporaries live immediately after node n
  - a variable is live immediately after node n if it is live before any of n's successors

$$\text{LiveOut}(n) = \bigcup_{s \in \text{succ}(n)} \text{LiveIn}(s)$$

However, writing this out for each node generates a system of simultaneous equations.

3rd February 2020

This lecture starts with almost half an hour on using *ANTLR4*. For anyone reading this, please use parser combinators for WACC.

Semantic Analysis

Ideally, we want to check the legality of the program at compile time. However, there will be some checks that we cannot do, such as checking array bounds, and those checks will still need to be done at runtime.

The idea of semantic analysis is to take in an AST, and output errors (if any) or give back semantic attributes (such as type information) augmented to the AST, as well as a symbol table (identifier map). In many compilers, the goal is to do as much work as this stage as possible, allowing for much easier code generation.

The rest of this lecture was him discussing one slide. Literally just going through what needs to be checked for six lines of code (and it's quite obvious).

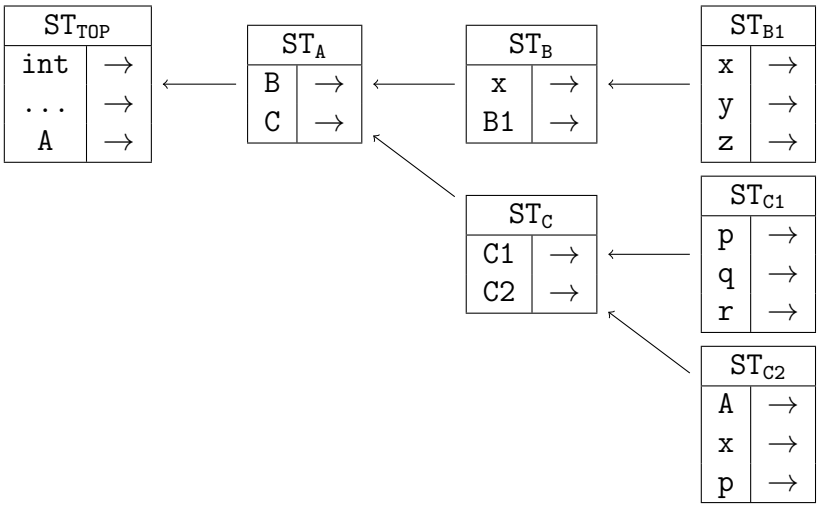
- in an assignment, check if the identifier is declared as a variable
- check that the RHS of the assignment can be assigned to the LHS

5th February 2020

Symbol Tables

Consider the symbol table as a dictionary or map (requires fast lookup) to hold data on identifiers. However it is represented, it should allow for scoping (thus allowing for redeclaration in an inner scope). One method for doing this is to have a global symbol table, with each entry holding auxiliary data on the scope. Another approach is to have a tree of symbol tables, with one for each lexical scope, pointing to the symbol table for the parent scope. The idea behind this is to first search in the local scope, then the parent scope, and so on, as it attempts to find the closest match for the identifier. A global scope is also added, holding predefined identifiers, such as functions, data types, and so on. When semantic analysis is done, there is also a global variable indicating the symbol table for the current lexical scope. For example, the code below has the following symbol table tree (note that the arrows ($\rightarrow$ in each row represents additional information about that identifier - this can be predefined, in the case of predefined identifiers, or may refer to AST nodes if it is programmer defined);

```
1  package A
2    class B
3      int x
4      method B1
5        object x, y, z
6    class C
7      method C1
8        object p, q, r
9      method C2
10       object A, x, p
```



In the example above, when we perform semantic analysis, we add `A` to the symbol table for the `TOP` scope, and then add auxiliary information into the entry, specifying that it is a package **and** points to the new symbol table for `A`'s scope. From here, when we see the class `B`, we add an entry into `A`'s symbol table, specifying that `B` is a class, and pointing to `B`'s symbol table. Note that the referencing to the parent symbol table is done as expected. In `B`, we see the integer declaration for `x`. This is added to the symbol table as a variable, and the type of the variable references the `int` type declared in the global scope.

In general, the symbol table is used by the AST nodes as follows;

- variable declaration

  In the case of a variable declaration AST node, given the name of the type, and the name of the variable, we can check if type exists in **all accessible** scopes, check if the variable exists in the **local** scope. The error cases are as follows;

  - identifier exists in the local scope (already declared)
  - type not found (unknown type)
  - type identifier found, but isn't a type
  - type found but cannot be declared

  Otherwise, add it to the current scope's symbol table, and give back this newly derived attribute to the AST.

- assignment

Note that in this case, we also want to recursively validate the RHS of the assignment (it also assumes that this augments the type of the RHS expression to the RHS' AST node). The error cases are as follows;

  - the identifier is not found (hence it hasn't been declared or isn't accessible)
  - the identifier is found, but isn't a variable
  - the identifier is found, and is a variable, but the types aren't compatible

  Otherwise, add the value in the symbol table to the AST node.

- function declaration

  This starts off similar to the variable declaration check, but also must perform additional checks due to the scoping. After performing the declaration checks (almost the same, assuming no overloading is allowed), it then creates a new symbol table for the new scope, it also adds the new symbol table to the function object augmented to the AST (and added into the current symbol table). It then validates each of its parameters (into the new symbol table), and links the new objects (the "formals") into the function object.

- function call

  On a function call, it must check if the function exists, and is a function (similar to how variable assignment checks all scopes). In terms of the parameters, it also checks that the number of parameters expected (formals), matches with the number of parameters supplied (actuals). Each of the supplied parameters are then validated, and then checked that the types match up; if everything is successful, then the function object is augmented onto the call AST node. This does **not** check the return type, that should be handled by the enclosing expression.

It's also important to note that, in practice, type checking should also allow for asymmetry (such as letting an integer be assigned to a double, but not the other way around). This can also include single inheritance (for example, if the LHS expected class `A`, but got class `B`, it should allow it if `A` is a superclass of `B`);

```
1 def compatible(TYPE lhs, TYPE rhs)
2   if lhs == rhs: return true
3   if rhs instanceof CLASS: return compatible(lhs, rhs.superclass)
4   ...
5   else: return false
```