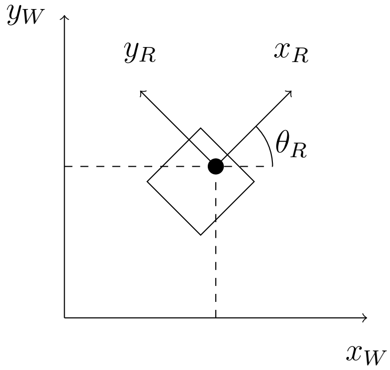
CO333 - Robotics (60019)

Lecture 1 - Introduction to Robotics

Lecture 2 - Robot Motion

A definition of a robot is something that can **move** and **sense**, and uses some sort of information processing to link the two. Robots might want to move in the water, fly in the air, walk (legged) on land, or work in space. The course will focus on wheeled robots that work on generally flat surfaces.

Coordinate Frames



We will be mostly focused on 2D coordinates, on the flat (close to planar) ground. The world frame W is anchored in the world, and the robot frame R is anchored to the robot (consider one point and orientation as the centre of the robot) - consider a set of axis carried by the robot. Often we are interested in the robot's location; the transformation between the world frame W and the robot frame R.

Degrees of Motion Freedom

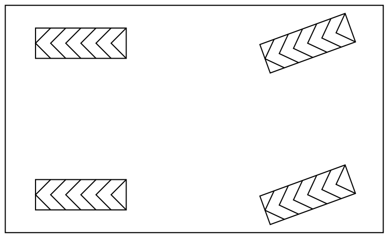
This is to do with how many parameters we need to specify the aforementioned transformation, generally related to the number of dimensions the robot is moving in. The simplest is a single degree of freedom, where a train moves along the x-axis - this position can be specified in one parameter. A rigid body which moves on a ground plane, such as an AV or robot vacuum cleaner has 3 DoF; two translational (x, y) and one rotational (typically θ). On the other hand, a rigid body which moves in 3D space has 6 degrees of freedom; three rotational and three rotational.

A **holonomic robot** is able to move instantaneously in any direction in its space of DoF, otherwise it is **non-holonomic**. Most are non-holonomic, but some holonomic robots do exist; ground-based robots can be made with omnidirectional wheels.

Standard wheel configurations (both non-holonomic; each has two motors but has three degrees of movement freedom, the number of control inputs are lower than the DoF) include;

- **drive and steer** (car) - not implemented in this course

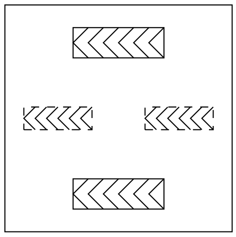
  The combination of acceleration and braking determines how fast it moves forwards, and the orientation of wheel determines direction.



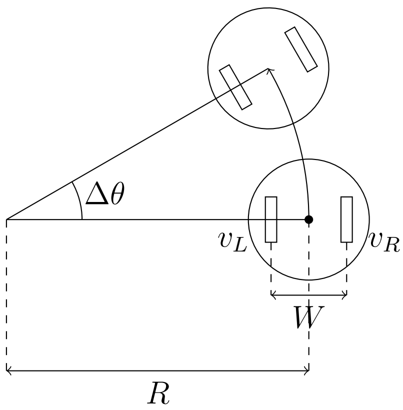
Rear wheels need a differential, variable (Ackerman) linkage for steering wheels.

- **differential drive** (robot vacuum)

  Has two driving wheels, both pointing forwards, and maybe castors which keep it balanced. Robot moves in different ways depending on the different speeds the wheels are turning at.



The caster wheels (dashed) are passive, and simply support the robot. The active wheels have one motor each. If the active wheels are running at equal speeds, the robot moves in a straight line, and wheels running at equal and opposite speeds turn on the spot. On the other hand, in general, other combinations lead to motion in circular arcs / curves.



Consider the left wheel at speed v_L and the right wheel at v_R (linear velocities over th ground, hence $v_L = r_L\omega_L$, where r_L is the radius of the wheel and ω_L is the angular velocity). We also assume no slipping (the intersection is the centre of rotation).

We want to determine R, which is the radius of the circle formed by the robot's centre moving. Consider a small period of time Δt, and an angle of movement $\Delta\theta$;

$$\Delta\theta = \frac{v_L \Delta t}{R - \frac{W}{2}} = \frac{v_R \Delta t}{R + \frac{W}{2}} \Rightarrow v_L \left(R + \frac{W}{2}\right) = v_R \left(R - \frac{W}{2}\right) \Rightarrow \frac{W}{2}(v_L + v_R) = R(v_R - v_L)$$

By rearranging the above, and substituting back in, we have the following;

$$R = \frac{W(v_R + v_L)}{2(v_R - v_L)}$$

$$\Delta\theta = \frac{(v_R - v_L)\Delta t}{W}$$

Actuation (DC Motors)

DC motors are controlled by a power signal, using **PWM (pulse width modulation)** and a fixed voltage. A gearing system is typically used to **gear down** the end effector to be slower with higher torque (compared to the DC motor's rapid rotation, but low torque). Many motors have a built in encoders (connected to the motor), which can be read to measure angular position by counting steps. This is required as a running motor's rotations depend on many conditions (including the load it's driving); for example a heavier robot moving on rough ground will move slower. To measure the rate the motor is moving at, we can feed the rotation back. We can then use feedback control (servo control) to adjust the motor to make it do what we want. In principle, we want to determine where the motor is, and where it actually is. At a high rate, we want to send pulses to reduce the error between the target location and actual location.

PID (Proportional / Integral / Differential) Control

The PID expression sets the power as a function of error;

$$P(t) = k_p e(t) + k_i \int_0^t e(\tau)\,\mathrm{d}\tau + k_d \frac{\mathrm{d}e(t)}{\mathrm{d}t}$$

In this, we have the following terms;

- $e(t)$ — demand minus position (error)

  For example, at each time step this is the angle we want the motor to be at minus the actual angle of the motor measured.
- k_p — main term (high values give rapid response)

  Continuing with the example above, $k_p e(t)$ will be higher if k_p is large. If there's a large error, send a large signal, and vice versa for small errors.
- k_i — integral term (increased to reduce steady state error)

  Imagine the motor is close to where it should be; this allows for the 'gap' to be closed.
- k_d — differential term (reduced settling time / oscillation)

Wheel Rotation Speed to Velocity

Consider a wheel of radius r_w rotating at ω (in radians per second). The speed of the wheel, in theory, would be $v = r_w\omega$. However, in practice, there are factors such as uneven ground and tyre softness, the radius may be hard to measure accurately. Another consideration would be possible slipping; in general it will be better to **calibrate** this for the surface by considering some constant of proportionality.

State in 2D

If a robot is moving on a plane, we can define the robot with a state vector $\boldsymbol{x}$, with three parameters;

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \begin{array}{l} x \text{ component of robot centre point in world frame} \\ y \text{ component of robot centre point in world frame} \\ \text{rotation angle between coordinate frames (angle between } x_W \text{ and } x_R \text{ axes)} \end{array}$$

The two frames coincide when the robot is at the origin ($x = y = \theta = 0$). Note that $-\pi < \theta \leq \pi$. During a straight line period of motion of distance D, we have;

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x + D\cos\theta \\ y + D\sin\theta \\ \theta \end{bmatrix}$$

On the other hand, during a pure rotation of angle α we have (rotation to the left is positive by convention);

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta + \alpha \end{bmatrix}$$

Lecture 3 - Sensors (Behaviours)

Lecture 4 - Probabilistic Robotics

Lecture 5 - Monte Carlo Localisation

Lecture 6 - Advanced Sonar Sensing

Lecture 7 - SLAM