CO142 - Discrete Structures

Prelude

The content discussed here is part of CO142 - Discrete Structures (Computing MEng); taught by Steffen van Bakel, in Imperial College London during the academic year 2018/19. The notes are written for my personal use, and have no guarantee of being correct (although I hope it is, for my own sake). This should be used in conjunction with the (extremely detailed) notes.

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Recommended Books

- K.H. Rosen. *Discrete Mathematics and its Applications*
- J.L. Gersting. *Mathematical Structures for Computer Science*
- J.K. Truss. *Discrete Mathematics for Computer Science*
- R. Johsonbaugh. *Discrete Mathematics*
- C. Schumacher. *Fundamental Notions of Abstract Mathematics*

However, these books don't cover the same content. Learn his notation.

Logical Formula, and Notation

This notation will be shared with **CO140**.

$A \wedge B$	A and B both hold
$A \vee B$	A or B holds (or both)
$\neg A$	A does not hold
$A \Rightarrow B$	if A holds, then so does B
$A \Leftrightarrow B$	A holds if and only if B holds
$\forall x(A)$	the predicate A holds for all x
$\exists x(A)$	the predicate A holds for some x
$a \in A$	the object a is in the set A (a is an element of A)
$a \notin A$	the object a is not in the set A
$=_A$	tests whether two elements of A are the same

Sets

Sets are like data types in Haskell: Haskell data type declaration;

```
data Bool = False | True
```

`{false, true}`	set of boolean values
`[true, false, true, false]`	list of boolean values
`{false, true} = {true, false}`	set equality (note that order doesn't matter)

A set is a collection of objects from a pool of objects. Each object is an *element*, or a *member* of the set. A set *contains* its elements. Sets can be defined in the following ways;

$\{a_1, ..., a_2\}$	as a collection of n distinct elements
$\{x \in A \mid P(x)\}$	for all the elements in A, where P holds
$\{x \mid P(x)\}$	for all elements, where P holds (dangerous - Russel's paradox)

Use of "triangleq"

The use of $\triangleq$ is for "is defined by". Hence the empty set, $\varnothing \triangleq \{\}$. The difference between $\triangleq$ and $=$, is that the former cannot be proven, it is fact, whereas the latter takes work to prove.

Russel's paradox

Not everything we write as $\{x \mid P(x)\}$ is automatically a set. Assume $R = \{X \mid X \notin X\}$ is a set, the set of all sets which don't contain themselves. As R is a set, then $R \in R$, or $R \notin R$ (law of excluded middle), and thus we can do a case by case analysis.

- Assume $R \in R$. By the definition of R, it then follows that $R \notin R$ (if $R \in R$, then it doesn't satisfy the definition of R) - which is a contradiction.
- Assume $R \notin R$. It then follows that $R \in R$, as it follows the definition of R, hence it is another contradiction.

As both assumptions lead to contradictions, it's possible to write sets which aren't defined. We should only select from a set that we know is defined; $\{x \in A \mid P(x)\}$ - where A is a well-defined set.