

CO140 - Logic

Introduction

A logic system consists of 3 things:

1. Syntax - formal language used to express concepts
2. Semantics - meaning for the syntax
3. Proof theory - syntactic way of identifying valid statements of language

Considering the basic example in a program, we can then see the features;

```
if count > 0 and not found then
    decrement count;
    look for next entry;
end if
```

1. basic (**atomic**) statements (**propositions**) are either \top or \perp depending on circumstance;
 - i. `count > 0`
 - ii. `found`
2. **boolean operations**, such as **and**, **or**, **not**, etc. are used to build complex statements from **atomic propositions**
3. the final statement `count > 0 and not found` evaluates to either \top or \perp

Syntax

The formal language of logic consists of three ingredients;

1. Propositional atoms (propositional variables), evaluate to a truth value of either \top or \perp . These are represented with letters; $p, p', p_0, p_1, p_2, p_n, q, r, s, \dots$
2. Boolean connectives;
 - **and** is written as $p \wedge q$ p and q both hold
 - **or** is written as $p \vee q$ p or q holds (or both)
 - **not** is written as $\neg p$ p does not hold
 - **if-then / implies** is written as $p \rightarrow q$ if p holds, then so does q
 - **if-and-only-if** is written as $p \leftrightarrow q$ p holds if and only if q holds
 - **truth**, and **falsity** are written as \top , and \perp respectively. logical constants
3. Punctuation. Similar to arithmetic, the lack of brackets can make an expression ambiguous. For example, $p_0 \vee p_1 \wedge p_2$ can be read as either $(p_0 \vee p_1) \wedge p_2$ or $p_0 \vee (p_1 \wedge p_2)$, which are different. The latter is the correct interpretation due to binding conventions.

Formulas

Something is a **well-formed formula** only if it is built from the following rules;

1. a propositional atom $(p, p', p_0, p_1, p_2, p_n, q, r, s, \dots)$ is a propositional formula
2. \top , and \perp are both formulas
3. if A is a formula, then $(\neg A)$ is also a formula
4. if A , and B are both formulas, then $(A \wedge B), (A \vee B), (A \rightarrow B), (A \leftrightarrow B)$ are also formulas