TAIPEI TECH

國立臺北科技大學

電路學 Circuit Theory

Lecture 1

Basic Concepts of Circuit Analysis

Week 2, Fall 2019

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Contents

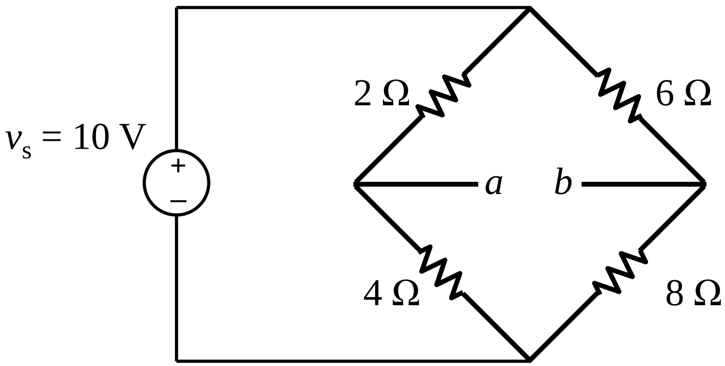
Lecture 1:
Basic Concepts of Circuit Analysis

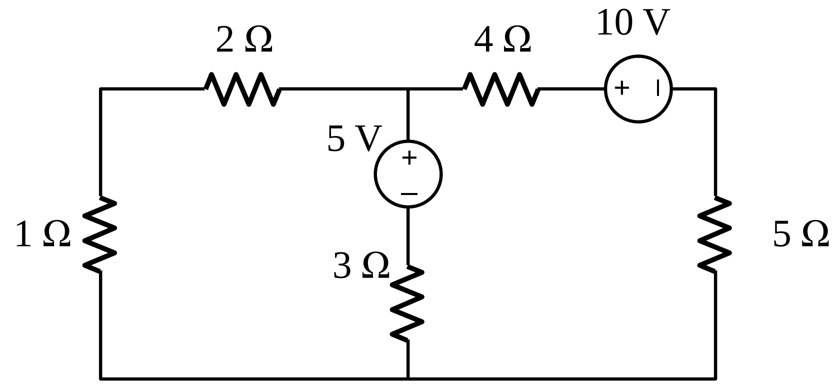
Contents

1.1 Circuit Variables

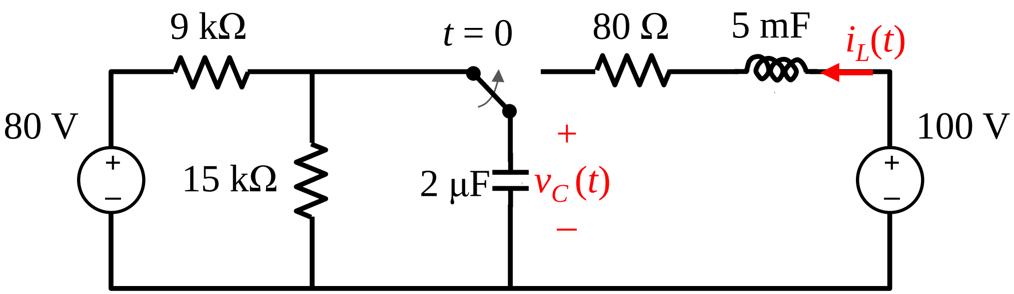
What Are We Going to Do in This Course?

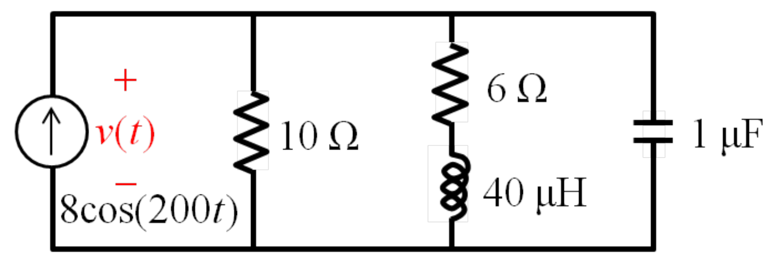
Midterm:





Final:





Give you a circuit and ask you to solve the voltage, current, and power at some components

Why do You Learn Circuit Theory?

The areas of EE:

- Communication systems
- Signal-processing systems
- Computer systems
- Control systems
- Power systems
- Electromagnetic devices
- Integrated circuits
- …

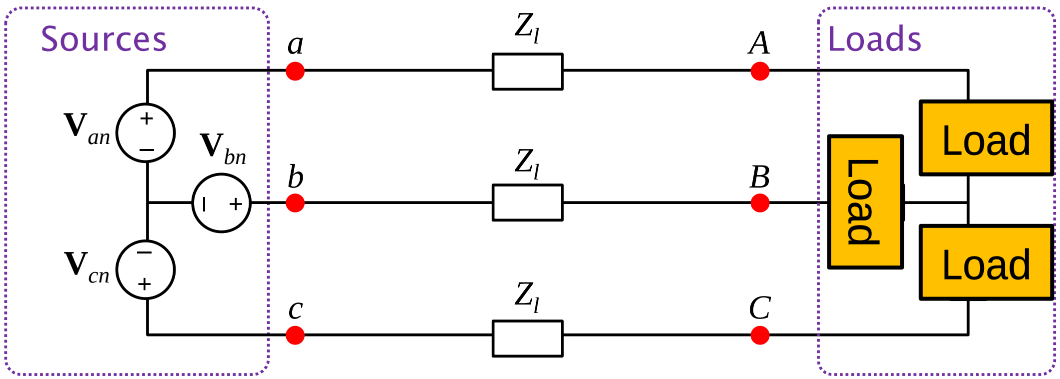
Whether all of these branches have anything in common?

YES!!

—Electric circuit theory!

Circuit Theory in EE

Power systems:

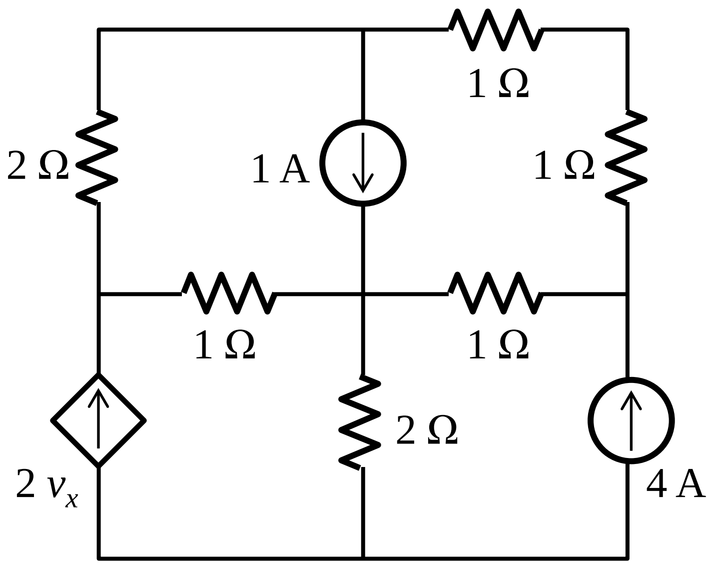


- A power distribution circuit
- The objective is to calculate the cost of consuming power on Load
- How?

Integrated circuits:

- The input source and the required output voltage are assigned
- The objective is to design a proper Intermediate circuit
- How?

The First Thing in This Course



What are we dealing with?

- We are going to solve a circuit
- In order to solve a problem, the problem must have some **unknowns**
- The unknowns = The **circuit variables**

The circuit variables in this course:

- Voltage
- Current
- Power
- Energy

We'd like to find these 4 quantities on circuit components. Why?

- Power systems: **Power ⟹ Revenue**
- Electronic analysis: **Voltage and current ⟹ Interesting responses**
- Communication systems: **Voltage and current ⟹ Signals**

Presumptions in Circuit Analysis

No Time Delay

- Time delay leads to transmission line theory
- What is a more practical model of “short circuit”?

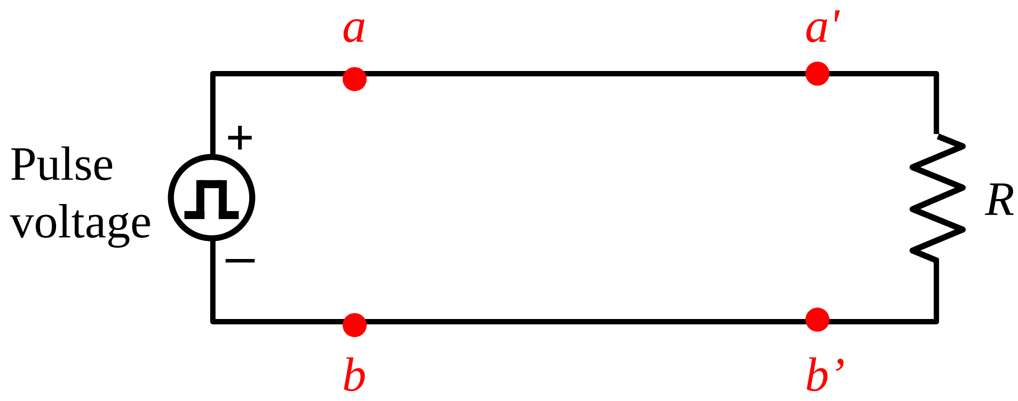
No Radiation

- Radiation loss
- Is radiation important for circuit analysis?

Linearity

- Linear components throughout this course
- Does a resistance remain constant in high-power scenarios?

Is the Assumption in *Circuit* Reasonable?



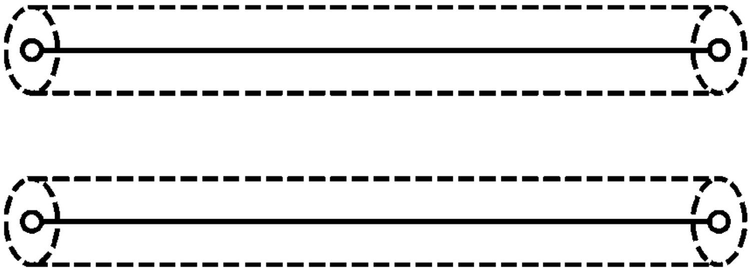
- The voltage source gives an impulse signal
- The impulse arrives at the load immediately

Is this assumption reasonable?

- Signal transmission has a number of delay
- But if the size of the circuity is ***small enough***, such a delay is omitted

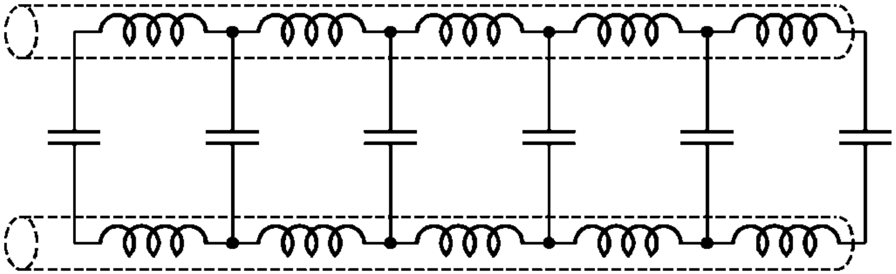
Equivalent Circuits of the Whole Line

Ideal: short circuit



- Voltage and current are equivalent everywhere

Real: capacitor and inductor



- Voltage and current are different everywhere

Charges on the conductors ∝ the potential difference $C = \frac{Q}{v}$

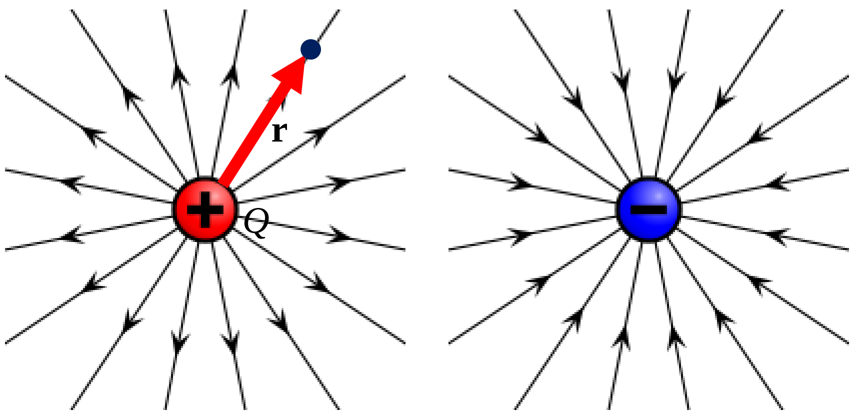
Magnetic flux around the loop ∝ the current on conductor $L = \frac{\phi_m}{i}$

$$\left.\begin{array}{l} C = \text{Shunt capacitance per unit length, in F/m} \\ L = \text{Series inductance per unit length in H/m} \end{array}\right\} \text{Distributed-parameter network}$$

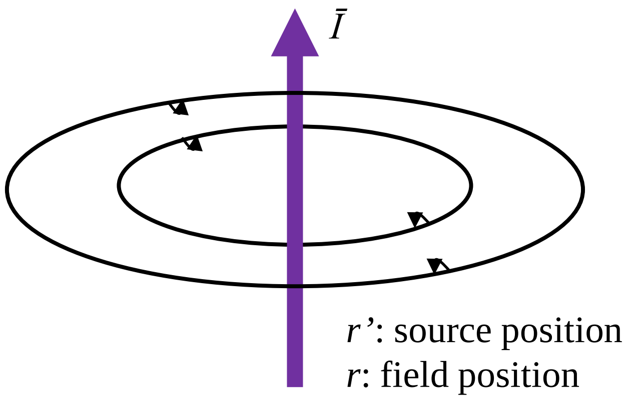
Static Electric and Magnetic Fields

The scenarios that **DO NOT** produce radiation:

Charges are not moving



Charges are moving with constant velocity



- Only producing static E-field

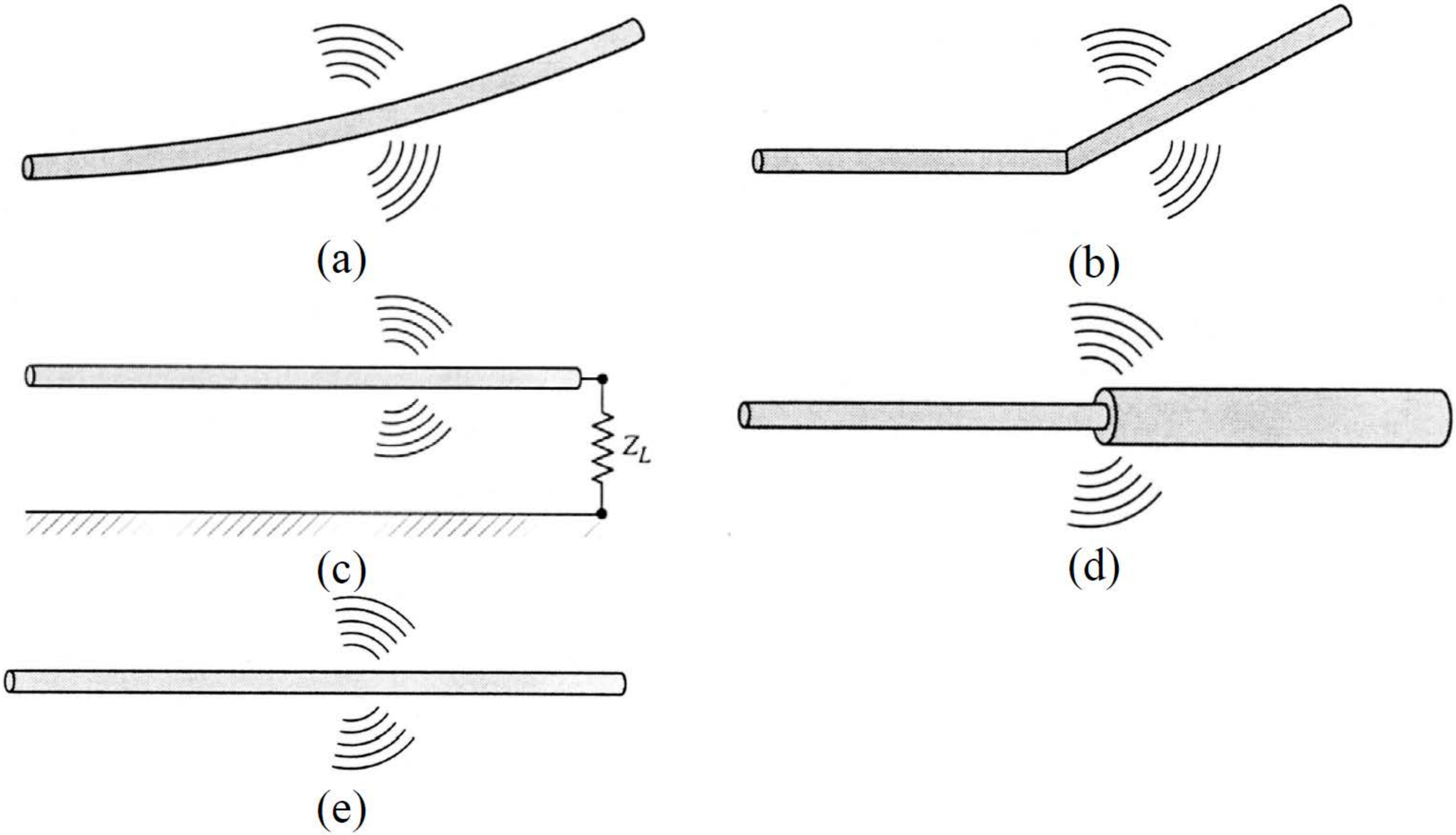
$$\mathbf{E}(r)=\frac{1}{4\pi\varepsilon_0}\frac{Q}{|r|^2}\hat{r}$$

- Only producing static magnetic field

$$\mathbf{B}(r)=\frac{\mu_0\bar{I}}{4\pi}\int\frac{r-r'}{|r-r'|^3}dl'$$

Wikipedia. [Online] Available: http://en.wikipedia.org/wiki/Electric_charge

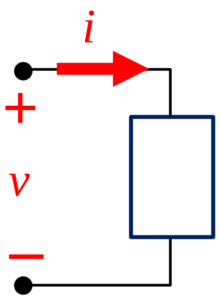
Radiation from Simple Sources



Current

- Definition: Electric current is the rate of charge flow
- SI unit: ampere (A)

$$i = \frac{dq}{dt}$$



i = the current in amperes

q = the charge in coulombs

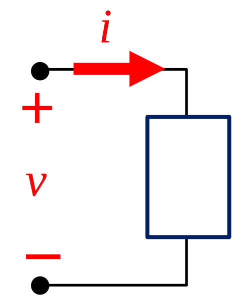
t = the time in seconds

- Current is the motion of charge creates an electric fluid
- Although current is made up of discrete electrons, we consider them as one smoothly flowing entity
- The current has a sign; it indicates the direction of current flow

Voltage

- Definition: Voltage is the energy per unit charge created by charge separation
- SI unit: volt (V)

$$v = \frac{dw}{dq}$$



v = the voltage in volts

w = the energy in joules

q = the charge in coulombs

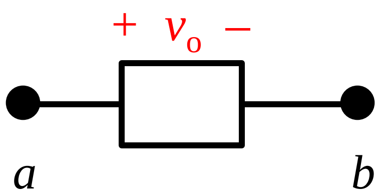
- It is the electric potential difference between two points

$$\Delta v = v_B - v_A$$

- The voltage has polarity; it implies the voltage drop or voltage rise

The Sign of Voltage and Current

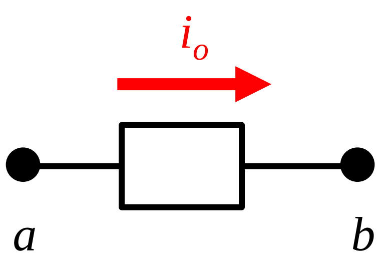
Voltage:



- If v_o is the unknown, and in your expressions somehow you find that $v_b - v_a = 3$ V
- How would you write down your answer?

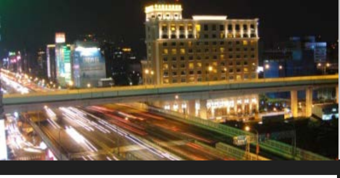
  $v_o = ?$

Current:



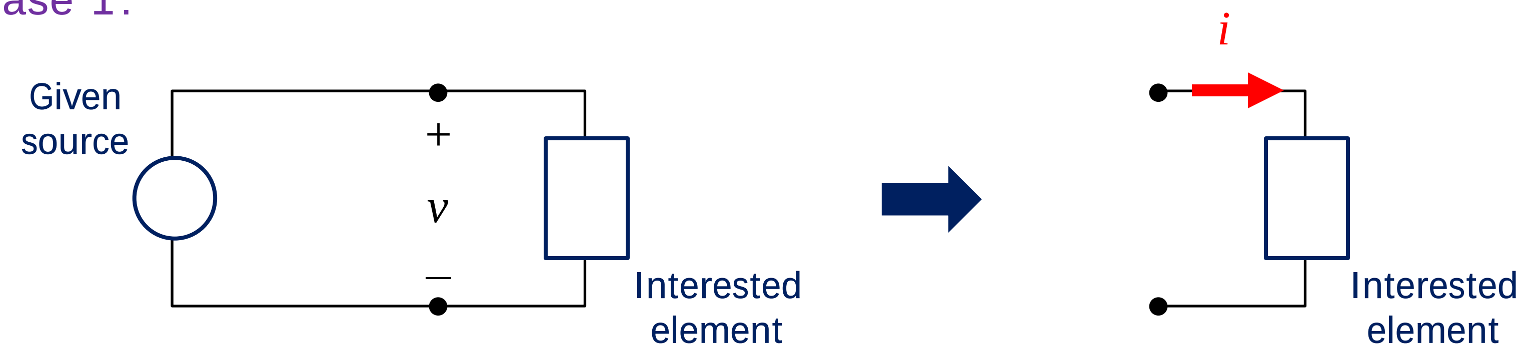
- If i_o is the unknown, and in your expression somehow you find that $i_{ba} = 2$ A
- How would you write down your answer?

  $i_o = ?$



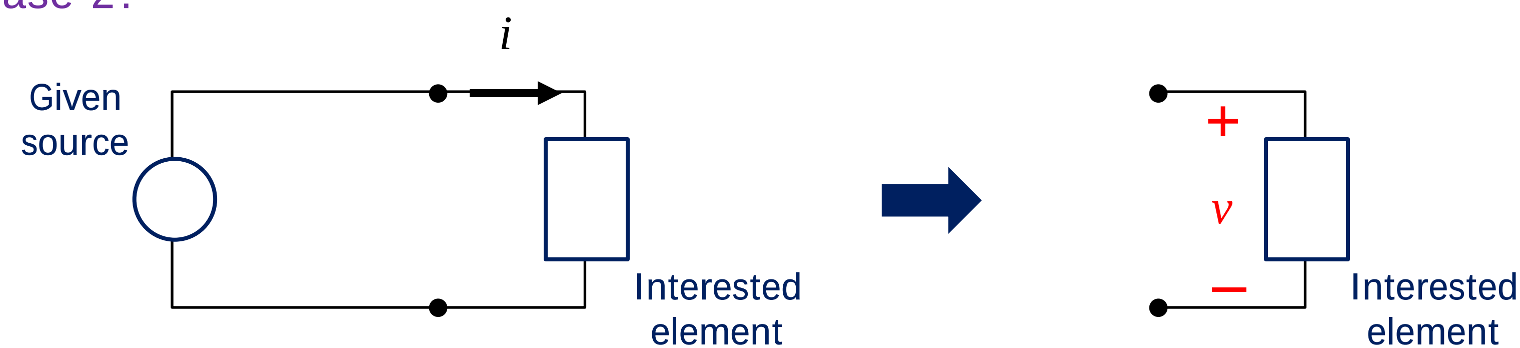
But When We Deal With Power…

Case 1:



If we choose voltage polarity reference… The current must go this way

Case 2:



If we choose current direction reference… The voltage must have such a polarity

Power

- Definition: from the notations of v and i:

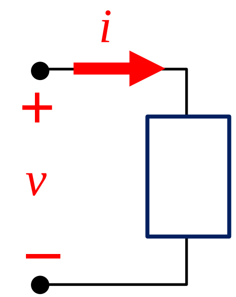
$$p = \frac{dw}{dt} = \frac{dw}{dq} \times \frac{dq}{dt} = vi$$

SI unit: watt (W)

p = the power in watts

v = the voltage in volts

i = the current in amperes

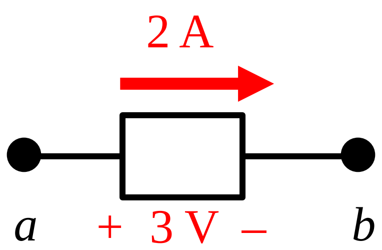


- Why do we calculate power?
  - It represents the “cost” in power systems
  - Practical devices have limitations on the amount of power that they can handle
  - The output capacity of an electrical system is often expressed in terms of power or energy

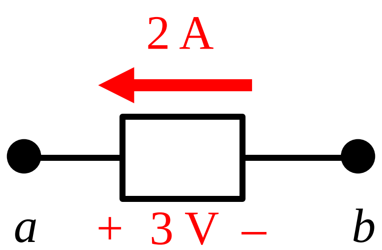
Passive Sign Convention

Passive sign convention:

Whenever the reference direction for the current in an element is in the direction of the reference voltage drop across the element



- The power = 3 × 2 = 6 W
- If the power $p = vi > 0$, then the power is dissipated in the component
- The component doesn't generate power, so we call it a passive component



- The power = 3 × (–2) = –6 W
- If the power $p = vi < 0$, then the power is extracted from the component
- Such a component do generate power, so we call it an active component

Energy

- Energy is calculated by integrating the associated power:

$$w = \int p \, dt$$

SI unit: Joule (J)

p = the power in watts

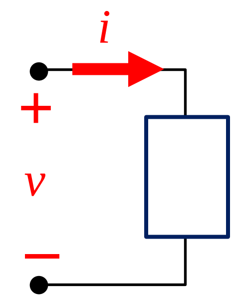
w = the energy in joules

t = the time in seconds

- With this definition, we can calculate the consuming energy of the passive component or the generating energy of the active component

Ex. 1.1 Calculation of Power and Energy

- Assume the element in the figure has terminal voltage and terminal current likes:



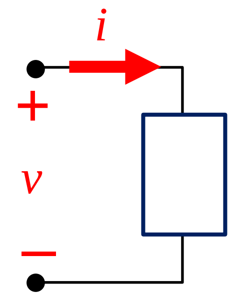
$$v = 0,\ i = 0, \qquad t < 0$$
$$v = 10e^{-5000t}\ \text{kV},\ i = 20e^{-5000t}\ \text{A}, \qquad t \geq 0$$

1. Calculate the power supplied to the element at 1 ms
2. Calculate the total energy (in J) delivered to the circuit element

Ex. 1.2 Calculation of Power and Energy

- The voltage and current at the terminals of the element are

$$v = 250\cos 800\pi t \text{ V}$$
$$i = 8\sin 800\pi t \text{ A}$$



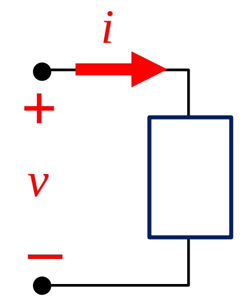
1. Find the maximum value of the power being delivered to the element
2. Find the maximum value of the power being extracted from the element
3. Find the average value of p in the interval $0 \le t \le 2.5$ ms
4. Find the average value of p in the interval $0 \le t \le 15.625$ ms

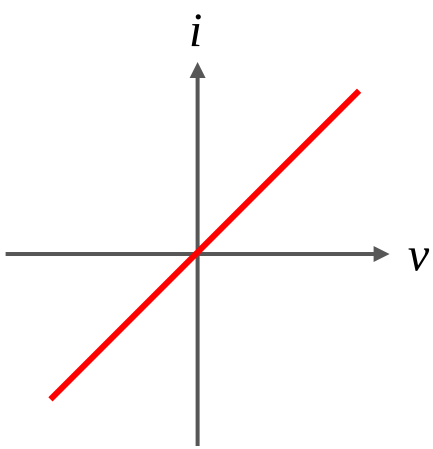
Contents

1.2 Circuit Components: Active Components

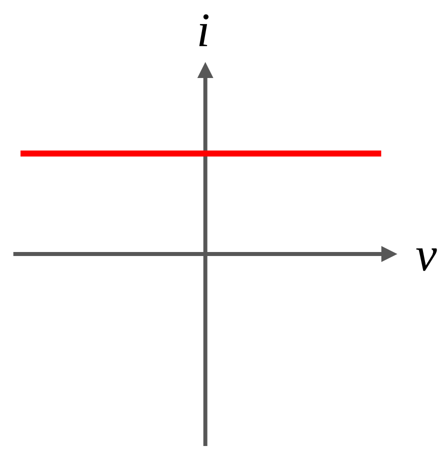
How to Describe a Component?

- A component is defined by the voltage and current **at its terminal**
- Even though two components are physically different, if they have the same relationship between terminal voltage and current, they are identical in circuit analysis

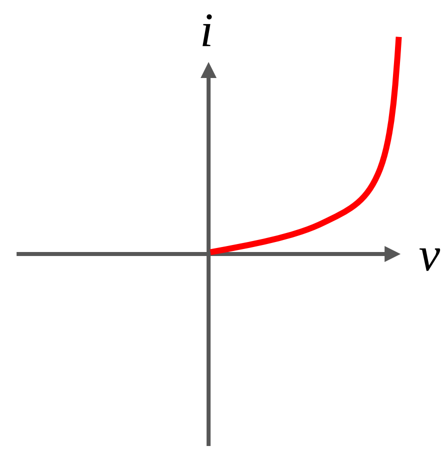




Component 1

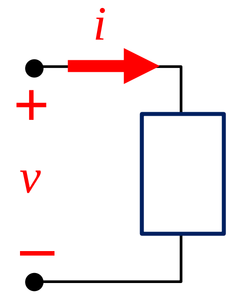


Component 2



Component 3

Ideal Basic Circuit Component



Definition of ideal basic circuit component:

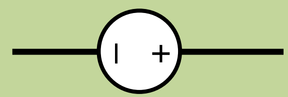
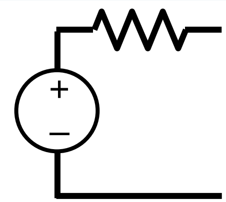
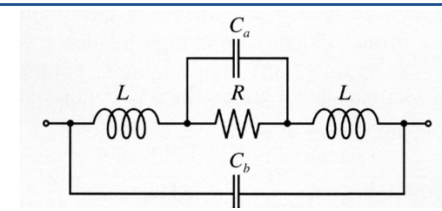
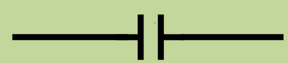
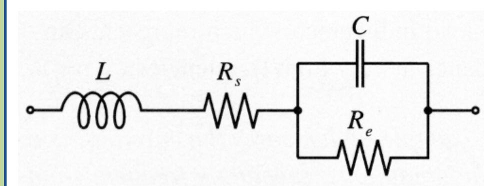
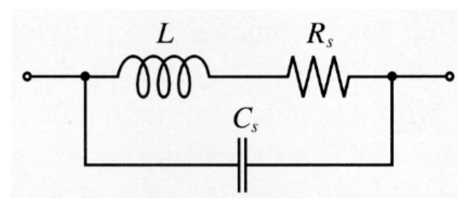
1. Only two terminals, which are points of connection to other circuit components
2. Described mathematically in terms of current and/or voltage
3. Cannot be subdivided into other elements

A component, or an element, will be named as a “branch” in the following lectures:

- They are ideal models; they do not exist in real world
- They are basic circuit elements; they construct circuit model, and they can’t be modeled with any other types of element

Mathematical Model of Circuit Component

All physical components are transformed into mathematical representation

	Physical world	Mathematical model	More realistic model
Power supply			
Resistor			
Capacitor			
Inductor			

Component Models

Circuit component

Active component:

- Independent voltage source
- Independent current source
- Dependent voltage source
- Dependent current source

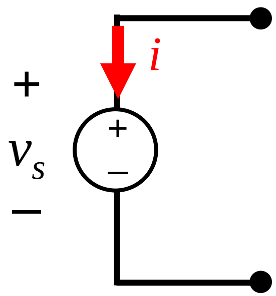
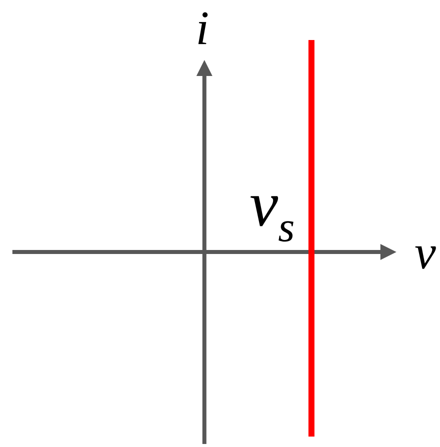
Passive component:

- Resistor
- Inductor
- Capacitor
- Mutual inductor

Component model:

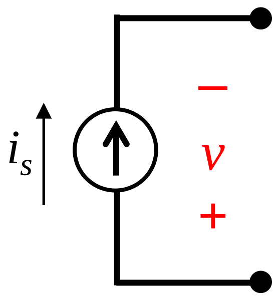
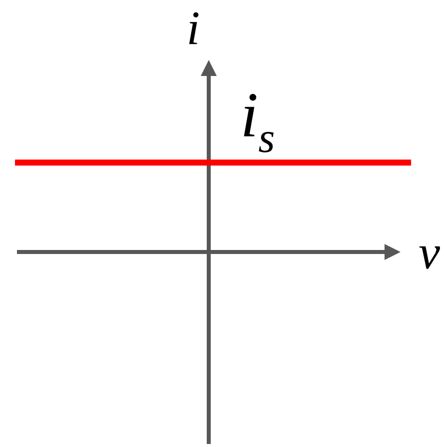
- Only one degree of freedom is set as the variable
- For example, if you set the voltage as the variable, then the current must be written as the function of the voltage
- All the active components and passive components have their own component models

Ideal Independent Voltage Source

Symbol	Model
	 $i = \infty$ $i = 10\ \mathrm{A}$ $i = 0\ \mathrm{A}$ $i = -10\ \mathrm{A}$ $i = -\infty$

- It maintains a prescribed voltage regardless of the current across the device
- In other words, the current is the function of the connected circuit
- What is the meaning that we turn off the independent voltage source?

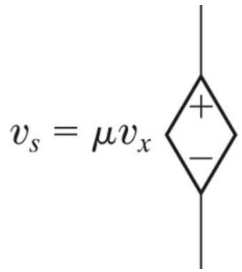
Ideal Independent Current Source

Symbol	Model
	 $v = \infty$ $v = 5\text{ V}$ $v = 0\text{ V}$ $v = -20\text{ V}$ $v = -\infty$

- It maintains a prescribed current regardless of the voltage across the device
- In other words, the voltage is the function of the connected circuit
- What is the meaning that we turn off the independent current source?

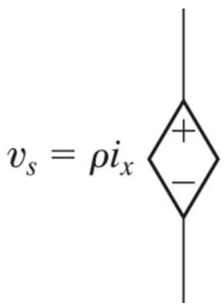
Ideal Dependent Voltage Source

Voltage-controlled voltage source (VCVS)



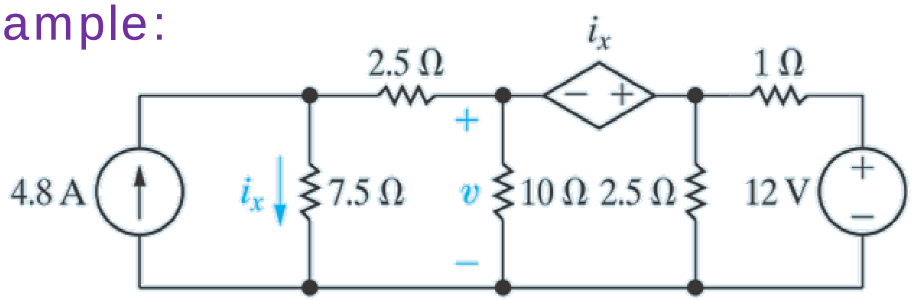
- v_x: controlling parameter;
  it comes from the voltage of other component
- Express v_x first
  → then we can express the source voltage
  → then we may calculate the source current

Current-controlled voltage source (CCVS)



- i_x: controlling parameter;
  it comes from the current of other component
- Express i_x first
  → then we can express the source voltage
  → then we may calculate the source current

Example:



Ideal Dependent Current Source

Voltage-controlled current source (VCCS)

$i_s = \alpha v_x$

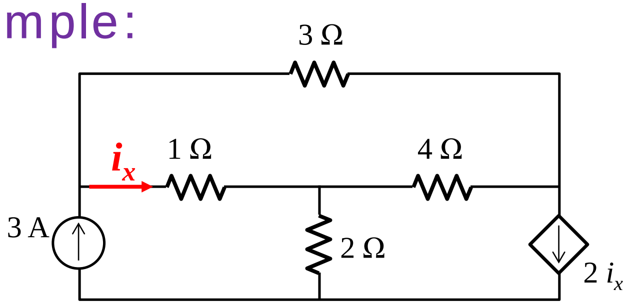
- v_x: controlling parameter;
  it comes from the voltage of other component
- Express v_x first
  → then we can express the source current
  → then we may calculate the source voltage

Current-controlled current source (CCCS)

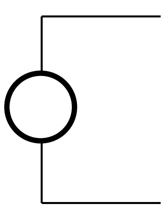
$i_s = \beta i_x$

- i_x: controlling parameter;
  it comes from the current of other component
- Express i_x first
  → then we can express the source current
  → then we may calculate the source voltage

Example:



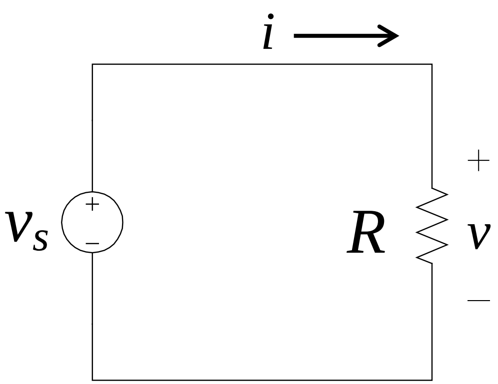
Dependent source



Independent source

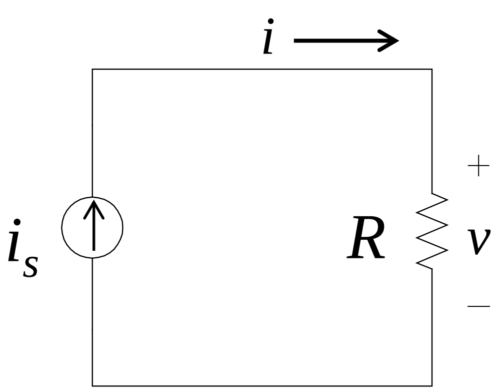
Another Viewpoint for "Ideal" Components

Ideal sources are convenient for modeling transistors,
<u>but they don't exist in reality</u>



$$If\ \ R \to 0,\ then\ i = \frac{v}{R} \to \infty,\ and\ \ v \to v_s,\ so\ p = vi \to \infty$$

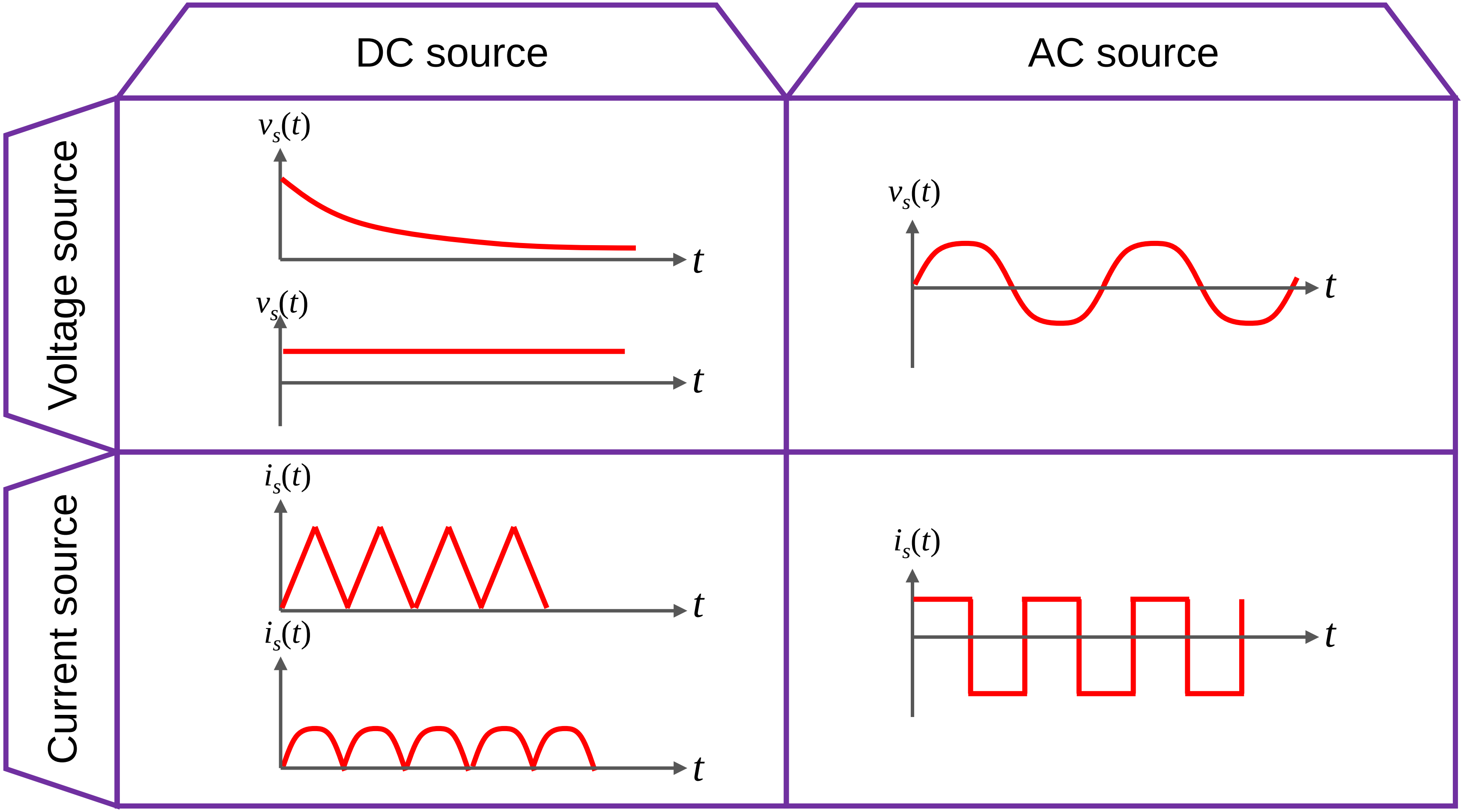
(But the power cannot be infinite!)



$$If\ \ R \to \infty\ then\ v = iR \to \infty,\ and\ \ i \to i_s,\ so\ p = vi \to \infty$$

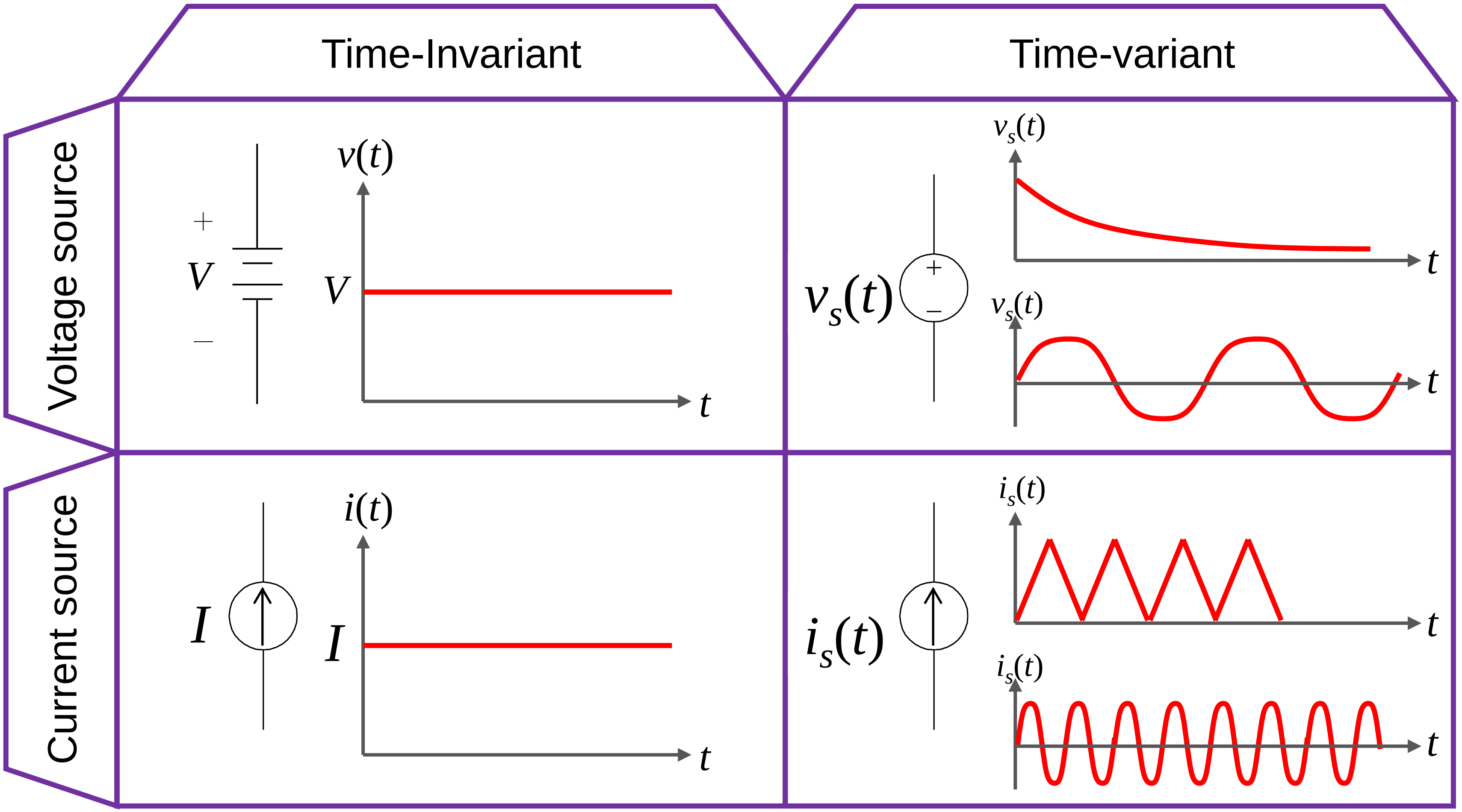
(But the power cannot be infinite!)

DC vs. AC



DC source
AC source
Voltage source
Current source
$v_s(t)$
t
$v_s(t)$
t
$v_s(t)$
t
$i_s(t)$
t
$i_s(t)$
t
$i_s(t)$
t

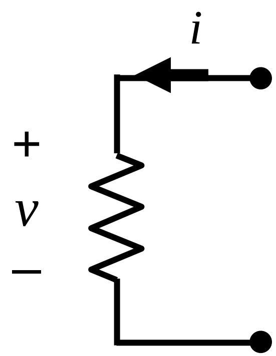
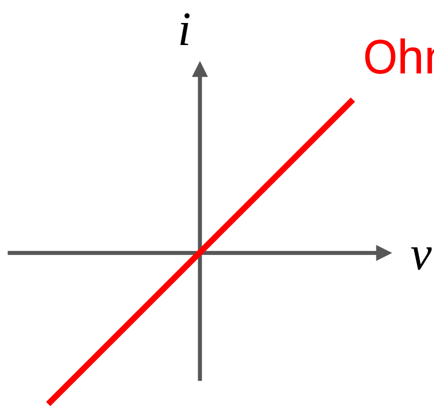
Time-Invariant vs. Time-Variant



Contents

1.3 Circuit Components: Passive Components

Resistor

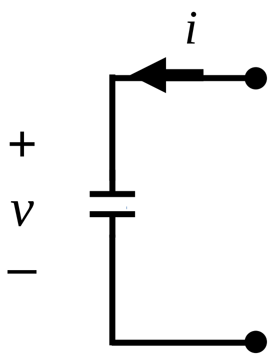
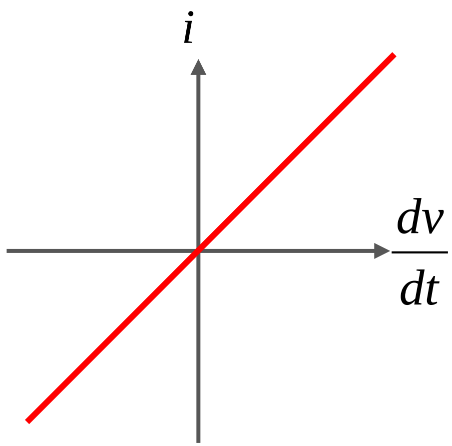
Symbol	Model
	

$$v = Ri$$
$$i = Gv$$

R: resistance (ohms, Ω)
G: conductance (siemens, S)

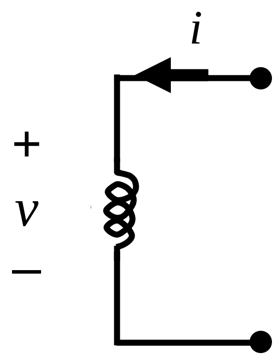
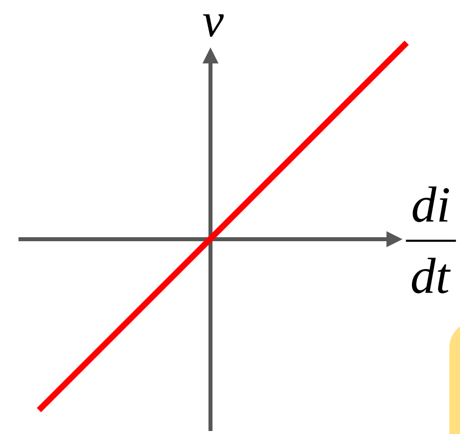
- Resistance: The capacity of materials to impede the flow of current
- Resistor: The circuit element used to model this behavior
- Why is resistor a passive component?

Capacitor

Symbol	Model
	 $i = C\dfrac{dv}{dt}$ C: capacitance (Farad, F)

- Capacitor: The electrical element which consists of two conductors separated by an insulator or dielectric material
- Capacitance: A linear circuit parameter that relates the current induced by a time-varying electric field to the voltage producing the field
- Why is capacitor a passive component?

Inductor

Symbol	Model
	

L: inductance (Henry, H)

$$v = L\frac{di}{dt}$$

- Inductor: The electrical element which is composed of a coil of wire wound around a supporting core
- Inductance: A linear circuit parameter that relates the voltage induced by a time-varying magnetic field to the current producing the field
- Why is inductor a passive component?

Contents

1.4 How to Solve a Circuit?

Three Laws in Circuit Analysis

Circuit analysis

KCL

- Nodal analysis (next lecture)

KVL

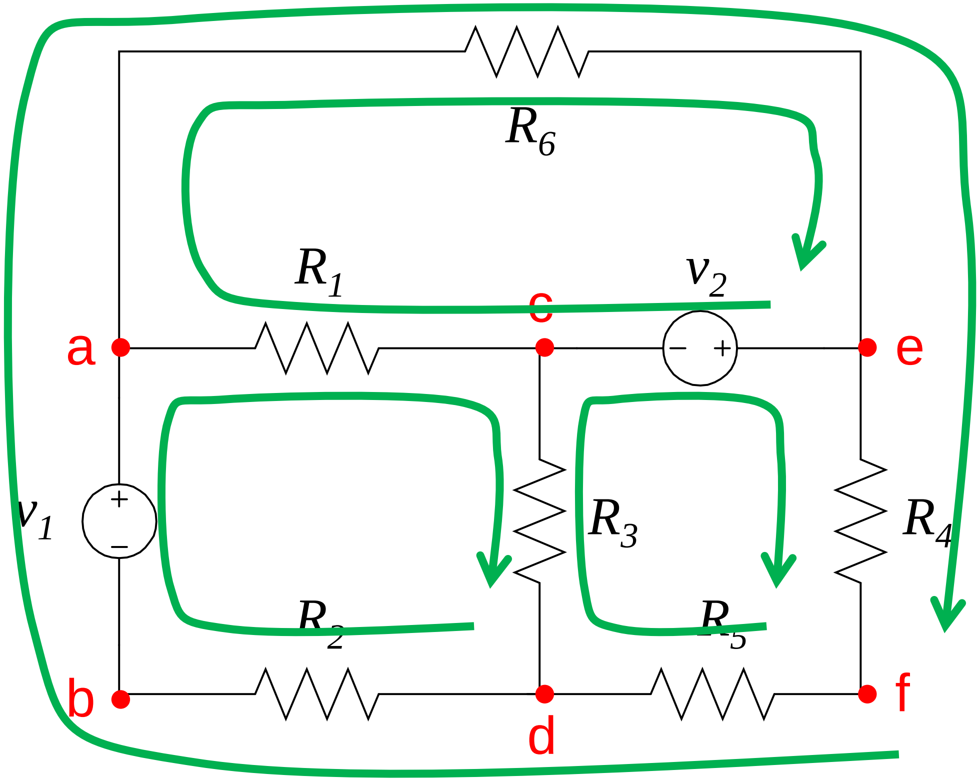
- Mesh analysis (next lecture)

Component models

- Voltage source
- Current source
- Resistor
- Inductor
- Capacitor

Node, Branch, Loop, and Mesh

- Node: A point where two or more components join
- Branch: A single two-terminal component or element
- Loop: A closed path in a circuit without passing through any intermediate node more than once
- Mesh: A loop that does not enclose any other loops



Node:

- a, b, c, d, e, f
- 6 nodes

Branch:

- $v_1, v_2, R_1, R_2, R_3, R_4, R_5, R_6$
- 8 branches

Loop:

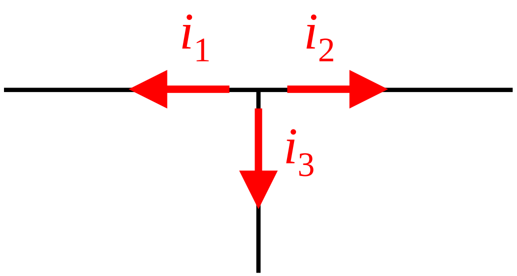
- bacdb, dcefd, ecae, baefdb
- There are many other loops, but only 3 meshes exist

Kirchhoff's Current Law (KCL)

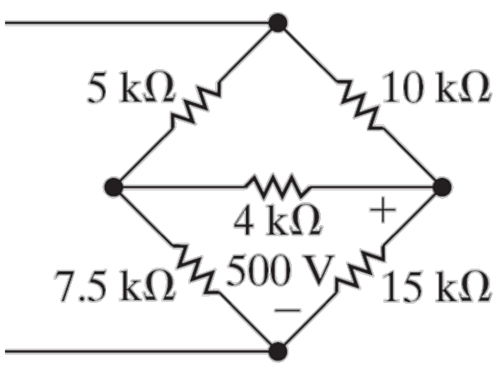
KCL:

- The algebraic sum of all the currents **leaving** any node in a circuit equals zero

$$\sum_{n=1}^{N} i_n = 0, \ \ for \ \ any \ \ node$$



N: number of elements connected to this node
i_n: the n^{th} element current leaving this node



- KCL is based on the law of conservation of charge
- KCL can be applied to a "super node"

J. Nilsson and S. Riedel, *Electric Circuits*, 9th Edition, Pearson, NJ, 2011

Other Forms of KCL

- The algebraic sum of all the current **entering** any node in a circuit equals zero

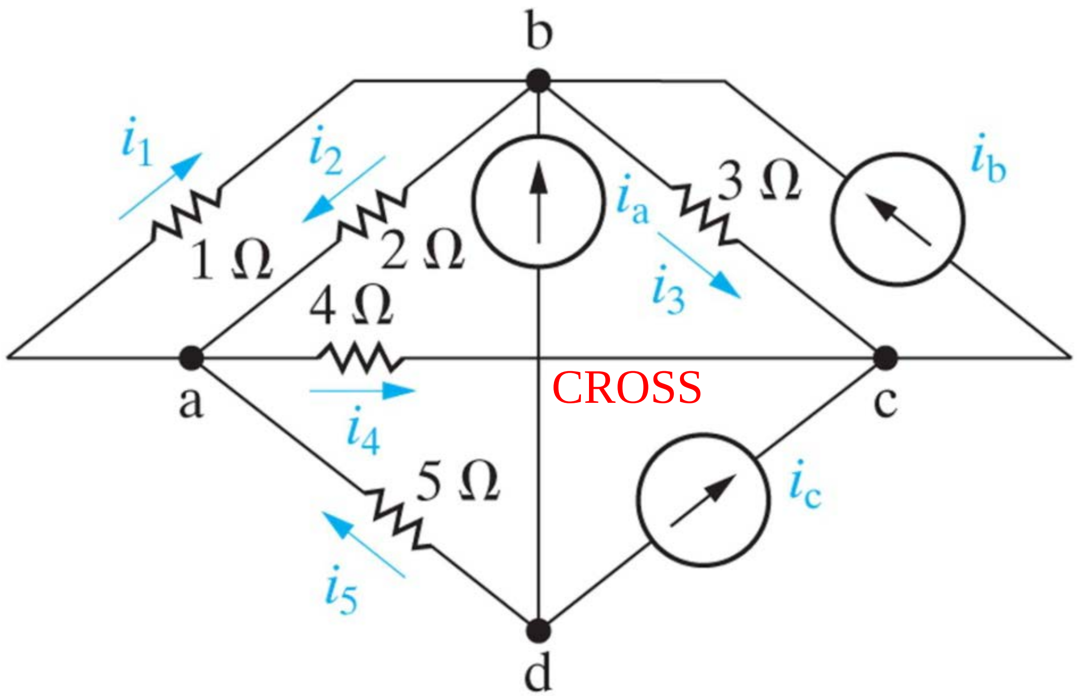
$$\sum_{n=1}^{N}\left(-i_n\right)=0,\ for\ any\ node$$

- The sum of all the current **entering** the node = The sum of all the current **leaving** the node

$$\sum_{L} i_l = \sum_{E}\left(-i_e\right),\ for\ any\ node$$

L : set of l with i_l leaving this node
E: set of e with i_e entering this node

Ex. 1.3 KCL



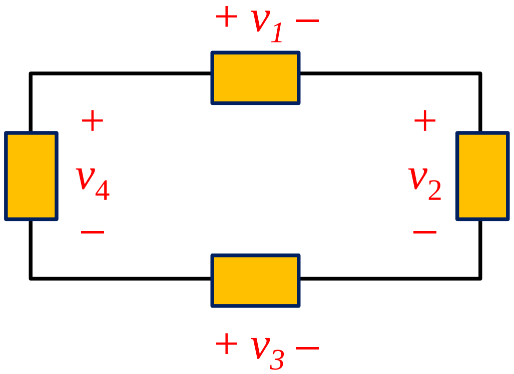
1. Write down the KCL equations at each node

Kirchhoff's Voltage Law (KVL)

KVL:

- The algebraic sum of all the element voltage **drops** around any loop in a circuit equals zero

$$\sum_{m=1}^{M} v_m = 0, \ for \ any \ loop$$



M: number of elements located in this loop
v_m: the voltage drops of the m^{th} element around the loop

- KVL is based on the law of conservation of energy

Other Forms of KVL

- The algebraic sum of all the element voltage **rises** around any loop in a circuit equals zero

$$\sum_{m=1}^{M} \left(-v_m\right) = 0, \; for \; any \; loop$$

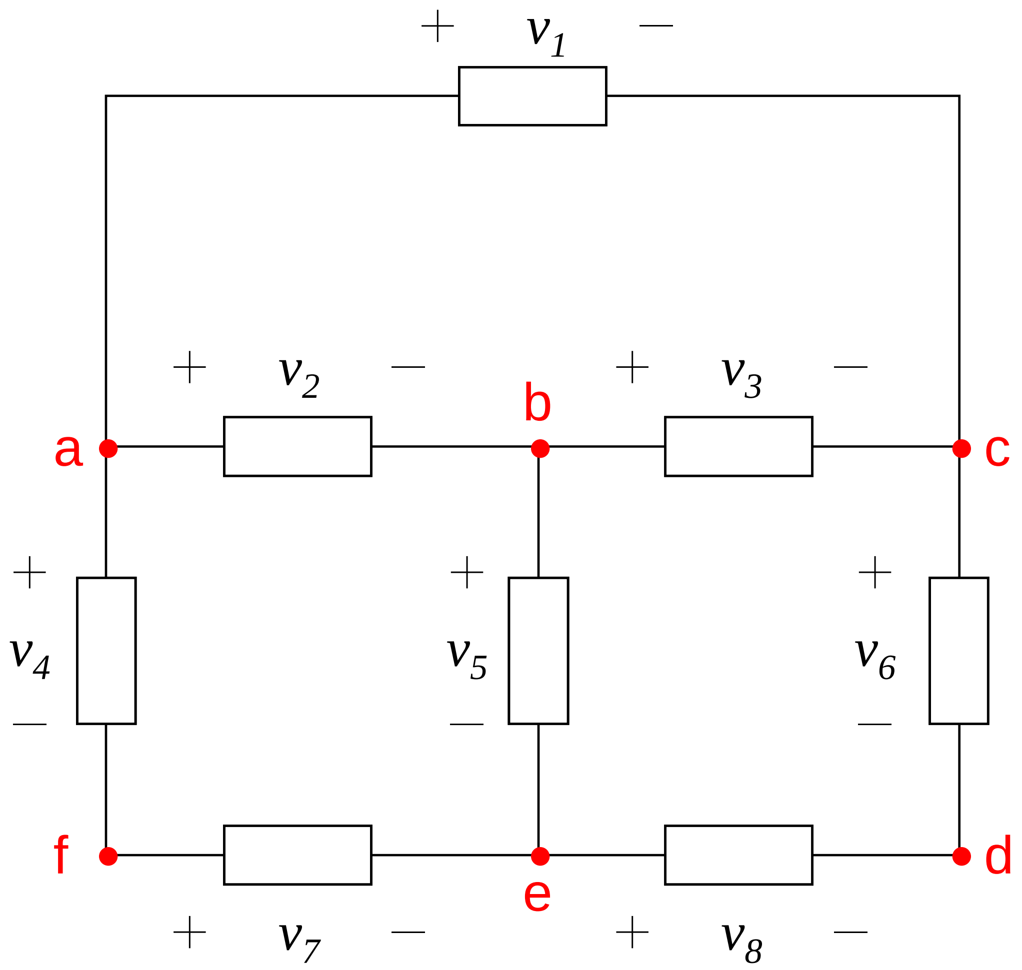
- The sum of all the voltage **drops** around the loop = The sum of all the voltage **rises** around the loop

$$\sum_{D} v_d = \sum_{R} \left(-v_r\right), \; for \; any \; loop$$

D : set of d with v_d drops around this loop
R: set of r with v_r rises around this loop

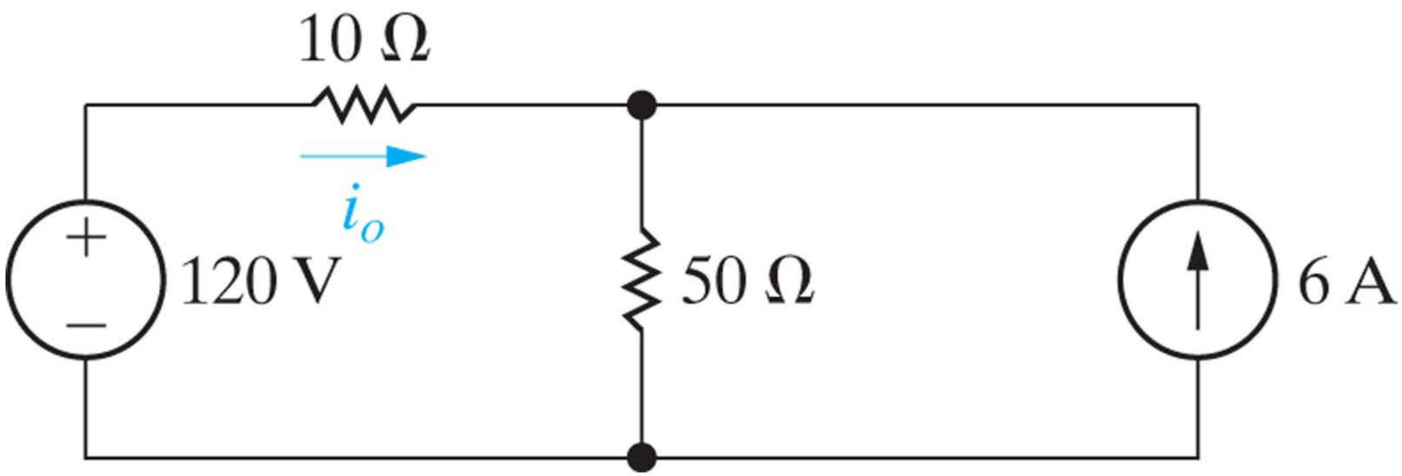
KVL

1. Write down the KVL around the loop a-b-c-d-e-f-a



Ex. 1.5

Basic Approach to Solve a Circuit



1. Use Kirchhoff's Laws and Ohm's Law to find i_0 in the above circuit
2. Test the solution for i_O by verifying that the total power generated equals the total power dissipated

Solving a Circuit More Systematically!

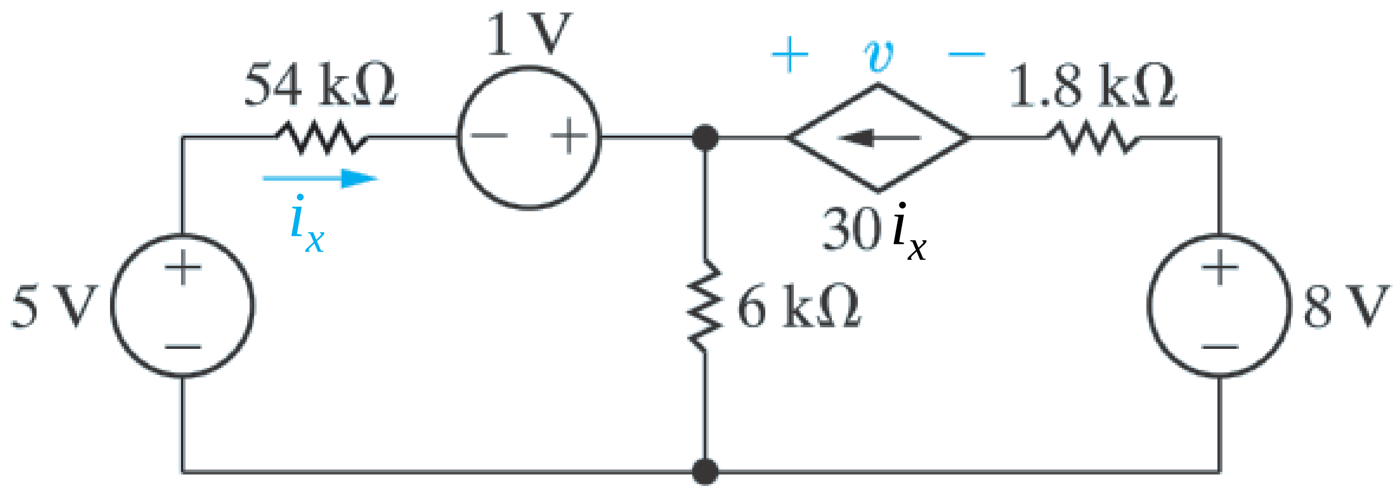
2B Method:

- It is a very systematic method
- More importantly, it is a programmable method

What are we interested in a given circuit?

- The number of unknowns: $2B$
- The number of equations:
  - The number of KCL equations: $N-1$
  - The number of KVL equations: $B-(N-1)$
  - The number of component model: B
  - So there are $2B$ equations for solving $2B$ unknowns!

Ex. 1.6 Basic Approach to Solve a Circuit

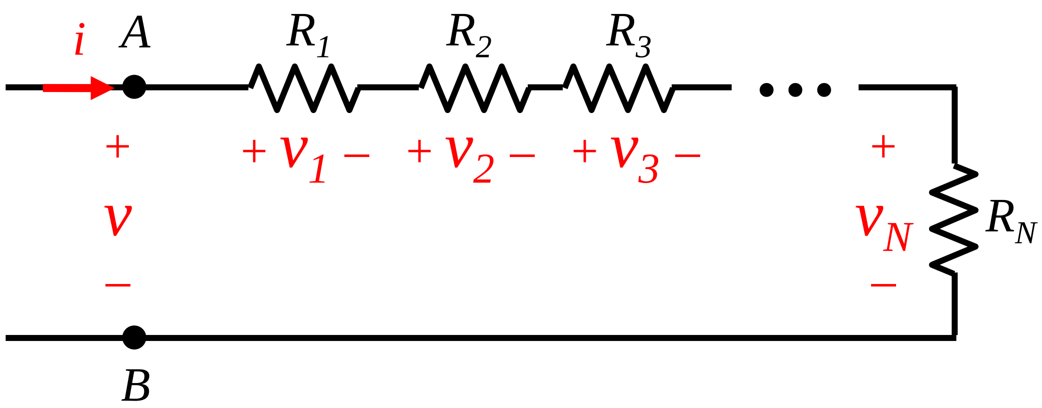


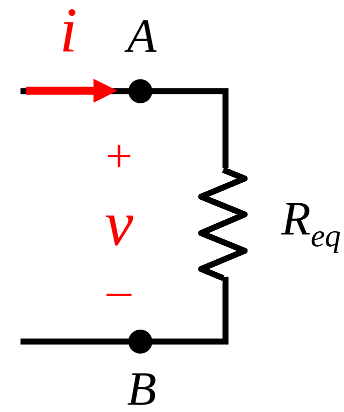
1. Find the current i_x (μA)
2. Find the voltage v (V)
3. Find the total power generated
4. Find the total power absorbed

Contents

1.5 Simple Resistive Circuits

Equivalent Resistance for Series Connection



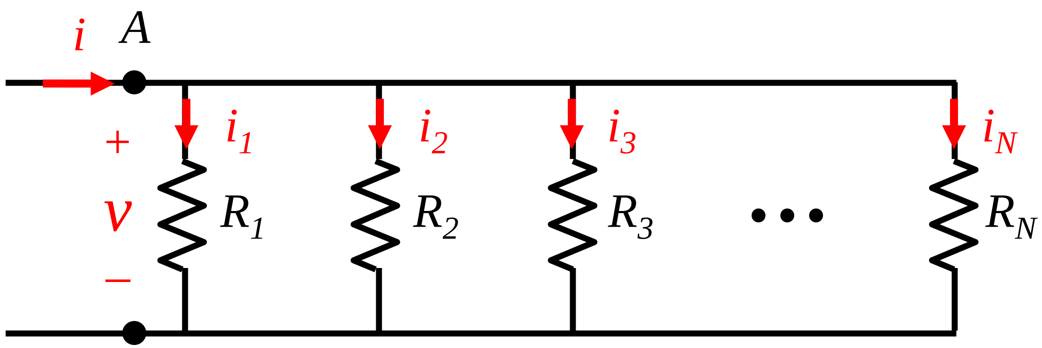


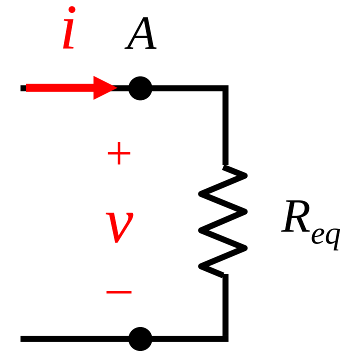
$$
\begin{aligned}
v &= v_1 + v_2 + v_3 + \ldots + v_N \\
&= R_1 i + R_2 i + R_3 i + \ldots + R_N i \\
&= \left(R_1 + R_2 + R_3 \ldots + R_N \right) i
\end{aligned}
$$

$$v = R_{eq} i$$

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \ldots + R_N}$$

Equivalent Resistance for Parallel Connection





$$i = i_1 + i_2 + i_3 + \ldots + i_N$$

$$= \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} + \ldots + \frac{v}{R_N}$$

$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} .. + \frac{1}{R_N} \right) v$$

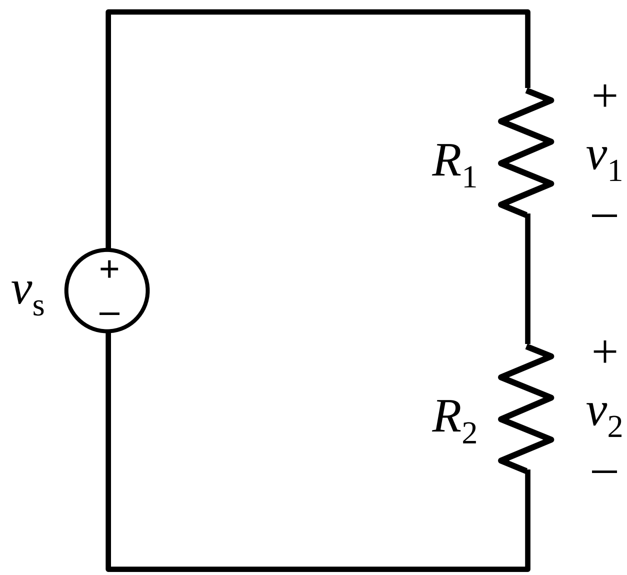
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}}$$

$$i = \frac{1}{R_{eq}} v$$

- Special case: for two parallel-connected resistance

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Voltage-Divider Circuit

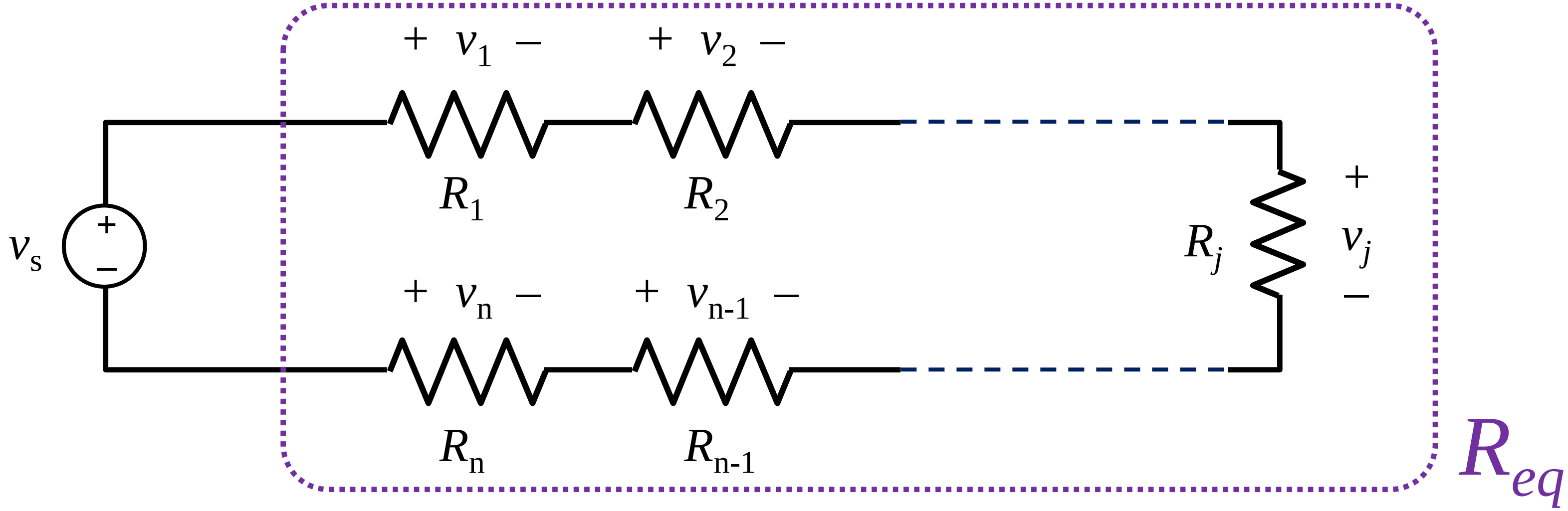


$$v_1 = \frac{R_1}{R_1 + R_2} v_s$$

$$v_2 = \frac{R_2}{R_1 + R_2} v_s$$

- The voltage across the series combination of resistors is divided up between the individual resistors in a predictable way
- This circuit demonstrates the principle of voltage division, and the circuit is called a **voltage divider**

Generalized Voltage Divider



- Equivalent resistance: $R_{eq} = R_1 + R_2 + \cdots + R_n = \sum_{j=1}^{n} R_j$
- Loop current:

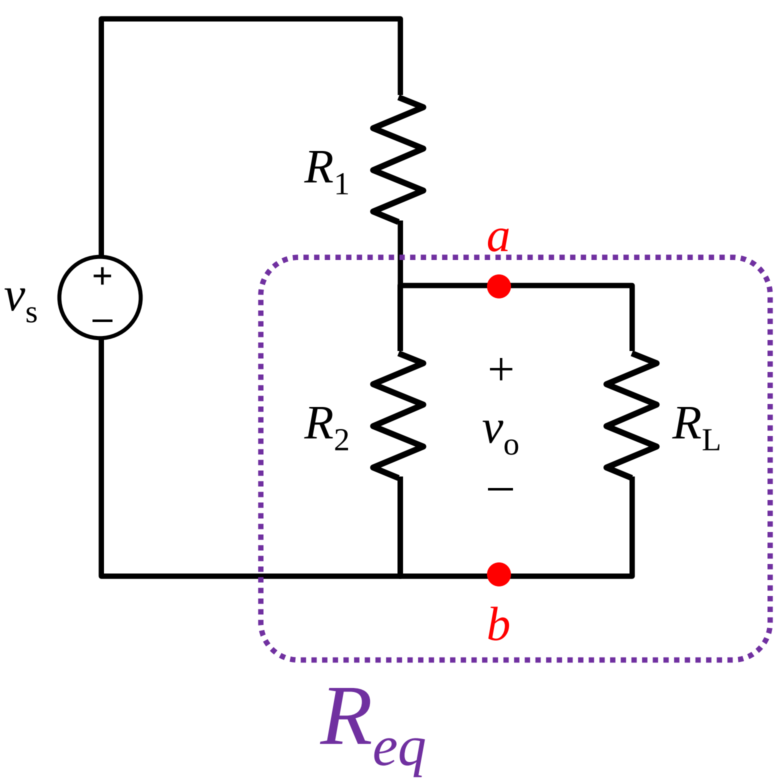
$$i = \frac{v_s}{R_1 + R_2 + \cdots + R_n} = \frac{v_s}{R_{eq}} \text{ (By KVL)}$$

- Voltage on R_j: $v_j = iR_j = \frac{R_j}{R_{eq}} v_s$

Application of Voltage Divider

Loading effect of voltage-divider circuit:

- Load: an element (or combination of elements) that draw power from the circuit

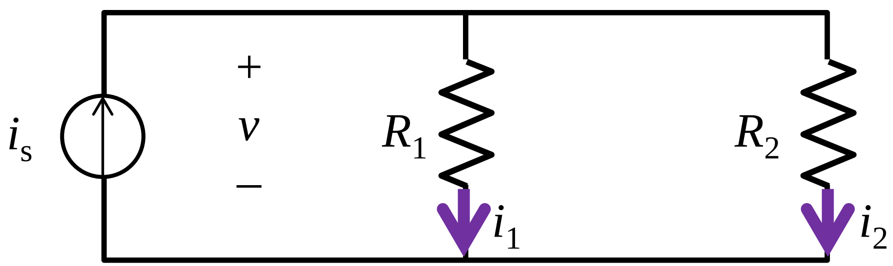


- The output voltage: $v_o = \dfrac{R_{eq}}{R_1 + R_{eq}} v_s$
- R_{eq} can be computed as $R_{eq} = \dfrac{R_2 R_L}{R_2 + R_L}$
- Substituting it into v_o:

$$v_o = \frac{R_2}{R_1\left(1+\dfrac{R_2}{R_L}\right)+R_2} v_s$$

- This is where a voltmeter comes from

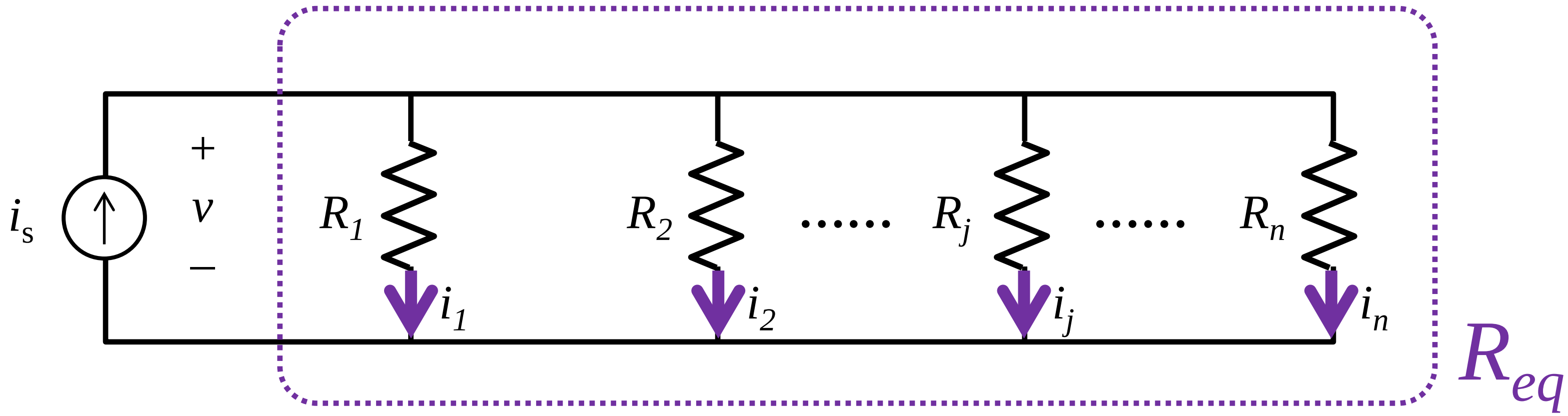
Current-Divider Circuit



$$i_1 = \frac{R_2}{R_1 + R_2} i_s, \quad i_2 = \frac{R_1}{R_1 + R_2} i_s$$

- The circuit is called a **current-divider circuit** because it divides the source current
- The current of the source divides between resistors R_1 and R_2 in proportional to their conductance values

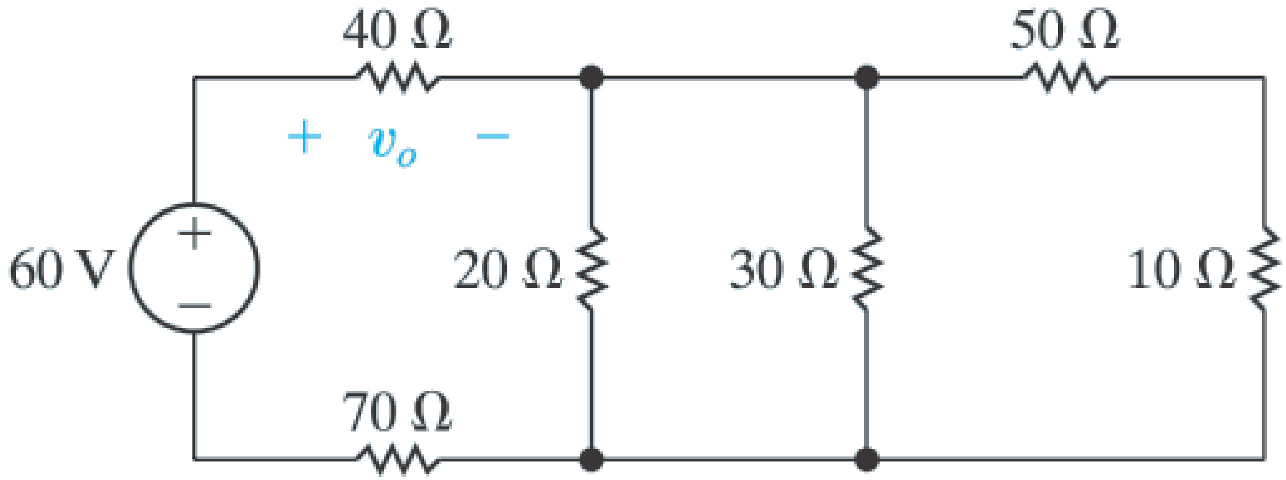
Generalized Current Divider



- Equivalent resistance: $R_{eq} = R_1 \| R_2 \| \cdots \| R_n$
- The voltage v is given by component model: $v = i_s R_{eq}$
- Component model on R_j:

$$i_j = \frac{v}{R_j} = \frac{R_{eq}}{R_j} i_s$$

Ex. 1.7 Voltage Divider & Current Divider



1. Use voltage division to determine the voltage v_0 across the 40 Ω resistor
2. Use v_0 to determine the current through the 40 Ω resistor, and use this current and current division to calculate the current in the 30 Ω resistor
3. How much power is absorbed by the 50 Ω resistor?

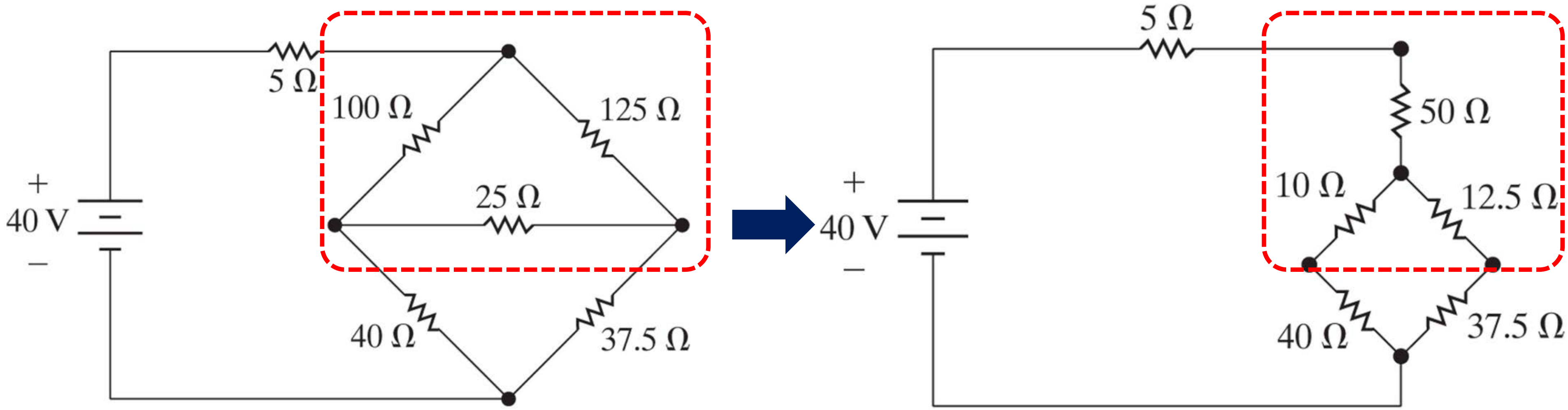
J. Nilsson and S. Riedel, *Electric Circuits*, 9th Edition, Pearson, NJ, 2011

Δ-Y Equivalent Circuits

Objective of this section:

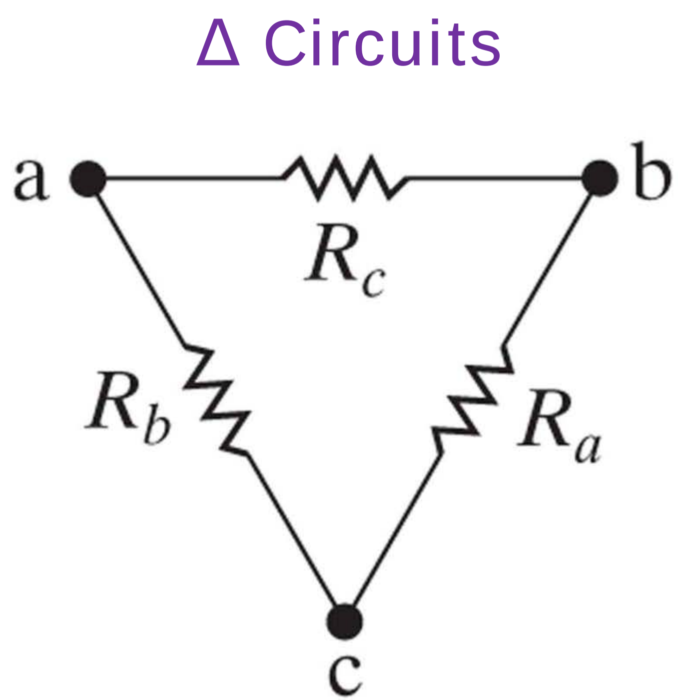
By changing the topology of a circuit, the analysis procedure can be simplified

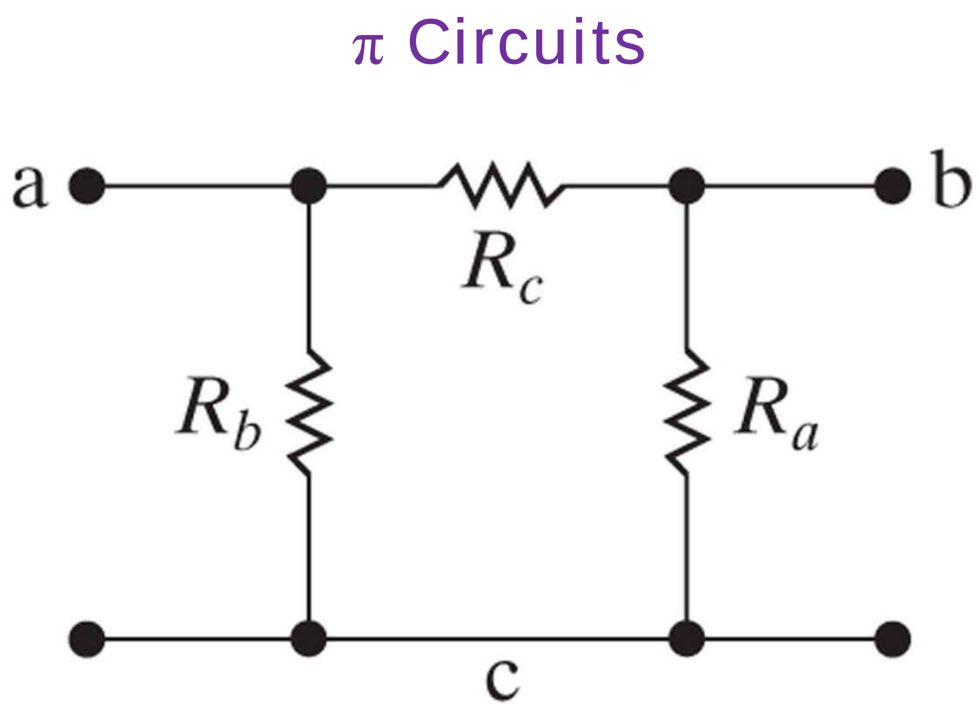
Find the current supplied by the 40 V source



J. Nilsson and S. Riedel, *Electric Circuits*, 9th Edition, Pearson, NJ, 2011

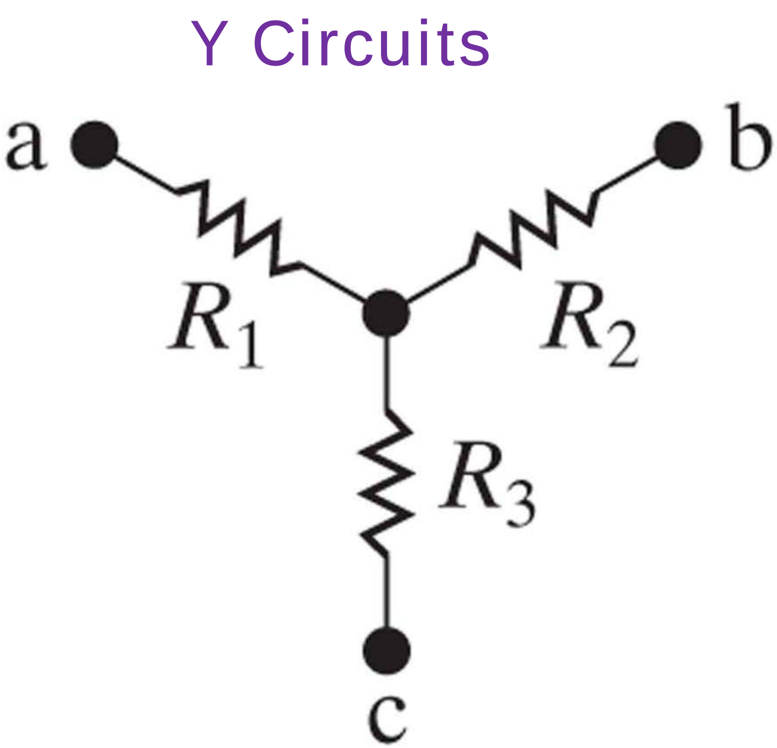
Δ Circuits or π Circuits

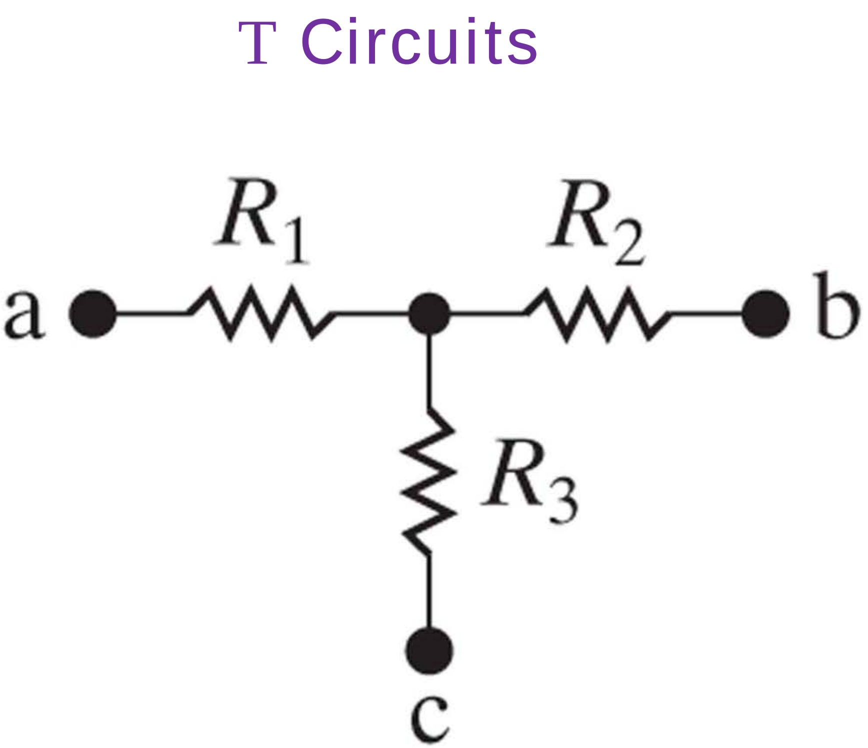




The Δ can be shaped into a π without disturbing the electrical equivalence of the two configurations

Y Circuits or T Circuits

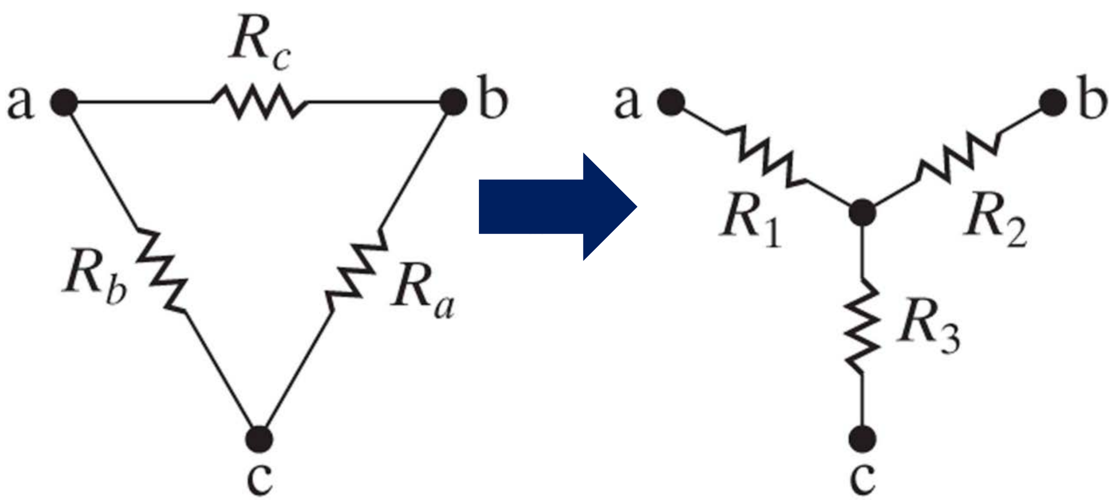




The Y can be shaped into a T without disturbing the electrical equivalence of the two configurations

Transformation Formula (1/2)

Δ-Y transformation



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

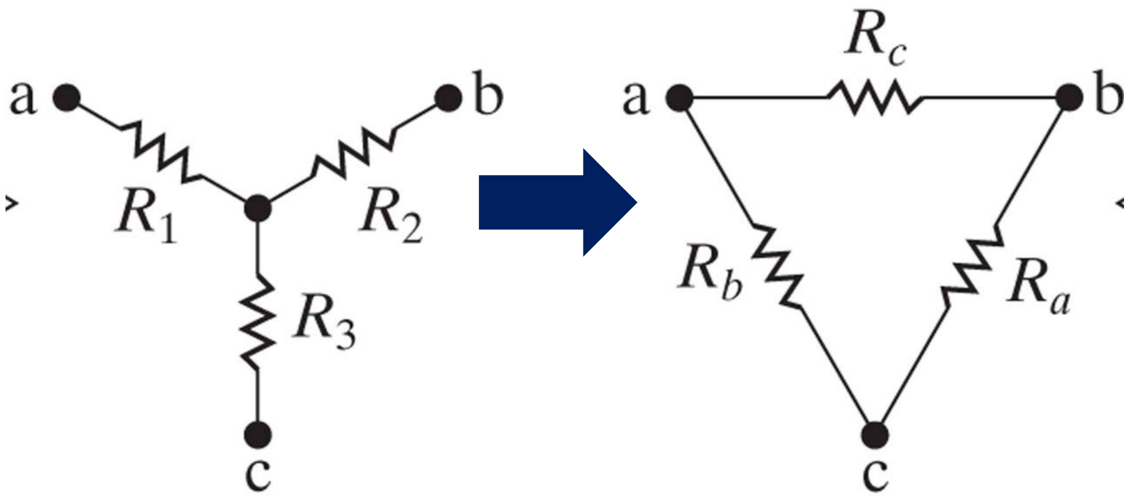
- Special case: If $R_a = R_b = R_c = R$

$\Rightarrow$ $R_1 = R_2 = R_3 = R/3$

J. Nilsson and S. Riedel, *Electric Circuits*, 9th Edition, Pearson, NJ, 2011

Transformation Formula (2/2)

Y-Δ transformation



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

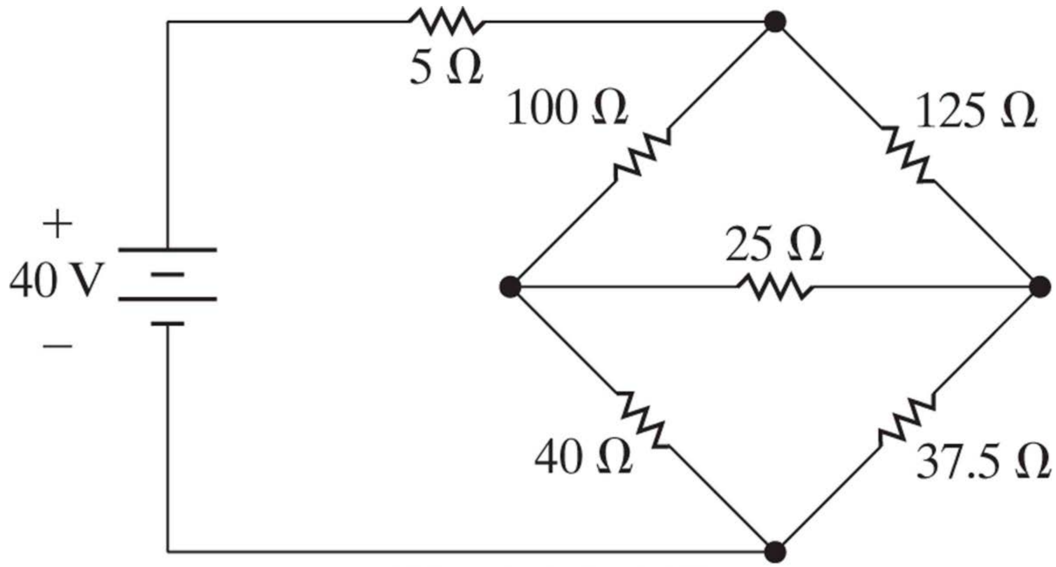
$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

- Special case: If $R_1 = R_2 = R_3 = R$

$\Rightarrow$ $R_a = R_b = R_c = 3R$

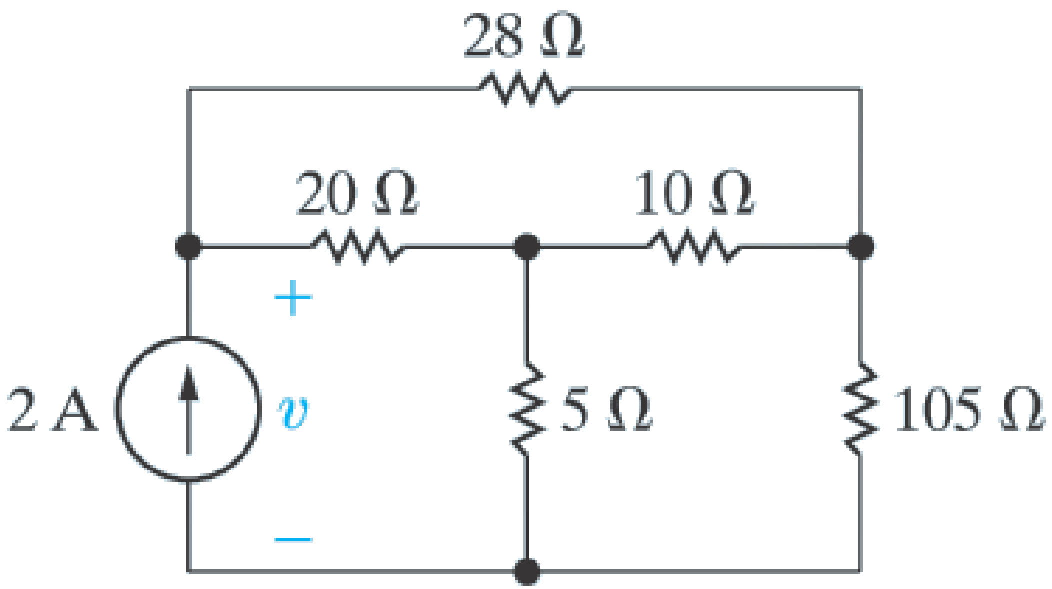
Ex. 1.8 Δ-Y Equivalent Circuits



1. Find the current and power supplied by the 40 V source

J. Nilsson and S. Riedel, *Electric Circuits*, 9th Edition, Pearson, NJ, 2011

Δ-Y Equivalent Circuits



1. Find the voltage *v* supplied by the current source

J. Nilsson and S. Riedel, *Electric Circuits*, 9th Edition, Pearson, NJ, 2011