Chapter 1

Basic Principles of Digital Systems

Analog vs. Digital

- Analog:
  - A way of representing a physical quantity by a proportional continuous voltage or current.

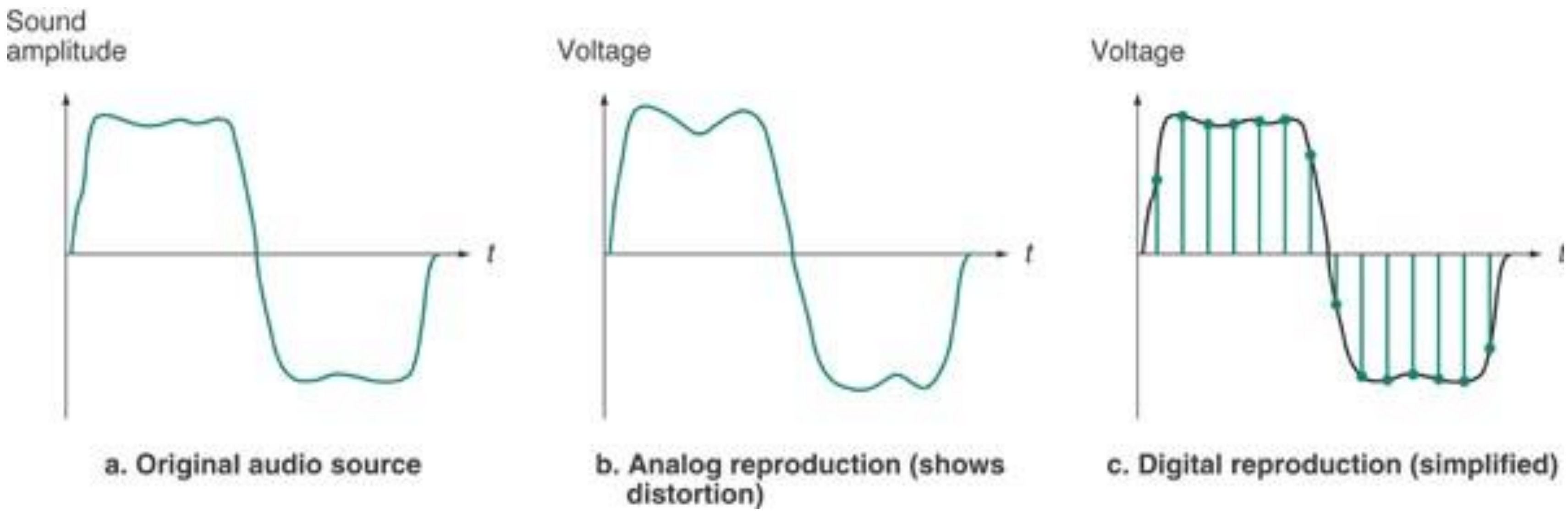
- Digital:
  - A way of representing a physical quantity in discrete voltage steps.

Analog Electronics

- Values are continuously variable between defined values.
- Can have any value within a defined range.

Analog Electronics

Voltage



Measured at microphone output

Digital audio system
: Precise intervals
: converted to binary number

Digital Electronics

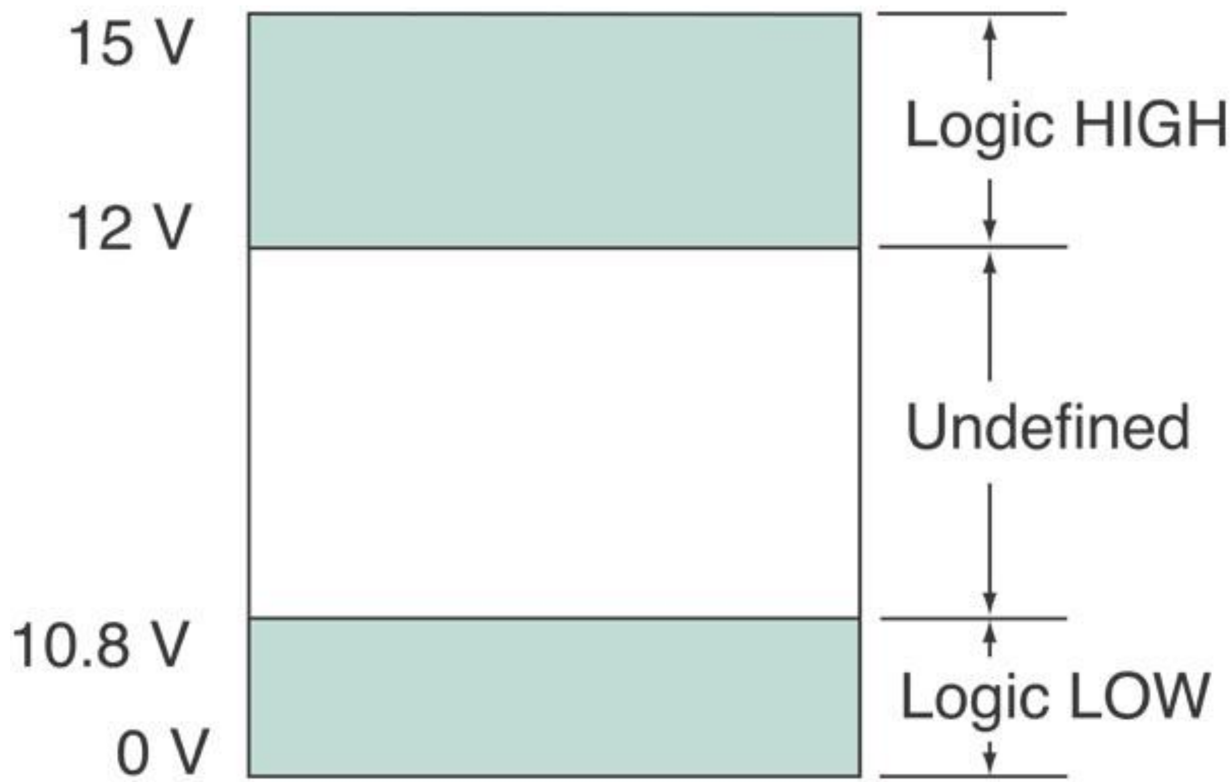
- Values can vary only by distinct(與其他不同的), or discrete(不連接的), steps.
- Can only have two values.

1.2 Digital Logic Levels

- Logic HIGH is the higher voltage and represented by binary digit ‘1’.

- Logic LOW is the lower voltage and represented by binary digit ‘0’.

Digital Logic Levels



1.3 Binary Number System

- Uses two digits, 0 and 1.
- Represents any number by using the positional notation.

Positional Notation

- The value of a digit depends on its placement within a number.
- In base 10, the positional values are (starting to the left of the decimal) –
  1 (10^0), 10 (10^1), 100 (10^2), 1000 (10^3), etc.
- In base 2, the positional values are
  1 (2^0), 2 (2^1), 4 (2^2), 8 (2^3), etc.

Decimal Equivalence of Binary Numbers

$$
\begin{aligned}
1101 &= (1 \times 2^{3}) + (1 \times 2^{2}) + (0 \times 2^{1}) + (1 \times 2^{0}) \\
&= (1 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1) \\
&= 8 + 4 + 0 + 1 \\
&= 13
\end{aligned}
$$

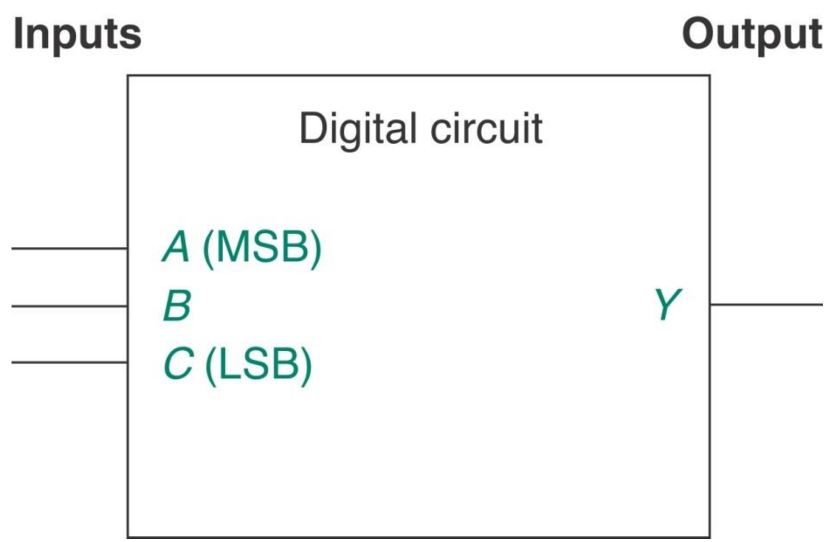
Followed by Example 1.1

Bit

- Shorthand for binary digit, a logic 0 or 1.
- The most significant bit (MSB) is the leftmost bit of a binary number.
- The least significant bit (LSB) is the rightmost bit of a binary number.

Binary Inputs

- Digital circuits operate by accepting logic levels (0,1) at their inputs.
- The corresponding outputs logic level will change (0,1).



Truth Table (真值表)

- A list of output logic levels corresponding to all possible input combinations.
- The number of input combinations is 2^n, where n is the number of inputs.
- A logic circuit with 3 inputs will have 2^3 or 8 possible input conditions.
- For this logic circuit there would also be 8 possible output conditions.

Constructing a Binary Sequence For a Truth Table – 1

- Two methods:
  - Learn to count in binary
  - Follow a simple repetitive pattern

- Memorize the binary numbers from 0000 to 1111 and their decimal equivalents (0 to 15).

- Use the weighted values of binary bits.

Binary Sequence for a Truth Table – 1

Logic Level			***Binary Value***			***Decimal Equivalent***
A	***B***	***C***	***A***	***B***	***C***	
L	L	L	0	0	0	0
L	L	H	0	0	1	1
L	H	L	0	1	0	2
L	H	H	0	1	1	3
H	L	L	1	0	0	4
H	L	H	1	0	1	5
H	H	L	1	1	0	6
H	H	H	1	1	1	7

Follow a Simple Repetitive Pattern

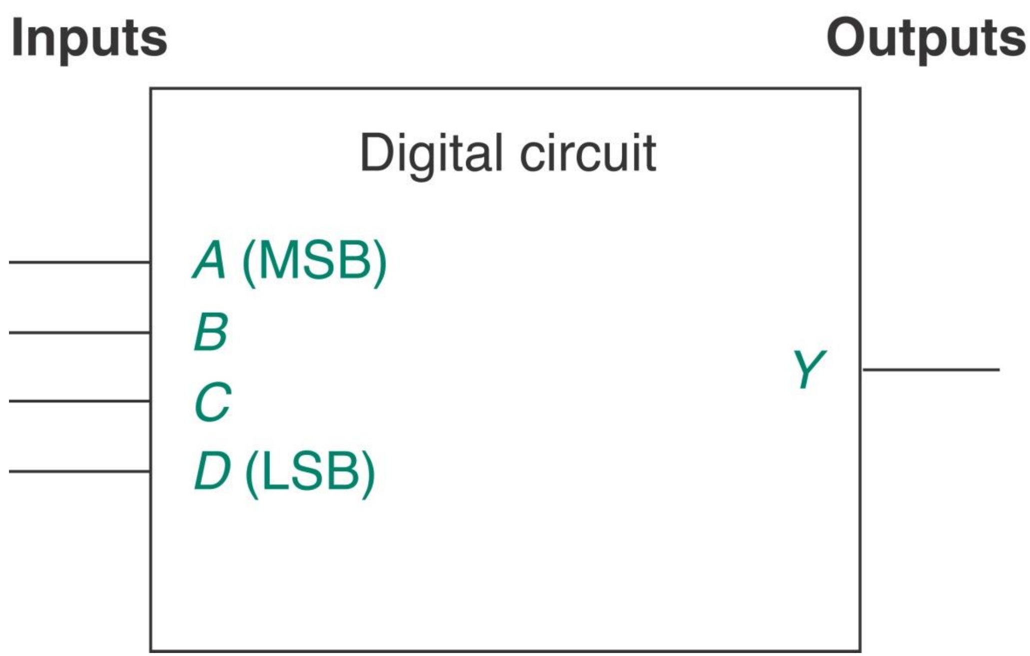
- The LSB of any binary number alternates between 0 and 1 with every line.
- The next bit alternates every two lines.
- The next bit alternates every four lines, and so on.

Follow a Simple Repetitive Pattern (Cont'd)

3 Input Truth Table			
Decimal Value	***Binary Value***		
Base 10	2^2	2^1	2^0
0	***0***	***0***	***0***
1	***0***	***0***	***1***
2	***0***	***1***	***0***
3	***0***	***1***	***1***
4	***1***	***0***	***0***
5	***1***	***0***	***1***
6	***1***	***1***	***0***
7	***1***	***1***	***1***

Example 1.2: 4-Input Digital Circuit

- $2^4 = 16$ possible input conditions.



Example 1.2: 4-Input Digital Circuit (Cont'd)

A	*B*	*C*	*D*	*Decimal*	*A*	*B*	*C*	*D*	*Decimal (Cont)*
0	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*8*
0	*0*	*0*	*1*	*1*	*1*	*0*	*0*	*1*	*9*
0	*0*	*1*	*0*	*2*	*1*	*0*	*1*	*0*	*10*
0	*0*	*1*	*1*	*3*	*1*	*0*	*1*	*1*	*11*
0	*1*	*0*	*0*	*4*	*1*	*1*	*0*	*0*	*12*
0	*1*	*0*	*1*	*5*	*1*	*1*	*0*	*1*	*13*
0	*1*	*1*	*0*	*6*	*1*	*1*	*1*	*0*	*14*
0	*1*	*1*	*1*	*7*	*1*	*1*	*1*	*1*	*15*

Binary Weights

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

Decimal-to-Binary Conversion (十進位-二進位轉換)

- Two methods:
  - Sum of powers(次方) of 2
  - Repeated division by 2

Sum of Powers of 2

- Step 1:
  - Determine the largest power of 2 less than or equal to the number to be converted.
  - Place a 1 in that positional location.
- Step 2:
  - Subtract the number found in Step 1 from the number to be converted.
  - For the new number, determine if the next lowest power of 2 is less than or equal to that number.

Sum of Powers of 2

- Step 3:
  - If the new power of two from Step 2 is larger, place a 0 in that positional location.
  - If the new value is less than or equal, place a 1 in that positional location.
- Step 4:
  - Repeat Steps 2 and 3 until there is nothing left to subtract.
  - All remaining bits are set to 0.

Followed by Example 1.3

Repeated Division by 2

- Step 1:
  - Take the number to be converted, and divide it by 2.
  - The remainder(餘) (0 or 1) is the LSB of the binary value.
- Step 2:
  - Divide the quotient (商) from Step 1 by 2.
  - The remainder (0 or 1) is the next most significant bit.
- Step 3:
  - Continue to execute Step 2 until the quotient is 0.
  - The last remainder is the MSB.

Followed by Example 1.4

Fractional (小數) Binary Numbers

- Radix point (小數點):
  - The generalized form of a decimal point(十進位小數點). The dividing line between positive and negative powers for positional multipliers.
- Binary point (二進位小數點):
  - The radix point for binary numbers.

Fractional Binary Values

- The value immediately to the right of the binary point is $2^{-1} = 0.5$.
- The next value to the right is $2^{-2} = 0.25$.
- The next value to the right is $2^{-3} = 0.125$, and so on.

Fractional Binary Weights

2^{-1}	2^{-2}	2^{-3}	2^{-4}
1/2	1/4	1/8	1/16
0.5	0.25	0.125	0.0625

Each digit is multiplied by a positional factor that is a negative power of 2. The first four multipliers on either side of the binary point are:

				binary point				
2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	2^{-4}
= 8	= 4	= 2	= 1		= 1/2	= 1/4	= 1/8	= 1/16

Example 1.5: Binary Fraction

Write the binary fraction 0.101101 as a decimal fraction.

- **Solution**

Fractional-Decimal-to-Fractional-Binary Conversion **(十進位小數轉二進位小數)**

- Step 1:
  - Multiply the decimal fraction by 2.
  - The integer part, 0 or 1, is the first bit to the right of the binary point.
- Step 2:
  - Discard the integer part from Step 1 and repeat Step 1 until the fraction repeats or terminates.

***Fractional-Decimal-to-Fractional-Binary Conversion* (十進位小數轉二進位小數)**

Example 1.6 Convert 0.95_{10} to its binary equivalent

1.4 Hexadecimal Numbers

- Base 16 number system.
- Primarily used as a shorthand form of binary numbers.
- Values range from 0 to F with the letters A to F used to represent the values 10 to 15 respectively.
- Positional multipliers are powers of 16: $16^0 = 1$, $16^1 = 16$, $16^2 = 256$, etc.

Hexadecimal vs. Decimal Numbers

TABLE 1.4 Hex Digits and Their Binary and Decimal Equivalents

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Counting in Hexadecimal

0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

10,11,12,13,14,15,16,17,18,19,1A,1B,1C,1D,1E,1F

20,21,22,23,24,25,26,27,28,29,2A,2B,2C,2D,2E,2F

30,31,32,33,34,35,36,37,38,39,3A,3B,3C,3D,3E,3F

Example

Example 1.9   Convert 7C6H to decimal.

Decimal-to-Hexadecimal Conversion (十進位轉十六進位)

- Two methods:
  - Sum of Weighted Hexadecimal Digits.
  - Repeated division by 16.

Decimal-to-Hexadecimal Conversion

- **Sum of weighted hexadecimal digits.**

Ex: Convert 135_{10} to hexadecimal

- 16^2 16^1

$256_{10} > 135_{10} > 16_{10}$

16^2	16^1	16^0	
0	8		$135-8*16^1=135-128=7$
0	8	7	$7-7=0$

$\therefore\ 135_{10}=87_{16}$

Followed by Example 1.11 and 1.12

Decimal-to-Hexadecimal Conversion

- **Sum of weighted hexadecimal digits.**

  Example 1.11: Convert 175_{10} to hexadecimal

Decimal-to-Hexadecimal Conversion

- ❑ **Repeated division by 16**

  Example 1.12: Convert 31581_{10} to hexadecimal

Conversion Between Hexadecimal and Binary (十六進位和二進位之間轉換)

❑ Each hexadecimal digit represents 4 binary bits.

Example: Converting FD69H to Binary

HEX	F	D	6	9
BIN	1111	1101	0110	1001
DEC	15	13	6	9

Conversion Between Hexadecimal and Binary **(十六進位和二進位之間轉換)**

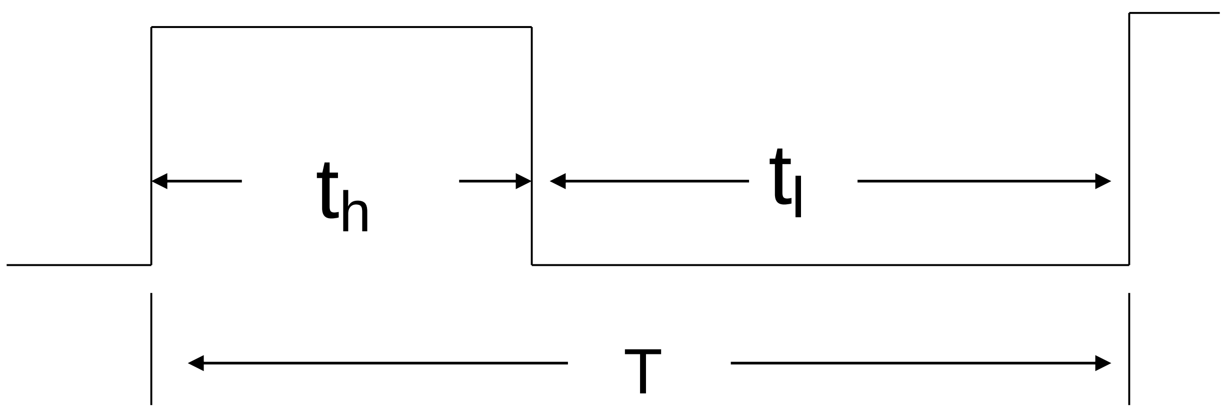
Example 1.13 Convert 7EF8H to its binary equivalent.

1.5 Periodic Digital Waveforms

- A periodic digital waveform is a time-varying sequence of logic HIGHs and LOWs that repeat over some period of time.
- Period (T) is the time required for the pattern to repeat.
- Frequency (*f*) is the number of times per second a signal repeats and is the reciprocal of period.
- $f = 1/T$

Waveform Definitions

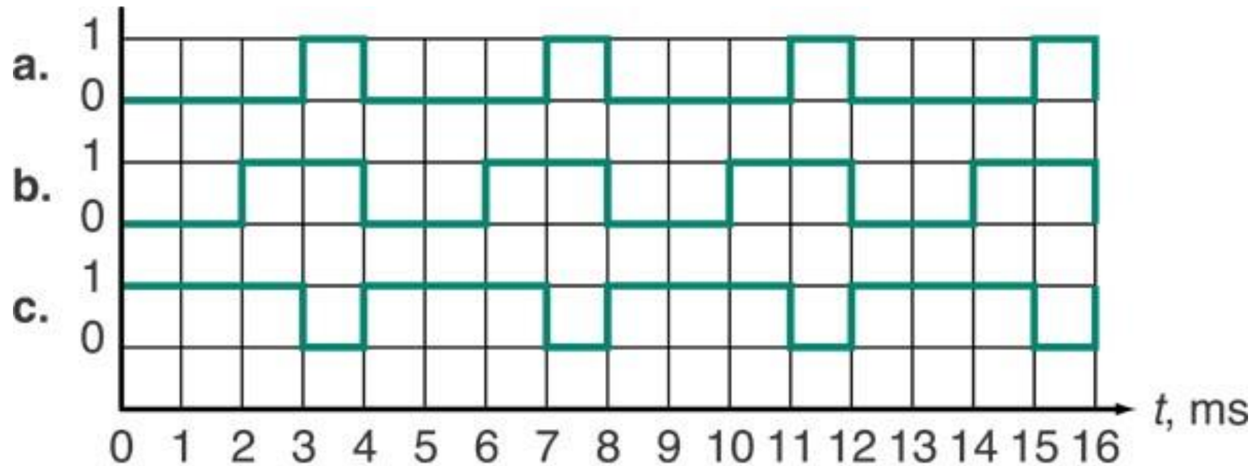
- Time HIGH (t_h) is the time a logic signal is in its HIGH state.
- Time LOW (t_l) is the time a logic signal is in its LOW state.
- Duty cycle is the ratio of the time a logic signal is HIGH (t_h) to the period (T).



Duty Cycle = t_h/T

Example 1.14: Periodic Digital Waveforms

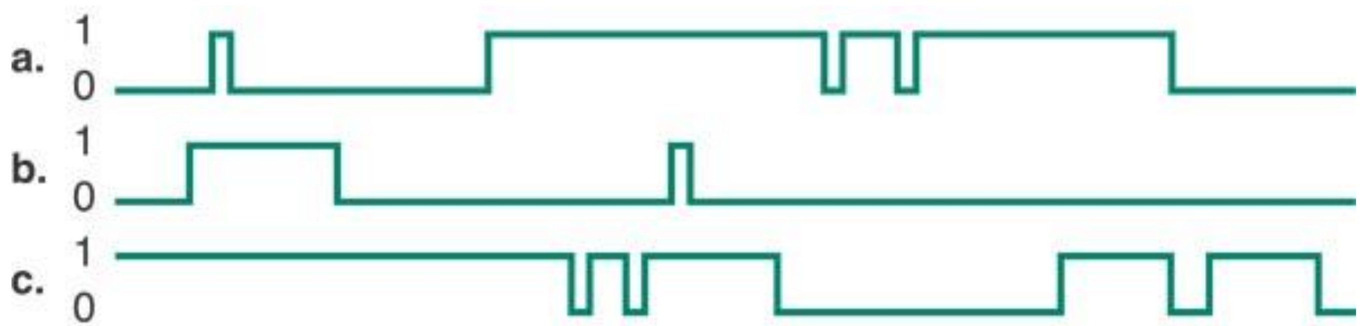
Calculate the **time LOW, time HIGH, period, frequency,** and percent **duty cycle** for each of the periodic waveforms



Sol:

Aperiodic Digital Waveforms

- An aperiodic digital waveform is a time-varying sequence of logic HIGHs and LOWs that does not repeat.



Example 1.15

A digital circuit generates the following strings of 0s and 1s:

a. 0011111101101011010000110000
b. 0011001100110011001100110011
c. 0000000011111111000000001111
d. 1011101110111011101110111011

Question:

The time between two bits is always the same. Sketch the resulting digital waveform for each string of bits. Which waveforms are periodic and which are aperiodic?

Answer: ?

Pulse Waveforms

- A pulse is a momentary (短暫的)variation of voltage from one logic level to the opposite level and back again.

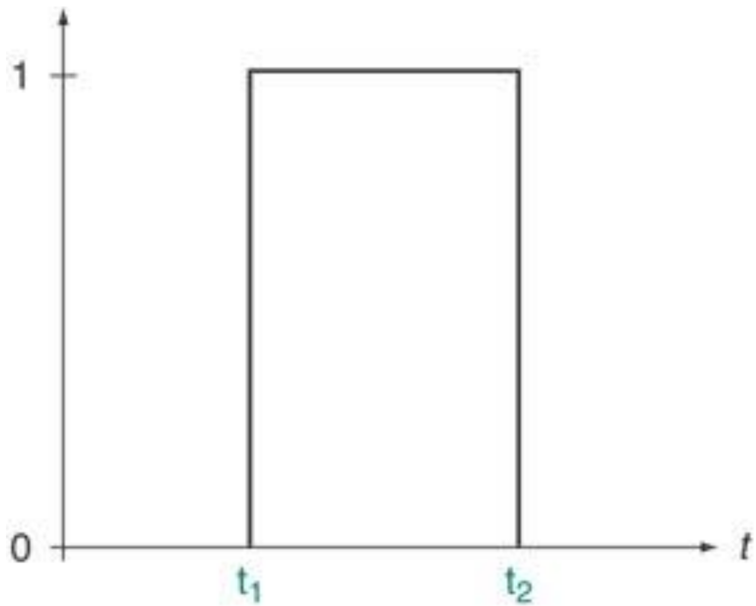
- Amplitude is the voltage magnitude of a pulse.

- Edge is the part of a pulse representing the transition from one logic level to the other.

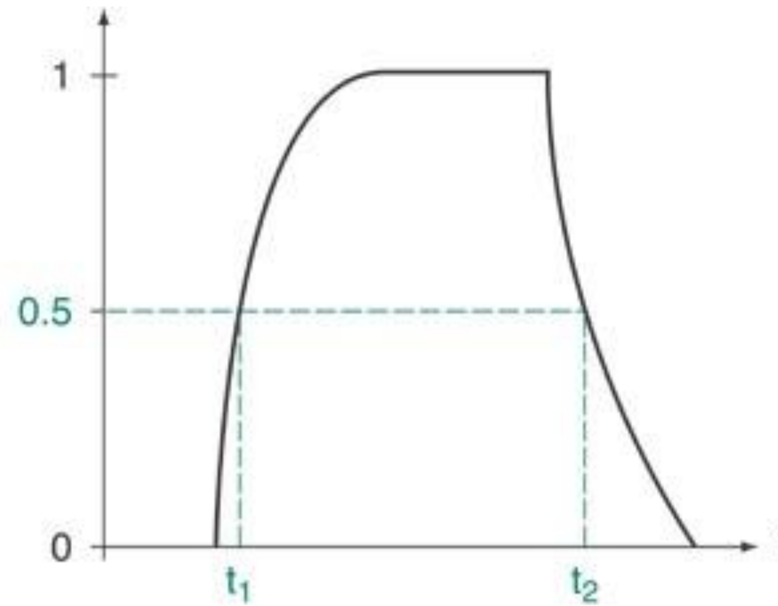
Pulse Waveform Characteristics

- Rising edge is the transition from LOW to HIGH.
- Falling edge is the transition from HIGH to LOW.
- Leading edge is the earliest transition.
- Trailing edge is the latest transition.

Pulse Waveforms

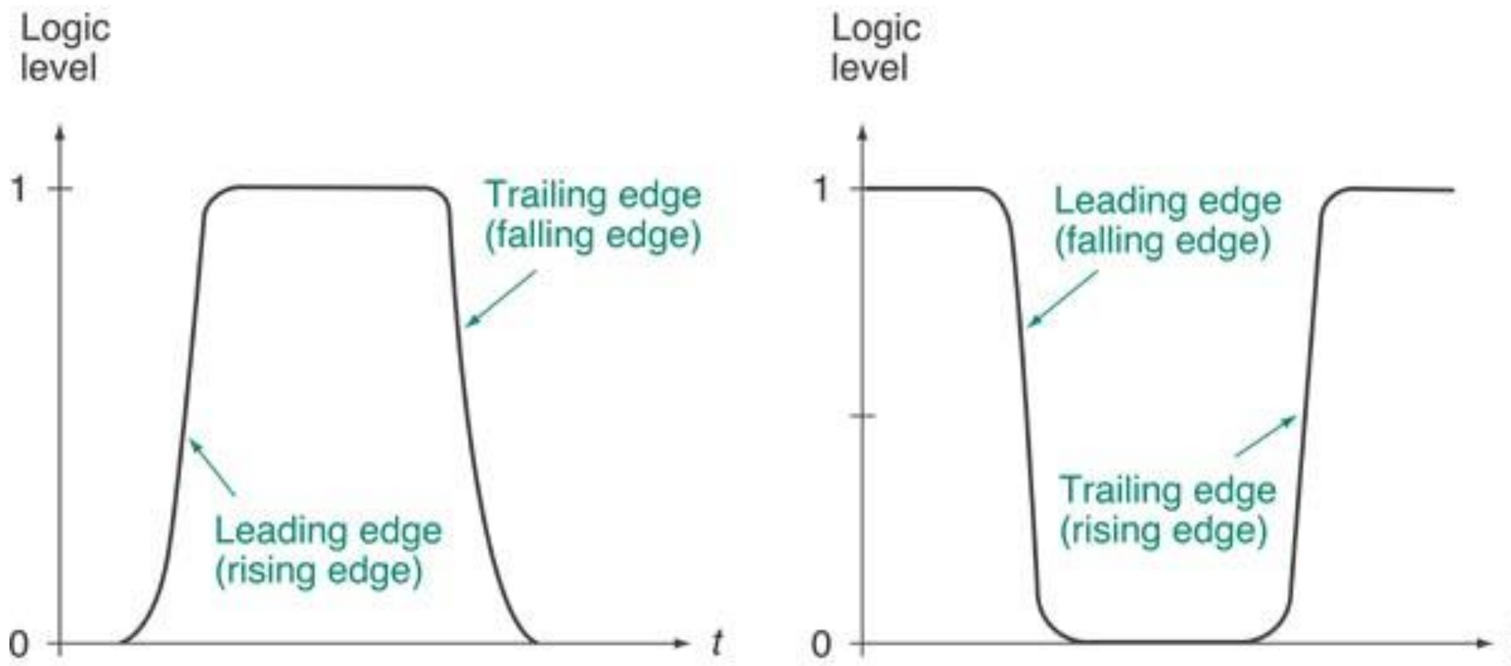


a. Ideal pulse (instantaneous transitions)



b. Nonideal pulse (1)

Pulse Waveform Characteristics



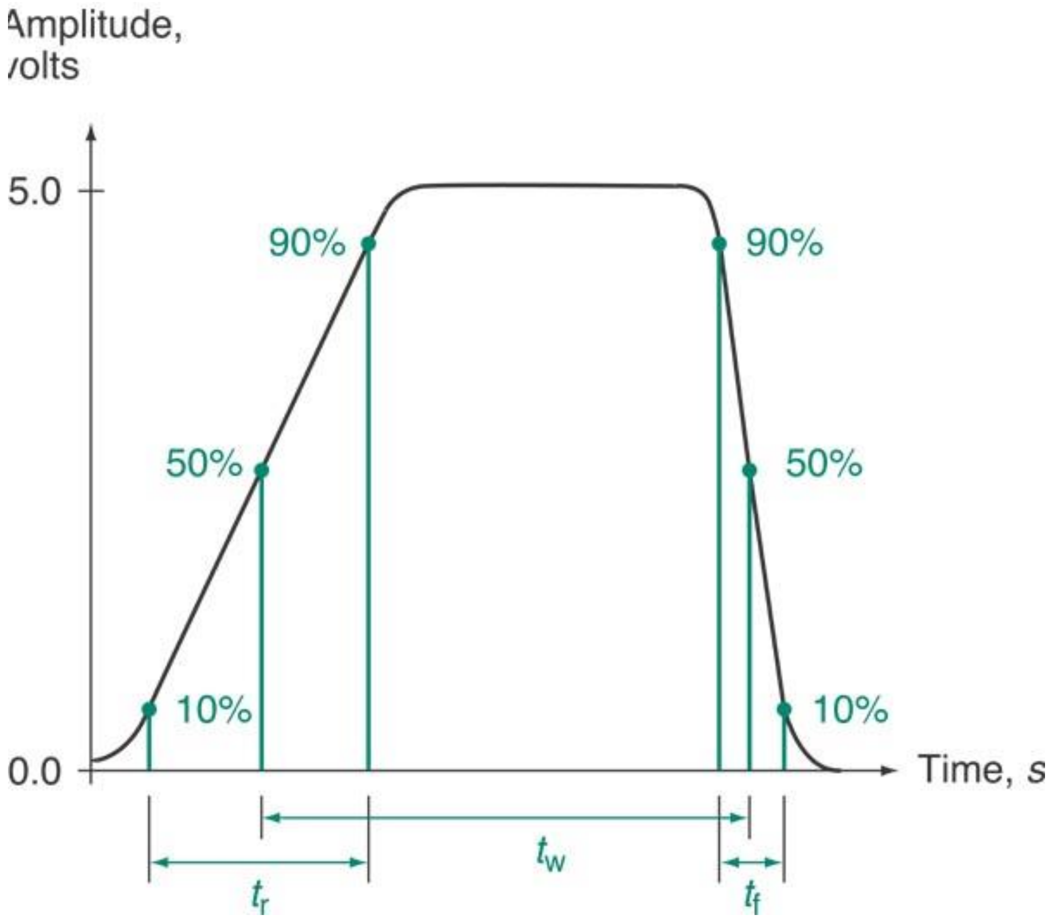
a. Positive-going pulse

b. Negative-going pulse

Pulse Waveform Timing

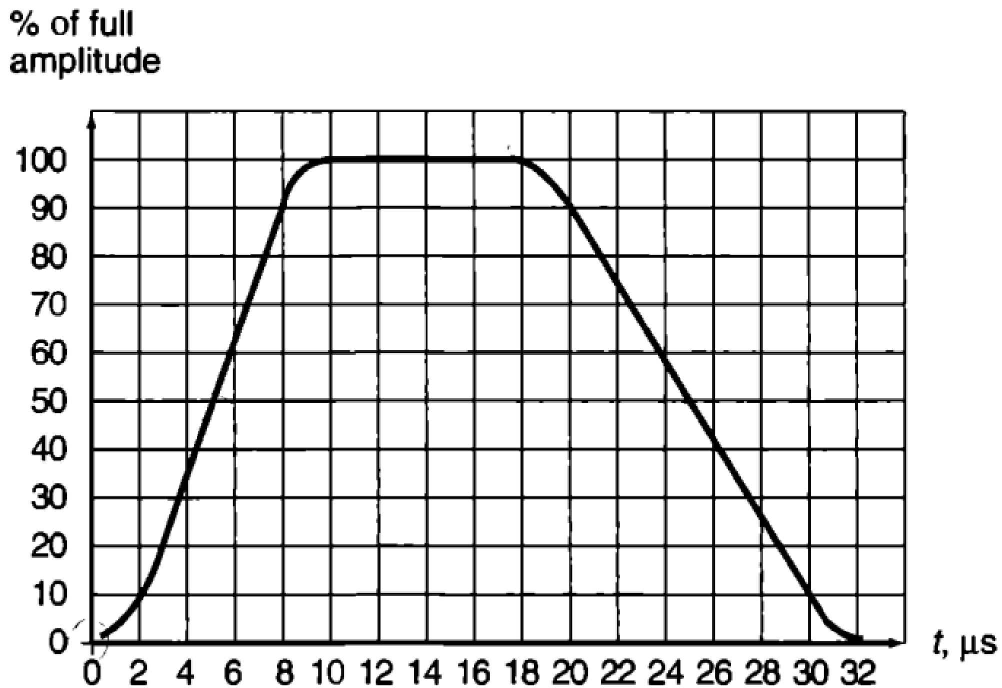
- Pulse width (t_w) is the time from the 50% point of the leading edge to the 50% point of the trailing edge.
- Rise time is the time from 10% to 90% amplitude of the rising edge.
- Fall time is the time from 90% to 10% amplitude of the falling edge.

Pulse Waveform Timing



Example 1.16

Calculate the pulse width, rise time, and fall time of the pulse



Answer: ?

Homework