Algebra - Solutions to Paul's Online Math Notes Assignment Problems

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1. Preliminaries

1.1. Integer Exponents

For problems 1 - 10, evaluate the given expression and write the answer as a single number with no exponents.

1. $2 \cdot 5^2 + (-4)^2$

$$\begin{aligned} 2 \cdot 5^2 + (-4)^2 &= 2 \cdot 25 + 16 \\ &= 50 + 16 \\ &= 66 \end{aligned}$$

2. $6^0 - 3^5$

$$\begin{aligned} 6^0 - 3^5 &= 1 - 243 \\ &= -242 \end{aligned}$$

3. $3 \cdot 4^3 + 2 \cdot 3^2$

$$\begin{aligned} 3 \cdot 4^3 + 2 \cdot 3^2 &= 3 \cdot 64 + 2 \cdot 9 \\ &= 192 + 18 \\ &= 210 \end{aligned}$$

4. $(-1)^4 + 2(-3)^4$

$$\begin{aligned} (-1)^4 + 2(-3)^4 &= 1 + 2(81) \\ &= 1 + 162 \\ &= 163 \end{aligned}$$

5. $7^0 \left(4^2 \cdot 3^2\right)^2$

$$\begin{aligned} 7^0 \left(4^2 \cdot 3^2\right)^2 &= 1(16 \cdot 9)^2 \\ &= (144)^2 \\ &= 20,736 \end{aligned}$$

6. $-4^3 + (-4)^3$

$$\begin{aligned} -4^3 + (-4)^3 &= -64 + (-64) \\ &= -64 - 64 \\ &= -128 \end{aligned}$$

7. $8 \cdot 2^{-3} + 16^0$

$$\begin{aligned} 8 \cdot 2^{-3} + 16^0 &= 8 \cdot \frac{1}{8} + 1 \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

8. $\left(2^{-1} + 3^{-1}\right)^{-1}$

$$\begin{aligned} \left(2^{-1} + 3^{-1}\right)^{-1} &= \left(\frac{1}{2} + \frac{1}{3}\right)^{-1} \\ &= \frac{1}{\frac{1}{2} + \frac{1}{3}} \\ &= \frac{1}{\frac{3}{6} + \frac{2}{6}} \\ &= \frac{1}{\frac{5}{6}} \\ &= \frac{6}{5} \end{aligned}$$

9. $\frac{3^2 \cdot (-2)^3}{6^{-2}}$

$$\begin{aligned} \frac{3^2 \cdot (-2)^3}{6^{-2}} &= \frac{9 \cdot -8}{\frac{1}{36}} \\ &= -72 \cdot 36 \\ &= -2,592 \end{aligned}$$

10. $\frac{4^{-2} \cdot 5^3}{3^{-4}}$

$$\begin{aligned} \frac{4^{-2} \cdot 5^3}{3^{-4}} &= \frac{\left(\frac{1}{16}\right) \cdot 125}{\frac{1}{81}} \\ &= \left(\frac{125}{16}\right) \cdot 81 \\ &= \frac{10125}{16} \end{aligned}$$

For problems 11 - 18, simplify the given expression and write the answer with only positive exponents.

11. $\left(3x^{-2}y^{-4}\right)^{-1}$

$$\begin{aligned}\left(3x^{-2}y^{-4}\right)^{-1} &= 3x^{-2\cdot -1}y^{-4\cdot -1}\\ &= 3x^2y^4\end{aligned}$$

12. $\left[\left(2a^2\right)^{-3}b^4\right]^{-3}$

$$\begin{aligned}\left[\left(2a^2\right)^{-3}b^4\right]^{-3} &= \left[2^{-3}a^{2\cdot(-3)}b^4\right]^{-3}\\ &= 2^{-3\cdot -3}a^{-6\cdot -3}b^{4\cdot -3}\\ &= 2^9a^{18}b^{-12}\\ &= \frac{512a^{18}}{b^{12}}\end{aligned}$$

13. $\frac{c^{-6}b^{10}}{b^9c^{-11}}$

$$\begin{aligned}\frac{c^{-6}b^{10}}{b^9c^{-11}} &= c^{(-6)-(-11)}b^{10-9}\\ &= c^{-6+11}b^1\\ &= c^5b\end{aligned}$$

14. $\frac{4a^3\left(b^2a\right)^{-4}}{c^{-6}a^2b^{-7}}$

$$\begin{aligned}\frac{4a^3\left(b^2a\right)^{-4}}{c^{-6}a^2b^{-7}} &= \frac{4a^3b^{2\cdot -4}a^{1\cdot -4}}{c^{-6}a^2b^{-7}}\\ &= \frac{4a^{3-4}b^{-8}}{c^{-6}a^2b^{-7}}\\ &= \frac{4a^{-1-2}b^{-8-(-7)}}{c^{-6}}\\ &= \frac{4a^{-3}b^{-1}}{c^{-6}}\\ &= \frac{4c^6}{a^3b}\end{aligned}$$

15. $\frac{\left(6v^2\right)^{-1}w^{-4}}{(2v)^{-3}w^{10}}$

$$\begin{aligned}\frac{\left(6v^2\right)^{-1}w^{-4}}{(2v)^{-3}w^{10}} &= \frac{6^{-1}v^{2(-1)}w^{-4}}{2^{-3}v^{-3}w^{10}} \\ &= \frac{\left(\frac{1}{6}\right)v^{-2}w^{-4}}{\left(\frac{1}{8}\right)v^{-3}w^{10}} \\ &= \left(\frac{8}{6}\right)\cdot v^{-2-(-3)}w^{-4-10} \\ &= \left(\frac{4}{3}\right)\cdot v^{1}\cdot w^{-14} \\ &= \frac{4v}{3w^{14}}\end{aligned}$$

16. $\left[\frac{\left(8x^{21}\right)^0 y^{-3}x^8}{y^{-9}x^{-1}}\right]^6$

$$\begin{aligned}\left[\frac{\left(8x^{21}\right)^0 y^{-3}x^8}{y^{-9}x^{-1}}\right]^6 &= \left[\frac{8^0x^{21\cdot 0}y^{-3}x^8}{y^{-9}x^{-1}}\right]^6 \\ &= \left[1\cdot y^{-3-(-9)}\cdot x^{8-(-1)}\right]^6 \\ &= \left(y^6x^9\right)^6 \\ &= y^{6\cdot 6}x^{9\cdot 6} \\ &= y^{36}x^{54}\end{aligned}$$

17. $\left(\frac{a^2b^{-4}c^{-1}}{b^{-9}c^8a^{-4}}\right)^{-2}$

$$\begin{aligned}\left(\frac{a^2b^{-4}c^{-1}}{b^{-9}c^8a^{-4}}\right)^{-2} &= \left(a^{2-(-4)}b^{-4-(-9)}c^{-1-8}\right)^{-2} \\ &= \left(a^6b^5c^{-9}\right)^{-2} \\ &= a^{6(-2)}b^{5(-2)}c^{-9(-2)} \\ &= a^{-12}b^{-10}c^{18} \\ &= \frac{c^{18}}{a^{12}b^{10}}\end{aligned}$$

18. $\left[\frac{p^{-6}q^7\left(p^2q\right)^{-3}}{\left(p^{-1}q^{-4}\right)^2p^{10}}\right]^3$

$$\begin{aligned}
\left[\frac{p^{-6}q^{7}\left(p^{2}q\right)^{-3}}{\left(p^{-1}q^{-4}\right)^{2}p^{10}}\right]^{3} &= \left[\frac{p^{-6}q^{7}p^{2\cdot(-3)}q^{-3}}{p^{-1\cdot(2)}q^{-4\cdot(2)}p^{10}}\right]^{3} \\
&= \left[\frac{p^{-6}p^{-6}q^{7}q^{-3}}{p^{-2}p^{10}q^{-8}}\right]^{3} \\
&= \left(\frac{p^{-6+(-6)}q^{7+(-3)}}{p^{-2+10}q^{-8}}\right)^{3} \\
&= \left(\frac{p^{-12}q^{4}}{p^{8}q^{-8}}\right)^{3} \\
&= \left(p^{-12-8}q^{4-(-8)}\right)^{3} \\
&= \left(p^{-20}q^{12}\right)^{3} \\
&= p^{-20\cdot 3}q^{12\cdot 3} \\
&= p^{-60}q^{36} \\
&= \frac{q^{36}}{p^{60}}
\end{aligned}$$

For problems 19 - 23, determine if the statement is true of false. If it is false, explain why it is false and give a corrected version of the statement.

19. $\frac{1}{6x} = 6x^{-1}$

The statement is false because given the property:

$$\begin{aligned}
(ab)^{-n} &= \frac{1}{(ab)^{n}} \quad \text{therefore} \\
6x^{-1} &= \frac{6}{x} \qquad \text{thus} \\
\frac{1}{6x} &\neq \frac{6}{x}
\end{aligned}$$

A corrected version of the statement should look like this:

$$\frac{1}{6x} = (6x)^{-1}$$

20. $\left(x^{3}\right)^{7} = x^{10}$

The statment is false because given the property:

$$\begin{aligned}(a^n)^m &= a^{nm} \text{ therefore}\\ \left(x^3\right)^7 &= x^{3\cdot 7}\\ &= x^{21} \quad \text{thus}\\ \left(x^3\right)^7 &\neq x^{10}\end{aligned}$$

A corrected version of the statement should look like this:

$$\left(x^3\right)^7 = x^{21}$$

21. $\left(m^3n^4\right)^2 = m^{12}n^8$

The statement is false because given the property:

$$(a^nb^m)^k = a^{nk}b^{mk}$$

Therefore, the corrected version of the statement should be:

$$\left(m^3n^4\right)^2 = m^6n^8$$

22. $\left[\left(z^2\right)^3\right]^4 = z^{24}$

The statement is true.

23. $(x+y)^3 = x^3 + y^3$

The statement is false because expanding that expression gives us:

$$\begin{aligned}(x+y)^3 &= (x+y)(x+y)(x+y)\\ &= (x^2+2xy+y^2)(x+y)\\ &= x^3 + yx^2 + 2x^2y + 2xy^2 + yx^2 + y^3\\ &= x^3 + 4x^2y + 2xy^2 + y^3\end{aligned}$$

The corrected version of the statment is:

$$(x+y)^3 = x^3 + 4x^2y + 2xy^2 + y^3$$