HW 7 - Chaoran Lin

1

(note: all proofs assumed that (A, B) = $(A \wedge B)$)

Prove: $Pr(A, B|K) = Pr(A|B, K)Pr(B|K)$

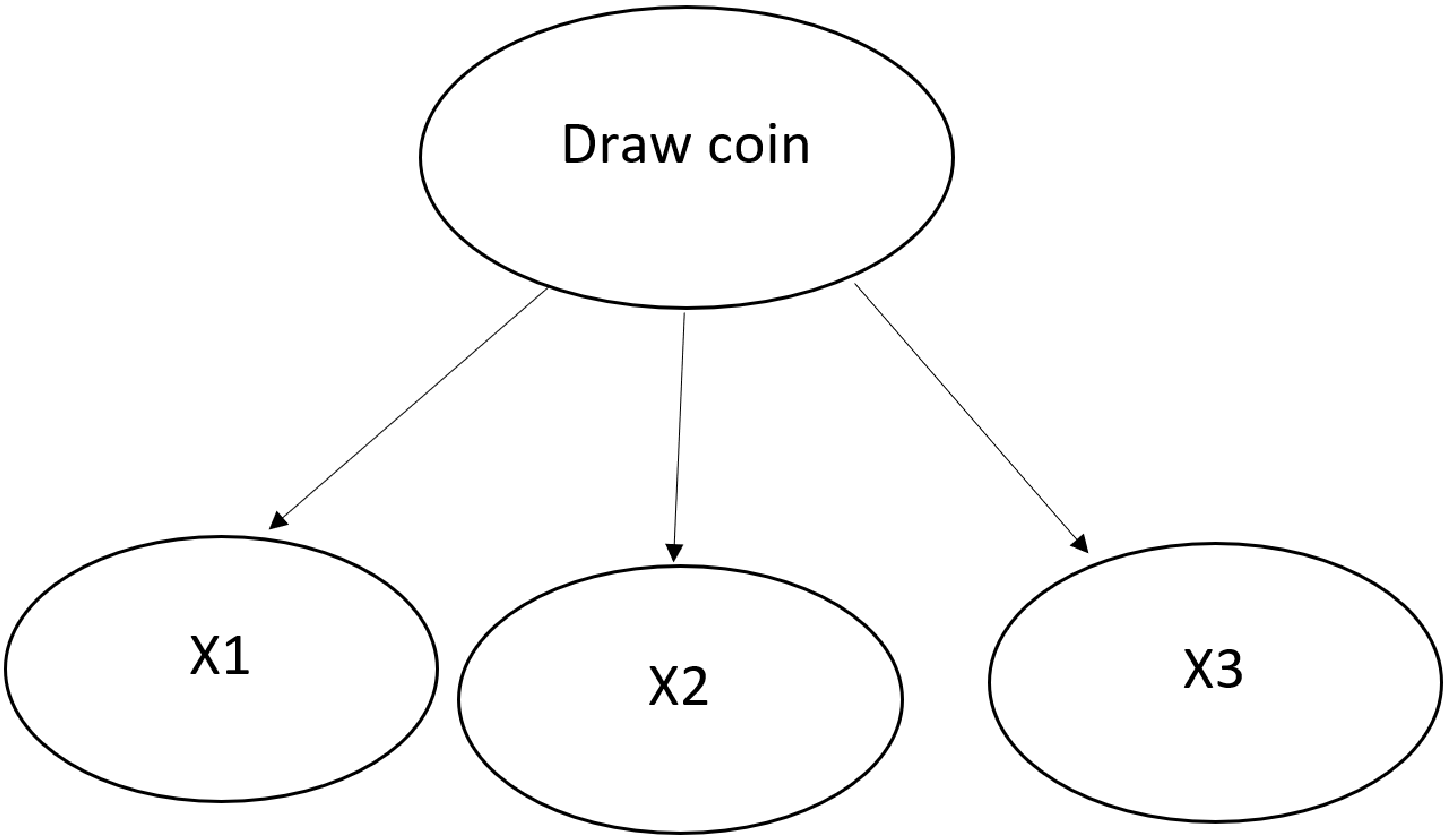
> $Pr(A, B \mid K)$
>
> $= Pr(A \wedge B \mid K)$
>
> $= \frac{Pr(A \wedge B \wedge K)}{Pr(K)}$
>
> $= Pr(A \mid B \ \wedge \ K) \ \frac{Pr(B \mid K) \ Pr(K)}{Pr(K)}$
>
> $= Pr(A \mid B \ \wedge \ K) \ Pr(B \mid K)$

Prove: $Pr(A|B, K) = Pr(B|A, K)Pr(A|K)/Pr(B|K)$

> $Pr(A \mid B, K)$
>
> $= Pr(A \mid B \wedge K)$
>
> $= \frac{Pr(A \wedge B \wedge K)}{Pr(B \wedge K)}$
>
> $= \frac{Pr(B \mid A \wedge K)Pr(A \wedge K)}{Pr(B \wedge K)}$
>
> $= Pr(B \mid A \wedge K) \frac{Pr(A \mid K) \ Pr(K)}{Pr(B \mid K) \ Pr(K)}$
>
> $= Pr(B \mid A \wedge K) \ Pr(A \mid K) \ / \ Pr(B \mid K)$

2

Bayesian network



CPTs

Coin	$Pr(C)$
a	$1/3$
b	$1/3$
c	$1/3$

Coin	X1	$Pr(X1 \| C)$
a	Head	0.2
a	Tail	0.8
b	Head	0.6
b	Tail	0.4
c	Head	0.8
c	Tail	0.2

Coin	X2	$Pr(X2\|\|C)$
a	Head	0.2
a	Tail	0.8
b	Head	0.6
b	Tail	0.4
c	Head	0.8
c	Tail	0.2

Coin	X3	$Pr(X3\|\|C)$
a	Head	0.2
a	Tail	0.8
b	Head	0.6
b	Tail	0.4
c	Head	0.8
c	Tail	0.2

3

Joint probability distribution

W	Black	Square	1	$Pr(W)$
1	T	T	T	$2/13$
2	T	T	F	$4/13$
3	T	F	T	$1/13$
4	T	F	F	$2/13$
5	F	T	T	$1/13$
6	F	T	F	$1/13$
7	F	F	T	$1/13$
8	F	F	F	$1/13$
				1

From the above table, we can conclude the probabilties below:

- $Pr(\alpha_1)\ or\ Pr(black)$

  = $Pr(W=1)$ + $Pr(W=2)$ + $Pr(W=3)$ + $Pr(W=4)$

  = $2/13$ + $4/13$ + $1/13$ + $2/13$

  = $9/13$

  ≈ 0.69

- $Pr(\alpha_2)\ or\ Pr(square)$

  = $Pr(W=1)$ + $Pr(W=2)$ + $Pr(W=5)$ + $Pr(W=6)$

  = $2/13$ + $4/13$ + $1/13$ + $1/13$

  = $8/13$

  ≈ 0.62

- $Pr(\alpha_3)\ or\ Pr(square \mid one \vee black)$

  = $Pr(W=1)$ + $Pr(W=2)$ + $Pr(W=5)$

  = $2/13$ + $4/13$ + $1/13$

  = $7/13$

  ≈ 0.54

With respect to the constructed distribution, we can identify two instances of sentences α, β, γ such that α is independent of β given γ.

We know that this relation can be expressed as:

$Pr(\alpha, \beta \mid \gamma) = Pr(\alpha \mid\gamma)\ Pr(\beta \mid\gamma)$

$=> \frac{Pr(\alpha\wedge\beta\wedge\gamma)}{Pr(\gamma)} = \frac{Pr(\alpha\wedge\gamma)}{Pr(\gamma)}\ *\ \frac{Pr(\beta\wedge\gamma)}{Pr(\gamma)}$

$=> Pr(\alpha\wedge\beta\wedge\gamma) = \frac{Pr(\alpha\wedge\gamma)Pr(\beta\wedge\gamma)}{Pr(\gamma)}$

Using this relation as a principle, we obtain the following two sentences:

- α = square

  β = two

  γ = black

  $Pr(\alpha\wedge\beta\wedge\gamma) = 4/13$

  $\frac{Pr(\alpha\wedge\gamma)Pr(\beta\wedge\gamma)}{Pr(\gamma)} = \frac{\frac{36}{169}}{\frac{9}{13}} = \frac{36}{169} * \frac{13}{9} = 4/13$

  Thus, square is independent of two given black.

- α = square

  β = two

  γ = white

  $Pr(\alpha \wedge \beta \wedge \gamma) = 1/13$

  $\frac{Pr(\alpha \wedge \gamma)Pr(\beta \wedge \gamma)}{Pr(\gamma)} = \frac{\frac{4}{169}}{\frac{4}{13}} = \frac{4}{169} * \frac{13}{4} = 1/13$

  Thus, square is independent of two given white.

4

(a)

- $I(A, \emptyset, BE)$
- $I(B, \emptyset, AC)$
- $I(C, A, BDE)$
- $I(D, AB, CE)$
- $I(E, B, ACDFG)$
- $I(F, CD, ABE)$
- $I(G, F, ABCDEH)$
- $I(H, EF, ABCDG)$

(b)

- $d - separated(A, BH, E)$:

Are A and E separated given BH? False. There is a path $A => C => F => H => E$

- $d - separated(G, D, E)$:

False. There is a path $G => F => C => A => D => B => E$

- $d - separated(AB, F, GH)$:

False. There is a path $B => E => H$

(c)

$Pr(a, b, c, d, e, f, g, h)$

$= Pr(a) * Pr(b) * Pr(c|a) * Pr(d|a, b) * Pr(e|b) * Pr(f|c, d) * Pr(g|f) * Pr(h|e, f)$

(d)

- $Pr(A = 0, B = 0) = 0.8 \, * \, 0.3 = 0.24$

Since A and B are independent events, the probability that A = 0 and B = 0 can simply be obtained through the product of those two individual probabilities.

- $Pr(E = 1|A = 1)$

  = $\frac{Pr(E=1 \wedge A=1)}{Pr(A=1)}$

  = $\frac{Pr(E=1) * Pr(A=1)}{Pr(A=1)}$

  = $Pr(E = 1)$

  = $Pr(E = 1 \wedge B = 0) + Pr(E = 1 \wedge B = 1)$

  = $Pr(E = 1|B = 0) * Pr(B = 0) + Pr(E = 1|B = 1) * Pr(B = 1)$

  = $0.9 * 0.3 + 0.1 * 0.7$

  = 0.34

We first compute the probability using the joint probability distribution, which we simplify to $Pr(E = 1)$. We realize that this probability is essentially the sum of $Pr(E = 1 \wedge B = 0)$ and $Pr(E = 1 \wedge B = 1)$, which can be rewritten using the product rule to obtain the final value.