

# HW 5 - Chaoran Lin

## 1

a.  $P \Rightarrow \neg Q, Q \Rightarrow \neg P$

$P$	$Q$	$\neg P$	$\neg Q$	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

Therefore,  $P \Rightarrow \neg Q$  is equivalent to  $Q \Rightarrow \neg P$

b.  $P \Leftrightarrow \neg Q, ((P \wedge \neg Q) \vee (\neg P \wedge Q))$

$P$	$Q$	$\neg P$	$\neg Q$	$P \wedge \neg Q$	$\neg P \wedge Q$	$P \Leftrightarrow \neg Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
T	T	F	F	F	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	F	F	F

Therefore,  $P \Leftrightarrow \neg Q$  is equivalent to  $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

## 2

a.  $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

$Smoke$	$Fire$	$\neg Smoke$	$\neg Fire$	$Smoke \Rightarrow Fire$	$\neg Smoke \Rightarrow \neg Fire$	$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Since the clause is true in some worlds but not all, it is **neither valid nor unsatisfiable**.

b.  $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$

$Smoke$	$Fire$	$Smoke \Rightarrow Fire$	$Smoke \vee Heat$	$((Smoke \vee Heat) \Rightarrow Fire)$	$(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	T	T

Since the clause is true in all worlds, it is **valid**.

c.  $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$

$Smoke$	$Heat$	$Fire$	$Smoke \wedge Heat$	$Smoke \wedge Heat \Rightarrow Fire$	$Smoke \Rightarrow Fire$	$Heat \Rightarrow Fire$	$((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$	$((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	F	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Since the clause is true in all worlds, it is **valid**.

## 3

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

a.

- $Y$  = "the unicorn is mythical"
- $I$  = "the unicorn is immortal"
- $M$  = "the unicorn is mortal"
- $A$  = "the unicorn is a mammal"
- $H$  = "the unicorn is horned"
- $G$  = "the unicorn is magical"

b.

$$\begin{aligned} &(Y \Rightarrow I) \wedge (\neg Y \Rightarrow (M \wedge A)) \wedge ((I \vee A) \Rightarrow H) \wedge (H \Rightarrow G) \\ &= (\neg Y \vee I) \wedge (Y \vee (M \wedge A)) \wedge (\neg(I \vee A) \vee H) \wedge (\neg H \vee G) \\ &= (\neg Y \vee I) \wedge (Y \vee M) \wedge (Y \vee A) \wedge (\neg(I \vee A) \vee H) \wedge (\neg H \vee G) \\ &= (\neg Y \vee I) \wedge (Y \vee M) \wedge (Y \vee A) \wedge ((\neg I \wedge \neg A) \vee H) \wedge (\neg H \vee G) \\ &= (\neg Y \vee I) \wedge (Y \vee M) \wedge (Y \vee A) \wedge (\neg I \vee H) \wedge (\neg A \vee H) \wedge (\neg H \vee G) \end{aligned}$$

c.

- Prove that the unicorn is mythical ( $Y$ )

	Statement	Reason
1	$\neg Y \vee I$	Given
2	$Y \vee M$	Given
3	$Y \vee A$	Given
4	$\neg I \vee H$	Given
5	$\neg A \vee H$	Given
6	$\neg H \vee G$	Given
7	$\neg Y$	Assume opposite of conclusion
8	$M$	<i>Resolution</i> (2, 7)
9	$A$	<i>Resolution</i> (3, 7)
10	$H$	<i>Resolution</i> (5, 9)
11	$G$	<i>Resolution</i> (6, 10)
12	$Y \vee H$	<i>Resolution</i> (3, 5)
13	$H$	<i>Resolution</i> (6, 9)

We see that there is a cyclic derivation with steps 10 and 13. After testing the other resolution possibilities, we see that we have exhausted all the possible options and it is impossible to derive a contradiction from assuming  $\neg Y$ . Thus, we cannot prove that  $Y$  must be true, i.e. the unicorn is mythical.

- Prove that the unicorn is magical ( $G$ )

	Statement	Reason
1	$\neg Y \vee I$	Given
2	$Y \vee A$	Given
3	$\neg I \vee H$	Given
4	$\neg A \vee H$	Given
5	$\neg H \vee G$	Given
6	$\neg G$	Assume opposite of conclusion
7	$\neg H$	<i>Resolution</i> (5, 6)
8	$\neg A$	<i>Resolution</i> (4, 7)
9	$\neg I$	<i>Resolution</i> (3, 7)
10	$Y$	<i>Resolution</i> (2, 8)
11	$\neg Y$	<i>Resolution</i> (1, 9)
12	FALSE	<i>Contradiction</i> (10, 11)

$G$  is false.

Therefore, G must be true, i.e. the unicorn is magical.

- Prove that the unicorn is horned ( $H$ )

	Statement	Reason
1	$\neg Y \vee I$	Given
2	$Y \vee A$	Given
3	$\neg I \vee H$	Given
4	$\neg A \vee H$	Given
5	$\neg H \vee G$	Given
6	$\neg H$	Assume opposite of conclusion
7	$\neg I$	<i>Resolution</i> (3, 6)
8	$\neg A$	<i>Resolution</i> (4, 6)
9	$\neg Y$	<i>Resolution</i> (1, 7)
10	$Y$	<i>Resolution</i> (2, 8)
11	FALSE	<i>Contradiction</i> (9, 10)

$H$  is false.

Therefore, H must be true, i.e. the unicorn is horned.