

For the Example I machine,

$$\text{Des}(\mathcal{M})$$

is an abbreviation for:

$$\begin{aligned} & \text{Inst}\{q_1 S_0 S_1 R q_2\} \\ & \& \text{Inst}\{q_2 S_0 S_1 R q_3\} \\ & \& \text{Inst}\{q_3 S_0 S_2 R q_4\} \\ & \& \text{Inst}\{q_4 S_0 S_0 R q_1\} \end{aligned}$$

which is an abbreviation for:

$$\begin{aligned} & \forall x \forall y \forall x' \forall y' \{ R_0 x y \& I x y \& K_1 x \& F x x' \& F y y' \\ & \quad \rightarrow I x' y' \& R_1 x' y \& K_2 x' \\ & \quad \& \forall z [F z y' \vee [(R_0 x z \rightarrow R_0 x' z) \& (R_1 x z \rightarrow R_1 x' z) \& (R_2 x z \rightarrow R_2 x' z)]] \} \& \\ & \forall x \forall y \forall x' \forall y' \{ R_0 x y \& I x y \& K_2 x \& F x x' \& F y y' \\ & \quad \rightarrow I x' y' \& R_1 x' y \& K_3 x' \\ & \quad \& \forall z [F z y' \vee [(R_0 x z \rightarrow R_0 x' z) \& (R_1 x z \rightarrow R_1 x' z) \& (R_2 x z \rightarrow R_2 x' z)]] \} \& \\ & \forall x \forall y \forall x' \forall y' \{ R_0 x y \& I x y \& K_3 x \& F x x' \& F y y' \\ & \quad \rightarrow I x' y' \& R_2 x' y \& K_4 x' \\ & \quad \& \forall z [F z y' \vee [(R_0 x z \rightarrow R_0 x' z) \& (R_1 x z \rightarrow R_1 x' z) \& (R_2 x z \rightarrow R_2 x' z)]] \} \& \\ & \forall x \forall y \forall x' \forall y' \{ R_0 x y \& I x y \& K_4 x \& F x x' \& F y y' \\ & \quad \rightarrow I x' y' \& R_0 x' y \& K_1 x' \\ & \quad \& \forall z [F z y' \vee [(R_0 x z \rightarrow R_0 x' z) \& (R_1 x z \rightarrow R_1 x' z) \& (R_2 x z \rightarrow R_2 x' z)]] \} \end{aligned}$$

And

$$\mathcal{Q}$$

is an abbreviation for:

$$\begin{aligned} & \forall x \exists w \forall y \forall z \{ F x w \& (F x y \rightarrow G x y) \\ & \quad (F x z \& G z y \rightarrow G x y) \\ & \quad [G z x \vee (G x y \& F y z) \vee (F x y \& F z y) \rightarrow \sim F x z] \} \end{aligned}$$

And now

$$\text{Un}(\mathcal{M})$$

is an abbreviation for:

$$\begin{aligned}
& \exists u \{ \\
& \forall x \exists w \forall y \forall z \{ Fxw \ \& \ (Fxy \rightarrow Gxy) \\
& \quad (Fxx \ \& \ Gzy \rightarrow Gxy) \\
& \quad [Gzx \vee (Gxy \ \& \ Fyz) \vee (Fxy \ \& \ Fzy) \rightarrow \sim Fxz] \} \ \& \ \forall y R_0uy \ \& \ Iuu \ \& \ K_1u \ \& \\
& \forall x \forall y \forall x' \forall y' \{ R_0xy \ \& \ Ixy \ \& \ K_1x \ \& \ Fxx' \ \& \ Fyy' \\
& \quad \rightarrow Ix'y' \ \& \ R_1x'y \ \& \ K_2x' \\
& \quad \ \& \ \forall z [Fzy' \vee [(R_0xz \rightarrow R_0x'z) \ \& \ (R_1xz \rightarrow R_1x'z) \ \& \ (R_2xz \rightarrow R_2x'z)]] \} \ \& \\
& \forall x \forall y \forall x' \forall y' \{ R_0xy \ \& \ Ixy \ \& \ K_2x \ \& \ Fxx' \ \& \ Fyy' \\
& \quad \rightarrow Ix'y' \ \& \ R_1x'y \ \& \ K_3x' \\
& \quad \ \& \ \forall z [Fzy' \vee [(R_0xz \rightarrow R_0x'z) \ \& \ (R_1xz \rightarrow R_1x'z) \ \& \ (R_2xz \rightarrow R_2x'z)]] \} \ \& \\
& \forall x \forall y \forall x' \forall y' \{ R_0xy \ \& \ Ixy \ \& \ K_3x \ \& \ Fxx' \ \& \ Fyy' \\
& \quad \rightarrow Ix'y' \ \& \ R_2x'y \ \& \ K_4x' \\
& \quad \ \& \ \forall z [Fzy' \vee [(R_0xz \rightarrow R_0x'z) \ \& \ (R_1xz \rightarrow R_1x'z) \ \& \ (R_2xz \rightarrow R_2x'z)]] \} \ \& \\
& \forall x \forall y \forall x' \forall y' \{ R_0xy \ \& \ Ixy \ \& \ K_4x \ \& \ Fxx' \ \& \ Fyy' \\
& \quad \rightarrow Ix'y' \ \& \ R_0x'y \ \& \ K_1x' \\
& \quad \ \& \ \forall z [Fzy' \vee [(R_0xz \rightarrow R_0x'z) \ \& \ (R_1xz \rightarrow R_1x'z) \ \& \ (R_2xz \rightarrow R_2x'z)]] \} \\
& \} \rightarrow \exists s \exists t R_1st
\end{aligned}$$

This is one of the set of formulas that can't be mechanically decided.