ML Programming assignment II

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1 Sigmoid Function and the Hyperbolic Tangent Function

1.1 Definition of the Sigmoid Function

The **sigmoid function**, also known as the logistic function, is defined as

$$\sigma(x) = \frac{1}{1+e^{-x}}.$$

Its range is $(0, 1)$, and it is commonly used as an activation function in neural networks due to its smooth and monotonic properties.

1.2 Definition of the Hyperbolic Tangent Function

The **hyperbolic tangent function** is defined as

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

Its range is $(-1, 1)$, and it is another widely used activation function. Notice that both functions are expressed in terms of exponentials.

1.3 Deriving the Relationship

We start by rewriting $\tanh(x)$ in a similar exponential form:

$$\tanh(x) = \frac{e^{2x} - 1}{e^{2x} + 1}.$$

Meanwhile, the sigmoid function can also be written as

$$\sigma(x) = \frac{e^x}{e^x + 1} = \frac{1}{1+e^{-x}}.$$

Now consider $\sigma(2x)$:

$$\sigma(2x) = \frac{1}{1+e^{-2x}}.$$

Then,

$$2\sigma(2x) - 1 = 2\left(\frac{1}{1+e^{-2x}}\right) - 1 = \frac{1-e^{-2x}}{1+e^{-2x}} = \tanh(x).$$

Hence, we obtain the fundamental relation:

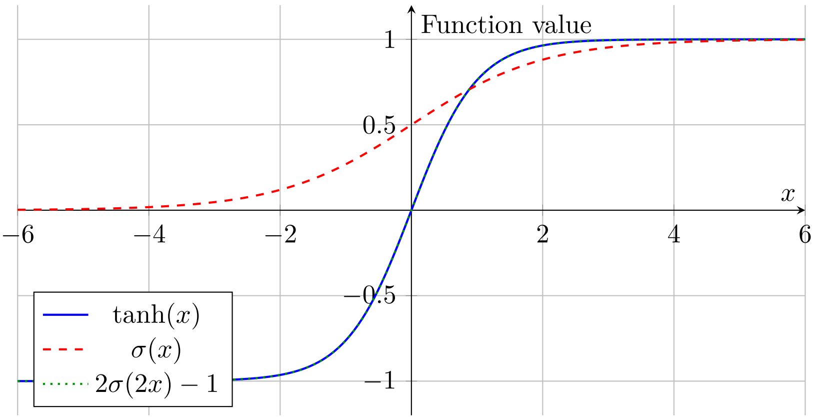
$$\boxed{\tanh(x) = 2\sigma(2x) - 1.}$$

1.4 Inverse Relation

By rearranging the above expression, we can express the sigmoid function in terms of the hyperbolic tangent function:

$$\sigma(x) = \frac{1 + \tanh\left(\frac{x}{2}\right)}{2}.$$

2 Graphical Comparison



3 Conclusion

From the derivation above, we conclude that the sigmoid and hyperbolic tangent functions are closely related. Specifically, $\tanh(x)$ is a **scaled and shifted** version of the sigmoid function. While $\sigma(x)$ maps inputs to $(0, 1)$, $\tanh(x)$ rescales this range to $(-1, 1)$, making it symmetric about the origin and often more suitable for neural network training.