ML Assignment II
Approximation / Regression to $f(x) = \frac{1}{1+25x^2}$

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1 Description of the Problem

In this assignment, we need to be given data to find $H(x)$ approximation / regression to $f(x) = \frac{1}{1+25x^2}$, and $x \in [-1, 1]$. Here $H(x)$ could be chosen as a polynomial, neural network, etc.

2 Programming process

I chose $H(x)$ as a polynomial for approximation, and for data initialization, I uniformly selected 200 points in $x \in [-1, 1]$ to train the model, and compared the MSE between the final results and the true function under different choices of degree.Additionally, I carefully examined how the choice of polynomial degree affects the accuracy: a low-degree polynomial might underfit the data, failing to capture the sharp curvature of $f(x)$ near the endpoints Figure (1), while a very high-degree polynomial can overfit the training points and exhibit oscillatory behavior between them Figure (4).

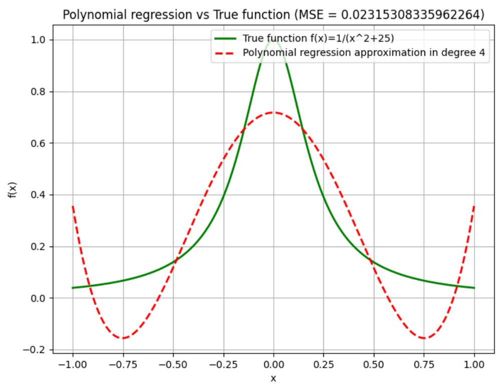

Figure 1: Degree is 4

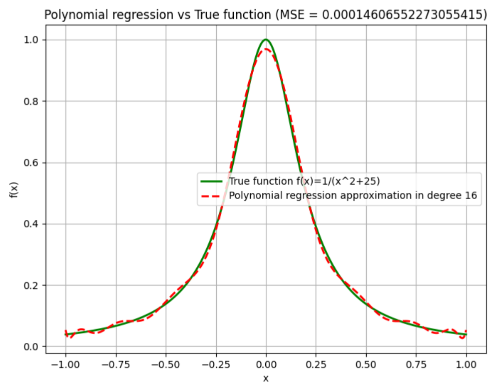

Figure 2: Degree is 16

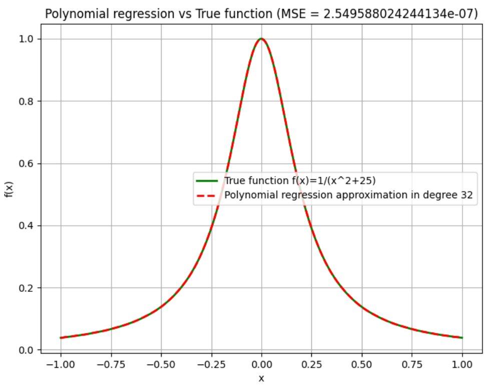

Figure 3: Degree is 32

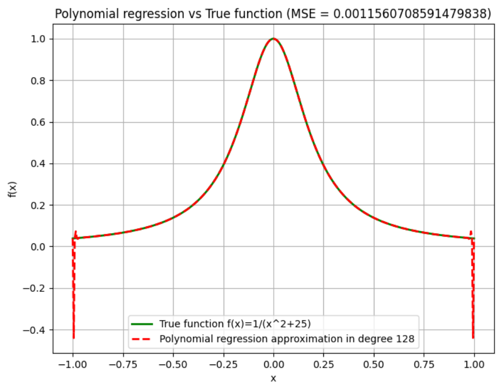

Figure 4: Degree is 128

From the image, it can be seen that as the degree of the polynomial $H(x)$ increases, it is better able to capture the detailed structure and shape of the true function. Higher-degree polynomials can approximate the curvature and variations of the target function more closely, resulting in smaller numerical discrepancies as measured by the mean squared error (MSE). This observation aligns with our theoretical expectations. However, it should be noted that excessively high-degree polynomials may introduce oscillations between training points, so a balance must be struck between approximation accuracy and stability. Overall, the trend that higher-degree polynomials yield better fits and lower MSE in this setting is clearly observed.