University of Waterloo
Department of Mechanical and Mechatronics Engineering

# MTE360 Group 20 Project 4

MTE 360
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## 1.2 Gain Selection

By only adjusting the gain of the system it can be found that to achieve the desired phase margin of 0 or a system on the verge of instability, a gain of C = **0.092094** is required which is the critical loop gain. At C = 0.092095, the system becomes unstable. The plots generated by the SISO tool for the barely stable case are shown in figure 1.

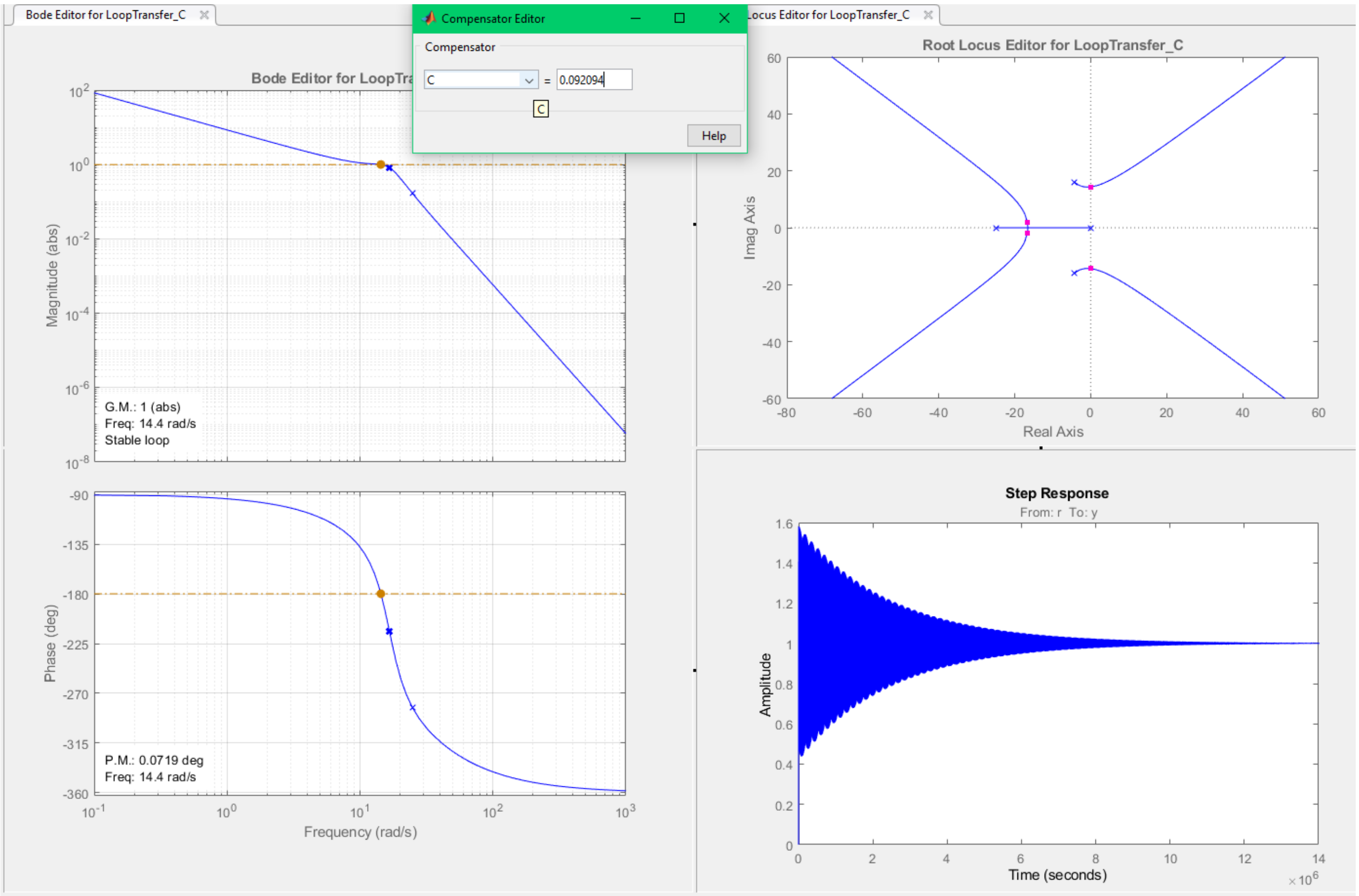


Figure 1: Plots showing system after gain adjustment creating a barely stable case

The crossover frequency value ($\omega_c$) occurs at 14.4 rad/s. This was analytically confirmed by plugging in 14.4rad/s directly into the transfer function resulting in an output of G(14.4) = -0.9983 + 0.0098i. The magnitude of this value is about 0.9984, which is approximately equal to 1, the gain at which crossover frequency is measured by definition.

By making further modifications to the initial system of gain alone to get a phase margin (PM) of 30 degrees, the required gain is **0.08715**.  A gain margin of 1.06 is achieved and a crossover frequency of 11.6 rad/sec is produced as can be seen in figure 2. Thus, the first two specifications are not met. In using only P-control to stabilize the flexible drive, it is clear that proportional control is not enough to meet the design specifications required. The phase margin requirement and gain margin requirement cannot be simultaneously satisfied with only P-control. It can further be observed that the step response is much closer to being reasonable. There is still significant overshoot and oscillation, but it is still significantly reduced compared to the verge of instability case.

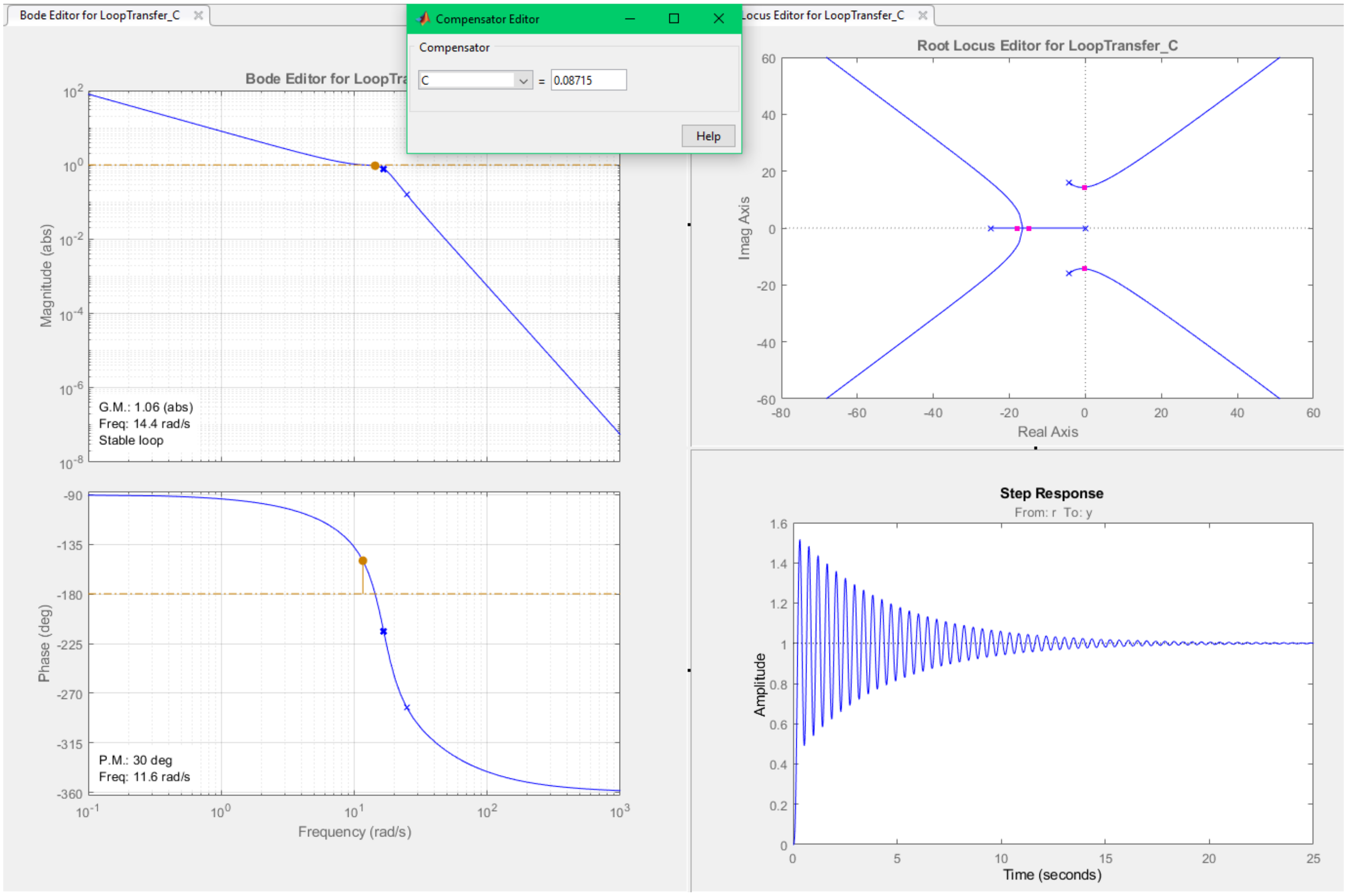


Figure 2: System modified to achieve a phase margin of 30 degrees

## 1.3 Notch Filter Design

Comparing the notch filter controller and p-control, the notch filter reduces the crossover frequency from 11.6 rad/sec to 12 rad/s for a PM of 30 degrees. The GM after the notch filter was implemented increased from 1.06 to 1.39 (abs). The main contribution of the notch filter was reducing the resonance encountered near, and thus increasing the stability near the crossover frequency. This reduction can be seen in the magnitude plot shown in figure 3, where the area around the crossover frequency has been smoothened. Since a lower gain is required (0.0887) to achieve the same phase margin as before, the notch filter also reduced step response overshoot and oscillations which now attenuate in just a few seconds.

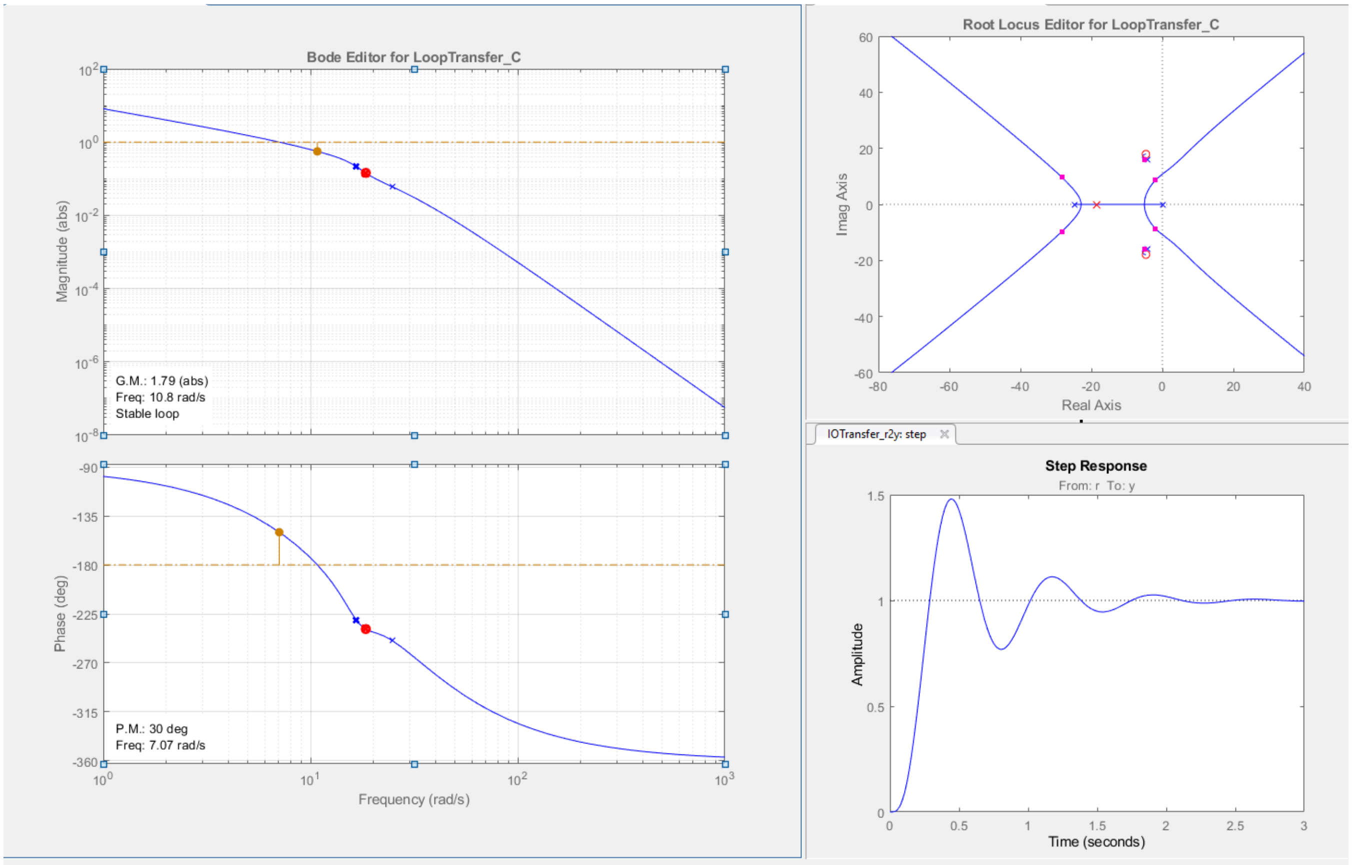


Figure 3: Bode Plots after the effects of implementing a Notch Filter

## 1.4 Lead Filter Design

By observation of the bode plot, the phase is approximately -255 degrees at 25 rad/s. For a phase margin of 30 degrees, and an additional buffer of 10 degrees, the total change required is 115 degrees.

After notch filter $\angle\{G_2(j\omega_c)\, G_{notch}(j\omega_c)\} \approx 255^\circ$

Desired $= 30$ with $10^\circ$ buffer $\therefore$ $\Phi_{max} = 115$

Thus with a double filter each contributes $\frac{115}{2} = 57.5^\circ$

$$\alpha = \frac{1 - \sin(57.5)}{1 + \sin(57.5)} \approx 0.084956$$

$$\omega_{max} = \frac{a}{\sqrt{\alpha}}$$

$$25 = \frac{a}{\sqrt{0.084956}}$$

$a = 7.286$ $\therefore$ $b = 85.77$

$1/\alpha = 11.7708$

After adding the two lead filters and readjusting gain, the crossover frequency is 31.2 rad/s as can be observed in figure 4.

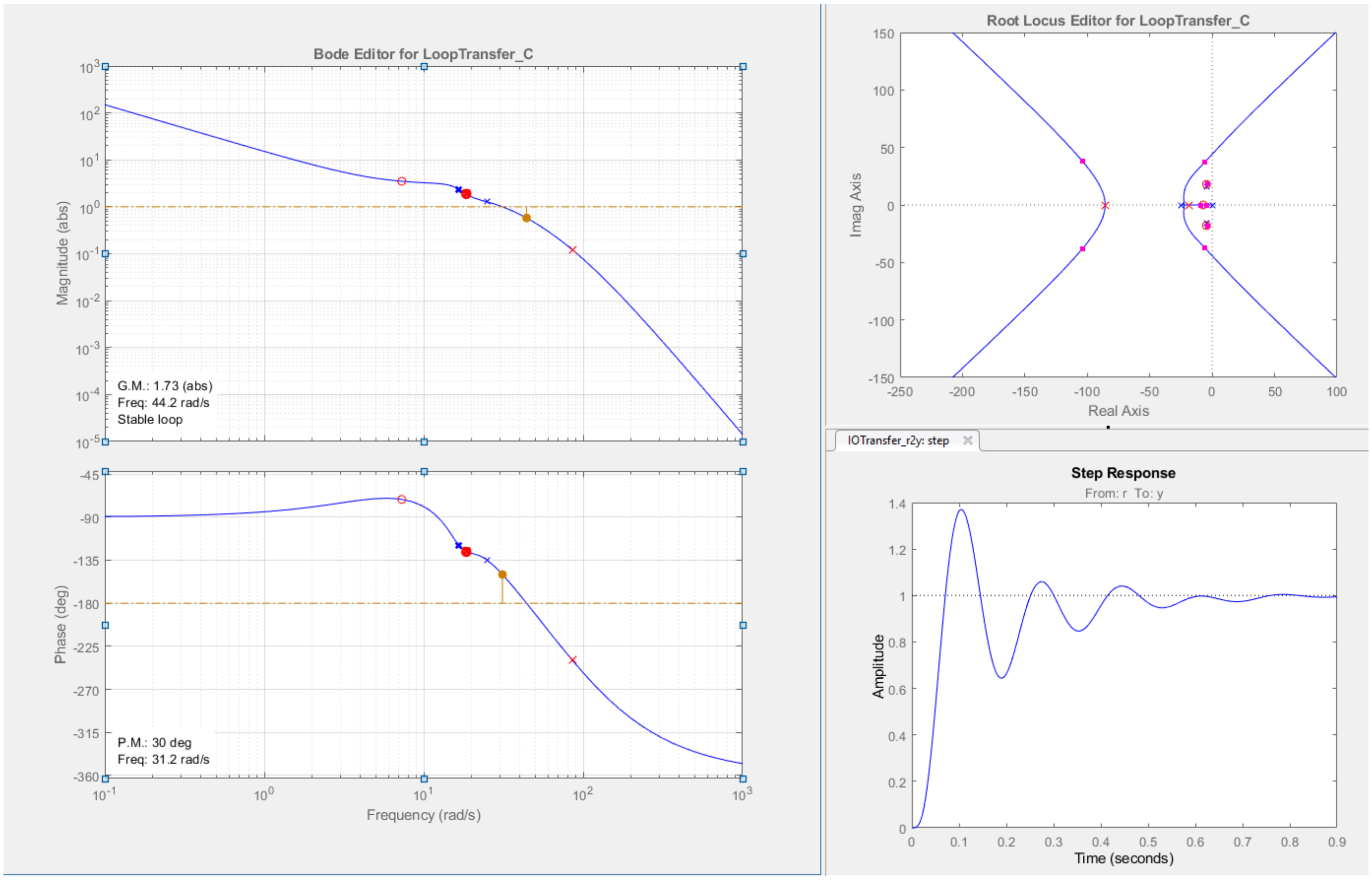

Figure 4: Bode Plots generated by the SISO tool after implementation of a double lead filter.

By calculating the gains produced by each element in the system and comparing it to desired gain at 25 rad/s, $K_p$ is calculated to be $K_p$ =17.6981 for the Simulink model.

## 1.5 Lag Filter Design

Through extensive iteration, the final values for the lag filter were established as c = 4, d = 0.045. The plots generated by the SISO tool are shown in figure 5. Observing the changes to the bode plot from the lag filter, gain margin has increased from 1.73 to 1.99 (abs). Phase margin has increased from 30 to 36.1 degrees at a frequency of 25 rad/s. The properties of the step response have not changed very drastically, with overshoot and rise time being relatively similar after the implementation of a lag filter. However, error created by the disturbance is significantly reduced from about 6 to nearly zero. The largest magnitude of the step disturbance is approximately 4.8, but drops off after the peak.

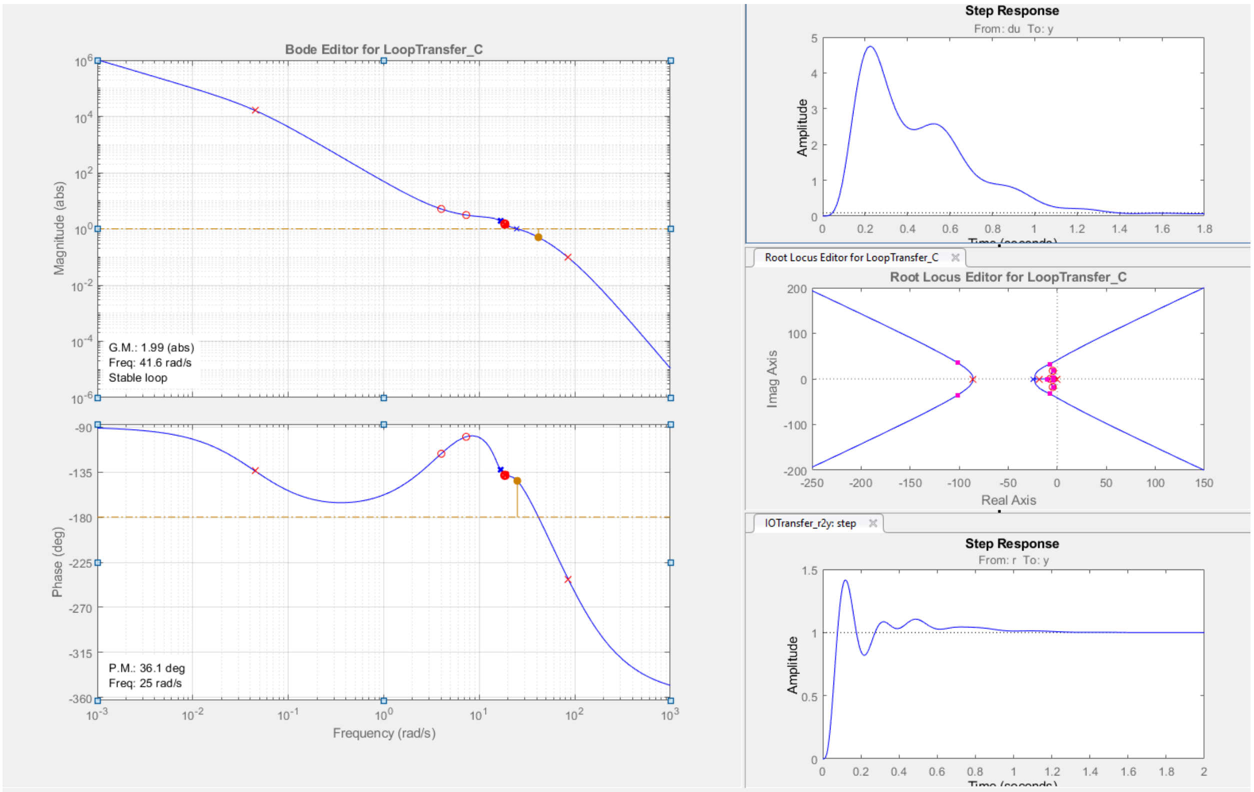


Figure 5: Lag Filter Bode Plots

With the addition of a lag filter, $K_p$ now has a value of 17.4758. The lag filter has a relatively small effect on $K_p$ as its contribution to gain when the input frequency is 25 rad/s is only 1.0127.

## 1.6 Feedforward Controller Design

m1 = 3.5065e-04

m2 = 1.8935e-04

b1 = 0.0099

b2 = 9.8814e-04

## 2.1 Control system with lead and notch filters

Non-collocated: The maximum cart 2 tracking error is 11.6256 mm and occurs at a time of 1.511 seconds. Collocated: The maximum cart 2 tracking error is 15.3013 mm and occurs at a time of 1.339 seconds. The graphs for non-collocated control are shown in Figure 6 and the graphs for collocated control are shown in Figure 7. In both models, it is observed that there is significant steady state error between xr and x2. While both non-collocated and collocated feedback models have position error, the non-collocated model has a better overall tracking accuracy. The maximum tracking accuracy for the non-collocated system is less than that of the collocated system. This is an expected result because cart 2 is the cart whose position that the system is intended to control. Therefore, when the feedback error measurement is taken from the cart 2, the system is able to achieve more accurate error tracking.

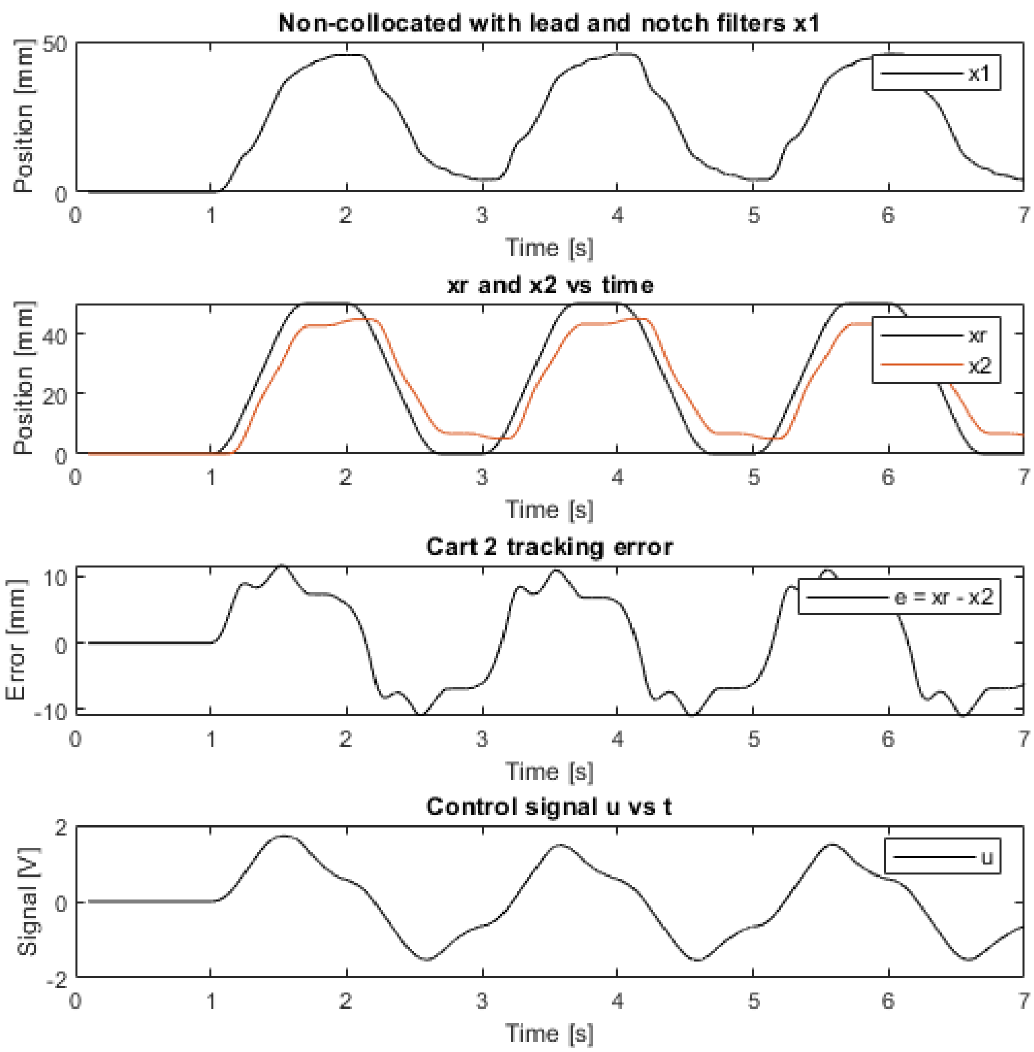


Figure 6: Simulated data when the carts are controlled via non-collocated control with a notch and double lead filters

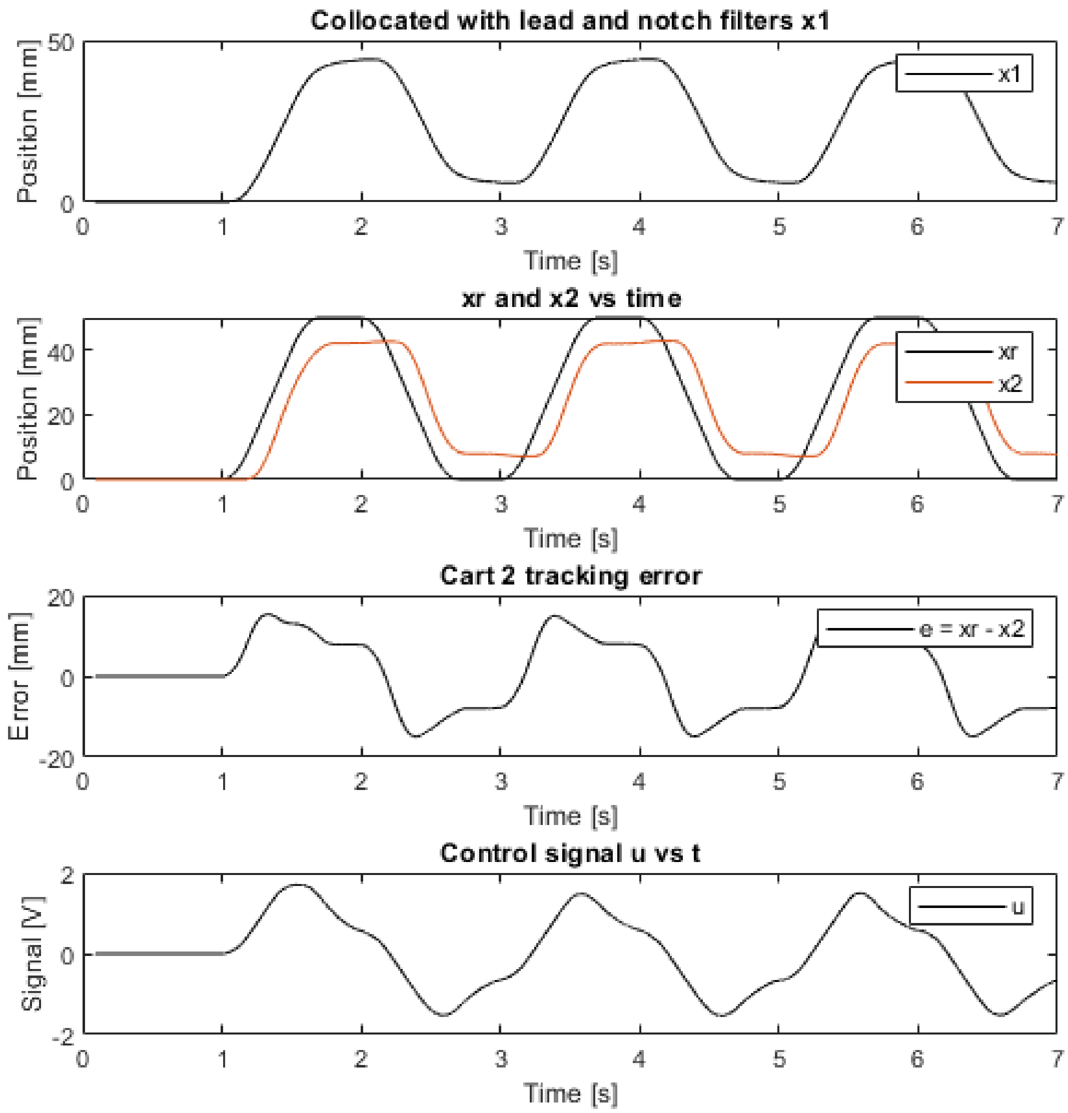


Figure 7: Simulated data when the carts are controlled via non-collocated control with a notch and double lead filters

## 2.2 Control system with lead, notch and lag filters

As can be seen in figure 8, for non-collocated control maximum tracking error occurs at: 3.241 seconds with a value of 11.91 mm. As can be seen in figure 9, for collocated control maximum tracking error occurs at: 4.328 seconds with a value of -17.5765 mm. These maximum error conditions occur just after a rapid change in the command position, essentially during the rise time.

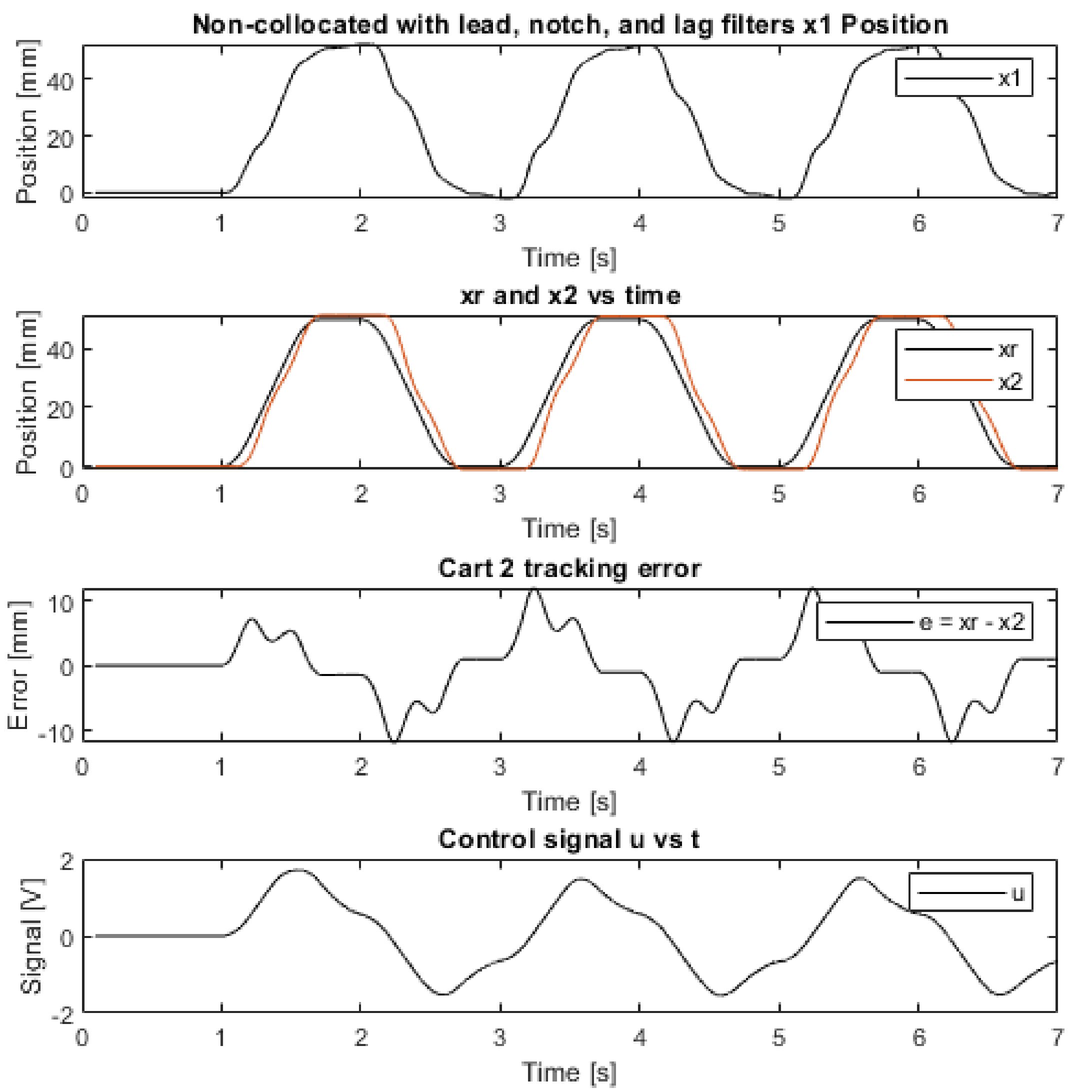


Figure 8: Simulated data when the carts are controlled via non-collocated control with a notch, double lead, and lag filter.

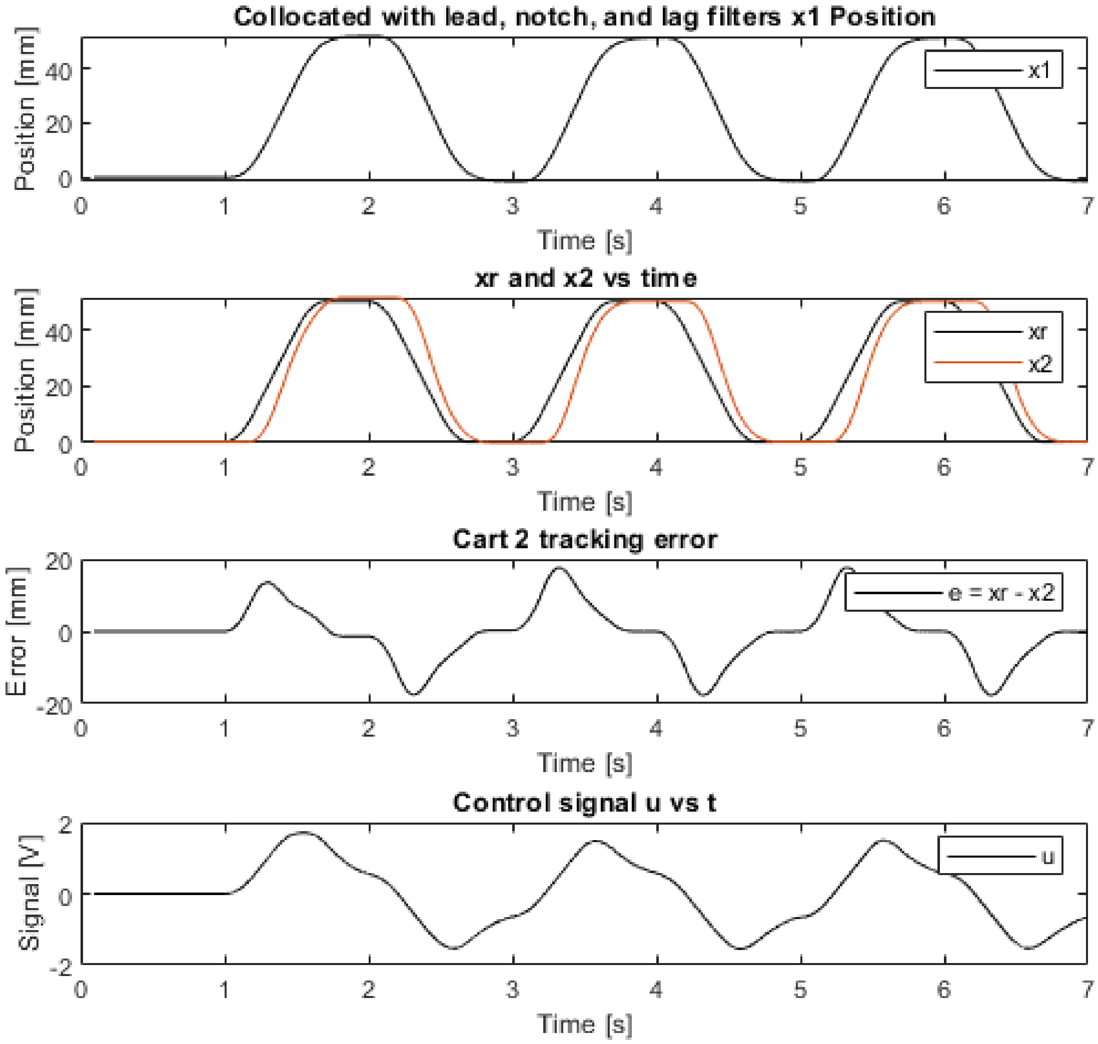


Figure 9: Simulated data when the carts are controlled via collocated control with a notch, double lead, and lag filter.

Compared to the system with only double lead and notch filters, the system with the added lag filter performs significantly better at steady state or close to steady state conditions. The system has approximately the same error during the initial rise, but has close to zero error at the final commanded position before the system starts moving again.

## 2.3 Control system with lead, notch, lag filters and feed forward loop

Figures 10 and 11 show the behaviour of both the collocated and non-collocated controllers with lead, notch, lag filters, and feedforward.

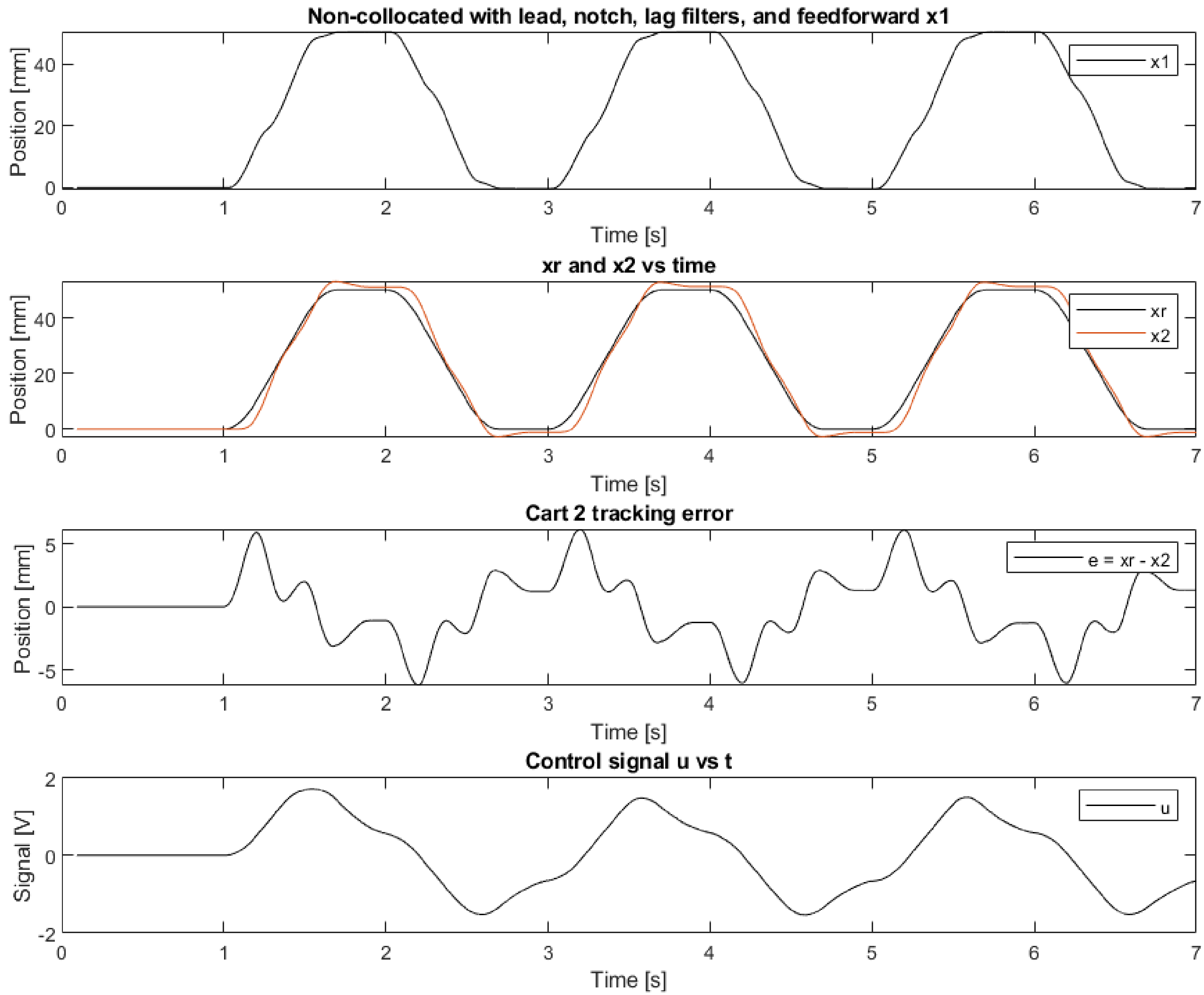


Figure 10: Simulated data when the carts are controlled via non-collocated control with a notch, double lead, lag filters, and feedforward.

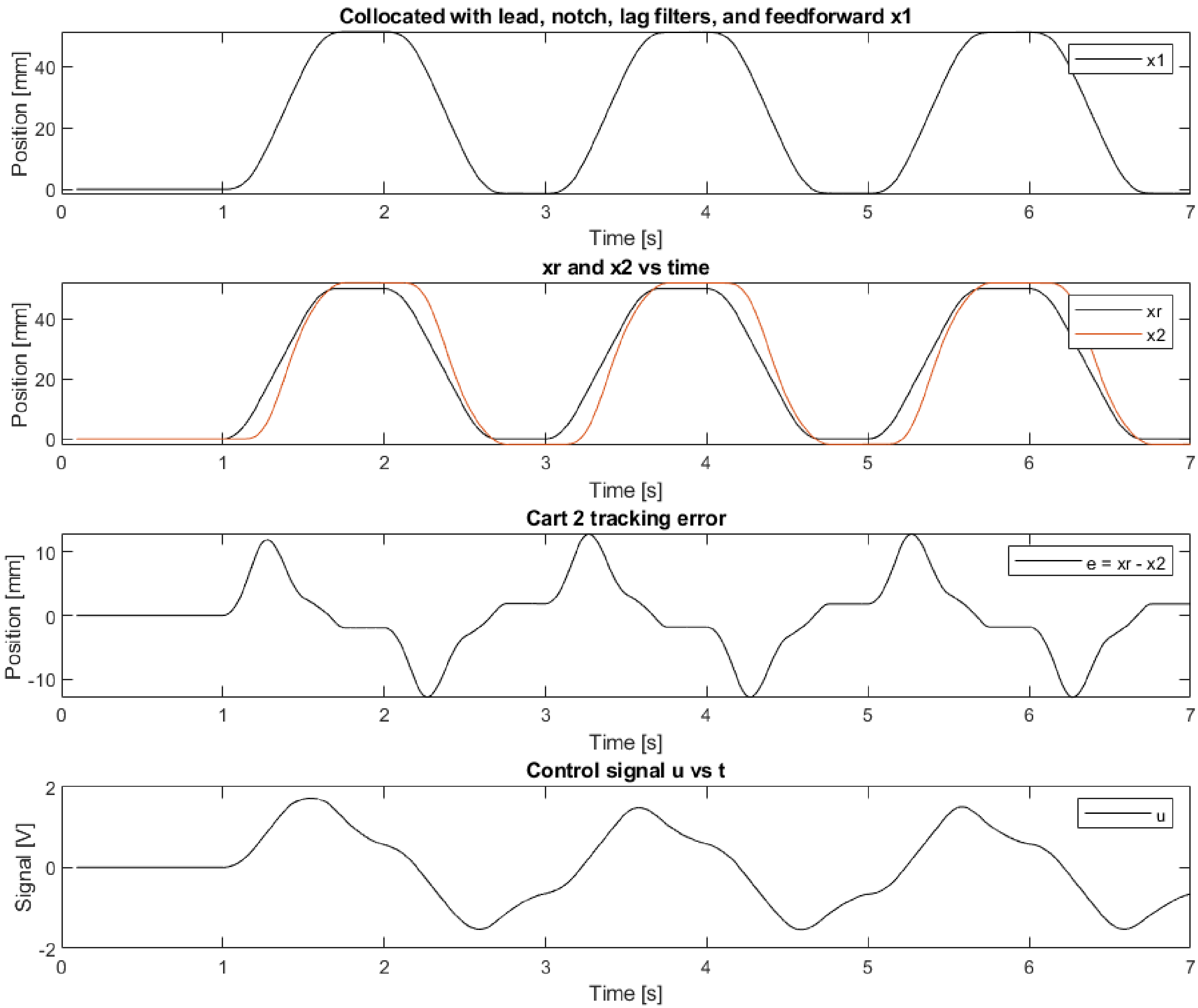


Figure 11: Simulated data when the carts are controlled via collocated control with a notch, double lead, lag filters, and feedforward.

Observing the changes in tracking error between feedforward and non feedforward plots, the magnitude of the tracking error is significantly reduced. The maximum tracking error without feedforward for collocated is -17.5765mm, compared to 12.7863mm with feedforward. The maximum tracking error without feedforward for non-collocated is 11.91mm, compared to 6.033mm with feedforward. This demonstrates that the feedforward implementation reduced tracking error by cancelling out anticipated inertial and viscous friction forces. Implementing feedforward allowed the controller to meet design specification #5.

Table 1: Time and magnitude of max tracking error for lead, notch, lag filters, and feedforward simulations

| Feedback type | Lead x2 and Notch Filters | | Lead x2, Notch, and Lag Filters | | Lead x2, Notch, Lag Filters and Feedforward | |
|---|---|---|---|---|---|---|
| | Max error (mm) | Time (s) | Max error (mm) | Time (s) | Max error (mm) | Time (s) |
| Collocated | 15.3013 | 1.339 | -17.5765 | 4.328 | 12.7863 | 5.263 |
| Non-collocated | 11.6256 | 1.511 | 11.91 | 3.241 | 6.033 | 3.184 |

Overall, as can be observed each new control system provides better and better accuracy, reducing the maximum tracking error experienced by cart 2. In all cases, non-collocated control is more effective than collocated control, which makes sense as we are measuring effectiveness by the positional error of cart 2, not cart 1.

## 2.4 Conclusions

Throughout the course of this project, knowledge was gained on the process behind designing a controls system. Throughout the course of this project we learned to use the SISO tool and were able to make real-time observations of the behaviour of our system when various filters were applied. We were thus able to directly observe the real world effects of notch filters, lead filters, and lag filters on step response behaviour and the system's frequency response. The project increased our intuition of dynamic systems and their stability.