Name: Tianpeng Zhang Date: Nov 13.

ES 155 Lab 2: Inverted Pendulum on a Cart

Objectives:
- ☐ Model an inverted pendulum cart system
- ☐ Use Simulink to implement open loop system
- ☐ Analyze stability of system
- ☐ Implement Controller on Hardware

NOTE: PART 1 SHOULD BE DONE OUTSIDE OF LAB
PART 2 WILL BE COMPLETED IN LAB

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MATLAB Values for Model Setup

```
g = 9.81;              % g accel [m/s^2]
mp = 0.230;            % mass of pendulum [kg]
l = 0.6413;            % length of pendulum [m]
r = l/2;               % radius to COM of pendulum [m]
J = (1/3)*mp*l^2;      % inertia of pendulum rotating about 1 end [kg-m^2]
γ = 0.0024;            % pendulum damping [N-m*s]
mc = 0.38;             % mass of cart [kg]
c = 0.90;              % cart damping [N-s/m]
```

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1.a Modeling The System

Using Newtonian or Lagrangian mechanics, we can find the equations of motion for our inverted pendulum system. We can construct two equations that consist of terms involving the desired states $x, \theta, \dot{x}, and\ \dot{\theta}$ for our system, where x is the horizontal displacement of the cart and θ is the angular displacement of the pendulum.

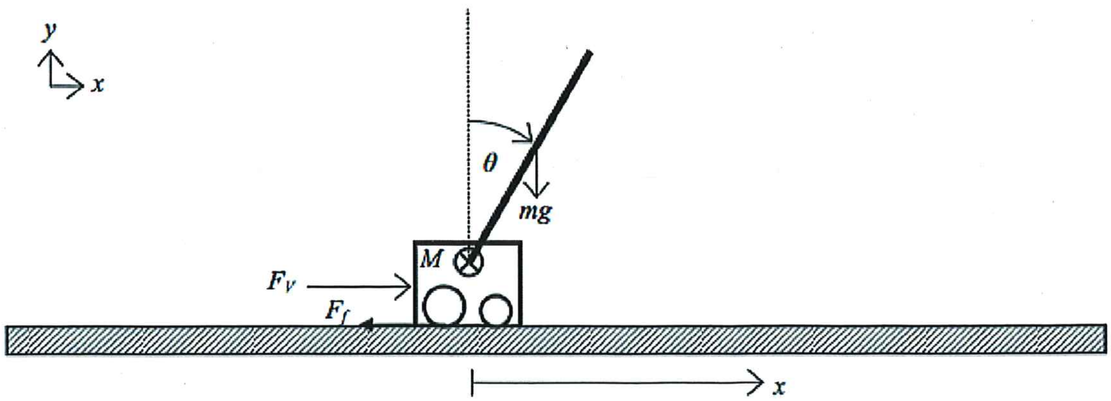


Figure 1: Inverted pendulum and cart system.

The nonlinear equations of motion for the cart-pendulum are:

$$(m_c + m_p)\ddot{x} + c\dot{x} = F + m_p r(sin\theta\ \dot{\theta}^2 - cos\theta\ \ddot{\theta})$$

$$(J + m_p r^2)\ddot{\theta} + \gamma\dot{\theta} = -m_p rgsin\theta - m_p r\ddot{x}cos\theta$$

where m_c and m_p are the mass of the cart and the pendulum respectively, r is the length of the pendulum, J is the moment of inertia about the center of mass of the pendulum, g the acceleration due to gravity, c and γ are damping coefficients. θ is defined as the deviation of the pendulum from the vertical ($\theta = 0$ is in the inverted standing position).

To model this system in state-space form, we linearize the system about the equilibrium point (θ = 0). Thus, the equations of motion become:

$$\overset{M}{\underline{(m_c + m_p)}}\ddot{x} + \overset{R}{\underline{m_p r}}\ddot{\theta} + c\dot{x} = \underline{f(t)}\ \boxed{u}$$

$$m_p r\ddot{x} + \underbrace{(J + m_p r^2)}_{K}\ddot{\theta} + \gamma\dot{\theta} - m_p gr\theta = 0$$

If we choose our state vector to be:

$$\vec{x} = [x\ \theta\ \dot{x}\ \dot{\theta}]^T$$

What are the open-loop A, B, C, and D matrices? The hardware on the inverted cart pendulum limits us to observing the position of the cart and the angle of the pendulum.

$\rightarrow$ C matrix

$$\dot{\vec{X}} = \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix}, \quad \vec{y} = [x\ \theta] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\vec{X} + 0\vec{u}$$

$\rightarrow$ D matrix

Let $(m_c+m_p) = M$, $m_p r = R$, $J + m_p r^2 = K$

Then $M\ddot{x} + R\ddot{\theta} + cx_3 = u$

$R\ddot{x} + K\ddot{\theta} + \gamma x_4 - gRx_2 = 0$

$$\ddot{\theta} = \frac{1}{R^2 - MK}\left[[0\ -MgR\ \ -cR\ \ M\gamma]\vec{X} + uR\right]$$

$$\ddot{x} = \frac{1}{R^2 - MK}\left[[0\ \ R^2 g\ \ Kc\ \ -\gamma R]\vec{X} + (-uK)\right]$$

$$\therefore \dot{\vec{X}} = \frac{1}{m_p^2 r^2 - (m_c+m_p)(J+m_p r^2)}\begin{bmatrix} 0 & 0 & m_p^2 r^2 - (m_c+m_p)(J+m_p r^2) & 0 \\ 0 & 0 & 0 & m_p^2 r^2 - (m_c+m_p)(J+m_p r^2) \\ 0 & m_p^2 r^2 g & c(J+m_p r^2) & -\gamma m_p r \\ 0 & -(m_c+m_p)m_p rg & -cm_p r & (m_c+m_p)m_p r \end{bmatrix}\vec{X} + \frac{1}{m_p^2 r^2 - (m_c+m_p)(J+m_p r^2)}\begin{bmatrix} 0 \\ 0 \\ -J - m_p r^2 \\ m_p r \end{bmatrix}\vec{u}$$

$\rightarrow$ A matrix

B matrix

1.b Stability Analysis

Open up a script in MATLAB and create a state-space model using the A, B, C, and D matrices you calculated above. Use the numerical values for the relevant parameters found at the end of this handout.

You can create a state-space model using the following command:

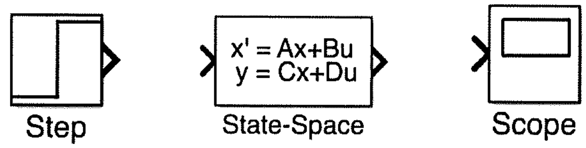
```
sys = ss(A, B, C, D);
```

Is the system stable or unstable at the equilibrium point ($\theta = 0$)?

Using Matlab we can calculate the eigenvalues of A. Which are $0, 3.7832, -4.4718, -1.3950$

Since $3.7832 > 0$, the system is unstable.

Implement this state-space model in Simulink. Use the state-space block to implement your model. All relevant blocks can be found in the Simulink Library browser:



In your state-space block, set your A, B, C, and D matrices to the variables “A”, “B”, “C”, and “D.” Write a MATLAB script to generate these matrices. When you run your Simulink model, the state-space block will automatically retrieve these matrices from your MATLAB workspace.

Provide a step force input with reasonable values and observe the step response using a scope. Save plots of (1) the step input, (2) the position of the cart, and (3) the angle of the pendulum.

How do the resulting step response plots correspond to your stability analysis above? Show your plots.

The system output goes unbounded, which is consistent with our analysis that the system is unstable.

1.c Full State Feedback

In order to stabilize the inverted pendulum, we need to implement a feedback control system. Try implementing and simulating a full state feedback controller (**u = -Kx**) using either Simulink or ODE 45. Write out your state space model with full state feedback below. Try different values for your K matrix (the 'place' function in Matlab will be helpful).

What K matrices worked well and which ones did not? Attach plots of the angle of the pendulum and the position of the cart.

$$\dot{x} = Ax + Bu$$
$$= Ax + (-Bkx)$$
$$\dot{x} = (A - BK)x.$$

$$y = Cx + Du$$
$$y = Cx$$

$$\therefore \begin{cases} \dot{x} = (A - BK)x \\ y = Cx \end{cases}$$ is the state space model with full state feed back.

Using only place function without looking at the Bode diagram does not yield very meaningful pole placement. Although the system is stable, there is significant overshoot. The K values we used include [-0.572, -15.47, -2.12, -4.02], [-0.572, -16.22, -2.17, -4.76], [-0.0006, -6.52, -0.91, -0.99], [-4.576, -36.55, -6.78, -10.25].

PART 2 : HARDWARE IMPLEMENTATION

Objectives:

- Implement a controller in real hardware
- Gain intuition about how the controller gains affect system performance

Background:

In this lab, you will implement a Proportional-Derivative (PD) controller on a real inverted pendulum on a cart system. By changing the controller gains and filters used to reconstruct states of the system, you will gain intuition about how these aspects affect the performance of the system. The control overall schemes and the detailed schemes are shown in Figure 1 and 2 separately. As shown in the figures, there are two measurements (position measurement and angle measurement) and one control input (force applied on the cart). Roughly, the PD controller implemented is in the form of

$$F(t) = K_p^x \cdot x(t) + K_d^x \cdot \dot{x}(t) + K_p^\theta \cdot \theta(t) + K_d^\theta \cdot \dot{\theta}(t)$$

where $K_p^x, K_d^x, K_p^\theta, K_d^\theta$ are the proportional and derivative control gain on the position and angle respectively.

You may notice that this controller structure looks the same as the full state controller that we designed in Part 1. However, note that in Figure 2 there is a filter before the derivative controller components. The reason is that in practice, it is hard to obtain accurate derivative measurements. As a result, a filter is applied to approximate the derivative measurements using the position (/angle) measurements. (We will talk about this in the later of the class.)

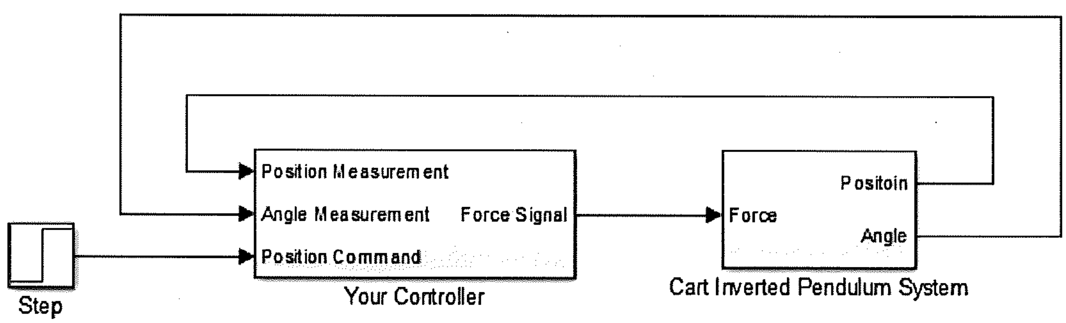


Figure 1: Control Overall Schemes

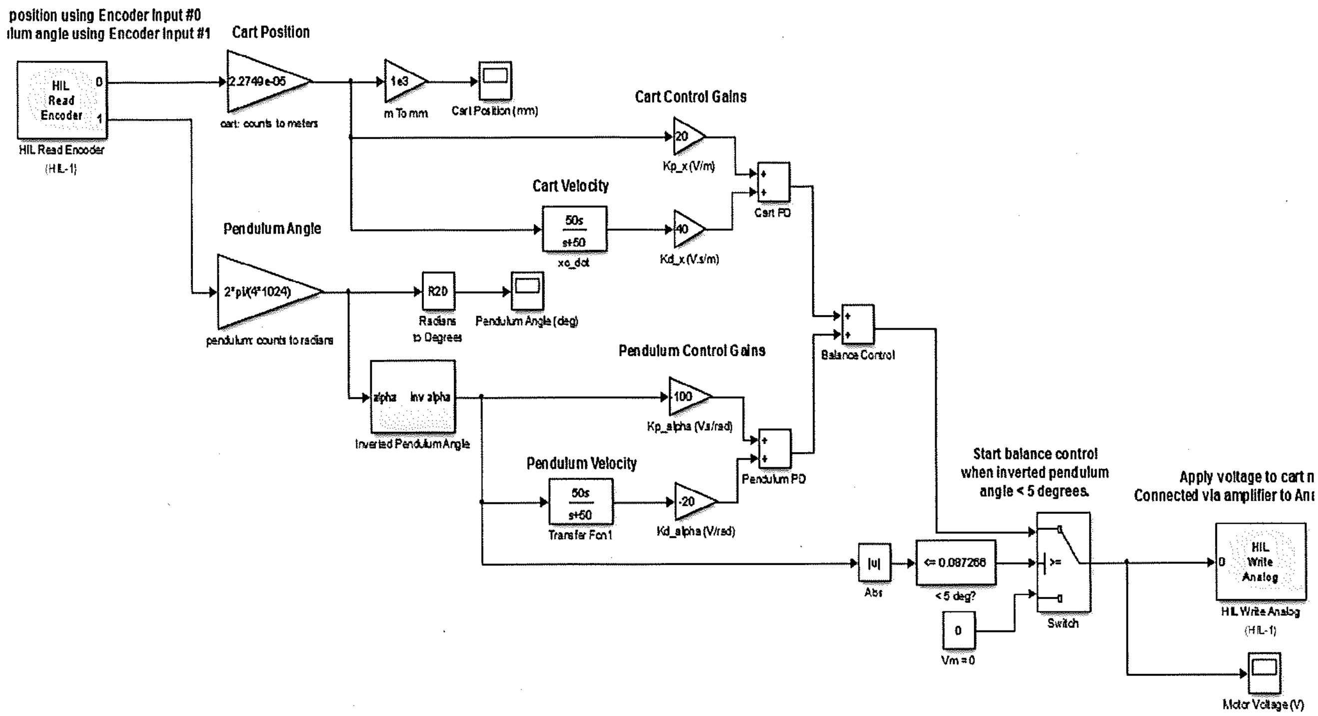


Figure 2: Detailed Schemes of Cart-Inverted Pendulum Controllers

Procedures:

Turn on the computer and login. Turn on the hardware power (for amplifier and sensor). Make sure all the instruments are in their proper positions and the pendulum swing area is cleared for safety.

To run the experiment:
Start MATLAB, open – C:\courses\es158\test\testpen\ip02_quick_start_good

Cart Gains K_p^x (V/m)	Cart Gains K_d^x (V.s/m)	Pendulum Gain K_p^0 (V/rad)	Pendulum Gain K_d^0 (V/rad)
20	40	-100	-20

To run the model (no change to any of the parameters):

a. Quarc – build
b. Quarc – run

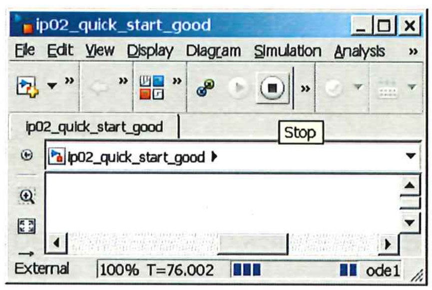
c. Slowly swing up the pendulum stick into the upward vertical position (note – when it is within ± 5 degs of the vertical line, then the controller will kick in)
d. What do you observe about the behavior of the pendulum? In response to providing a small disturbance into the system (like lightly tapping the stick), what do you observe?

Very cool, **isn't** it?

To stop the controller:

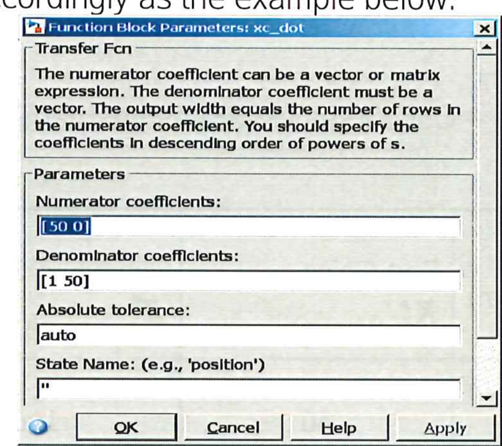
a. One member of your lab group can make a circle with his/her fingers around the pendulum (but do not touch it).

b. Then another member can click on the "**stop**" button (the square button)



c. The pendulum will fall momentarily; catch it and slowly swing it down to the downward vertical (pointing floor) position.

To change the cart or pendulum velocity parameters, double click on the block and change the values accordingly as the example below:



Now **let's** change some of the controller parameters per the table below, one line at a time: Observe the pendulum behavior, record what you see and explain the resulting effects based on what you learned so far:

Make sure to follow the above procedure when starting the controller. Implement the controller with the different parameters listed below, and record your observations as to the performance of the system.

Cart Gains K_p^x (V/m)	Cart Gains K_d^x (V.s/m)	Pendulum Gain K_p^θ (V/rad)	Pendulum Gain K_d^θ (V/rad)	Observation	Explain possible effects b/c of the parameter changes
20	40	-100	-20	Balanced & Very smooth	☺ factory setting
30	40	-100	-20	Balanced. The cart responds more violently	$K_p^x \uparrow$
50	40	-100	-20	Balanced, shaky The cart responds even more.	$K_p^x \uparrow$ Does better of staying at Cart x_0.
10	40	-100	-20	similar. Balanced. The cart responds less violently when pushed.	$K_p^x \downarrow$ Does worse of maintaining x_0 position.
5	40	-100	-20		
20	40	-100	-20		
20	40	-50	-20	Not as well balanced, It takes some oscillations before stablizes.	$K_p^\theta \downarrow$. Cart positioning is good.
20	40	-150	-20	It catches the pendulum well. But the cart's positioning is compromised	
20	40	-100	-30	Balance well But the cart encounters high freq oscillations around equilibrium point.	$K_d^\theta \uparrow$ with noise