Лабораторная работа №7
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Вариант №8

In[13]:=
```
fSRC := -6 * x * y + 4 * y * z + 3 * z ^ 2 - x - y + z - 10
TraditionalForm[fSRC]
```

Out[14]//TraditionalForm=

$$-6\,x\,y - x + 4\,y\,z - y + 3\,z^2 + z - 10$$

In[15]:=
```
A = {
{0, -3, 0},
{-3, 0, 2},
{0, 2, 3}
};
MatrixForm[A]
```

Out[16]//MatrixForm=

$$\begin{pmatrix} 0 & -3 & 0 \\ -3 & 0 & 2 \\ 0 & 2 & 3 \end{pmatrix}$$

In[17]:=
```
(*Матричное уравнение*)
l =.
AE = A - IdentityMatrix[3] * l;
MatrixForm[AE]
```

Out[19]//MatrixForm=

$$\begin{pmatrix} -l & -3 & 0 \\ -3 & -l & 2 \\ 0 & 2 & 3-l \end{pmatrix}$$

In[21]:=
```
myCharPoly = Det[AE]
```

Out[21]= $-27 + 13\,l + 3\,l^2 - l^3$

In[22]:=
```
wolframCharPoly = CharacteristicPolynomial[A, l]
```

Out[22]= $-27 + 13\,l + 3\,l^2 - l^3$

In[23]:=
```
(*Проверка равенства характеристических уравнений*)
FullSimplify[myCharPoly == wolframCharPoly]
```

Out[23]= True

In[105]:=
```
(*Собственные значения*)
sol = Solve[myCharPoly == 0, l]
```

Out[105]= $\{\{l \to -3.33\ldots\}, \{l \to 1.78\ldots\}, \{l \to 4.55\ldots\}\}$

In[25]:=
```
myEigenVals = l /. sol
```
Out[25]= $\{-3.33\ldots, 1.78\ldots, 4.55\ldots\}$

In[26]:=
```
% // N
```
Out[26]= {-3.33241, 1.77962, 4.5528}

In[106]:=
```
(*Проверка собственных значений*)
wolframEigenVals = Eigenvalues[A];
Sort[myEigenVals] == Sort[wolframEigenVals]
```
Out[107]= True

In[79]:=
```
X = {x, y, z};
myD1 = AE /. l → myEigenVals[[1]];
myD1.X // MatrixForm
myZ1 = myD1.X
myD2 = AE /. l → myEigenVals[[2]];
myD2.X // MatrixForm
myZ2 = myD2.X
myD3 = AE /. l → myEigenVals[[3]];
myD3.X // MatrixForm
myZ3 = myD3.X
```
Out[81]//MatrixForm=
$$\begin{pmatrix} -3y - x(-3.33\ldots) \\ -3x + 2z - y(-3.33\ldots) \\ 2y + z(3 - (-3.33\ldots)) \end{pmatrix}$$

Out[82]= $\{-3y - x(-3.33\ldots), -3x + 2z - y(-3.33\ldots), 2y + z(3 - (-3.33\ldots))\}$

Out[84]//MatrixForm=
$$\begin{pmatrix} -3y - x(1.78\ldots) \\ -3x + 2z - y(1.78\ldots) \\ 2y + z(3 - (1.78\ldots)) \end{pmatrix}$$

Out[85]= $\{-3y - x(1.78\ldots), -3x + 2z - y(1.78\ldots), 2y + z(3 - (1.78\ldots))\}$

Out[87]//MatrixForm=
$$\begin{pmatrix} -3y - x(4.55\ldots) \\ -3x + 2z - y(4.55\ldots) \\ 2y + z(3 - (4.55\ldots)) \end{pmatrix}$$

Out[88]= $\{-3y - x(4.55\ldots), -3x + 2z - y(4.55\ldots), 2y + z(3 - (4.55\ldots))\}$

In[89]:=
```
myEigenVec1 = Solve[myZ1 == 0 /. z → 1]
myEigenVec1 = {x, y, 1} /. myEigenVec1 [[1]]
myEigenVec2 = Solve[myZ2 == 0 /. z → 1]
myEigenVec2 = {x, y, 1} /. myEigenVec2 [[1]]
myEigenVec3 = Solve[myZ3 == 0 /. z → 1]
myEigenVec3 = {x, y, 1} /. myEigenVec3 [[1]]
```

Out[89]= $\left\{\left\{x \to \frac{1}{6} \times \left(4 + 3\sqrt{-3.33\ldots} - \left(\sqrt{-3.33\ldots}\right)^2\right),\ y \to \frac{1}{2} \times \left(-3 + \sqrt{-3.33\ldots}\right)\right\}\right\}$

Out[90]= $\left\{\frac{1}{6} \times \left(4 + 3\sqrt{-3.33\ldots} - \left(\sqrt{-3.33\ldots}\right)^2\right),\ \frac{1}{2} \times \left(-3 + \sqrt{-3.33\ldots}\right),\ 1\right\}$

Out[91]= $\left\{\left\{x \to \frac{1}{6} \times \left(4 + 3\sqrt{1.78\ldots} - \left(\sqrt{1.78\ldots}\right)^2\right),\ y \to \frac{1}{2} \times \left(-3 + \sqrt{1.78\ldots}\right)\right\}\right\}$

Out[92]= $\left\{\frac{1}{6} \times \left(4 + 3\sqrt{1.78\ldots} - \left(\sqrt{1.78\ldots}\right)^2\right),\ \frac{1}{2} \times \left(-3 + \sqrt{1.78\ldots}\right),\ 1\right\}$

Out[93]= $\left\{\left\{x \to \frac{1}{6} \times \left(4 + 3\sqrt{4.55\ldots} - \left(\sqrt{4.55\ldots}\right)^2\right),\ y \to \frac{1}{2} \times \left(-3 + \sqrt{4.55\ldots}\right)\right\}\right\}$

Out[94]= $\left\{\frac{1}{6} \times \left(4 + 3\sqrt{4.55\ldots} - \left(\sqrt{4.55\ldots}\right)^2\right),\ \frac{1}{2} \times \left(-3 + \sqrt{4.55\ldots}\right),\ 1\right\}$

In[108]:=
```
(*В первом списке - собственные числа,
дальше последовательные вектора для каждого из этих чисел.*)
wolframEigenSys = Eigensystem [A]
```

Out[108]= $\left\{\left\{\sqrt{4.55\ldots},\ \sqrt{-3.33\ldots},\ \sqrt{1.78\ldots}\right\},\right.$
$\left.\left\{\left\{\sqrt{-0.512\ldots},\ \sqrt{0.776\ldots},\ 1\right\},\ \left\{\sqrt{-2.85\ldots},\ \sqrt{-3.17\ldots},\ 1\right\},\ \left\{\sqrt{1.03\ldots},\ \sqrt{-0.610\ldots},\ 1\right\}\right\}\right\}$

In[95]:=
```
myEigenVals [[1]]
myEigenVals [[2]]
myEigenVals [[3]]
```

Out[95]= $\sqrt{-3.33\ldots}$

Out[96]= $\sqrt{1.78\ldots}$

Out[97]= $\sqrt{4.55\ldots}$

In[98]:=
```
(*Проверка собственных векторов*)
wolframEigenSys [[2, 2]] == N[myEigenVec1 ]
wolframEigenSys [[2, 3]] == N[myEigenVec2 ]
wolframEigenSys [[2, 1]] == N[myEigenVec3 ]
```

Out[98]= True

Out[99]= True

Out[100]= True

In[109]:=
```
(*Нормализация вектора*)
Normalize [myEigenVec1 ]
```

Out[109]=
$$\left\{\frac{4+3\sqrt{-3.33\ldots}-\left(\sqrt{-3.33\ldots}\right)^2}{6\sqrt{1+\frac{1}{4}\left(3-\sqrt{-3.33\ldots}\right)^2+\frac{1}{36}\left(-4-3\sqrt{-3.33\ldots}+\left(\sqrt{-3.33\ldots}\right)^2\right)^2}},\right.$$
$$\frac{-3+\sqrt{-3.33\ldots}}{2\sqrt{1+\frac{1}{4}\left(3-\sqrt{-3.33\ldots}\right)^2+\frac{1}{36}\left(-4-3\sqrt{-3.33\ldots}+\left(\sqrt{-3.33\ldots}\right)^2\right)^2}},$$
$$\left.\frac{1}{\sqrt{1+\frac{1}{4}\left(3-\sqrt{-3.33\ldots}\right)^2+\frac{1}{36}\left(-4-3\sqrt{-3.33\ldots}+\left(\sqrt{-3.33\ldots}\right)^2\right)^2}}\right\}$$

In[110]:=
```
(*Нормализация квадратичной формы*)
S = {Normalize [wolframEigenSys [[2, 1]]]
 , Normalize [wolframEigenSys [[2, 2]]]
  , Normalize [wolframEigenSys [[2, 3]]]
};
(*Матрица перехода в канонический базис*)
N[S] // MatrixForm
```

Out[111]//MatrixForm=
$$\begin{pmatrix} -0.374665 & 0.568592 & 0.732345 \\ -0.651363 & -0.723537 & 0.228519 \\ 0.659812 & -0.391404 & 0.641444 \end{pmatrix}$$

In[47]:=
```
a = {1, 1, 1};
a1 = Transpose[S].a;
p = wolframEigenSys [[1, 1]] * x1 ^ 2 + wolframEigenSys [[1, 2]] * y1 ^ 2 +
    wolframEigenSys [[1, 3]] * z1 ^ 2 + 2 * a1[[1]] * x1 + 2 * a1[[2]] * y1 + 2 * a1[[3]] * z1 - 10;
p = FullSimplify [p];
N[p] // TraditionalForm
p /. {x1 → x, y1 → y, z1 → z}
```

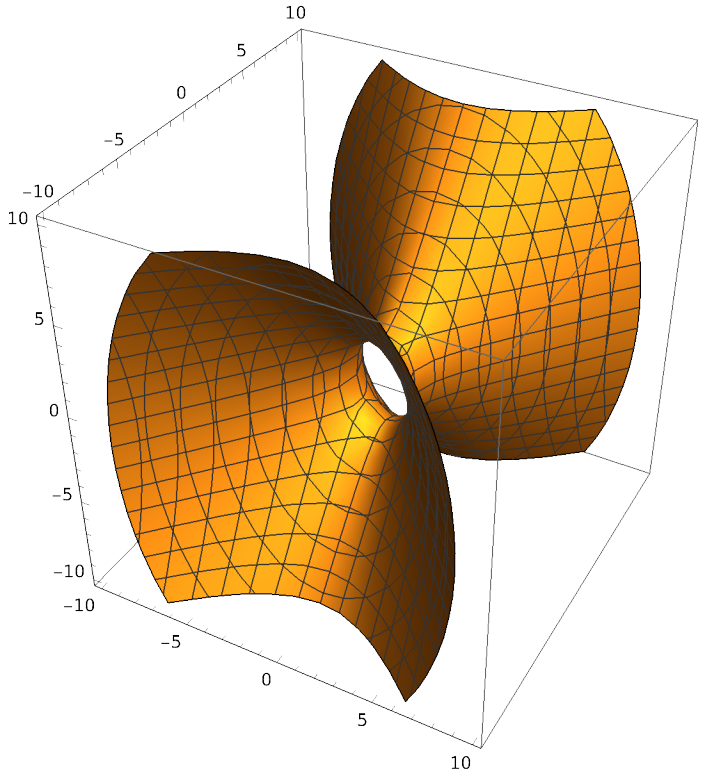
Out[51]//TraditionalForm=

$$4.5528\,(\text{x1}-1.\,\text{y1})\,(\text{x1}+\text{y1})-0.732432\,\text{x1}+1.77962\left(\text{z1}^2-1.\,\text{y1}^2\right)+3.\,\text{y1}^2-1.0927\,\text{y1}+3.20461\,\text{z1}-10.$$

Out[52]=
$$-10+3\,y^2+\left(-y^2+z^2\right)\boxed{1.78\ldots}+(x-y)\,(x+y)\boxed{4.55\ldots}+$$
$$2\,x\boxed{-0.366\ldots}+2\,y\boxed{-0.546\ldots}+2\,z\boxed{1.60\ldots}$$

In[58]:=
```
x =.
y =.
z =.
ft[a_, b_, c_] := p /. {x1 → a, y1 → b, z1 → c}
ContourPlot3D [ft[x, y, z] == 0, {x, -10, 10}, {y, -10, 10}, {z, -10, 10}]
```

Out[62]=



In[102]:=
```
p
```

Out[102]=
$$-10+3\,\text{y1}^2+\left(-\text{y1}^2+\text{z1}^2\right)\boxed{1.78\ldots}+(\text{x1}-\text{y1})\,(\text{x1}+\text{y1})\boxed{4.55\ldots}+$$
$$2\,\text{x1}\boxed{-0.366\ldots}+2\,\text{y1}\boxed{-0.546\ldots}+2\,\text{z1}\boxed{1.60\ldots}$$

In[66]:= **fSRC[x, y, z]**

Out[66]= $(-10 - x - y - 6\,x\,y + z + 4\,y\,z + 3\,z^2)[x, y, z]$