Понедельник (нижний) 13:00

Контрольная работа

Вариант-28

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1. $2x\cos^2 y dx + (2y - x^2 \sin 2y) dy = 0$
2. $y = (y')^2 + 2(y')^3$
3. $xy'' = y'(\ln y - \ln x)$
4. $yy'' + (y')^2 = 2yy'$

1. $2x\cos^2 y dx + (2y - x^2\sin 2y) dy = 0$

$M(x,y) = 2x\cos^2 y$

$N(x,y) = 2y - x^2 \sin 2y$

$\left.\begin{aligned} \frac{\partial M}{\partial y} &= 2x(-2\cos y \sin y) = -2x\sin 2y \\ \frac{\partial N}{\partial x} &= -2x\sin 2y \end{aligned}\right\} \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ - ур-ние в полных диф

$I = \int 2x\cos^2 y\, dx = x^2\cos^2 y + C(y)$

$\frac{\partial I}{\partial y} = -x^2 \cdot 2 \cdot \cos y \cdot \sin y + C'(y)$

$-x^2 \cdot 2 \cdot \cos y \cdot \sin y + C'(y) = -x^2 \sin 2y + 2y$

$\cancel{-x^2\sin 2y} + C'(y) = 2y \cancel{- x^2 \sin 2y}$

$C'(y) = 2y$

$\int C'(y) = \int 2y\, dy$

$C = y^2 + \tilde{C}$

$$\boxed{\begin{aligned} I &= x^2\cos^2 y + y^2 + \tilde{C} \\ \tilde{C} &= x^2\cos^2 y + y^2 \end{aligned}}$$

2. $y = (y')^2 + 2(y')^3$

$y' = p = \frac{dy}{dx}$ $\left(\frac{dy}{dx} = \frac{dy}{dp} \cdot \frac{dp}{dx}\right)$

$dx = \frac{dy}{p}$

$y = p^2 + 2p^3$

$y = 2p + 6p^2$

$dx = \frac{2p + 6p^2}{p} = 2 + 6p$

$x = \int (2 + 6p) dp = 2p + \frac{6p^2}{2} = 2p + 3p^2 + C$

$$\boxed{\begin{cases} y = p^2 + 2p^3 \\ x = 2p + 3p^2 + C \end{cases}}$$

3. $xy'' = y'(\ln y - \ln x)$

$y' = p,\ y'' = p'$

$xp' = p(\ln p - \ln x)$

$$xp' = p \cdot \ln\frac{p}{x}$$
$$p' = \frac{p}{x} \cdot \ln\frac{p}{x} \quad (x \neq 0)$$
$$\frac{p}{x} = z \Rightarrow p = z \cdot x \Rightarrow p' = z + z'x$$
$$z + z'x = z \cdot \ln z$$
$$z'x = z \cdot \ln z - z$$
$$z'x = z(\ln z - 1)$$
$$\frac{dz}{dx} = \frac{1}{x} \cdot z(\ln z - 1)$$
$$\frac{dz}{z(\ln z - 1)} = \frac{dx}{x}$$
$$\int \frac{dz}{z(\ln z - 1)} = \int \frac{dx}{x}$$
$$\int \frac{dz}{z(\ln z - 1)} = \int \frac{d(\ln z)}{\ln z - 1} = \int \frac{d(\ln z - 1)}{\ln z - 1} = \ln|\ln z - 1|$$
$$\ln|\ln z - 1| = \ln|x| + \ln|C|$$
$$\ln z - 1 = Cx$$
$$\ln z = Cx + 1$$
$$z = e^{Cx+1}$$
$$\frac{p}{x} = e^{Cx+1}$$
$$p = x \cdot e^{Cx+1}$$
$$\frac{dy}{dx} = xe^{Cx+1}$$
$$dy = xe^{Cx+1}\,dx$$
$$\int dy = \int xe^{Cx+1}\,dx$$
$$\int xe^{Cx+1}\,dx = e\int xe^{Cx}\,dx = e\left(\frac{xe^{Cx}}{C} - \int \frac{e^{Cx}}{C}\,dx\right) =$$
$$= e\left(\frac{xe^{Cx}}{C} - \frac{1}{C^2}\int e^{Cx}\,d(Cx)\right) = e\left(\frac{xe^{Cx}}{C} - \frac{1}{C^2} \cdot e^{Cx}\right) =$$
$$= \frac{xe^{Cx+1}}{C} - \frac{e^{Cx+1}}{C^2} + \tilde{C}$$
$$y = \frac{e^{Cx+1}(Cx - 1)}{C^2} + \tilde{C}$$
$$\boxed{y = \frac{e^{Ax+1}(Ax - 1)}{A^2} + B}$$

4. $yy'' + (y')^2 = 2yy'$

$$y'' = y''_{xx} = (y'_x)'_x = \frac{d}{dx}(y'_x) = \frac{d}{dy}\underbrace{(y'_x)}_{p} \cdot \underbrace{\frac{dy}{dx}}_{p} =$$
$$= p'_y \cdot p$$

$$y' = p, \quad y'' = p'_y \cdot p$$
$$y \cdot p'_y \cdot p + p^2 = 2yp$$
$$yp'_y + p = 2y \quad \text{и} \quad p = 0 \Rightarrow \boxed{y = C} \text{ решение}$$
$$p'_y + \frac{1}{y} \cdot p = 2 \quad (y \neq 0)$$

метод Бернулли.

$p = u \cdot v$

$p' = u'v + uv'$

$u'v + uv' + \frac{1}{y} \cdot u \cdot v = 2$

$\begin{cases} u'v + \frac{1}{y} \cdot u \cdot v = 0 \\ uv' = 2 \end{cases}$

$u'x + \frac{1}{y} \cdot u \cdot x = 0 \Rightarrow \frac{du}{dy} = -\frac{u}{y} \Rightarrow \frac{du}{u} = -\frac{dy}{y} \Rightarrow \ln|u| = -\ln|y| \Rightarrow$

$\Rightarrow u = \frac{1}{y}.$

$\frac{1}{y} \cdot v' = 2 \Rightarrow \frac{dv}{dy} = 2y \Rightarrow dv = 2y\,dy \Rightarrow v = y^2 + C$

$p = u \cdot v = \frac{1}{y}(y^2 + C) = y + \frac{C}{y}$

$p = y' = \frac{dy}{dx} = y + \frac{C}{y}$

$dx = \frac{dy}{y + \frac{C}{y}}$

$\int dx = \int \frac{y\,dy}{y^2 + C} = \frac{1}{2} \int \frac{d(y^2 + C)}{y^2 + C} = \frac{1}{2} \ln|y^2 + C|$

$x = \frac{1}{2} \ln|y^2 + C| + \tilde{C} \longrightarrow x = \frac{1}{2} \ln|y^2 + A| + B.$

(должно быть тоже самое, только там выражен y)

$x = \frac{1}{2} \ln|y^2 + A| + B$

$x - B = \frac{1}{2} \ln|y^2 + A|$

$2(x - B) = \ln|y^2 + A|$

$y^2 + A = e^{2(x-B)}$

$y^2 = e^{2(x-B)} - A$

$\boxed{\begin{array}{l} y = \pm\sqrt{e^{2(x-B)} - A} \\ y = C \end{array}}$