1 вариант

1. y''+4y=1/sin(2x)

Вар 51

① $y''+4y=\frac{1}{\sin 2x}$

1. $y''+4y=0$

$\lambda^2+4=0$

$\lambda^2-(2i)^2=0$

$(\lambda-2i)(\lambda+2i)=0$

$\lambda_1=2i,\ \lambda_2=-2i \Rightarrow e^{0x}\cos 2x,\ e^{0x}\sin 2x$

$\qquad\qquad\qquad\qquad \cos 2x \qquad \sin 2x$

$y=C_1\underbrace{\cos 2x}_{\varphi_1}+C_2\underbrace{\sin 2x}_{\varphi_2}$

2. $y=C_1(x)\cos(2x)+C_2(x)\sin(2x)$

$$\begin{pmatrix}\cos(2x) & \sin(2x)\\ -2\sin(2x) & 2\cos(2x)\end{pmatrix}\begin{pmatrix}C_1'\\ C_2'\end{pmatrix}=\begin{pmatrix}0\\ \frac{1}{\sin 2x}\end{pmatrix}$$

$$\Delta=\begin{vmatrix}\cos(2x) & \sin(2x)\\ -2\sin(2x) & 2\cos(2x)\end{vmatrix}=2\cos^2(2x)+2\sin^2(2x)=2$$

$$\Delta_1=\begin{vmatrix}0 & \sin(2x)\\ \frac{1}{\sin(2x)} & 2\cos(2x)\end{vmatrix}=-1$$

$$\Delta_2=\begin{vmatrix}\cos(2x) & 0\\ -2\sin(2x) & \frac{1}{\sin(2x)}\end{vmatrix}=\mathrm{ctg}\,2x$$

$C_1'(x)=\frac{\Delta_1}{\Delta}=-\frac{1}{2}\Rightarrow C_1(x)=-\frac{1}{2}\int dx=-\frac{1}{2}x+A$

$C_2'(x)=\frac{\Delta_2}{\Delta}=\frac{\mathrm{ctg}\,2x}{2}\Rightarrow C_2(x)=\frac{1}{2}\int \mathrm{ctg}\,2x\,dx=$

$=\frac{1}{2}\left(\frac{1}{2}\ln|\sin 2x|+B\right)=\frac{1}{4}\ln|\sin 2x|+B$

Ответ: $y=\left(-\frac{1}{2}x+A\right)\cos 2x+\left(\frac{1}{4}\ln|\sin 2x|+B\right)\sin 2x$

2. x’=x+y
   y’=-10x-y, x(0)=1, y(0)=-1

② $\begin{cases} \dot{x} = x+y \\ \dot{y} = -10x - y \end{cases}$ $x(0)=1,\ y(0)=-1$

$A = \begin{pmatrix} 1 & 1 \\ -10 & -1 \end{pmatrix}$ $\begin{vmatrix} 1-\lambda & 1 \\ -10 & -1-\lambda \end{vmatrix} = 0$

$-1-\lambda+\lambda+\lambda^2+10=0$

$\lambda^2+9=0$

$\lambda^2-(3i)^2=0$

$(\lambda-3i)(\lambda+3i)=0$

$\lambda_1=3i \quad \lambda_2=-3i$

Выберем $\lambda=-3i$

$\left(\begin{array}{cc|c} 1+3i & 1 & 0 \\ -10 & -1+3i & 0 \end{array}\right) \to \bar{S} = \begin{pmatrix} -1+3i \\ 10 \end{pmatrix}$

$\bar{S}e^{\lambda t} = \begin{pmatrix} -1+3i \\ 10 \end{pmatrix} e^{-3it} = \begin{pmatrix} -1+3i \\ 10 \end{pmatrix} e^{(-3t)i} =$

$= \left[\begin{pmatrix} -1 \\ 10 \end{pmatrix} + i\begin{pmatrix} 3 \\ 0 \end{pmatrix}\right](\cos(-3t)+i\sin(-3t)) = \begin{pmatrix} -1 \\ 10 \end{pmatrix}\cos(-3t) +$

$+ i\begin{pmatrix} 3 \\ 0 \end{pmatrix}\cos(-3t) + i\begin{pmatrix} -1 \\ 10 \end{pmatrix}\sin(-3t) - \begin{pmatrix} 3 \\ 0 \end{pmatrix}\sin(-3t) =$

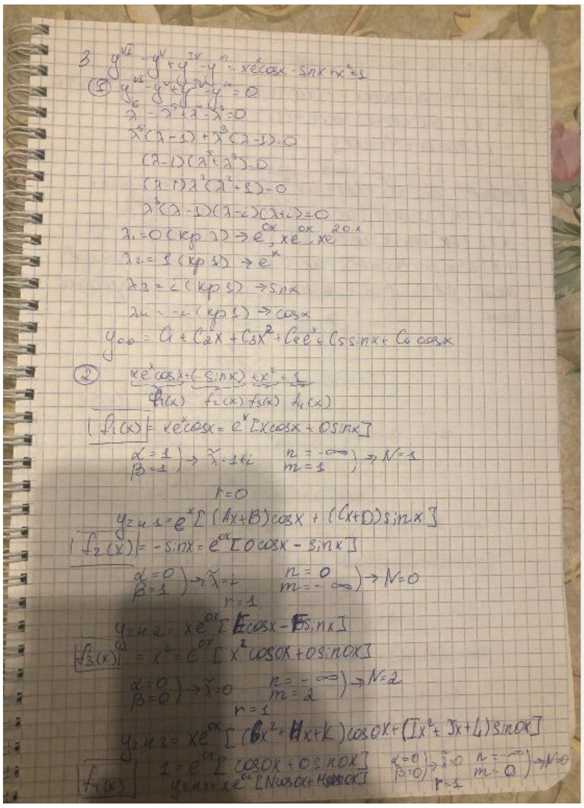
$= \begin{pmatrix} -\cos(-3t)-3\sin(-3t) \\ 10\cos(-3t) \end{pmatrix} + i\begin{pmatrix} 3\cos(-3t)-\sin(-3t) \\ 10\sin(-3t) \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = C_1\begin{pmatrix} -\cos(-3t)-3\sin(-3t) \\ 10\cos(-3t) \end{pmatrix} + C_2\begin{pmatrix} 3\cos(-3t)-\sin(-3t) \\ 10\sin(-3t) \end{pmatrix}$

$\begin{cases} x(0) = C_1(-3\sin 0) + C_2(-\sin 0) = -3C_1 - C_2 \\ y(0) = C_2(10\sin 0) = 10C_2 \end{cases}$

$\begin{cases} -3C_1 - C_2 = 1 \to C_1 = -0{,}3 \\ 10C_2 = -1 \to C_2 = -0{,}1 \end{cases}$

3. y(6)-y(5)+y(5)-y(3)=xe^(x)cosx-sinx+x^2+1



3\. $y^{VI}-y^{V}+y^{IV}-y^{III}=xe^{x}\cos x-\sin x+x^2+1$

(1) $y^{VI}-y^{V}+y^{IV}-y^{III}=0$

$\lambda^6-\lambda^5+\lambda^4-\lambda^3=0$

$\lambda^5(\lambda-1)+\lambda^3(\lambda-1)=0$

$(\lambda-1)(\lambda^5+\lambda^3)=0$

$(\lambda-1)\lambda^3(\lambda^2+1)=0$

$\lambda^3(\lambda-1)(\lambda-i)(\lambda+i)=0$

$\lambda_1=0\ (\text{кр } 3)\Rightarrow e^{0x}, xe^{0x}, x^2e^{0x}$

$\lambda_2=1\ (\text{кр } 1)\Rightarrow e^{x}$

$\lambda_3=i\ (\text{кр } 1)\Rightarrow \sin x$

$\lambda_4=-i\ (\text{кр } 1)\Rightarrow \cos x$

$y_{o.o}=C_1+C_2x+C_3x^2+C_4e^x+C_5\sin x+C_6\cos x$

(2) $\underbrace{xe^x\cos x}_{f_1(x)}+\underbrace{(-\sin x)}_{f_2(x)}+\underbrace{x^2}_{f_3(x)}+\underbrace{1}_{f_4(x)}$

$\boxed{f_1(x)}=xe^x\cos x=e^x[x\cos x+0\sin x]$

$\left.\begin{matrix}\alpha=1\\ \beta=1\end{matrix}\right)\to\tilde{\lambda}=1+i\quad\left.\begin{matrix}n=-\infty\\ m=1\end{matrix}\right)\to N=1$

$r=0$

$y_{r.н.1}=e^x[(Ax+B)\cos x+(Cx+D)\sin x]$

$\boxed{f_2(x)}=-\sin x=e^{0x}[0\cos x-\sin x]$

$\left.\begin{matrix}\alpha=0\\ \beta=1\end{matrix}\right)\to\tilde{\lambda}=i\quad\left.\begin{matrix}n=0\\ m=-\infty\end{matrix}\right)\to N=0$

$r=1$

$y_{r.н.2}=xe^{0x}[E\cos x-F\sin x]$

$\boxed{f_3(x)}=x^2=e^{0x}[x^2\cos 0x+0\sin 0x]$

$\left.\begin{matrix}\alpha=0\\ \beta=0\end{matrix}\right)\to\tilde{\lambda}=0\quad\left.\begin{matrix}n=-\infty\\ m=2\end{matrix}\right)\to N=2$

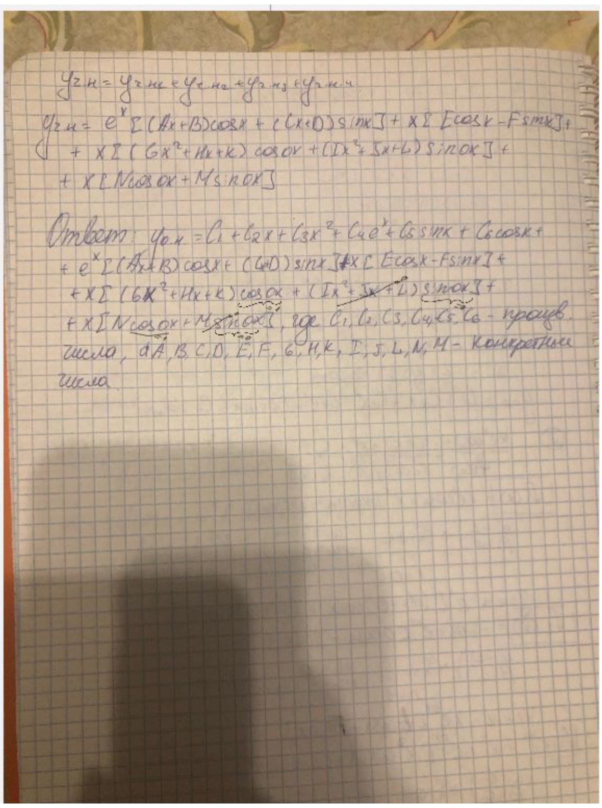
$r=1$

$y_{r.н.3}=xe^{0x}[(Gx^2+Hx+K)\cos 0x+(Ix^2+Jx+L)\sin 0x]$

$\boxed{f_4(x)}$ $1=e^{0x}[\cos 0x+0\sin 0x]$ $\left.\begin{matrix}\alpha=0\\ \beta=0\end{matrix}\right)\to\tilde{\lambda}=0\quad\left.\begin{matrix}n=-\infty\\ m=0\end{matrix}\right)\to N=0$

$r=1$

$y_{r.н.4}=xe^{0x}[N\cos 0x+M\sin 0x]$



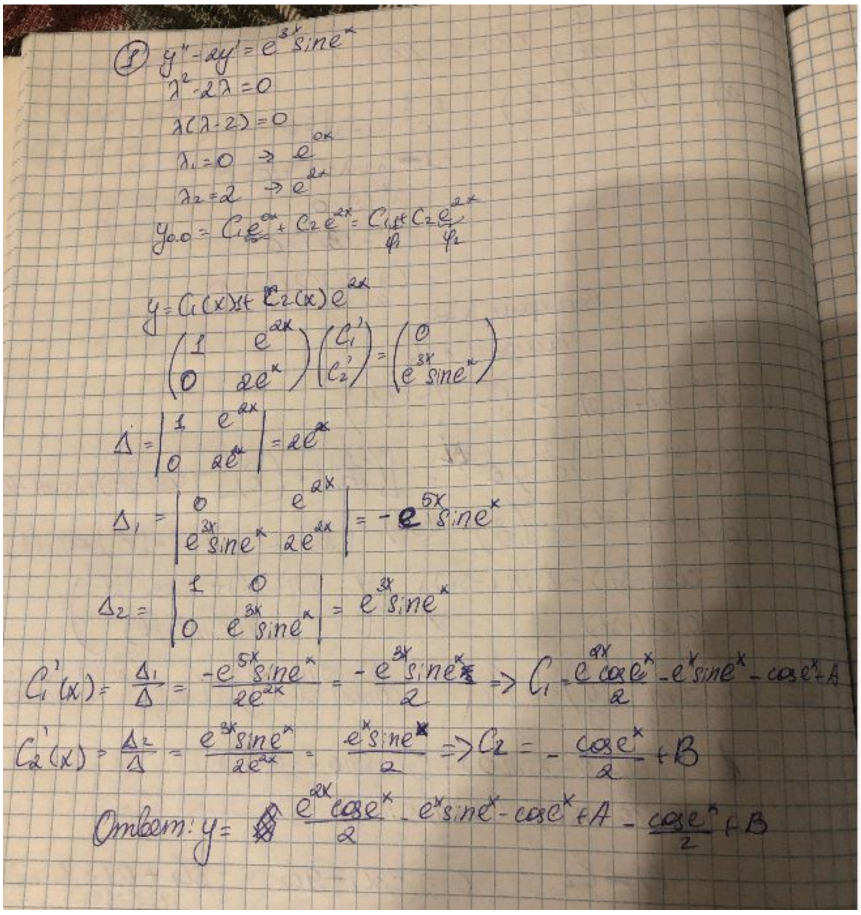
$y_{r.н}=y_{r.н1}+y_{r.н2}+y_{r.н3}+y_{r.н4}$

$y_{r.н}=e^x[(Ax+B)\cos x+(Cx+D)\sin x]+x[E\cos x-F\sin x]+$
$+x[(Gx^2+Hx+K)\cos 0x+(Ix^2+Jx+L)\sin 0x]+$
$+x[N\cos 0x+M\sin 0x]$

Ответ: $y_{o.н}=C_1+C_2x+C_3x^2+C_4e^x+C_5\sin x+C_6\cos x+$
$+e^x[(Ax+B)\cos x+(Cx+D)\sin x]+x[E\cos x-F\sin x]+$
$+x[(Gx^2+Hx+K)\cos 0x+(Ix^2+Jx+L)\sin 0x]+$
$+x[N\cos 0x+M\sin 0x]$, где $C_1, C_2, C_3, C_4, C_5, C_6$ - произв. числа, A, B, C, D, E, F, G, H, K, I, J, L, N, M - конкретные числа

2 вариант

1. y’’-2y’=e^(3x)sin(e^x)



(1) $y''-2y'=e^{3x}\sin e^x$

$\lambda^2-2\lambda=0$

$\lambda(\lambda-2)=0$

$\lambda_1=0\ \Rightarrow e^{0x}$

$\lambda_2=2\ \Rightarrow e^{2x}$

$y_{o.o}=C_1e^{0x}+C_2e^{2x}=C_1\underset{\varphi_1}{}+C_2\underset{\varphi_2}{e^{2x}}$

$y=C_1(x)+C_2(x)e^{2x}$

$$\begin{pmatrix}1 & e^{2x}\\ 0 & 2e^{2x}\end{pmatrix}\begin{pmatrix}C_1'\\ C_2'\end{pmatrix}=\begin{pmatrix}0\\ e^{3x}\sin e^x\end{pmatrix}$$

$$\Delta=\begin{vmatrix}1 & e^{2x}\\ 0 & 2e^{2x}\end{vmatrix}=2e^{2x}$$

$$\Delta_1=\begin{vmatrix}0 & e^{2x}\\ e^{3x}\sin e^x & 2e^{2x}\end{vmatrix}=-e^{5x}\sin e^x$$

$$\Delta_2=\begin{vmatrix}1 & 0\\ 0 & e^{3x}\sin e^x\end{vmatrix}=e^{3x}\sin e^x$$

$C_1'(x)=\frac{\Delta_1}{\Delta}=\frac{-e^{5x}\sin e^x}{2e^{2x}}=-\frac{e^{3x}\sin e^x}{2}\Rightarrow C_1=\frac{e^{2x}\cos e^x}{2}-e^x\sin e^x-\cos e^x+A$

$C_2'(x)=\frac{\Delta_2}{\Delta}=\frac{e^{3x}\sin e^x}{2e^{2x}}=\frac{e^x\sin e^x}{2}\Rightarrow C_2=-\frac{\cos e^x}{2}+B$

Ответ: $y=\frac{e^{2x}\cos e^x}{2}-e^x\sin e^x-\cos e^x+A-\frac{\cos e^x}{2}+B$

2. x’=5x+4y
y’=4x+5y, x(0)=-4, y(0)=1

② $\begin{cases} \dot{x} = 5x+4y \\ \dot{y} = 4x+5y \end{cases}$ $x(0) = -4,\ y(0) = 1$

$A = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$ $\begin{vmatrix} 5-\lambda & 4 \\ 4 & 5-\lambda \end{vmatrix} = 0$

$(5-\lambda)^2 - 16 = 0$

$25 - 10\lambda + \lambda^2 - 16 = 0$

$\lambda^2 - 10\lambda + 9 = 0$

$\lambda_{1,2} = \frac{10 \pm 8}{2} = \left[\begin{array}{l} 9 \\ 1 \end{array}\right.$

$\lambda_1 = 9: \left(\begin{array}{cc|c} -4 & 4 & 0 \\ 4 & -4 & 0 \end{array}\right) \to S_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = 1: \left(\begin{array}{cc|c} 4 & 4 & 0 \\ 4 & 4 & 0 \end{array}\right) \to S_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{9t} + C_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t}$

$x(t) = C_1 e^{9t} - C_2 e^{t} \to x(0) = C_1 - C_2 = -4$

$y(t) = C_1 e^{9t} + C_2 e^{t} \to y(0) = C_1 + C_2 = 1$

$\begin{cases} C_1 + C_2 = 1 \\ C_1 - C_2 = -4 \end{cases} \to \begin{cases} C_1 = C_2 - 4 \\ C_2 - 4 + C_2 = 1 \end{cases} \to \begin{cases} 2C_2 = 5 \\ C_1 = C_2 - 4 \end{cases} \to \begin{cases} C_2 = 2{,}5 \\ C_1 = -1{,}5 \end{cases}$

3. y'''+4y''+5y'=e^(-2x)sinx+cosx+xe^(-2x)+x

3. $y''' + 4y'' + 5y' = e^{-2x}\sin x + \cos x + xe^{-2x} + x$

① $y''' + 4y'' + 5y' = 0$

$\lambda^3 + 4\lambda^2 + 5\lambda = 0$

$\lambda(\lambda^2 + 4\lambda + 5) = 0$

$\lambda_1 = 0 \to e^{0x}$ $\quad \lambda^2 + 4\lambda + 5 = 0$

$D = 16 - 20 = -4 = (2i)^2$

$\lambda_{2,3} = \frac{-4 \pm 2i}{2} = -2 \pm i = \left[\begin{array}{l} -2-i \to e^{-2x}\sin x \\ -2+i \to e^{-2x}\cos x \end{array}\right.$

$y_{oo} = C_1 + C_2 e^{-2x}\sin x + C_3 e^{-2x}\cos x$

② $\underbrace{e^{-2x}\sin x}_{f_1(x)} + \underbrace{\cos x}_{f_2(x)} + \underbrace{xe^{-2x}}_{f_3(x)} + \underbrace{x}_{f_4(x)}$

$\boxed{f_1(x)} = e^{-2x}\sin x = e^{-2x}[0\cos x + \sin x]$

$\left.\begin{array}{l} \alpha = -2 \\ \beta = 1 \end{array}\right) \to \tilde{\lambda} = -2+i \qquad \left.\begin{array}{l} n = 0 \\ m = -\infty \end{array}\right) \to N = 0$

$\downarrow$

$r = 1$

$y_{ч.н.1} = xe^{-2x}[A\cos x + B\sin x]$

$\boxed{f_2(x)} = \cos x = e^{0x}[\cos x + 0\sin x]$

$\left.\begin{array}{l} \alpha = 0 \\ \beta = 1 \end{array}\right) \to \tilde{\lambda} = i \qquad \left.\begin{array}{l} n = -\infty \\ m = 0 \end{array}\right) \to N = 0 \quad r = 0$

$y_{ч.н.2} = C\cos x + D\sin x$

$\boxed{f_3(x)} = xe^{-2x} = e^{-2x}[x\cos 0x + 0\sin 0x]$

$\left.\begin{array}{l} \alpha = -2 \\ \beta = 0 \end{array}\right) \to \tilde{\lambda} = -2 \qquad \left.\begin{array}{l} n = -\infty \\ m = 1 \end{array}\right) \to N = 1 \quad r = 0$

$y_{ч.н.3} = e^{-2x}[(Ex + F)\cos 0x + (Gx + H)\sin 0x]$

$\boxed{f_4(x)} = x = e^{0x}[x\cos 0x + 0\sin 0x]$

$\left.\begin{array}{l} \alpha = 0 \\ \beta = 0 \end{array}\right) \to \tilde{\lambda} = 0 \qquad \left.\begin{array}{l} n = -\infty \\ m = 1 \end{array}\right) \to N = 1 \qquad r = 1$

$y_{ч.н.4} = x[(Jx + K)\cos 0x + (Lx + M)\sin 0x]$

Ответ: $y_{он} = xe^{-2x}[A\cos x + B\sin x] + (C\cos x + D\sin x) + e^{-2x}[(Ex+F)\cos 0x + (Gx+H)\sin 0x] + x[(Jx+K)\cos 0x + (Lx+M)\sin 0x] + C_1 + C_2 e^{-2x}\sin x + C_3 e^{-2x}\cos x$, где C_1, C_2, C_3 – произв. числа, $A, B, C, D, E, F, G, H, J, K, L, M$ – конкретные числа.

3 вариант

1. y''+8y'+16y=e^(4x)/(x^2+9)

$$y''+8y'+16y=\frac{e^{4x}}{x^2+9}$$

$$\lambda^2+8\lambda+16=0$$

$$\lambda=-4\ (\text{кр } 2)$$

$y'' + 8y' + 16y = \frac{e^{4x}}{x^2+9}$

$$y_{o.g}=C_1e^{-4x}+C_2xe^{-4x}$$

$$\Delta=\begin{vmatrix} e^{-4x} & xe^{-4x} \\ -4e^{-4x} & e^{-4x}-4xe^{-4x} \end{vmatrix}=e^{-8x}-4xe^{-8x}+4xe^{-8x}=e^{-8x}$$

$$\Delta_1=\begin{vmatrix} 0 & xe^{-4x} \\ \frac{e^{4x}}{x^2+9} & e^{-4x}-4xe^{-4x} \end{vmatrix}=-\frac{x}{x^2+9}$$

$$\Delta_2=\begin{vmatrix} e^{-4x} & 0 \\ -4e^{-4x} & \frac{e^{4x}}{x^2+9} \end{vmatrix}=\frac{1}{x^2+9}$$

$$\frac{\Delta_2}{\Delta}=\frac{e^{8x}}{(x^2+9)}=C_2'(x)$$

$$C_1(x)=\frac{\Delta_1}{\Delta}=\int-\frac{xe^{8x}}{x^2+9}dx$$

$$C_2(x)=\int\frac{e^{8x}}{(x^2+9)}dx=$$

Решение

$$y(x)=c_1e^{-4x}+c_2e^{-4x}x-\frac{1}{2}e^{72i-4(x+9i)}\,\mathrm{Ei}(12(x-3i))-\frac{1}{6}ie^{72i-4(x+9i)}x\,\mathrm{Ei}(12(x-3i))-\frac{1}{2}e^{-4(x+9i)}\,\mathrm{Ei}(12(x+3i))+\frac{1}{6}ie^{-4(x+9i)}x\,\mathrm{Ei}(12(x+3i))$$

2. x’=4x+5y
   y’=-4x-4y, x(0)=0, y(0)=1

$$\begin{cases} \dot{x} = 4x+5y \\ \dot{y} = -4x-4y \end{cases} \quad \begin{matrix} x(0)=0, \\ y(0)=1 \end{matrix}$$

$$\begin{vmatrix} 4-\lambda & 5 \\ -4 & -4-\lambda \end{vmatrix} = (4-\lambda)(-4-\lambda)+20=0$$

$$\lambda^2+4=0$$

$$\lambda = \pm 2i \quad \text{к}\,\text{р}\ 1$$

$$\begin{pmatrix} 4-2i & 5 \\ -4 & -4-2i \end{pmatrix} \qquad \begin{pmatrix} 2+i \\ -2 \end{pmatrix} \text{с.в.}$$

$$\begin{pmatrix} 2+i \\ -2 \end{pmatrix} e^{2ti} \qquad (\cos(2t)+i\sin(2t))\left[\begin{pmatrix} 2 \\ -2 \end{pmatrix} + i\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] = \cos(2t)\begin{pmatrix} 2 \\ -2 \end{pmatrix} + i\cos(2t)\begin{pmatrix} 1 \\ 0 \end{pmatrix} + i\sin(2t)\begin{pmatrix} 2 \\ -2 \end{pmatrix} -$$

$$-\sin(2t)\begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$x = (2\cos(2t)-\sin(2t))C_1 + C_2(\cos(2t)+2\sin(2t))$$

$$y = -2\cos(2t)C_1 - 2\sin(2t)C_2$$

$$x_0 = 2C_1 + C_2 = 0$$

$$y_0 = -2C_1 = 1$$

$$\begin{cases} C_1 = -\frac{1}{2} \\ C_2 = 1 \end{cases} \qquad \begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 2\cos(2t)-\sin(2t) \\ -2\cos(2t) \end{pmatrix} + \begin{pmatrix} \cos(2t)+2\sin(2t) \\ -2\sin(2t) \end{pmatrix}$$

3. y'''+2y''+2y'=e^(-x)sinx+cosx+2xe^(-x)-1

$y''' + 2y'' + 2y' = e^{-x}\sin x + \cos x + 2xe^{-x} - 1$

$\lambda(\lambda^2 + 2\lambda + 2) = 0$

$\left[\begin{array}{l} \lambda = 0 \\ \lambda = -1 \pm i \end{array}\right.$

$y_{o.o} = C_1 + C_2 e^{-x}\sin x + C_3 e^{-x}\cos x$

$e^{-x}\sin x = e^{-x}(0\cos x + \sin x)$

$\alpha = -1, \beta = 1, m = -\infty, n = 0$

$\tilde{\lambda} = -1 + i \quad \nu = 0$

$r = 1$

$\underline{e^{-x}x(A\cos x + B\sin x) = Axe^{-x}\cos x + Bxe^{-x}\sin x}$ (1)

$\cos x = e^{0x}(1\cdot\cos x + 0\sin x)$

$\alpha = 0, \beta = 1, m = 0, n = -\infty$

$\tilde{\lambda} = i, \nu = 0, r = 0$

$\underline{e^{0}(A\cos x + B\sin x) = A\cos x + B\sin x}$ (2)

$2xe^{-x} = e^{-x}(2x\cos 0x + 0\sin 0x)$

$\alpha = -1, \beta = 0, m = 1, n = -\infty$

$\tilde{\lambda} = -1, \nu = 1, r = 0$

$\underline{e^{-x}((Ax + B)\cos 0x + (Cx + D)\sin 0x) = e^{-x}Ax + e^{-x}B}$ (3)

$-1 = e^{0x}(-1\cos 0x + 0\sin 0x)$

$\alpha = 0, \beta = 0, m = 0, n = -\infty$

$\tilde{\lambda} = 0, \nu = 0, r = 1$

$\underline{e^{0x}x(A\cos 0x + B\sin 0x) = Ax}$ (4)

$y = C_1 + C_2 e^{-x}\sin x + C_3 e^{-x}\cos x + Axe^{-x}\cos x + Bxe^{-x}\sin x + C\cos x + D\sin x + EAx + Fe^{-x} + Gx$

4 вариант

1. y’’-10y’+25y=e^(5x)sqrt(3x+2)

$$y''-10y'+25y=e^{5x}\cdot\sqrt{3x+2}$$

$$\lambda^2-10\lambda+25=0$$

$$D=100-100=0$$

$$\lambda=5$$

$$y_{одн}=C_1(x)e^{5x}+C_2(x)xe^{5x}$$

$$\Delta=\begin{vmatrix} e^{5x} & xe^{5x} \\ 5e^{5x} & e^{5x}+5xe^{5x} \end{vmatrix}=e^{10x}+5xe^{10x}-5xe^{10x}=e^{10x}$$

$$\Delta_1=\begin{vmatrix} 0 & xe^{5x} \\ e^{5x}\sqrt{3x+2} & e^{5x}+5xe^{5x} \end{vmatrix}=-xe^{10x}\sqrt{3x+2}$$

$$\Delta_2=\begin{vmatrix} e^{5x} & 0 \\ 5e^{5x} & e^{5x}\sqrt{3x+2} \end{vmatrix}=e^{10x}\sqrt{3x+2}$$

$$C_1'(x)=-x\sqrt{3x+2}$$

$$C_1(x)=\frac{2(3x+2)^{\frac{3}{2}}(9x-4)}{135}+C$$

$$C_2'(x)=\sqrt{3x+2}\,dx$$

$$C_2(x)=\frac{2(3x+2)^{\frac{3}{2}}}{9}+C$$

$$y=\frac{e^{5x}2(\ \ -2)^{\frac{3}{2}}(9x-4)}{135}+C_1e^{5x}+\frac{e^{5x}x\,2(3x+2)^{\frac{3}{2}}}{9}+C_2e^{5x}x$$

2. x'=x-2y
   y'=6x-6y, x(0)=0, y(0)=2

$$x' = x - 2y \qquad x(0) = 2$$
$$y' = 6x - 6y \qquad y(0) = 0$$

$$\begin{pmatrix} 1 & -2 \\ 6 & -6 \end{pmatrix} \quad \begin{vmatrix} 1-\lambda & -2 \\ 6 & -6-\lambda \end{vmatrix} = 0 \quad (1-\lambda)(-6-\lambda)+12=0$$

$$\lambda^2 + 5\lambda + 6 = 0$$
$$(\lambda+2)(\lambda+3)=0$$

$\lambda_1 = -2$: $\begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix} \Rightarrow S_1 = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ $\quad$ $\lambda = -3$: $\begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix} \Rightarrow S_3 = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$$C_1 \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{-2t} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-3t}$$

$$\begin{cases} x = C_1 2 e^{-2t} + C_2 e^{-3t} \\ y = C_1 3 e^{-2t} + C_2 \cdot 2 e^{-3t} \end{cases} \Rightarrow \begin{cases} 2C_1 + C_2 = 2 \\ 3C_1 + 2C_2 = 0 \end{cases}$$

$$3C_1 + 2(1 - 2C_1) = 0$$
$$3C_1 + 2 - 4C_1 = 0$$
$$C_1 = 2$$
$$C_2 = 1 - 4 = -3$$

Ответ: $C_1 = 2$
$C_2 = -3$

3. y(6)-81y''=xcos(3x)+e^(3x)sin(3x)-3e^(-3x)-3

$$(\lambda^2 - 9)(\lambda^2 + 9)$$
$$(\lambda - 3)(\lambda + 3)(\lambda^2 + 9)$$
$\lambda = 3$ $\quad \lambda = -3 \quad \lambda = \pm 3i$

$$y^{VI} - 81y'' = x\cos 3x + e^{3x}\sin 3x - 3e^{-3x} - 3$$

$$\lambda^6 - 81\lambda^2 = 0 \qquad y_{o.o} = C_1 e^{0x} + C_2 e^{0x} x + C_3 \cdot e^{3x} + C_4 e^{-3x} + C_5 \sin 3x + C_6 \cos 3x$$

$$\lambda^2(\lambda^4 - 81) = 0 \qquad (\lambda^2+9)(\lambda^2-9) = (\lambda+3i)(\lambda-3i)(\lambda+3)(\lambda-3)$$

$\lambda = 0$ (кр 2) $\lambda = 3$ (кр 4) $\quad$ $\lambda = \pm 3i$ кр 1, $\lambda = 3$ кр 1, $\lambda = -3$ кр 1

$x\cos 3x = e^{0x}[x\cos 3x + 0\sin 3x] \longrightarrow \alpha = 0, \beta = 3, N = 1, \bar{\lambda} = 3i$

$e^{3x}\sin 3x = e^{3x}[0\cos 3x + \sin 3x] \longrightarrow \alpha = 3, \beta = 3, N = 0, \bar{\lambda} = 3 + 3i$

$-3e^{-3x} = e^{-3x}[-3\cos 0x + 0\sin 0x] \longrightarrow \alpha = -3, \beta = 0, N = 0, \bar{\lambda} = -3$

$-3 = e^{0x}[-3\cos 0x + 0\sin 0x] \longrightarrow \alpha = 0, \beta = 0, N = 0, \bar{\lambda} = 0$

$$y_{ч.н_1} = x e^{0x}[(Ax+B)\cos 3x + (Cx+D)\sin 3x]$$
$$y_{ч.н_2} = e^{3x}[E\cos 3x + G\sin 3x]$$
$$y_{ч.н_3} = x e^{-3x}[H\cos 0x + M\sin 0 \cdot x]$$
$$y_{ч.н_4} = x^2[N\cos 0x + P\sin 0 \cdot x]$$

Ответ: $y = y_{o.o} + y_{ч.н}$

5 вариант

1. y''+2y'+y=e^(-x)sqrt(16-x)

1) $y''+2y'+y=e^{-x}\sqrt{16-x}$

$y''+2y'+y=0$

$\lambda^2+2\lambda+1=0 \Rightarrow (\lambda+1)^2=0$

$\lambda_{1,2}=-1 \quad k=2 \Rightarrow e^{-x},\ xe^{-x}$

$y_{o.o}=C_1e^{-x}+C_2xe^{-x}$

$\varphi_1=e^{-x} \rightarrow \varphi_1'=-e^{-x}$

$\varphi_2=xe^{-x} \rightarrow \varphi_2'=e^{-x}-xe^{-x}$

$$\Delta=\begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & e^{-x}(1-x) \end{vmatrix}=e^{-2x}\begin{vmatrix} 1 & x \\ -1 & 1-x \end{vmatrix}=e^{-2x}(1-x+x)=e^{-2x}$$

$$\Delta_1=\begin{vmatrix} 0 & xe^{-x} \\ e^{-x}\sqrt{16-x} & e^{-x}(1-x) \end{vmatrix}=e^{-2x}x\sqrt{16-x}$$

$$\Delta_2=\begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & e^{-x}\sqrt{16-x} \end{vmatrix}=e^{-2x}\sqrt{16-x}$$

$$C_1'=\frac{\Delta_1}{\Delta}=x\sqrt{16-x} \rightarrow C_1 \underset{k=16-x}{=} \int\sqrt{k}(16-k)dk=\int k^{\frac{3}{2}}dk-\int 16\sqrt{k}\,dk=$$

$$=\frac{2}{5}k^{\frac{5}{2}}-16\cdot\frac{2}{3}k^{\frac{3}{2}}=2(16-x)^{\frac{3}{2}}\left(\frac{(16-x)}{5}-\frac{16}{3}\right)=\frac{2(-3x-32)(16-x)^{\frac{3}{2}}}{15}+B$$

$$C_2'=\frac{\Delta_2}{\Delta}=\sqrt{16-x} \rightarrow C_2=-\frac{2}{3}(16-x)^{\frac{3}{2}}+A$$

$$\boxed{y=e^{-x}\left(\frac{2(-3x-32)(16-x)^{\frac{3}{2}}}{15}+B-\frac{2}{3}x(16-x)^{\frac{3}{2}}+Ax\right)}$$

2. x’=x-2y
y’=x-y, x(0)=0, y(0)=-1

$$\begin{cases} \dot{x} = x - 2y \\ \dot{y} = x - y \end{cases} \qquad A = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}$$

$$\det(A - \lambda E) = \begin{pmatrix} 1-\lambda & -2 \\ 1 & -1-\lambda \end{pmatrix} = (\lambda-1)(\lambda+1) + 2 = \lambda^2 + 1$$

$\lambda = \pm i$

$$\boxed{\lambda = i} : \begin{pmatrix} 1-i & -2 \\ 1 & -1-i \end{pmatrix} \Rightarrow \vec{s} = \begin{pmatrix} 1+i \\ 1 \end{pmatrix}$$ (методом червяка)

$$se^{\lambda t} = \begin{pmatrix} 1+i \\ 1 \end{pmatrix} e^{it} = \begin{pmatrix} 1+i \\ 1 \end{pmatrix} (\cos t + i \sin t) =$$

$$= \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} + i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] (\cos t + i \sin t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cos t + i \begin{pmatrix} 1 \\ 1 \end{pmatrix} \sin t +$$

$$+ i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t = \begin{pmatrix} \cos t - \sin t \\ \cos t \end{pmatrix} + i \begin{pmatrix} \cos t + \sin t \\ \sin t \end{pmatrix}$$

$$\begin{cases} x(t) = C_1(\cos t - \sin t) + C_2(\cos t + \sin t) \\ y(t) = C_1(\cos t) + C_2 \sin t \end{cases}$$

$$\begin{cases} x(0) = C_1 \cdot 1 + C_2 \cdot 1 = 0 \\ y(0) = C_1 = -1 \end{cases}$$

$$\boxed{\begin{cases} C_1 = -1 \\ C_2 = 1 \end{cases}}$$

3. y(4)-2y''+y=e^(-x)sinx+xcosx-xe^x+2

$y^{(4)}-2y^{(2)}+y=e^{-x}\sin x+x\cos x-xe^{x}+2$

$\lambda^4-2\lambda^2+1=0$

$(\lambda-1)^2=0$

$\lambda=1 \quad k=2 \longrightarrow e^x,\ xe^x$

$y_{o.o}=C_1e^x+C_2xe^x$

1) $e^{-x}\sin x=e^{-1\cdot x}[0\cdot\cos x+1\cdot\sin x]$

$\underbrace{\alpha=-1,\ \beta=1}_{\bar\lambda=-1+i} \quad \underbrace{m=-\infty,\ n=0}_{N=0}$

$r=0$

$y_{ч.н}=x^0e^{-1x}[A_1\cdot\cos x+A_2\sin x]$

$y'_{ч.н}=(A_2-A_1)\cos x\, e^{-x}-(A_1+A_2)\sin x\cdot e^{-x}$

$y''_{ч.н}=2A_1\sin x\cdot e^{-x}-2A_2\cos x e^{-x}$

$y'''_{ч.н}=2(A_1+A_2)\cos x\, e^{-x}+2(A_2-A_1)\sin x e^{-x}$

$y^{(4)}_{ч.н}=-4A_1\cos x e^{-x}-4A_2\sin x\cdot e^{-x}$

$-4(A_1\cos x e^{-x}+A_2\sin x e^{-x})-2\cdot2(A_1\sin x\cdot e^{-x}-$

$-A_2\cos x\cdot e^{-x})+A_1\cos x\cdot e^{-x}+A_2\sin x\cdot e^{-x}=$

$=e^{-x}\sin x$

$\begin{cases}\cos x(-4A_1+4A_2+A_1)=0\\ \sin x(-4A_2-4A_1+A_2)=1\end{cases}$

$\begin{cases}-3A_1+4A_2=0\\ -3A_2-4A_1=1\end{cases} \longrightarrow \begin{cases}A_1=\frac{4}{3}A_2\\ -3A_2-\frac{16}{3}A_2=1\end{cases}$

$\begin{cases}A_1=\frac{4}{3}A_2\\ A_2=-\frac{3}{25}\end{cases} \quad \begin{cases}A_1=-\frac{4}{25}\\ A_2=-\frac{3}{25}\end{cases}$

$y_{ч.н_1}=e^{-x}\left(-\frac{4}{25}\cos x-\frac{3}{25}\sin x\right)$

2) $x\cdot\cos x=e^{0\cdot x}[x\cdot\cos x+0\cdot\sin x]$

$\underbrace{\alpha=0,\ \beta=1}_{\bar\lambda=i} \quad \underbrace{m=1,\ n=-\infty}_{N=1}$

$r=0$

$y_{ч.н.}=(A_1x+B_1)\cos x+(A_2x+B_2)\sin x$

$y'_{ч.н.}=(A_1+A_2x+B_2)\cos x+(A_2-A_1x+B_1)\sin x$

$y''_{ч.н.}=(2A_2-A_1x+B_1)\cos x-(2A_1+A_2x+B_2)\sin x$

$y'''_{ч.н.}=(3A_1+A_2x+B_2)\cos x-(3A_2-A_1x+B_1)\sin x$

$y^{(4)}_{ч.н.}=-(2A_2-A_1x+B_1)\cos x-(2A_1+A_2x+B_2)\sin x$

$-(2A_2-A_1x+B_1)\cos x-(2A_1+A_2x+B_2)\sin x-$

$-2(2A_2-A_1x+B_1)\cos x+2(2A_1+A_2x+B_2)\sin x+$

$+(A_1x+B_1)\cos x+(A_2x+B_2)\sin x=x\cdot\cos x$

$\begin{cases}\cos x(-2A_1+A_1x-\cancel{B_1}-4A_2+2A_1x-2B_1+\\ \quad +A_1x+\cancel{B_1})=x\cdot\cos x\\ \sin x(-2A_1-\cancel{A_2x}-\cancel{B_2}+4A_1+2A_2x+2B_2+\cancel{A_2x}+\cancel{B_2})=\\ =0\end{cases}$

$\begin{cases}-6A_2+4A_1x-2B_1=x\\ 2A_1+2A_2x+2B_2=0\end{cases} \quad \begin{cases}4A_1=1\\ -6A_2-2B_1=0\\ A_2=0\\ 2A_1+2B_2=0\end{cases}$

$\begin{cases}A_1=\frac{1}{4}\\ A_2=0\\ B_1=0\\ B_2=-\frac{1}{4}\end{cases}$

$y_{ч.н_2}=\frac{1}{4}x\cdot\cos x-\frac{1}{4}\sin x$

3) $-xe^x=e^x[-x\cdot\cos0+0\cdot\sin0]$

$\underbrace{\alpha=1\ \ \beta=0}_{\bar\lambda=1} \quad \underbrace{m=1,\ n=-\infty}_{N=1}$

$r=2$

$y_{ч.н.}=x^2e^x[(A_1x+B_1)\cos0+(A_2x+B_2)\sin0]=$

$=x^2e^x(A_1x+B_1)=e^x(Ax^3+Bx^2)$

$y'_{ч.н.}=e^x(Ax^3+(B+3A)x^2+2Bx)$

$y''_{ч.н.}=e^x(Ax^3+(B+6A)x^2+(4B+6A)x+2B)$

$y'''_{ч.н.}=e^x(Ax^3+(B+9A)x^2+(6B+18A)x+(6B+6A))$

$y^{(4)}_{ч.н.}=e^x(Ax^3+(B+12A)x^2+(8B+36A)x+(12B+24A))$

$e^x(\cancel{Ax^3}+(\cancel{B}+\cancel{12A})x^2+(\cancel{8B}+36A)x+(12B+24A)-$

$-\cancel{2Ax^3}-(\cancel{2B}+\cancel{12A})x^2-(\cancel{8B}+12A)x-4B+$

$+\cancel{Ax^3}+\cancel{Bx^2})=e^x\cdot x$

$\begin{cases}24A=1\\ 8B+24A=0\end{cases} \quad \begin{matrix}A=\frac{1}{24}\\ B=-\frac{1}{8}\end{matrix}$

$y_{ч.н_3}=e^x\left(\frac{1}{24}x^3-\frac{1}{8}x^2\right)$

4) $2=e^{0x}[2\cdot\cos0+0\cdot\sin0]$

$\underbrace{\alpha=0,\ \beta=0}_{\bar\lambda=0} \quad \underbrace{m=0,\ n=-\infty}_{N=0}$

$r=0$

$y_{ч.н.}=x^0e^{0x}[A_1\cos0+A_2\sin0]=A_1$

$y''_{ч.н.}=0$

$y'''_{ч.н.}=0$

$0-2\cdot0+A_1=2$

$A_1=2$

$y_{ч.н_4}=2$

$$\boxed{y=C_1e^x+C_2xe^x+2+e^x\left(\frac{x^3}{24}-\frac{x^2}{8}\right)+\frac{1}{4}(x\cdot\cos x-\sin x)+e^{-x}\left(-\frac{4}{25}\cos x-\frac{3}{25}\sin x\right)}$$

6 вариант

1. y’’-3y’=e^(4x)sin(e^x)

$y'' - 3y' = e^{4x}\sin e^x$

$y'' - 3y' = 0 \Rightarrow \lambda^2 - 3\lambda = 0$

$\lambda_1 = 0 \quad k=1 \longrightarrow e^{0x} = \underline{1}$

$\lambda_2 = 3 \quad k=1 \longrightarrow \underline{\underline{e^{3x}}}$

$y_{o.o} = C_1 + C_2 \cdot e^{3x}$

$\varphi_1 = 1 \longrightarrow \varphi_1' = 0$

$\varphi_2 = e^{3x} \longrightarrow \varphi_2' = 3e^{3x}$

$$\Delta = \begin{vmatrix} 1 & e^{3x} \\ 0 & 3e^{3x} \end{vmatrix} = 3e^{3x}$$

$$\Delta_1 = \begin{vmatrix} 0 & e^{3x} \\ e^{4x}\sin e^x & 3e^{3x} \end{vmatrix} = -e^{7x}\sin e^x$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 \\ 0 & e^{4x}\sin e^x \end{vmatrix} = e^{4x}\sin e^x$$

$$C_1' = \frac{\Delta_1}{\Delta} = \frac{-e^{7x}\sin e^x}{3e^{3x}} = \frac{-e^{4x}\sin e^x}{3} \Rightarrow C_1 = \frac{e^x(e^{2x}-6)\cos e^x}{3} - (e^{2x}-2)\sin e^x + A$$

$$C_2' = \frac{\Delta_2}{\Delta} = \frac{e^{4x}\sin e^x}{3e^{3x}} = \frac{e^x\sin e^x}{3} \Rightarrow C_2 = \frac{-\cos e^x}{3} + B$$

$$y = 1\cdot C_1 + e^{3x}\cdot C_2 = \frac{e^x(e^{2x}-6)\cos e^x}{3} - (e^{2x}-2)\sin e^x + A - \frac{e^{3x}\cos e^x}{3} + e^{3x}B$$

$$\boxed{y = \frac{-6e^x\cos e^x}{3} - (e^{2x}-2)\sin e^x + A + e^{3x}B}$$

2. x'=-5x+2y
   y'=x-6y, x(0)=0, y(0)=2

$$\begin{cases}\dot{x} = -5x+2y \\ \dot{y} = x-6y\end{cases} \qquad \begin{array}{l} x(0)=0 \\ y(0)=-1 \end{array}$$

$$A = \begin{pmatrix} -5 & 2 \\ 1 & -6 \end{pmatrix}$$

$$\det(A-\lambda E)=0: \quad \begin{pmatrix} -5-\lambda & 2 \\ 1 & -6-\lambda \end{pmatrix} = (\lambda+5)(\lambda+6)-2 = \lambda^2+11\lambda+28$$

$$D = 121-4\cdot 28 = 121-80-32 = 41-32 = 9$$

$$\lambda_{1,2} = \frac{-11\pm 3}{2} \Rightarrow \lambda_1=-7 \quad k_1=1 \longrightarrow e^{-7t}$$

$$\lambda_2=-4 \quad k_2=1 \longrightarrow e^{-4t}$$

$$\lambda_1=-7: \left(\begin{array}{cc|c} 2 & 2 & 0 \\ 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow \vec{S_1} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\lambda_2=-4: \left(\begin{array}{cc|c} -1 & 2 & 0 \\ 1 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} -1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow \vec{S_2} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{7t} + C_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-4t}$$

$$x(t) = C_1 e^{-7t} + C_2\cdot 2\cdot e^{-4t} \longrightarrow x(0) = C_1 e^{1} + C_2\cdot 2e^{0} = 0$$

$$y(t) = -C_1 e^{-7t} + C_2 e^{-4t} \longrightarrow y(0) = -C_1 e^{0} + C_2 e^{0} = 2$$

$$\begin{cases} C_1 + 2C_2 = 0 \\ C_2 = 2 + C_1 \end{cases} \rightarrow \begin{cases} C_1 = -2C_2 \\ C_1 = C_2 - 2 \end{cases} \rightarrow \begin{cases} C_1 = -2C_2 \\ -2C_2 = C_2 - 2 \end{cases} \rightarrow \boxed{\begin{cases} C_1 = \frac{-4}{3} \\ C_2 = \frac{2}{3} \end{cases}}$$

3. y(7)+27y(4)=xcos(3x)-sin(3x)+x(e^(-3x)+3)

$$y^{(7)}+27y^{(4)}=x\cdot\cos 3x-\sin 3x+x(e^{-3x}+3)$$

$$\lambda^7+27\lambda^4=0$$

$$\lambda^4(\lambda^3+27)=0$$

$$\lambda^4(\lambda+3)(\lambda^2-3\lambda+9)=0$$

$$\begin{array}{r|l} -\lambda^3+27 & \lambda+3 \\ \lambda^3+3\lambda^2 & \lambda^2-3\lambda+9 \\ \hline -3\lambda^2+27 & \\ -\;-3\lambda^2-9\lambda & \\ \hline 9\lambda+27 & \\ -\;9\lambda+27 & \\ \hline 0 & \end{array}$$

$$\lambda^2-3\lambda+9=0$$

$$D=9-4\cdot 9=-3\cdot 9$$

$$\lambda_{1,2}=\frac{3\pm i3\sqrt3}{2}=\frac{3}{2}\pm i\,\frac{3\sqrt3}{2}$$

$\lambda_1=0 \quad k_1=4\longrightarrow 1, x, x^2, x^3$

$\lambda_2=-3 \quad k_2=1\longrightarrow e^{-3x}$

$\lambda_{3,4}=\frac{3\pm i3\sqrt3}{2} \quad k_{3,4}=1\longrightarrow e^{\frac32 x}\cdot\cos\frac{3\sqrt3}{2}x$

$e^{\frac32 x}\cdot\sin\frac{3\sqrt3}{2}x$

$$y_{oo}=C_1+C_2x+C_3x^2+C_4x^3+C_5e^{-3x}+C_6e^{\frac32 x}\cos\frac{3\sqrt3}{2}x+C_7e^{\frac32 x}\sin\frac{3\sqrt3}{2}x$$

1) $x\cdot\cos 3x=e^{0x}[x\cdot\cos 3x+0\cdot\sin 3x]$

$\underbrace{\alpha=0,\beta=3}_{\lambda=3i} \quad \underbrace{m=1, n=-\infty}_{N=1} \quad r=0$

$y_{ч.н}=x^0e^{0x}[(A_1x+B_1)\cos 3x+(A_2x+B_2)\sin 3x]$

$y^{(4)}_{ч.н}=(81A_2x+81B_2+108A_1)\sin 3x+(81A_1x-108A_2+81B_1)\cos 3x$

$y^{(7)}_{ч.н}=(2187A_1x-5103A_2+2187B_1)\sin 3x-(2187A_2x+2187B_2-5103A_1)\cos 3x$

$y^{(7)}_{ч.н}-27y^{(4)}_{ч.н}=x\cdot\cos 3x$

$$\begin{cases}\sin 3x(-5103A_2+2187B_1-2187B_2-2916A_1)=0\\ x\sin 3x(2187A_1-2187A_2)=0\\ \cos 3x(-2187B_2+5103A_1-2187B_1+2916A_2)=0\\ x\cos 3x(-2187A_2-2187A_1)=x\cos 3x\end{cases}$$

$$\begin{cases}-7A_2+3B_1-3B_2-4A_1=0\\ A_1=A_2\\ -3B_2+7A_1-3B_1+4A_2=0\\ -4374A_1=1\end{cases} \qquad \begin{cases}3B_1-3B_2-11A_1=0\\ A_1=A_2\\ -3B_1-3B_2+11A_1=0\\ A_1=\frac{-1}{4374}\end{cases}$$

$$\begin{cases}-6B_2=0\\ A_1=A_2=-\frac{1}{4374}\\ 3B_1-3B_2=11A_1\end{cases} \qquad \begin{array}{l}B_2=0\\ A_1=A_2=\frac{-1}{4374}\\ B_1=\frac{11}{3}\cdot\left(\frac{-1}{4374}\right)=\frac{-11}{13122}\end{array}$$

$$y_{ч.н_1}=\left(\frac{-1}{4374}x-\frac{11}{13122}\right)\cos 3x-\frac{1}{4374}x\cdot\sin 3x$$

2) $-\sin 3x=e^0[0\cdot\cos 3x-1\cdot\sin 3x]$

$\underbrace{\alpha=0\ \beta=3}_{\lambda=3i} \quad \underbrace{m=-\infty, n=0}_{N=0} \quad r=0$

$y_{ч.н}=x^0e^0[A_1\cdot\cos 3x+A_2\sin 3x]$

$y^{(4)}_{ч.н}=81(A_1\cos 3x+A_2\sin 3x)$

$y^{(7)}_{ч.н}=2187(A_1\sin 3x-A_2\cos 3x)$

$y^{(7)}_{ч.н}-27y^{(4)}_{ч.н}=-\sin 3x$

$$\begin{cases}\sin 3x(2187A_1-2187A_2)=-1\\ \cos 3x(-2187A_2-2187A_1)=0\end{cases}$$

$$\begin{cases}A_1=-A_2\\ 4374A_1=-1\end{cases} \qquad \begin{cases}A_2=\frac{1}{4374}\\ A_1=-\frac{1}{4374}\end{cases}$$

$$y_{ч.н_2}=-\frac{1}{4374}\cos 3x+\frac{1}{4374}\sin 3x$$

3) $xe^{-3x}=e^{-3x}[x\cdot\cos 0+0\cdot\sin 0]$

$\underbrace{\alpha=-3,\beta=0}_{\lambda=-3} \quad \underbrace{m=1, n=-\infty}_{N=1} \quad r=1$

$y_{ч.н}=xe^{-3x}[(A_1x+B_1)\cos 0+(A_2x+B_2)\sin 0]=$

$=xe^{-3x}(A_1x+B_1)=e^{-3x}(A_1x^2+B_1x)$

$y^{(4)}_{ч.н}=(81A_1x^2+(81B_1-216A_1)x-108B_1+108A_1)e^{-3x}$

$y^{(7)}_{ч.н}=-(2187A_1x^2+(2187B_1-10206A_1)x-5103B_1+10206A_1)e^{-3x}$

$y^{(7)}_{ч.н}-27y^{(4)}_{ч.н}=xe^{-3x}$

$$\begin{cases}e^{-3x}x^2(-2187A_1-2187A_1)=0\\ e^{-3x}(5103B_1-10206A_1+2916B_1-2916A_1)=0\\ e^{-3x}x(-2187B_1+10206A_1-2187B_1+5832A_1)=1\end{cases}$$

у мене тут дробинка

4) $3x=e^0[3x\cos 0+0\sin 0]$

$\underbrace{\alpha=0,\beta=0}_{\lambda=0} \quad \underbrace{m=1, n=-\infty}_{N=1}$

$r=4$

$y_{ч.н}=x^4e^0[(A_1x+B_1)\cos 0+(A_2x+B_2)\sin 0]=$

$=x^4(Ax+B)=Ax^5+Bx^4$

1) $5Ax^4+4Bx^3$
2) $20Ax^3+12Bx^2$
3) $60Ax^2+24Bx$
4) $120Ax+24B$
5) $120A$

$y^{(4)}_{ч.н}=120Ax+24B$

$y^{(7)}_{ч.н}=0$

$-27(120Ax+24B)=3x$

$B=0 \qquad A=\frac{3}{-27\cdot 120}=\frac{-1}{27\cdot 40}=\frac{-1}{1080}$

$$y_{ч.н_4}=\frac{-x^5}{1080}$$

7 вариант

1. y''-2y'+2y=e^x/sinx

$$y''-2y'+2y=\frac{e^x}{\sin x}$$

$$\lambda^2-2\lambda+2=0$$

$$(\lambda-1)^2=0$$

$$\lambda=1\ (\text{кр } 2)$$

$$y_{o.o}=C_1e^x+C_2e^x x$$

$$\Delta=\begin{vmatrix} e^x & e^x x \\ e^x & xe^x+e^x \end{vmatrix}=e^{2x}+e^{2x}x-e^{2x}x=e^{2x}$$

$$\Delta_1=\begin{vmatrix} 0 & e^x \\ \frac{e^x}{\sin x} & xe^x+e^x \end{vmatrix}=\frac{-e^{2x}}{\sin x}$$

$$\Delta_2=\begin{vmatrix} e^x & 0 \\ e^x & \frac{e^x}{\sin x} \end{vmatrix}=\frac{e^{2x}}{\sin x}$$

$$C_1'(x)=\frac{\frac{-e^{2x}}{\sin x}}{e^{2x}}=\frac{-1}{\sin x} \qquad C_1(x)=-\int\frac{1}{\sin x}$$

$$C_2'(x)=\frac{\frac{e^{2x}}{\sin x}}{e^{2x}}=\frac{1}{\sin x} \qquad C_2(x)=\int\frac{1}{\sin x}$$

Ответ: $-\int\frac{1}{\sin x}e^x+\int\frac{1}{\sin x}e^x x$.

2. x'=2x+3y
   y'=x+4y, x(0)=-1, y(0)=0

$$\begin{cases} x' = 2x+3y \\ y' = x+4y \end{cases} \qquad \begin{matrix} x = 0 \\ y = -1 \end{matrix}$$

$$\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \quad \begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = 0$$

$$8 + \lambda^2 - 6\lambda = 0$$

$$D = 36 - 4\cdot 8 = 4$$

$$\lambda_{1,2} = \frac{6 \pm 2}{2} \begin{matrix} \nearrow 4 \ (\text{k}=1) \\ \searrow 2 \ (\text{k}=1) \end{matrix} \qquad y_{o.o} = C_1 e^{4x} + C_2 e^{2x}$$

$$\lambda = 4: \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \overline{S}_1$$

$$\lambda = 2 \begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} -2 \\ 1 \end{pmatrix} = S_2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{4x} + C_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{2x}$$

$$\begin{cases} x = C_2(-2)\cdot e^{2x} \\ y = -C_1 e^{4x} + C_2 e^{2x} \end{cases}$$

$$\begin{cases} -2C_2 = 0 \\ -C_1 + C_2 = -1 \end{cases} \Rightarrow \begin{cases} C_2 = 0 \\ C_1 = 1 \end{cases} \text{ ответ}$$

3. y'''+3y''-10y'=x^2*sin(5x)-xe^(-5x)+x

$$y''' + 3y'' - 10y' = x^2 \sin 5x - xe^{-5x} + x$$

$$\lambda^3 + 3\lambda^2 - 10\lambda = 0$$

$$\lambda(\lambda^2 + 3\lambda - 10) = 0$$

$$\lambda_{2} = \frac{-3 \pm 7}{2} < {2 \atop -5}$$

$$\lambda(\lambda - 2)(\lambda + 5) = 0$$

$$y_{o.o} = C_1 e^{0x} + C_2 e^{2x} + C_3 e^{-5x}$$

$$x^2 \sin 5x = e^{0x}[0 \cdot \cos 5x + x^2 \sin 5x]$$

$\alpha = 0$ $\beta = 5$

$\bar{\lambda} = 5i$ $N = 2$

$$-xe^{-5x} = e^{-5x}[-x \cos 0 \cdot x + 0 \sin 0 x]$$

$\alpha = -5$ $\beta = 0$ $\bar{\lambda} = -5$

$N = 1$

$$x = e^{0x}[x \cos 0x + 0 \sin 0x]$$

$\alpha = 0$ $\beta = 0$ $\lambda = 0$

$N = 1$

$$y_{ч.н_1} = x^0 e^{0x}[(Ax^2 + Bx + C)\cos 0 \cdot x + (Dx^2 + Ex + F)\sin 0x]$$

$$y_{ч.н_2} = xe^{-5x}[(Mx + N)\cos 0x + (Px + Q)\sin 0x]$$

$$y_{ч.н_3} = xe^{0x}[(Lx + R)\cos 0x + (Ox + Y)\sin 0x]$$

Ответ: $y = y_{o.o} + y_{ч.н_1} + y_{ч.н_2} + y_{ч.н_3}$

8 вариант

1. y’’-6y’+9y=e^(3x)/x^2

$$y''-6y'+9y=\frac{e^{3x}}{x^2}$$

$$\lambda^2-6\lambda+9=0$$

$$(\lambda-3)(\lambda-3)=0$$

$$\lambda_1=3 \quad k_1=2 \to e^{3x},\ xe^{3x}$$

$$\varphi_1=e^{3x} \qquad \varphi_2=xe^{3x}$$

$$\varphi_1'=3e^{3x} \qquad \varphi_2'=3xe^{3x}+e^{3x}$$

$$\Delta=\begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & 3xe^{3x}+e^{3x} \end{vmatrix}=e^{6x}\begin{vmatrix} 1 & x \\ 3 & 3x+1 \end{vmatrix}=e^{6x}(3x+1-3x)=e^{6x}$$

$$\Delta_1=\begin{vmatrix} 0 & xe^{3x} \\ \frac{e^{3x}}{x^2} & 3xe^{3x}+e^{3x} \end{vmatrix}=e^{6x}\begin{vmatrix} 0 & x \\ \frac{1}{x^2} & 3x+1 \end{vmatrix}=-\frac{e^{6x}}{x}$$

$$\Delta_2=\begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & \frac{e^{3x}}{x^2} \end{vmatrix}=e^{6x}\begin{vmatrix} 1 & 0 \\ 3 & \frac{1}{x^2} \end{vmatrix}=\frac{e^{6x}}{x^2}$$

$$C_1'(x)=-\frac{1}{x} \qquad C_2'(x)=\frac{1}{x^2}$$

$$C_1(x)=-\ln|x|+A \quad C_2=-\frac{1}{x}+B$$

$$\boxed{y=(-\ln|x|+A)e^{-3x}+\left(-\frac{1}{x}+B\right)xe^{-3x}}$$

2. x'=2x-y
   y'=x+2y, x(0)=1, y(0)=2

$$\begin{cases} \dot{x} = 2x - y \\ \dot{y} = x + 2y \end{cases} \qquad x(0) = 1, \ y(0) = 2$$

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} : \begin{vmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 + 1 = 4 - 4\lambda + \lambda^2 + 1 =$$

$$= 5 - 4\lambda + \lambda^2 \qquad \lambda^2 - 4\lambda + 5 = 0$$

$$D = 16 - 20 = -4$$

$$\lambda_{1,2} = \frac{4 \pm 2i}{2} = 2 \pm i$$

$$\lambda = 2+i : \begin{vmatrix} -i & -1 \\ 1 & -i \end{vmatrix}$$

$$\Downarrow$$

$$\vec{\xi} = \begin{pmatrix} i \\ 1 \end{pmatrix}$$

$$\vec{\xi} e^{\lambda t} = \begin{pmatrix} i \\ 1 \end{pmatrix} e^{(2+i)t} = \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} + i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] e^{2t} \cdot e^{it} =$$

$$= \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} + i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] (\cos t + i \sin t) e^{2t} =$$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t \cdot e^{2t} + i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin t \cdot e^{2t} + i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t \cdot e^{2t} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t \cdot e^{2t} =$$

$$= e^{2t} \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} + i e^{2t} \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

$$\begin{cases} x(t) = C_2 e^{2t} \cos t - C_1 e^{2t} \sin t \\ y(t) = C_1 e^{2t} \cos t + C_2 e^{2t} \sin t \end{cases}$$

$$\begin{cases} x(0) = C_2 \\ y(0) = C_1 \end{cases} \qquad \boxed{\begin{cases} C_2 = 1 \\ C_1 = 2 \end{cases}}$$

3. y(6)+18y(4)+81y''=e^(3x)-xe^(-3x)+cos(3x)-x

9 вариант

1. y’’-2y’+y=e^x/x^2

Вариант 59 (9)

① $y'' - 2y' + y = e^x / x^2$ ← неодн. лин. ур-е с пост. коэф.

1) решим соотв. одн. ур-е

$$y'' - 2y' + y = 0$$

$$\lambda^2 - 2\lambda + 1 = 0$$

$$(\lambda - 1)^2 = 0$$

$$\lambda = 1 \quad (\text{кр. } 2)$$

$$y = C_1 e^x + C_2 x e^x$$

2) $y = C_1(x) e^x + C_2(x) \cdot x e^x$

3) $$\begin{pmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{pmatrix} \begin{pmatrix} C_1'(x) \\ C_2'(x) \end{pmatrix} = \begin{pmatrix} 0 \\ e^x / x^2 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{vmatrix} = e^{2x} \begin{vmatrix} 1 & x \\ 1 & 1+x \end{vmatrix} = e^{2x}$$

$$\Delta_1 = \begin{vmatrix} 0 & xe^x \\ \frac{e^x}{x^2} & e^x + xe^x \end{vmatrix} = e^{2x} \begin{vmatrix} 0 & x \\ \frac{1}{x^2} & 1 \end{vmatrix} = e^{2x}\left(-\frac{1}{x}\right)$$

$$\Delta_2 = \begin{vmatrix} e^x & 0 \\ e^x & e^x / x^2 \end{vmatrix} = e^{2x} \begin{vmatrix} 1 & 0 \\ 1 & 1/x^2 \end{vmatrix} = e^{2x} \cdot \frac{1}{x^2}$$

$$C_1' = -\frac{e^{2x}}{x} \cdot \frac{1}{e^{2x}} = -\frac{1}{x} \Rightarrow C_1 = -\ln|x| + A$$

$$C_2' = \frac{e^{2x}}{x^2} \cdot \frac{1}{e^{2x}} = \frac{1}{x^2} \Rightarrow C_2 = -\frac{1}{x} + B$$

Ответ: $y = Ae^x + Bxe^x - e^x(\ln|x| + e^x)$

2. x’=-2x-y
   y’=2x-4y, x(0)=2, y(0)=1

(2) $\begin{cases} \dot{x} = -2x - y \\ \dot{y} = 2x - 4y \end{cases}$ $x(0) = 2$, $y(0) = 1$

↳ однородн. сист. лин. ДУ с пост. коэф.

1) $A = \begin{pmatrix} -2 & -1 \\ 2 & -4 \end{pmatrix}$

$$\underbrace{(-2-\lambda)(-4-\lambda)}^{(\lambda+2)(\lambda+4)}$$

$$|A - \lambda E| = (-2-\lambda)(-4-\lambda) + 2 = \lambda^2 + 6\lambda + 10 =$$

$$= (\lambda+3)^2 - (i)^2 = (\lambda - (-3+i))(\lambda - (-3-i)) = 0$$

$$\lambda_{1,2} = -3 \pm i$$

2) $\lambda_1 = -3 + i$

$$\begin{pmatrix} 1-i & -1 \\ 2 & -1-i \end{pmatrix}; \quad \bar{S}_1 = \begin{pmatrix} 1+i \\ 2 \end{pmatrix}$$

3) $$S_1 \cdot e^{\lambda_1 t} = \begin{pmatrix} 1+i \\ 2 \end{pmatrix} e^{(-3+i)t} =$$

$$= \begin{pmatrix} 1+i \\ 2+0i \end{pmatrix} e^{-3t} \cdot e^{(t)i} = \left[\begin{pmatrix} 1 \\ 2 \end{pmatrix} + i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] \cdot$$

$$\cdot (\cos t + i \sin t) \cdot e^{-3t} = \begin{pmatrix} \cos t - \sin t \\ 2\cos t \end{pmatrix} e^{-3t} +$$

$$+ i \begin{pmatrix} \cos t + \sin t \\ 2\sin t \end{pmatrix} e^{-3t}$$

4) $$\begin{cases} x(t) = C_1 (\cos t - \sin t) e^{-3t} + C_2 (\cos t + \sin t) e^{-3t} \\ y(t) = C_1 (2\cos t) e^{-3t} + C_2 (2\sin t) e^{-3t} \end{cases}$$

5) $$\begin{cases} 2 = C_1 + C_2 \\ 1 = C_1 \cdot 2 \end{cases} \quad \begin{cases} C_1 = \frac{1}{2} \\ C_2 = 1\frac{1}{2} \end{cases}$$

Ответ: $$\begin{cases} x(t) = \frac{1}{2} (\cos t - \sin t) \cdot e^{-3t} + \frac{3}{2} (\cos t + \sin t) e^{-3t} \\ y(t) = \frac{2}{2} \cos t \cdot e^{-3t} + 3 \sin t \cdot e^{-3t} \end{cases}$$

3. y(5)-21y'''-100y'=e^(5x)cos(2x)+xe^(-5x)-5x

③ $y^{(5)} - 21y^{(3)} - 100y' = e^{5x}\cos 2x + xe^{-5x} - 5x$

1) решим соотв. однородное ур-е:

$y^{(5)} - 21y^{(3)} - 100y' = 0$

$\lambda^5 - 21\lambda^3 - 100\lambda = 0$

$\lambda(\lambda^4 - 21\lambda^2 - 100) =$

$= \lambda(\lambda^2+4)(\lambda^2-25) =$

$= \lambda(\lambda-5)(\lambda+5)(\lambda-2i)(\lambda+2i) = 0$

$D = 441 + 400 = 841$

$t_{1,2} = \frac{21 \pm 29}{2}$

$t_1 = 25 \quad t_2 = -4$

$\lambda_1 = 0$ (кр. 1) $\quad \lambda_{4,5} = \pm 2i$ (од. кр. 1)

$\lambda_2 = 5$

$\lambda_3 = -5$

$y_{oo} = C_1 + C_2 e^{5x} + C_3 e^{-5x} + C_4 \sin 2x + C_5 \cos 2x$

2) $\underbrace{e^{5x}\cos 2x}_{f_1(x)} + \underbrace{xe^{-5x}}_{f_2(x)} + \underbrace{(-5x)}_{f_3(x)}$

а) $f_1(x) = e^{5x}\cos 2x = e^{5x}[1 \cdot \cos 2x + 0 \sin 2x]$

$\left.\begin{matrix}\alpha = 5 \\ \beta = 2\end{matrix}\right\} \Rightarrow \tilde{\lambda} = 5+2i \quad N = \max\{0, -\infty\} = 0$

$r_1 = 0$

$y_{чн1} = e^{5x}[A\cos 2x + B\sin 2x]$

б) $f_2(x) = xe^{-5x} = e^{-5x}[x\cos 0x + 0 \sin 0x]$

$\left.\begin{matrix}\alpha = -5 \\ \beta = 0\end{matrix}\right\} \quad \tilde{\lambda} = -5 \quad N = \max\{1; -\infty\} = 1$

$r_2 = 1$

$y_{чн2} = xe^{-5x}[(Ax+B)\cos 0x + (Cx+D)\sin 0x] =$

$= (\tilde{A}x + \tilde{B})xe^{-5x}$

в) $f_3(x) = -5x = e^{0x}[-5x\cos 0x + 0 \cdot \sin 0x]$

$\tilde{\lambda} = 0 \quad N = \max\{1, -\infty\} = 1$

$r_3 = 1$

$y_{чн3} = xe^{0x}[(\bar{A}x + \bar{B})\cos 0x] = (\bar{A}x + \bar{B})x$

3) $y_{чн} = y_{чн1} + y_{чн2} + y_{чн3}$

Ответ: $y_{он} = y_{оо} + y_{чн}$,

$C_i \in R$, $\quad A, B, \tilde{A}, \tilde{B}, \bar{A}, \bar{B}$ - конкр. числа

10 вариант

1. y’’’+16y=1/(sin(4x)^5)

Вариант 60 (10)

① $y'' + 16y = \frac{1}{\sin^5 4x}$

1) решение соотв. однородное ур-е

$$y'' + 16y = 0$$
$$\lambda^2 + 16 = 0$$
$$(\lambda - 4i)(\lambda + 4i) = 0$$
$$\lambda_{1,2} = \pm 4i \text{ (оба кр. 1)}$$
$$y = C_1 \cos 4x + C_2 \cdot \sin 4x$$

2) $y = C_1(x)\cos 4x + C_2(x)\sin 4x$

3) $$\begin{pmatrix} \cos 4x & \sin 4x \\ -4\sin 4x & 4\cos 4x \end{pmatrix}\begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\sin^5 4x} \end{pmatrix}$$

$$\Delta = \begin{vmatrix} \cos 4x & \sin 4x \\ -4\sin 4x & 4\cos 4x \end{vmatrix} = 4$$

$$\Delta_1 = \begin{vmatrix} 0 & \sin 4x \\ \sin^{-5} 4x & 4\cos 4x \end{vmatrix} = -\frac{1}{\sin^4 4x}$$

$$\Delta_2 = \begin{vmatrix} \cos 4x & 0 \\ -4\sin 4x & \sin^{-5} 4x \end{vmatrix} = \frac{\cos 4x}{\sin^5 4x}$$

$$C_1' = \frac{\Delta_1}{\Delta} = -\frac{1}{4\sin^4 4x}$$

$$C_2' = \frac{\Delta_2}{\Delta} = \frac{\cos 4x}{4\sin^5 4x}$$

$$C_1 = \frac{1}{16}\int \frac{1}{\sin^2 4x} \cdot \frac{-1}{\sin^2 4x}\, d4x =$$

$$\left[\begin{array}{l|l} u = ctg 4x = \frac{\cos 4x}{\sin 4x} & u^2 = \frac{\cos^2 4x}{\sin^2 4x} \\ du = -\frac{1}{\sin^2 4x} \cdot 4 & u^2 = \frac{1 - \sin^2 4x}{\sin^2 4x} \\ & u^2 = \frac{1}{\sin^2 4x} - 1 \\ & u^2 + 1 = \frac{1}{\sin^2 4x} \end{array}\right]$$

$$= \frac{1}{64}\int u^2 + 1 \cdot du = \frac{1}{64}\left(\frac{u^3}{3} + u\right) =$$

$$= \frac{1}{64}\left(\frac{1}{3} ctg^3 4x + ctg 4x\right) + A$$

$$C_2 = \int \frac{\cos 4x}{4} \frac{1}{\sin^5 4x}\, dx = \frac{1}{16}\int \frac{1}{\sin^5 4x}\, d(\sin 4x) =$$

$$= \frac{1}{16} \cdot \left(-\frac{1}{4\sin^4 4x}\right) + B = -\frac{1}{64\sin^4 4x} + B$$

Ответ: $y = \left(\frac{1}{64}\left(\frac{ctg^3 4x}{3} + ctg 4x\right) + A\right)\cos 4x +$

$+ \left(-\frac{1}{64\sin^4 4x} + B\right)\sin 4x$

2. $x' = y$
   $y' = 6x + y$, $x(0)=0$, $y(0)=3$

Ответ: $y = \left(\frac{1}{64}\left(\frac{ctg^3 4x}{3} + ctg 4x\right) + A\right)\cos 4x + \left(-\frac{1}{64 \sin^4 4x} + B\right)\sin 4x$

② $\begin{cases} \dot{x} = y \\ \dot{y} = 6x + y \end{cases}$, $x(0) = 0$, $y(0) = 3$

1) $A = \begin{pmatrix} 0 & 1 \\ 6 & 1 \end{pmatrix}$

$|A - \lambda E| = -\lambda(1-\lambda) - 6 = \lambda^2 - \lambda - 6 = (\lambda - 3)(\lambda + 2) = 0$

$D = 1 + 24 = 25$

$\lambda_{1,2} = \frac{1 \pm 5}{2}$

$\lambda_1 = 3$, $\lambda_2 = -2$

$\lambda_1 = 3$ (кр. 1)

$\lambda_2 = -2$

а) $\lambda_1 = 3$

$\begin{pmatrix} -3 & 1 \\ 6 & -2 \end{pmatrix}$; $\bar{S}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

б) $\lambda_2 = -2$

$\begin{pmatrix} 2 & 1 \\ 6 & 3 \end{pmatrix}$, $\bar{S}_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

Ответ:

2) $\begin{cases} x(t) = C_1 e^{3t} - C_2 e^{-2t} \\ y(t) = C_1(3e^{3t}) + C_2(2e^{-2t}) \end{cases}$

3) $\begin{cases} 0 = C_1 - C_2 \\ 3 = 3C_1 + 2C_2 \end{cases}$ $\begin{cases} C_2 = \frac{3}{5} \\ C_1 = \frac{3}{5} \end{cases}$

Ответ: $\begin{cases} x(t) = \frac{3}{5} e^{3t} - \frac{3}{5} e^{-2t} \\ y(t) = \frac{9}{5} e^{3t} + \frac{6}{5} e^{-2t} \end{cases}$

③ $y^{(4)} - 256y = x \sin 4x + 4\cos 4x + 3e^{-4x} + 4$

1) решим соотв. одн-е ур-е:

$y^{(4)} - 256y = 0$

$(\lambda^4 - 4^4) = 0$

$(\lambda^2 - 4^2)(\lambda^2 - (4i)^2) = 0$

$(\lambda - 4)(\lambda + 4)(\lambda - 4i)(\lambda + 4i) = 0$

$\lambda_1 = 4$ (кр. 1)

$\lambda_2 = -4$

$\lambda_{3,4} = \pm 4i$

3. y(4)-256y=xsin(4x)+4cos(4x)+3e^(-4x)+4

$$y_{оо} = C_1 e^{4x} + C_2 e^{-4x} + C_3 \sin 4x + C_4 \cos 4x$$

2) $\underbrace{x \sin 4x + 4\cos 4x}_{f_1(x)} + \underbrace{3e^{4x}}_{f_2(x)} + \underbrace{4}_{f_3(x)}$

а) $f_1(x) = e^{0x}\left[x \sin 4x + 4\cos 4x\right]$

$\left.\begin{array}{l}\alpha = 0\\ \beta = 4\end{array}\right\}$ $\tilde{\lambda} = 4i \Rightarrow r_1 = 1$ $\quad N = \max\{0; 1\} = 1$

$$y_{чн1} = x\left[(Ax + B)\sin 4x + (Cx + D)\cos 4x\right]$$

б) $f_2(x) = e^{-4x}\left[3\cos 0x + 0 \cdot \sin 0x\right]$

$\left.\begin{array}{l}\alpha = -4\\ \beta = 0\end{array}\right\}$ $\tilde{\lambda} = -4 \Rightarrow r_2 = 1$ $\quad N = \max\{0; -\infty\} = 0$

$$y_{чн2} = x \cdot e^{-4x}\left[E \cos 0x\right] = Exe^{-4x}$$

в) $f_3(x) = e^{0x}\left[4\cos 0x + 0 \cdot \sin 0x\right]$

$\tilde{\lambda} = 0 \Rightarrow r_3 = 0$ $\quad N = \max\{0; -\infty\} = 0$

$$y_{чн3} = F$$

3) $y_{чн} = y_{чн1} + y_{чн2} + y_{чн3}$

Ответ: $y_{он} = y_{оо} + y_{чн}$

$C_i \in R$; A, B, C, D, E, F - конкр. числа

11 вариант

1. y’’+9y=ctg(3x)^2

Вар 61

$y''+9y=ctg^2 3x$

$\lambda^2+9=0$

$\lambda^2=-9$

$\lambda=\pm 3i$

$y_{оди}=C_1\sin 3x+C_2\cos 3x$

$$\begin{pmatrix}\sin 3x & \cos 3x\\ 3\cos 3x & -3\sin 3x\end{pmatrix}\begin{pmatrix}C_1'(x)\\ C_2'(x)\end{pmatrix}=\begin{pmatrix}0\\ ctg^2 3x\end{pmatrix}$$

$$\Delta=\begin{vmatrix}\sin 3x & \cos 3x\\ 3\cos 3x & -3\sin 3x\end{vmatrix}=-3\sin^2 3x-3\cos^2 3x=-3$$

$$\Delta_1=\begin{vmatrix}0 & \cos 3x\\ ctg^2 3x & -3\sin 3x\end{vmatrix}=-ctg^2 3x\cos 3x=-\frac{\cos^3 3x}{\sin^2 3x}$$

$$\Delta_2=\begin{vmatrix}\sin 3x & 0\\ 3\cos 3x & ctg^2 3x\end{vmatrix}=\frac{\cos^2 3x}{\sin 3x}$$

$$C_1'(x)=\frac{\Delta_1}{\Delta}=\frac{1}{3}\frac{\cos^3 3x}{\sin^2 3x}$$

$$C_2'(x)=\frac{\Delta_2}{\Delta}=-\frac{1}{3}\frac{\cos^2 3x}{\sin 3x}$$

$$C_1(x)=\frac{1}{3}\int\frac{\cos^3 3x}{\sin^2 3x}dx=\frac{1}{9}\int\frac{\cos^3 3x}{\sin^2 3x}d3x=\frac{1}{9}\int\cos 3x\,\frac{1-\sin^2 3x}{\sin^2 3x}d3x=$$

$$=\frac{1}{9}\left(\int\frac{ctg\,3x}{\sin 3x}d3x-\int\cos 3x\,d3x\right)=$$

$$=-\frac{1}{9}\frac{1}{\sin 3x}-\frac{\sin 3x}{9}+A$$

$$C_2(x)=-\frac{1}{3}\int\frac{\cos^2 3x}{\sin 3x}dx=-\frac{1}{9}\int\frac{\cos^2 3x}{\sin 3x}d3x=$$

$$=\frac{\ln\left(\frac{1}{\sin 3x}+ctg\,3x\right)-\cos 3x}{9}+B$$

$$y=-\frac{1}{9}-\frac{\sin^2 3x}{9}+A+\left(\frac{\ln\left(\frac{1}{\sin 3x}+ctg\,3y\right)-\cos 3x}{9}+B\right)\cos 3x$$

Answer:

$$y(x)=y_c(x)+y_p(x)=$$
$$c_1\cos(3x)+c_2\sin(3x)+\frac{1}{9}\left(\cos(3x)\log\left(\cot\left(\frac{3x}{2}\right)\right)-2\right)$$

2. x'=2x-y
   y'=x+2y, x(0)=1, y(0)=-1

2) $\begin{cases} \dot{x} = 2x - y \\ \dot{y} = x + 2y \end{cases}$ $\quad x(0)=1$, $y(0)=-1$

$A = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$

$\begin{vmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 + 1 = \lambda^2 - 4\lambda + 5 = 0$

$D = 16 - 20 = -4$

$\lambda = 2 \pm i$

$\left(\begin{array}{cc|c} i & -1 & 0 \\ 1 & i & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} i & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\quad x^2 = -ix_1$, $\vec{\xi} = \begin{pmatrix} 1 \\ -i \end{pmatrix}$

$\begin{pmatrix} 1 \\ -i \end{pmatrix} e^{(2-i)t} = \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{2t} e^{(-t)i} = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ -1 \end{pmatrix}\right] (\cos(-t) + i\sin(-t)) e^{2t} =$

$= \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ -1 \end{pmatrix}\right] (\cos t - i \sin t) e^{2t} = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t - i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t + i \begin{pmatrix} 0 \\ -1 \end{pmatrix} \cos t + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \sin t\right] e^{2t} =$

$= \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} e^{2t} + i \begin{pmatrix} -\sin t \\ -\cos t \end{pmatrix} e^{2t}$

$\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} e^{2t} - C_2 \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} e^{2t}$

$\begin{cases} x = C_1 \cos t\, e^{2t} - C_2 \cdot \sin t \cdot e^{2t} \\ y = -C_1 \cdot \sin t \cdot e^{2t} - C_2 \cdot \cos t \cdot e^{2t} \end{cases} \longrightarrow \begin{cases} 1 = C_1 \cos 0\, e^0 - C_2 \cdot \sin 0 \cdot e^0 \\ -1 = C_1 \sin 0 \cdot e^0 - C_2 \cdot \cos 0 \cdot e^0 \end{cases} \longrightarrow \begin{cases} 1 = C_1 \to C_1 = 1 \\ -1 = -C_2 \to C_2 = 1 \end{cases}$

$\boxed{\begin{array}{l} x(0) = 1 \\ y(0) = 1 \\ C_1 = 1 \ ? \\ C_2 = 1 \end{array}}$

Ответ: $\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} e^{2t} - C_2 \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} e^{2t}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} e^{2t} - \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} e^{2t}$

3. y(5)+3y(4)+3y'''+y''=xe^(-x)+e^(-x)cosx-1

3) $y^{(5)}+3y^{(4)}+3y^{(3)}+y^{(2)}=xe^{-x}+e^{-x}\cos x-1$

$\lambda^5+3\lambda^4+3\lambda^3+\lambda^2=0$

$\lambda^2(\lambda^3+3\lambda^2+3\lambda+1)=0$

$\lambda^2(\lambda+1)^3=0$

$\left[\begin{array}{l}\lambda=0 \text{ кр } 2\\ \lambda=-1 \text{ кр } 3\end{array}\right.$

$y_{og}=C_1+C_2x+C_3e^{-x}+C_4xe^{-x}+C_5x^2e^{-x}$

$y_{ч.1}$

$f_1=xe^{-x}=e^{-x}(x\cos 0x+0\sin 0x)$

$\alpha=-1,\ \beta=0,\ m=1,\ n=-\infty$

$\bar{\lambda}=-1,\ \mu=1$

$r=3$

$e^{-x}x^3((Ax+B)\cos 0x+(Cx+D)\sin 0x)=$

$=Ae^{-x}x^4+Be^{-x}x^3$

$(Ae^{-x}x^4)'=4Ae^{-x}x^3-Ae^{-x}x^4$

$(Ae^{-x}x^4)''=12Ae^{-x}x^2-4Ae^{-x}x^3-4Ae^{-x}x^3+Ae^{-x}x^4=12Ae^{-x}x^2-8Ae^{-x}x^3+Ae^{-x}x^4$

$(Ae^{-x}x^4)^{(3)}=24Ae^{-x}x-12Ae^{-x}x^2-24Ae^{-x}x^2+8Ae^{-x}x^3+4Ae^{-x}x^3-Ae^{-x}x^4=$

$=24Ae^{-x}x-36e^{-x}x^2+12Ae^{-x}x^3-Ae^{-x}x^4$

$(Ae^{-x}x^4)^{(4)}=24Ae^{-x}-24Ae^{-x}x-72e^{-x}x+36Ae^{-x}x^2+36e^{-x}x^2-12Ae^{-x}x^3-4Ae^{-x}x^3+Ae^{-x}x^4$

$=24Ae^{-x}-96Ae^{-x}x+72Ae^{-x}x^2-16Ae^{-x}x^3+Ae^{-x}x^4$

$(Ae^{-x}x^4)^{(5)}=-24Ae^{-x}-96Ae^{-x}+96Ae^{-x}x+144Ae^{-x}x-72Ae^{-x}x^2-48Ae^{-x}x^2+16e^{-x}x^3+4Ae^{-x}x^3-Ae^{-x}x^4$

$=-120Ae^{-x}+240Ae^{-x}x-120Ae^{-x}x^2+20Ae^{-x}x^3-Ae^{-x}x^4$

$(Be^{-x}x^3)'=3Be^{-x}x^2-Be^{-x}x^3$

$(Be^{-x}x^3)''=6Be^{-x}x-3Be^{-x}x^2-3Be^{-x}x^2+Be^{-x}x^3=6Be^{-x}x-6Be^{-x}x^2+Be^{-x}x^3$

$(Be^{-x}x^3)^{(3)}=6Be^{-x}-6Be^{-x}x-12Be^{-x}x+6Be^{-x}x^2+3Be^{-x}x^2-Be^{-x}x^3=$
$=6Be^{-x}-18Be^{-x}x+9Be^{-x}x^2-Be^{-x}x^3$

$(Be^{-x}x^3)^{(4)}=-6Be^{-x}-18Be^{-x}+18Be^{-x}x+18Be^{-x}x-9Be^{-x}x^2-3Be^{-x}x^2+Be^{-x}x^3=$
$=-24Be^{-x}+36Be^{-x}x-12Be^{-x}x^2+Be^{-x}x^3$

$(Be^{-x}x^3)^{(5)}=24Be^{-x}+36Be^{-x}-36Be^{-x}x-24Be^{-x}x+12Be^{-x}x^2+3Be^{-x}x^2-Be^{-x}x^3=$
$=60Be^{-x}-60Be^{-x}x+15Be^{-x}x^2-Be^{-x}x^3$

$(Ae^{-x}\cos x)'=(-Ae^{-x}\sin x-Ae^{-x}\cos x)$

$(Ae^{-x}\cos x)''=(-Ae^{-x}\cos x+Ae^{-x}\sin x+Ae^{-x}\sin x+Ae^{-x}\cos x=2e^{-x}\sin x$

$(Ae^{-x}\cos x)^{(3)}=2\cos x e^{-x}-2e^{-x}\sin x$

$(Ae^{-x}\cos x)^{(4)}=-2\sin x e^{-x}-2\cos x e^{-x}-2e^{-x}\cos x+2e^{-x}\sin x=-4e^{-x}\cos x$

$(Ae^{-x}\cos x)^{(5)}=4e^{-x}\sin x+4e^{-x}\cos x$

$(Be^{-x}\sin x)'=(Be^{-x}\cos x-Be^{-x}\sin x)$

$(Be^{-x}\sin x)''=(-Be^{-x}\sin x-Be^{-x}\cos x-Be^{-x}\cos x+Be^{-x}\sin x)=$
$=-2Be^{-x}\cos x$

$(Be^{-x}\sin x)^{(3)}=2Be^{-x}\sin x+2Be^{-x}\cos x$

$(Be^{-x}\sin x)^{(4)}=2Be^{-x}\cos x-2Be^{-x}\sin x-2Be^{-x}\sin x-2Be^{-x}\cos x$
$=-4Be^{-x}\sin x$

$(Be^{-x}\sin x)^{(5)}=-4Be^{-x}\cos x+4Be^{-x}\sin x$

$(Ax^2)'=2Ax$

$(Ax^2)''=2A$

$(2Ax)'''=0$

$-120Ae^{-x}+240Ae^{-x}x-120Ae^{-x}x^2+20Ae^{-x}x^3-Ae^{-x}x^4+60Be^{-x}-60Be^{-x}x+15Be^{-x}x^2-Be^{-x}x^3$
$72Ae^{-x}-288Ae^{-x}x+216Ae^{-x}x^2-48Ae^{-x}x^3+3Ae^{-x}x^4-72Be^{-x}+108Be^{-x}x-36Be^{-x}x^2+3Be^{-x}x^3$
$72Ae^{-x}x-108Ae^{-x}x^2+36Ae^{-x}x^3-3Ae^{-x}x^4+18Be^{-x}-54Be^{-x}x+27Be^{-x}x^2-3Be^{-x}x^3$
$12Ae^{-x}x^2-8Ae^{-x}x^3+Ae^{-x}x^4 \quad 6Be^{-x}x-6Be^{-x}x^2+Be^{-x}x^3=xe^{-x}$

$48Ae^{-x}+24Ae^{-x}x \quad 0 \quad 0 \quad 0 \quad 6Be^{-x} \quad 0 \quad 0 \quad 0 = xe^{-x}$

$48A+24Ax+6B=x$

$\begin{cases}A=\frac{1}{24}\\ B=-\frac{1}{3}\end{cases} \quad y_{ч.1}=\frac{1}{24}e^{-x}x^4-\frac{1}{3}e^{-x}x^3$

$f_2=e^{-x}\cos x=e^{-x}(\cos x+0\sin x)$

$\alpha=-1,\ \beta=1,\ m=0,\ n=-\infty$

$\bar{\lambda}=-1+i,\ \mu=0$

$e^{-x}(A\cos x+B\sin x)=Ae^{-x}\cos x+Be^{-x}\sin x$

1) $4Ae^{-x}\sin x+4Ae^{-x}\cos x-12Ae^{-x}\cos x+6\cos x e^{-x}A-6Ae^{-x}\sin x+2e^{-x}\sin x A=$

$=2Ae^{-x}\cos x$

2) $-4Be^{-x}\cos x+4Be^{-x}\sin x-12Be^{-x}\sin x+6Be^{-x}\sin x+6Be^{-x}\cos x-2Be^{-x}\cos x=$

$=-2Be^{-x}\sin x$

$2Ae^{-x}\cos x-2Be^{-x}\sin x=e^{-x}\cos x$

$\begin{cases}B=0\\ A=\frac{1}{2}\end{cases} \quad y_{ч.2}=\frac{1}{2}e^{-x}\cos x$

$f_3=-1=e^{0x}(-1\cos 0x+0\sin 0x)$

$\alpha=0,\ \beta=0,\ m=0,\ n=0$

$\bar{\lambda}=0,\ \mu=0$

$r=2$

$x^2e^{0x}(A\cos 0x+B\sin 0x)=Ax^2$

$2A=1 \qquad y_{ч.н.3}=\frac{1}{2}x^2$

$A=\frac{1}{2}$

$y=C_1+C_2x+C_3e^{-x}+C_4xe^{-x}+C_5x^2e^{-x}+\frac{1}{24}e^{-x}x^4-\frac{1}{3}x^3+\frac{1}{2}e^{-x}\cos x+\frac{1}{2}x^2$

12 вариант

1. y''-2y'=e^(5x)sin(e^(3x))

1) $y''-2y'=e^{5x}\sin e^{3x}$

$\lambda^2-2\lambda=0$

$\lambda(\lambda-2)=0$

$\left[\begin{array}{l}\lambda=0\ \text{кр}\ 1\\ \lambda=2\ \text{кр}\ 1\end{array}\right.$

$y_{oo}=C_1+C_2e^{2x}$

$\begin{pmatrix}1 & e^{2x}\\ 0 & 2e^{2x}\end{pmatrix}\begin{pmatrix}C_1'(x)\\ C_2'(x)\end{pmatrix}=\begin{pmatrix}0\\ e^{5x}\sin e^{3x}\end{pmatrix}$ $\quad \Delta=\begin{vmatrix}1 & e^{2x}\\ 0 & 2e^{2x}\end{vmatrix}=2e^{2x}$

$\Delta_1=\begin{vmatrix}0 & e^{2x}\\ e^{5x}\sin e^{3x} & 2e^{2x}\end{vmatrix}=-e^{7x}\sin e^{3x}$

$C_1'(x)=\frac{\Delta_1}{\Delta}=-\frac{1}{2}e^{5x}\sin e^{3x}$

$C_1(x)=-\frac{1}{2}\int e^{5x}\sin e^{3x}$? $\quad -\frac{i\left(\Gamma\left(\frac{5}{3},ie^{3x}\right)+\Gamma\left(\frac{5}{3},-ie^{3x}\right)\right)}{6(-1)^{\frac{5}{6}}}+C$

$\Delta_2=\begin{vmatrix}1 & 0\\ 0 & e^{5x}\sin e^{3x}\end{vmatrix}=e^{5x}\sin e^{3x}$

$C_2'(x)=e^{3x}\sin e^{3x}$

$C_2(x)=\int e^{3x}\sin e^{3x}dx=\frac{1}{3}\int e^{3x}\sin e^{3x}d3x=\frac{1}{3}\int\sin e^{3x}de^{3x}=-\frac{1}{3}\cos e^{3x}$

$y=C_1(x)?-\frac{e^{2x}}{3}\cos e^{3x}$

2. x'=3x
   y'=x+y, x(0)=-3, y(0)=2

2) $\begin{cases}\dot{x}=2x\\ \dot{y}=x+y\end{cases}$ $\quad x(0)=-3,\ y(0)=2$

$\begin{pmatrix}2 & 0\\ 1 & 1\end{pmatrix}$ $\quad \begin{vmatrix}2-\lambda & 0\\ 1 & 1-\lambda\end{vmatrix}=(2-\lambda)(1-\lambda)$

$\left[\begin{array}{l}\lambda=2\ \text{кр}\ 1\\ \lambda=1\ \text{кр}\ 1\end{array}\right.$

$\lambda=2\ \begin{pmatrix}0 & 0\\ 1 & -1\end{pmatrix}$ $\quad$ С.В $\begin{pmatrix}1\\ 1\end{pmatrix}$

$\lambda=1\ \begin{pmatrix}1 & 0\\ 1 & 0\end{pmatrix}$ $\quad$ С.В $\begin{pmatrix}0\\ 1\end{pmatrix}$

$\begin{pmatrix}x\\ y\end{pmatrix}=C_1\begin{pmatrix}1\\ 1\end{pmatrix}e^{2t}+C_2\begin{pmatrix}0\\ 1\end{pmatrix}e^{t}$

$\begin{cases}x=C_1e^{2t}\\ y=C_1e^{2t}+C_2e^{t}\end{cases}$ $\quad \begin{cases}-3=C_1\\ 2=-3+C_2\end{cases}$ $\quad \begin{cases}C_1=-3\\ C_2=5\end{cases}$

Ответ: $\begin{pmatrix}x\\ y\end{pmatrix}=-3\begin{pmatrix}1\\ 1\end{pmatrix}e^{2t}+5\begin{pmatrix}0\\ 1\end{pmatrix}e^{t}$

3. y(4)-8y’=x^2+1+sin(sqrt(3)x)-2e^(2x)

3) $y^{(4)}-8y'=x^2+1+\sin(\sqrt{3}x)-2e^{2x}$

$\lambda^4-8\lambda=0$

$\lambda(\lambda^3-8)=0$

$\lambda(\lambda-2)(\lambda^2+2\lambda+4)=0 \qquad D=4-16=-12$

$$\begin{cases} \lambda=0 & \text{кр } 1 \\ \lambda=2 & \text{кр } 1 \\ \lambda=-1\pm\sqrt{3}i & \text{кр } 1 \end{cases}$$

$y_{oo}=C_1+C_2e^{2x}+C_3e^{-x}\sin\sqrt{3}x+C_3e^{-x}\cos\sqrt{3}x$

$f_1=x^2+1=e^{0x}((x^2+1)\cos 0x+0\sin 0x)$

$\alpha=0,\ \beta=0,\ m=2,\ n=-\infty$

$\tilde{\lambda}=0,\ \nu=1$

$xe^{0x}((Ax^2+Bx+C)\cos 0x+(Dx^2+Ex+F)\sin 0x)=$

$=Ax^3+Bx^2+Cx$

$-24Ax^2-16Bx-8C=x^2+1$

$$\begin{cases} A=-\frac{1}{24} \\ B=0 \\ C=-\frac{1}{8} \end{cases} \qquad y_{r1}=-\frac{x^3}{24}-\frac{x}{8}$$

$f_2=\sin(\sqrt{3}x)=e^{0x}(0\cos\sqrt{3}x+1\sin\sqrt{3}x)$

$\alpha=0,\ \beta=\sqrt{3},\ m=-\infty,\ n=0$

$\tilde{\lambda}=\sqrt{3}i,\ \nu=0$

$e^{0x}(A\cos\sqrt{3}x+B\sin\sqrt{3}x)=A\cos\sqrt{3}x+B\sin\sqrt{3}x$

$9A\cos\sqrt{3}x+9B\sin\sqrt{3}x+8\sqrt{3}A\sin\sqrt{3}x-8\sqrt{3}B\cos\sqrt{3}x=\sin\sqrt{3}x$

$$\begin{cases} 9B+8\sqrt{3}A=1 \\ 9A-8\sqrt{3}B=0 \end{cases} \qquad 9A=8\sqrt{3}B \qquad A=\frac{8\sqrt{3}B}{9}$$

$9B+\frac{64}{3}B=1$

$30\frac{1}{3}B=1$

$$\begin{cases} B=\frac{3}{91} \\ A=\frac{8\sqrt{3}}{273} \end{cases} \qquad y_{r2}=\frac{8\sqrt{3}}{273}\cos\sqrt{3}x+\frac{3}{91}\sin\sqrt{3}x$$

$f_3=-2e^{2x}=e^{2x}(-2\cos 0x+0\sin 0x)$

$\alpha=2,\ \beta=0,\ m=0,\ n=-\infty$

$\tilde{\lambda}=2,\ \nu=0$

$r=1$

$e^{2x}x(A\cos 0x+B\sin 0x)=Ae^{2x}x$

$(Ax^3+Bx^2+Cx)'=3Ax^2+2Bx+C$

$(Ax^3+Bx^2+Cx)''=6Ax+2B$

$(Ax^3+Bx^2+Cx)^{(3)}=6A$

$(Ax^3+Bx^2+Cx)^{(4)}=0$

$(A\cos\sqrt{3}x+B\sin\sqrt{3}x)'=-A\sqrt{3}\sin\sqrt{3}x+B\sqrt{3}\cos\sqrt{3}x$

$(A\cos\sqrt{3}x+B\sin\sqrt{3}x)''=-A3\cos\sqrt{3}x-B3\sin\sqrt{3}x$

$(A\cos\sqrt{3}x+B\sin\sqrt{3}x)^{(3)}=3\sqrt{3}A\sin\sqrt{3}x-3\sqrt{3}B\cos\sqrt{3}x$

$(A\cos\sqrt{3}x+B\sin\sqrt{3}x)^{(4)}=9A\cos\sqrt{3}x+9B\sin\sqrt{3}x$

$(Ae^{2x}x)'=Ae^{2x}+2Ae^{2x}x$

$(Ae^{2x}x)''=2Ae^{2x}+2Ae^{2x}+4Ae^{2x}x=4Ae^{2x}+4Ae^{2x}x$

$(Ae^{2x}x)^{(3)}=8Ae^{2x}+4Ae^{2x}+8Ae^{2x}x=12Ae^{2x}+8Ae^{2x}x$

$(Ae^{2x}x)^{(4)}=24Ae^{2x}+8Ae^{2x}+16Ae^{2x}x=32Ae^{2x}+16Ae^{2x}x$

$32Ae^{2x}+16Ae^{2x}x-8Ae^{2x}-16Ae^{2x}x=e^{2x}$

$24A=1$

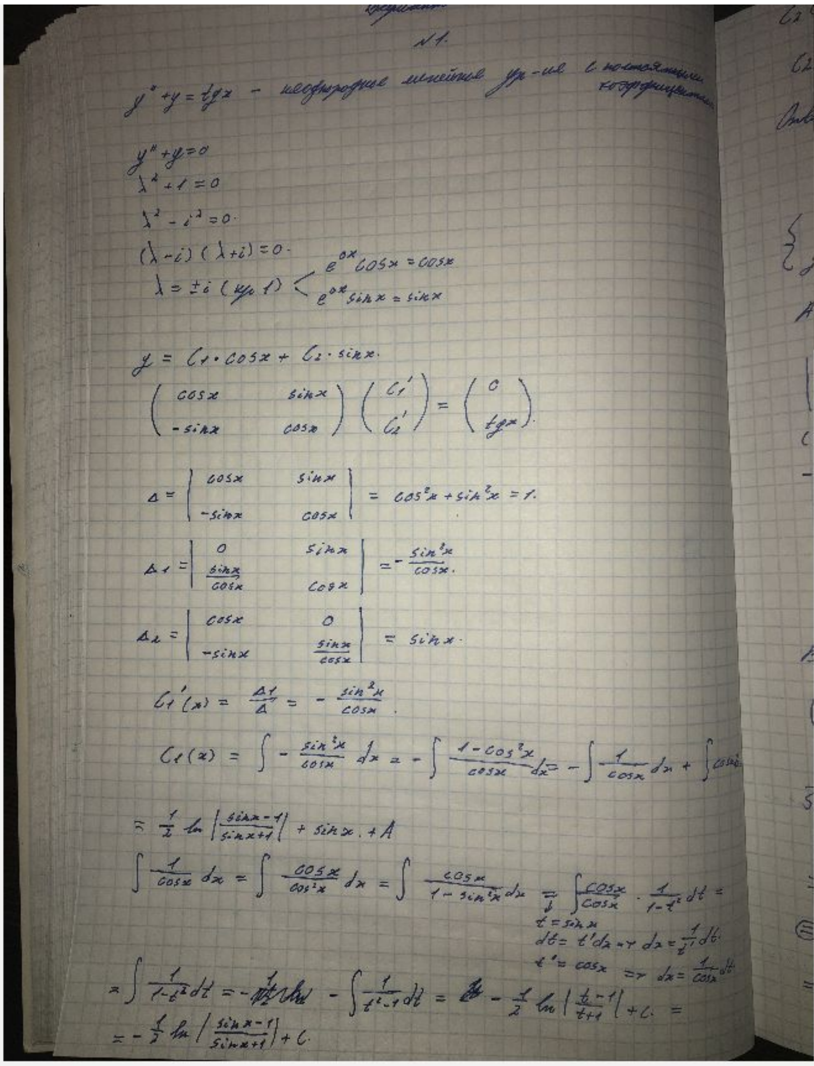
$A=\frac{1}{24}$

$y_{r3}=\frac{1}{24}e^{2x}x$

$$y=C_1+C_2e^{2x}+C_3\sin\sqrt{3}x+C_4\cos\sqrt{3}x-\frac{x^3}{24}-\frac{x}{8}+\frac{8\sqrt{3}}{273}\cos\sqrt{3}x+\frac{3}{91}\sin\sqrt{3}x+\frac{1}{24}e^{2x}x$$

13 вариант

1. y''+y=tgx



№1.

$y''+y=tgx$ – неоднородное линейное ур-ие с постоянными коэффициентами

$y''+y=0$

$\lambda^2+1=0$

$\lambda^2-i^2=0$

$(\lambda-i)(\lambda+i)=0$

$\lambda=\pm i$ (кр. 1) $<\begin{matrix} e^{0x}\cos x=\cos x \\ e^{0x}\sin x=\sin x\end{matrix}$

$y=C_1\cdot\cos x+C_2\cdot\sin x$

$$\begin{pmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{pmatrix}\begin{pmatrix}C_1'\\ C_2'\end{pmatrix}=\begin{pmatrix}0\\ tgx\end{pmatrix}$$

$$\Delta=\begin{vmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{vmatrix}=\cos^2x+\sin^2x=1$$

$$\Delta_1=\begin{vmatrix}0 & \sin x\\ \frac{\sin x}{\cos x} & \cos x\end{vmatrix}=-\frac{\sin^2x}{\cos x}$$

$$\Delta_2=\begin{vmatrix}\cos x & 0\\ -\sin x & \frac{\sin x}{\cos x}\end{vmatrix}=\sin x$$

$$C_1'(x)=\frac{\Delta_1}{\Delta}=-\frac{\sin^2x}{\cos x}$$

$$C_1(x)=\int-\frac{\sin^2x}{\cos x}dx=-\int\frac{1-\cos^2x}{\cos x}dx=-\int\frac{1}{\cos x}dx+\int\cos x\,dx$$

$$=\frac{1}{2}\ln\left|\frac{\sin x-1}{\sin x+1}\right|+\sin x+A$$

$$\int\frac{1}{\cos x}dx=\int\frac{\cos x}{\cos^2x}dx=\int\frac{\cos x}{1-\sin^2x}dx=\int\frac{\cos x}{\cos x}\cdot\frac{1}{1-t^2}dt=$$

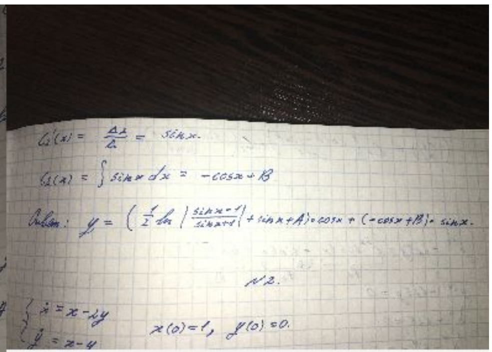
$t=\sin x$

$dt=t'dx\Rightarrow dx=\frac{1}{t'}dt$

$t'=\cos x\Rightarrow dx=\frac{1}{\cos x}dt$

$$=\int\frac{1}{1-t^2}dt=-\int\frac{1}{t^2-1}dt=-\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|+C=$$

$$=-\frac{1}{2}\ln\left|\frac{\sin x-1}{\sin x+1}\right|+C.$$



$$C_2'(x)=\frac{\Delta_2}{\Delta}=\sin x$$

$$C_2(x)=\int\sin x\,dx=-\cos x+B$$

Ответ: $y=\left(\frac{1}{2}\ln\left|\frac{\sin x-1}{\sin x+1}\right|+\sin x+A\right)\cdot\cos x+(-\cos x+B)\cdot\sin x.$

№2.

$\begin{cases}\dot{x}=x-2y\\ \dot{y}=x-y\end{cases}$ $x(0)=1,\ y(0)=0.$

2. x'=x-2y
   y'=x-y, x(0)=1, y(0)=0

$$\begin{cases} \dot{x} = x - 2y \\ \dot{y} = x - y \end{cases} \quad x(0)=1, \quad y(0)=0.$$

$$A = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}.$$

$$\begin{vmatrix} 1-\lambda & -2 \\ 1 & -1-\lambda \end{vmatrix} = 0.$$

$$(1-\lambda)(-1-\lambda)+2=0.$$

$$-1-\lambda+\lambda+\lambda^2+2=0.$$

$$\lambda^2+1=0.$$

$$\lambda^2-i^2=0.$$

$$(\lambda-i)(\lambda+i)=0$$

$\lambda = \pm i$ (кр. 1). – случай чуть посложнее

Выберем $\lambda = i$.

$$\begin{pmatrix} 1-i & -2 \\ 1 & -1-i \end{pmatrix} \to \begin{pmatrix} 1+i \\ 1 \end{pmatrix} \Rightarrow \bar{\xi} = \begin{pmatrix} 1+i \\ 1 \end{pmatrix}.$$

$$\xi \cdot e^{\lambda t} = \begin{pmatrix} 1+i \\ 1 \end{pmatrix} \cdot e^{i \cdot t} = \begin{pmatrix} 1+i \\ 1 \end{pmatrix} \cdot (\cos t + i \sin t) \ominus$$

Формула Эйлера: $e^{i\varphi} = \cos\varphi + i\sin\varphi$

$$\ominus \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} + i\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right](\cos t + i\sin t) =$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix}\cos t + i\begin{pmatrix} 1 \\ 1 \end{pmatrix}\sin t + i\begin{pmatrix} 1 \\ 0 \end{pmatrix}\cos t - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\sin t = \begin{pmatrix} \cos t - \sin t \\ \cos t \end{pmatrix} + i\begin{pmatrix} \sin t + \cos t \\ \sin t \end{pmatrix}.$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = C_1\begin{pmatrix} \cos t - \sin t \\ \cos t \end{pmatrix} + C_2\begin{pmatrix} \sin t + \cos t \\ \sin t \end{pmatrix}.$$

N 3.

$$y^{(4)} - 625y = e^{5x}\cos 5x - \sin 5x + 5xe^{-5x}$$

$$\begin{cases} x = C_1(\cos t - \sin t) + C_2(\sin t + \cos t) \\ y = C_1 \cos t + C_2 \sin t \end{cases}$$

$x(0)=1$

$y(0)=0$

$$\begin{cases} 1 = C_1(\cos 0 - \sin 0) + C_2(\sin 0 + \cos 0) \\ 0 = C_1 \cdot \cos 0 + C_2 \cdot \sin 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 1 = C_1 + C_2 \\ 0 = C_1 \end{cases} \Rightarrow \begin{cases} C_1 = 0 \\ C_2 = 1 \end{cases}$$

3. y(4)-625y=e^(5x)cos(5x)-sin(5x)+5xe^(-5x)

$$y^{(6)} + 8y^{(4)} + 16y^{(2)} = \underbrace{x\cos 2x}_{f_1} + \underbrace{xe^{2x}}_{f_2} \underbrace{-2x}_{f_3}$$

$$y^{(6)} + 8y^{(4)} + 16y^{(2)} = 0$$

$$\lambda^6 + 8\lambda^4 + 16\lambda^2 = 0$$

$$\lambda^2(\lambda^4 + 8\lambda^2 + 16) = 0$$

$$\lambda^2(\lambda^2+4)^2 = 0$$

$$\lambda^2(\lambda^2 - (2i)^2)^2 = 0$$

$$\lambda^2(\lambda - 2i)^2(\lambda + 2i)^2 = 0$$

$\lambda = 0$ (кр. 2) $\to e^{0x} = 1$, $xe^{0x} = x$

$\lambda = \pm 2i$ (кр. 2) $\to e^{0x}\cos 2x = \cos 2x$, $x\cos 2x$; $\sin 2x$, $x\sin 2x$

1) $x\cos 2x = e^{0x}[x\cos 2x + 0\sin 2x]$

$\left.\begin{matrix} \alpha = 0 \\ \beta = 2 \end{matrix}\right) \to \tilde{\lambda} = \alpha + \beta i = 2i \Rightarrow r = 2.$

$\left.\begin{matrix} m = 1 \\ n = -\infty \end{matrix}\right) \to N = 1$

2) $xe^{2x} = e^{2x}[x \cdot \cos 0x + 0\sin 0x]$

$\left.\begin{matrix} \alpha = 2 \\ \beta = 0 \end{matrix}\right) \to \tilde{\lambda} = \alpha + \beta i = 2. \Rightarrow r = 0$

$\left.\begin{matrix} m = 1 \\ n = -\infty \end{matrix}\right) \to N = 1$

14 вариант

1. y’’+6y’+9y=e^(-3x)/x

№1.

$y'' + 6y' + 9y = \frac{e^{-3x}}{x}$ – неодн. лин. диф. ур-ие. с пост. коэф.

$y'' + 6y' + 9y = 0$

$\lambda^2 + 6\lambda + 9 = 0.$

$(\lambda + 3)^2 = 0.$

$\lambda = -3$ (кр. 2) $\rightarrow e^{-3x}, \; xe^{-3x}.$

$y = C_1 \cdot e^{-3x} + C_2 \cdot xe^{-3x}.$

$$\begin{pmatrix} e^{-3x} & xe^{-3x} \\ -3e^{-3x} & e^{-3x} + x\cdot(-3)e^{-3x} \end{pmatrix} \begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = 0.$$

$$\Delta = \begin{vmatrix} e^{-3x} & xe^{-3x} \\ -3e^{-3x} & e^{-3x} - 3xe^{-3x} \end{vmatrix} = e^{-3x} \cdot e^{-3x} \begin{vmatrix} 1 & x \\ -3 & 1-3x \end{vmatrix} =$$

$$= e^{-6x}(1 - 3x + 3x) = e^{-6x}.$$

$$\Delta_1 = \begin{vmatrix} 0 & xe^{-3x} \\ \frac{e^{-3x}}{x} & e^{-3x} - 3xe^{-3x} \end{vmatrix} = e^{-3x} \cdot e^{-3x} \begin{vmatrix} 0 & x \\ \frac{1}{x} & 1-3x \end{vmatrix} =$$

$$= e^{-6x} \cdot \left(-x \cdot \frac{1}{x}\right) = -e^{-6x}.$$

$$\Delta_2 = \begin{vmatrix} e^{-3x} & 0 \\ -3e^{-3x} & \frac{e^{-3x}}{x} \end{vmatrix} = e^{-3x} \cdot e^{-3x} \begin{vmatrix} 1 & 0 \\ -3 & \frac{1}{x} \end{vmatrix} = e^{-6x} \cdot \frac{1}{x}.$$

$$C_1'(x) = \frac{\Delta_1}{\Delta} = \frac{-e^{-6x}}{e^{-6x}} = -1.$$

$$C_1(x) = -\int 1\,dx = -x + A.$$

$$C_2'(x) = \frac{\Delta_2}{\Delta} = \frac{e^{-6x}}{x} \cdot \frac{1}{e^{-6x}} = \frac{1}{x}.$$

$$C_2(x) = \int \frac{1}{x}\,dx = \ln|x| + B.$$

$$y = (-x + A)e^{-3x} + (\ln|x| + B)xe^{-3x}.$$

2. x’=-2x-y
y’=-3x-4y, x(0)=0, y(0)=4

$$\begin{cases} \dot{x} = -2x - y \\ \dot{y} = -3x - 4y \end{cases}$$

$$A = \begin{pmatrix} -2 & -1 \\ -3 & -4 \end{pmatrix}.$$

$$\begin{vmatrix} -2-\lambda & -1 \\ -3 & -4-\lambda \end{vmatrix} = 0 \qquad x(0)=0,\ y(0)=4.$$

$$(-2-\lambda)(-4-\lambda) - 3 = 0$$

$$8 + 2\lambda + 4\lambda + \lambda^2 - 3 = 0$$

$$\lambda^2 + 6\lambda + 5 = 0$$

$$(\lambda+1)(\lambda+5) = 0$$

$\lambda = -1$ (кр 1).

$\lambda = -5$ (кр 1).

$\lambda = -1$

$$\begin{pmatrix} -1 & -1 \\ -3 & -3 \end{pmatrix} \to \begin{pmatrix} 3 \\ -3 \end{pmatrix} \Rightarrow \bar{S}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

$\lambda = -5$

$$\begin{pmatrix} 3 & -1 \\ -3 & 1 \end{pmatrix} \to \begin{pmatrix} 1 \\ 3 \end{pmatrix} \Rightarrow \bar{S}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-5t}.$$

$$\begin{cases} x = C_1 \cdot e^{-t} + C_2 e^{-5t} \\ y = -C_1 e^{-t} + 3C_2 e^{-5t} \end{cases}$$

$$\begin{cases} 0 = C_1 e^0 + C_2 \cdot e^0 \\ 4 = -C_1 e^0 + 3C_2 \cdot e^0 \end{cases} \Rightarrow \begin{cases} 0 = C_1 + C_2 \\ 4 = -C_1 + 3C_2 \end{cases} \Rightarrow \begin{cases} C_1 = -C_2 \\ 4 = 4C_2 \Rightarrow C_2 = 1,\ C_1 = -1. \end{cases}$$

$$\begin{cases} C_1 = -1 \\ C_2 = 1. \end{cases}$$

3. y(5)+3y'''+2y'=e^x*cos(sqrt(2)x)-xsinx+x

15 вариант

1. y''-y'=e^(2x)cos(e^x)

$$y'' - y = e^{2x}\cos e^{x}$$

$$\lambda^2 - 1 = 0$$

$$\boxed{\lambda = 1 \quad \lambda = -1}$$

$$y = C_1 e^{x} + C_2 e^{-x}$$

$$\begin{pmatrix} e^{x} & e^{-x} \\ e^{x} & -e^{-x} \end{pmatrix} \begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = \begin{pmatrix} 0 \\ e^{2x}\cos e^{x} \end{pmatrix}$$

$$\Delta = \begin{vmatrix} e^{x} & e^{-x} \\ e^{x} & -e^{-x} \end{vmatrix} = -\frac{e^{x}}{e^{x}} - \frac{e^{x}}{e^{x}} = -2$$

$$\Delta_2 = \begin{vmatrix} e^{x} & 0 \\ e^{x} & e^{2x}\cos e^{x} \end{vmatrix} = e^{3x}\cos e^{x}$$

$$\Delta_1 = \begin{vmatrix} 0 & e^{-x} \\ e^{2x}\cos e^{x} & -e^{-x} \end{vmatrix} = -e^{x}\cos e^{x}$$

$$C_2' = \frac{\Delta_2}{\Delta} = -\frac{e^{3x}\cos e^{x}}{2}$$

$$C_1' = \frac{\Delta_1}{\Delta} = \frac{e^{x}\cos e^{x}}{2}$$

$$C_2 = -\int \frac{e^{3x}\cos e^{x}}{2} = -\frac{1}{2}\left((e^{2x}-2)\sin(e^{x}) + 2e^{x}\cos e^{x} + B\right)$$

$$C_1 = \frac{1}{2}\int e^{x}\cos e^{x}\,dx = \frac{1}{2}\left(\sin e^{x} + A\right)$$

$$y = \frac{1}{2}e^{x}\left(\sin e^{x} + A\right) - \frac{e^{-x}}{2}\left((e^{2x}-2)\sin(e^{x}) + 2e^{x}\cos e^{x} + B\right).$$

2. x'=x-2y
   y'=4x-3y, x(0)=-1, y(0)=0

$$\begin{cases} \dot{x} = x - 2y \\ \dot{y} = 4x - 3y, \end{cases} \quad \begin{matrix} x(0) = -1 \\ y(0) = 0 \end{matrix}$$

$$\begin{pmatrix} 1 & -2 \\ 4 & -3 \end{pmatrix} \Rightarrow \begin{pmatrix} 1-\lambda & -2 \\ 4 & -3-\lambda \end{pmatrix}$$

$$(1-\lambda)(-3-\lambda) + 8 = 0$$

$$-3 + \lambda^2 + 2\lambda + 8 = 0$$

$$\lambda^2 + 2\lambda + 5 = 0$$

$$D = 4 - 20 = -16$$

$$\boxed{\begin{matrix} \lambda_1 = -1 + 2i \\ \lambda_2 = -1 - 2i \end{matrix}} \leftarrow \text{Забить}$$

$$\begin{pmatrix} 2-2i & -2 \\ 4 & -2-2i \end{pmatrix} \to \begin{pmatrix} 1+i \\ 2 \end{pmatrix} \text{ св.}$$

$$\begin{pmatrix} 1+i \\ 2 \end{pmatrix} e^{(-1+2i)t}$$

$$e^{-t}(\cos(2t) + i\sin(2t))\left[\begin{pmatrix} 1 \\ 2 \end{pmatrix} + i\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] = e^{-t}\left[\cos(2t)\begin{pmatrix} 1 \\ 2 \end{pmatrix} + i\cos(2t)\begin{pmatrix} 1 \\ 0 \end{pmatrix} + i\sin(2t)\begin{pmatrix} 1 \\ 2 \end{pmatrix} - \sin(2t)\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] =$$

$$= e^{-t}\left[\begin{pmatrix} \cos(2t) - \sin(2t) \\ 2\cos(2t) \end{pmatrix} + i\begin{pmatrix} \cos(2t) + \sin(2t) \\ 2\sin(2t) \end{pmatrix}\right]$$

$$\begin{cases} x = e^{-t} C_1 (\cos(2t) - \sin(2t)) + e^{-t} C_2 (\cos(2t) + \sin(2t)) \\ y = e^{-t} C_1 2\cos(2t) + e^{-t} C_2 2\sin(2t) \end{cases} \quad \begin{cases} -1 = C_1 + C_2 \\ 0 = 2C_1 \end{cases} \quad \boxed{\begin{cases} C_1 = 0 \\ C_2 = -1 \end{cases}}$$

3. y(6)+8y(4)+16y''=xcos(2x)+xe^(2x)-2x

$y^{VI}+8y^{IV}+16y''=x\cos 2x+xe^{2x}-2x$

$\lambda^2(\lambda^4+8\lambda^2+16)=0$

$\lambda^2(\lambda^2+4)^2=0$

$\lambda_1=0$ к/ч 2

$\lambda_2=2i$ $\lambda_3=-2i$ к/ч 2

Ошибка

много решать

1) $x\cos 2x$

$\alpha=0$ $\beta=2$ $\tilde{\lambda}=2i$ $R=2$

$m=1$ $n=-\infty$ $N=1$

$y=x^2((Ax+B)\cos 2x+(Cx+D)\sin 2x)$

$y''=\ldots$ $y^{IV}=\ldots$ $y^{VI}=\ldots$

2) xe^{2x}

$\alpha=2$ $\beta=0$ $\tilde{\lambda}=2$ $R=0$

$n=1$ $m=-\infty$ $N=1$

$y=e^{2x}(Ax+B)$

$y'=Ae^{2x}+2e^{2x}(Ax+B)$

$y''=4e^{2x}A+4(Ax+B)e^{2x}$

$y'''=8e^{2x}A+4Ae^{2x}+8e^{2x}(Ax+B)$

$y^{IV}=24e^{2x}A+8e^{2x}A+16e^{2x}(Ax+B)$

$y^{V}=64e^{2x}A+16e^{2x}A+32e^{2x}(Ax+B)$

$y^{VI}=224e^{2x}A+64e^{2x}(Ax+B)$

3) $-2x$

$\alpha=0$ $\beta=0$ $\tilde{\lambda}=0$ $R=2$

$m=1$ $n=-\infty$ $N=1$

$y=x^2(Ax+A)$

$y'=2x(Ax+B)+Ax^2$

$y''=2(Ax+B)+2x^2+2xA$

$y'''=2x+4x+2A$

$y^{IV}=6$

$y^{V}=0$

16 вариант

1. y’’-6y’+9y=e^(3x)sqrt(x+4)

$$y''-6y'+9y=e^{3x}\sqrt{x+4}$$

$$\lambda^2-6\lambda+9=0$$

$$\boxed{\lambda=3} \quad k|2$$

$$y=C_1e^{3x}+C_2xe^{3x}$$

$$\begin{pmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & e^{3x}+3xe^{3x} \end{pmatrix}\begin{pmatrix} C_1' \\ C_2' \end{pmatrix}=\begin{pmatrix} 0 \\ e^{3x}\sqrt{x+4} \end{pmatrix}$$

$$\Delta=\begin{pmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & e^{3x}+3xe^{3x} \end{pmatrix}=e^{6x}+3xe^{6x}-3xe^{6x}=e^{6x}$$

$$\Delta_1=\begin{pmatrix} 0 & xe^{3x} \\ e^{3x}\sqrt{x+4} & e^{3x}+3xe^{3x} \end{pmatrix}=-xe^{6x}\sqrt{x+4}$$

$$\Delta_2=\begin{pmatrix} e^{3x} & 0 \\ 3e^{3x} & e^{3x}\sqrt{x+4} \end{pmatrix}=e^{6x}\sqrt{x+4}$$

$$C_1'=\frac{\Delta_1}{\Delta}$$

$$C_2'=\frac{\Delta_2}{\Delta}$$

$$C_1=-\int x\sqrt{x+4}=-\frac{2}{15}(x+4)^{3/2}(3x-8)$$

$$C_2=\int\sqrt{x+4}=\frac{2}{3}(x+4)^{3/2}$$

$$y=-e^{3x}\frac{2}{15}(x+4)^{3/2}(3x-8)+xe^{3x}\frac{2}{3}(x+4)^{3/2}$$

2. x’=-2y-x
y’=4y+3x, x(0)=1, y(0)=-2

$$\begin{cases} \dot{x} = -2y - x & x(0)=1 \\ \dot{y} = 4y + 3x & y(0)=-2 \end{cases}$$

$$\begin{pmatrix} -1 & -2 \\ 3 & 4 \end{pmatrix} \Rightarrow \begin{pmatrix} -1-\lambda & -2 \\ 3 & 4-\lambda \end{pmatrix}$$

$$(-1-\lambda)(4-\lambda)+6=0$$

$$\lambda^2-3\lambda+2=0$$

$$\boxed{\lambda_1=2 \quad \lambda_2=1}$$

$$\lambda_1 \begin{pmatrix} 1 & -2 \\ 3 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} -2 \\ 3 \end{pmatrix} \text{с.в.}$$

$$\lambda_2 \begin{pmatrix} 0 & -2 \\ 3 & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 \\ 1 \end{pmatrix} \text{с.в}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + C_2 \begin{pmatrix} -2 \\ 3 \end{pmatrix} e^{2t}$$

$$\begin{cases} 1 = -C_1 - 2C_2 \\ -2 = C_1 + 3C_2 \end{cases} \quad (+) \quad -1 = C_2$$

$$\begin{cases} C_1 = 1 \\ C_2 = -1 \end{cases}$$

3. y(6)+y'''=xe^(sqrt(3)*x/2)cosx-sqrt(5)x-sqrt(3)+e^(-x)

$$y^{VI}+y^{III}=xe^{\frac{x}{2}}+e^{\frac{\sqrt{3}}{2}x}\cos x-\sqrt{5}x-\sqrt{3}+e^{-x}$$

$\lambda^3(\lambda^3+1)=0$

$\lambda=0$ к.р. 3

$(\lambda+1)(\lambda^2-\lambda+1)$ $\lambda=-1$ к.р. 1

$D=1-4=-3$

$\lambda=\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$ к.р. 1

1) $xe^{\frac{x}{2}}$

$\alpha=\frac{1}{2}$ $\beta=0$ $\tilde{\lambda}=\frac{1}{2}$ $R=0$

$m=1$ $n=-\infty$ $N=1$

$y=e^{\frac{x}{2}}(Ax+B)$

$y'=\frac{1}{2}e^{\frac{x}{2}}(Ax+B)+e^{\frac{x}{2}}A$

$$y''=\frac{1}{4}e^{\frac{x}{2}}(Ax+B)+\underbrace{\frac{1}{2}e^{\frac{x}{2}}A+e^{\frac{x}{2}}A+\frac{1}{2}e^{\frac{x}{2}}A}_{2e^{\frac{x}{2}}A}$$

$y'''=\frac{1}{8}e^{\frac{x}{2}}(Ax+B)+\frac{5}{4}e^{\frac{x}{2}}A$

2) $e^{\frac{\sqrt{3}}{2}x}\cos x$

$\alpha=\frac{\sqrt{3}}{2}$ $\beta=1$ $\tilde{\lambda}=\frac{\sqrt{3}}{2}+i$ $R=0$

17 вариант

1. y’’-4y’+4y=e^(3x)sqrt(9-x)

$$y'' - 4y' + 4y = e^{2x}\sqrt{9-x}$$

1) $y''-4y'+4y=0$

$y=e^{\lambda x}$

$\lambda^2-4\lambda+4=0$

$(\lambda-2)^2=0$

$\lambda=2$ (кр.2): $y_1=e^{2x}$

$y_2=xe^{2x}$

$y_{o.o.}=C_1e^{2x}+C_2xe^{2x}$

2) $y_{o.н.}=C_1(x)e^{2x}+C_2(x)xe^{2x}$

$$\begin{cases} C_1'e^{2x}+C_2'xe^{2x}=0 \\ 2C_1'e^{2x}+C_2'(2xe^{2x}+e^{2x})=e^{2x}\sqrt{9-x} \end{cases}$$

$C_2'e^{2x}=e^{2x}\sqrt{9-x} \Rightarrow C_2'=\sqrt{9-x}$

$C_2=\int\sqrt{9-x}\,dx=-\int(9-x)^{\frac12}d(9-x)=-\frac{2}{3}(9-x)^{3/2}+\hat{C}_2$

$C_1'e^{2x}+\sqrt{9-x}\,xe^{2x}=0$

$C_1'=-\sqrt{9-x}\,x$

$C_1=-\int\sqrt{9-x}\,x\,dx=\frac{2x}{3}(9-x)^{\frac32}-\int\frac{2}{3}(9-x)^{\frac32}dx=\frac{2}{3}x(9-x)^{3/2}+\frac{2}{3}\cdot\frac{2}{5}(9-x)^{\frac52}+\hat{C}_1$

$y_{o.н.}=\left(\frac{2}{3}x(9-x)^{3/2}+\frac{4}{15}(9-x)^{\frac52}+\hat{C}_1\right)e^{2x}+\left(-\frac{2}{3}(9-x)^{\frac32}+\hat{C}_2\right)xe^{2x}$

2. x’=x+y
   y’=4y-2x, x(0)=0, y(0)=-1

$$\begin{cases} \dot{x} = x + y \\ \dot{y} = -2x + 4y \end{cases} \qquad \begin{matrix} x(0)=0 \\ y(0)=-1 \end{matrix}$$

Ищем реш. в виде:

$$\begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix} \qquad Y = \begin{pmatrix} x \\ y \end{pmatrix} = e^{\lambda t}\bar{\alpha}$$

$$\left|\begin{pmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{pmatrix}\right| = \lambda^2 - 5\lambda + 4 + 2 = \lambda^2 - 5\lambda + 6 = (\lambda-2)(\lambda-3)$$

1) $\lambda_1 = 2$: $\begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow \bar{\alpha}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$; $Y_1 = e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

2) $\lambda_2 = 3$: $\begin{pmatrix} -2 & 1 \\ -2 & 1 \end{pmatrix} \sim \begin{pmatrix} -2 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow \tilde{\alpha}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$Y_2 = e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

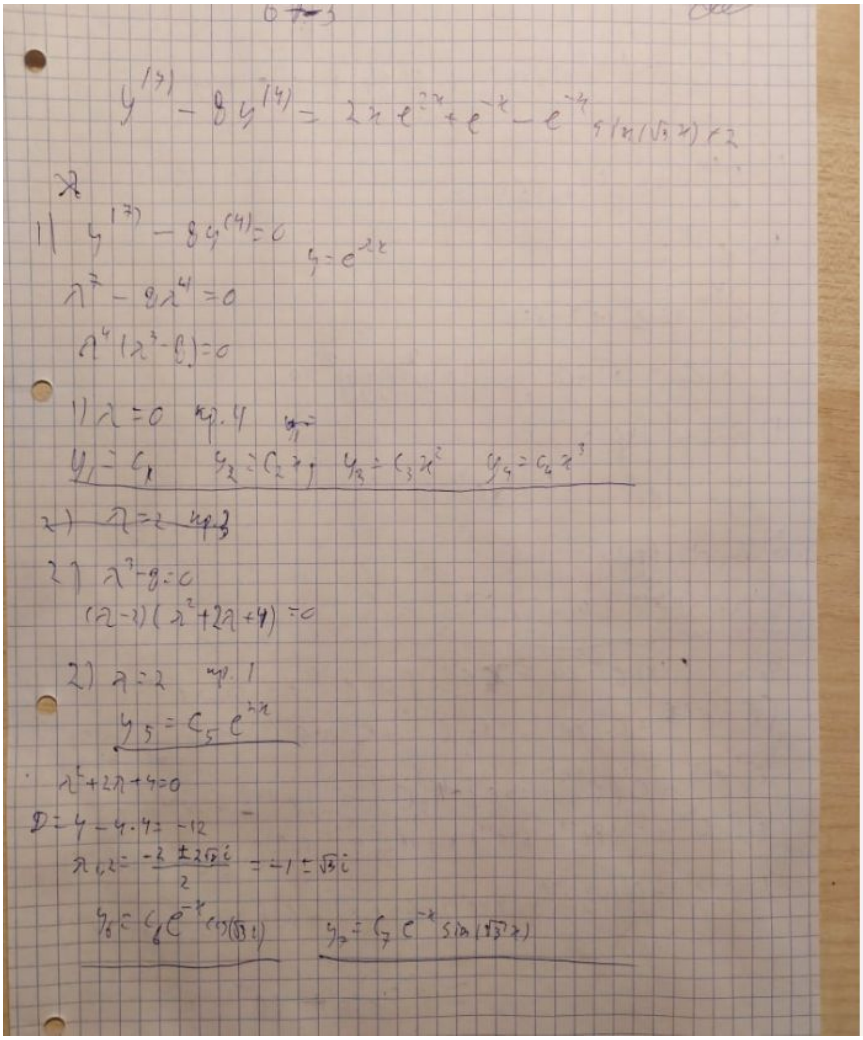
$$\Rightarrow Y_{o.o} = C_1 e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\cancel{Y\begin{pmatrix} 0 \\ 0 \end{pmatrix}} = Y(0) = \begin{pmatrix} C_1 \\ C_1 \end{pmatrix} + \begin{pmatrix} C_2 \\ 2C_2 \end{pmatrix} = \begin{pmatrix} C_1 + C_2 \\ C_1 + 2C_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{cases} C_1 + C_2 = 0 \\ C_1 + 2C_2 = -1 \end{cases} \Rightarrow \begin{cases} C_1 + C_2 = 0 \\ C_2 = -1 \end{cases} \Rightarrow C_1 = 1;\ C_2 = -1$$

$$\boxed{Y = e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{3t}\begin{pmatrix} -1 \\ -2 \end{pmatrix}; \quad \begin{matrix} x = e^{2t} - e^{3t} \\ y = e^{2t} - 2e^{3t} \end{matrix}}$$

3. y(7)-y(4)=2xe^(2x)+e^(-x)-e(-x)sin(sqrt(3)x)+2



$$y^{(7)} - 8y^{(4)} = 2xe^{2x} + e^{-x} - e^{-x}\sin(\sqrt{3}x) + 2$$

1) $y^{(7)} - 8y^{(4)} = 0$ $\qquad y = e^{-2x}$

$\lambda^7 - 8\lambda^4 = 0$

$\lambda^4(\lambda^3 - 8) = 0$

1) $\lambda = 0$ кр. 4

$y_1 = C_1 \quad y_2 = C_2 x, \quad y_3 = C_3 x^2 \quad y_4 = C_4 x^3$

~~2) $\lambda = 2$ кр. 3~~

2) $\lambda^3 - 8 = 0$

$(\lambda - 2)(\lambda^2 + 2\lambda + 4) = 0$

2) $\lambda = 2$ кр. 1

$y_5 = C_5 e^{2x}$

$\lambda^2 + 2\lambda + 4 = 0$

$D = 4 - 4 \cdot 4 = -12$

$\lambda_{1,2} = \frac{-2 \pm 2\sqrt{3}i}{2} = -1 \pm \sqrt{3}i$

$y_6 = C_6 e^{-x} \cos(\sqrt{3}x) \qquad y_7 = C_7 e^{-x} \sin(\sqrt{3}x)$

Надо дорешать!

18 вариант

1. y’’+6y’+9y=e^(-3x)/x

$$y''+6y'+9y=\frac{e^{-3x}}{x}$$

1) $y''+6y'+9y=0$

$y=e^{\lambda x}$

$\lambda^2+6\lambda+9=0$

$(\lambda+3)^2=0$

$\lambda=-3$ (кр.2) $\Rightarrow y_{о.о.}=C_1e^{-3x}+C_1xe^{-3x}$

2) $y_{о.н.}=C_1(x)e^{-3x}+C_2(x)xe^{-3x}$

$$\begin{cases} C_1'e^{-3x}+C_2'xe^{-3x}=0 & \Big|\cdot(-3) \\ -3C_1'e^{-3x}+C_2'(-3xe^{-3x}+e^{-3x})=\dfrac{e^{-3x}}{x} \end{cases}$$

$C_2'e^{-3x}=\dfrac{e^{-3x}}{x} \Rightarrow C_2'=\dfrac{1}{x} \Rightarrow C_2=\ln|x|+\hat{C}_2$

$C_1'=-\frac{1}{x}\cdot x\cdot e^{-3x}=-e^{-3x} \Rightarrow C_1=\dfrac{e^{-3x}}{3}+\hat{C}_1$

$y_{о.н.}=\left(\dfrac{e^{-3x}}{x}+\hat{C}_1\right)e^{-3x}+(\ln|x|+\hat{C}_2)xe^{-3x}$

2. x'=4x-5y
   y'=x, x(0)=0, y(0)=1

$$\begin{cases} x' = 4x - 5y \\ y' = x \end{cases} \qquad \begin{matrix} x(0) = 0 \\ y(0) = 1 \end{matrix}$$

$$\begin{pmatrix} 4 & -5 \\ 1 & 0 \end{pmatrix}$$

$$\begin{vmatrix} 4-\lambda & -5 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 4\lambda + 5 = \cancel{(\lambda+1)(\lambda-5)}$$

$$D = 16 - 20 = -4$$

$$\lambda_{1,2} = \frac{4 \pm i \cdot 2}{2} = 2 \pm i$$

$$\cancel{\Rightarrow y_{o.o} = C_1 e^{2x} \cos x + C_2 e^{2x} \sin x}$$

$$\cancel{\lambda = 2 + i} \qquad \cancel{y = \alpha e^{\lambda t}} \qquad Y = e^{\lambda t}$$

$$\lambda = 2 - i$$

$$\begin{pmatrix} 2+i & -5 \\ 1 & -2+i \end{pmatrix} \sim \begin{pmatrix} 2+i & -5 \\ 2+i & -1-4 \end{pmatrix} \sim \begin{pmatrix} 2+i & -5 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \bar{\alpha} = \begin{pmatrix} 5 \\ 2+i \end{pmatrix}$$

$$\cancel{Y_0 = \bar{\alpha} e^{(2-i)t} = \begin{pmatrix} 5 \\ 2+i \end{pmatrix} e^{2t}}$$

$$Y_0 = \begin{pmatrix} 5 \\ 2+i \end{pmatrix} e^{2t} (\cos t - i \sin t) = \begin{pmatrix} 5 \\ 2 \end{pmatrix} e^{2t} \cos t - \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{2t} \sin t +$$

$$+ \begin{pmatrix} 0 \\ i \end{pmatrix} e^{2t} \cos t - \begin{pmatrix} 5 \\ 2 \end{pmatrix} e^{2t} i \sin t =$$

$$= \begin{pmatrix} 5 \\ 2 \end{pmatrix} e^{2t} \cos t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t} \sin t +$$

$$+ i \left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t} \cos t - \begin{pmatrix} 5 \\ 2 \end{pmatrix} e^{2t} \sin t \right)$$

$$Y_{o.o} = C_1 \left(\begin{pmatrix} 5 \\ 2 \end{pmatrix} e^{2t} \cos t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t} \sin t \right) +$$

$$+ C_2 \left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t} \cos t - \begin{pmatrix} 5 \\ 2 \end{pmatrix} e^{2t} \sin t \right)$$

$$Y(0) = C_1 \begin{pmatrix} 5 \\ 2 \end{pmatrix} + C_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{cases} 5C_1 = 0 \\ 2C_1 + C_2 = 1 \end{cases} \Rightarrow \begin{matrix} C_1 = 0 \\ C_2 = 1 \end{matrix}$$

$$\Rightarrow Y = \begin{pmatrix} -5 \sin t \\ \cos t - 2 \sin t \end{pmatrix} e^{2t}$$

$$x = -5 \sin t \, e^{2t}$$

$$y = (\cos t - 2 \sin t) e^{2t}$$

3. y(4)-32y’’+256y=x^2*sin(4x)+xe^(-4x)-4

$$y^{(4)} - 32y^{(2)} + 256y = x^2 \sin 4x + xe^{-4x} - 4$$

1) $y^{(4)} - 32y^{(2)} + 256y = 0$

$y = e^{\lambda x}$:

$\lambda^4 - 32\lambda^2 + 256 = 0$

$(\lambda^2 - 16)^2 = 0$; $(\lambda-4)^2(\lambda+4)^2 = 0$

1) $\lambda_1 = 4$ кр. 2

$y_1 = C_1 e^{4x}$ $y_2 = C_2 x e^{4x}$

2) $\lambda_2 = -4$ кр. к

$y_3 = C_3 e^{-4x}$ $y_4 = C_4 x e^{-4x}$

$y_{oн} = C_1(x) e^{4x} + C_2(x) x e^{4x} + C_3(x) e^{-4x} + C_4(x) x e^{-4x}$

$$\begin{cases} C_1' e^{4x} + C_2' x e^{4x} + C_3' e^{-4x} + C_4' x e^{-4x} = 0 \\ 4C_1' e^{4x} + C_2'(e^{4x} + 4xe^{4x}) - 4C_3' e^{-4x} + C_4'(e^{-4x} - 4xe^{-4x}) = 0 \\ 16C_1' e^{4x} + C_2'(8e^{4x} + 16xe^{4x}) + 16C_3' e^{-4x} + C_4'(-8e^{-4x} + 16xe^{-4x}) = 0 \\ 64C_1' e^{4x} + C_2'(48e^{4x} + 64xe^{4x}) - 64C_3' e^{-4x} + C_4'(48e^{-4x} - 64xe^{-4x}) = \\ = x^2 \sin 4x + xe^{-4x} - 4 \end{cases}$$

32+16

~~32+16~~

Надо дорешать!