1 вариант

Вариант № 1.

1. $3x^2(1+\ln y)dx = (2y - \frac{x^3}{y})dy$
2. $y = (y')^3 + y'$
3. $xy'' = y'(\ln y' - \ln x)$
4. $yy'' - 2yy'\ln y = (y')^2$

1. 3x^2*(1+lny)dx=(2y-x^3/y)dy

1 вар

$$3x^2(1+\ln y)\,dx - (2y - \frac{x^3}{y})\,dy = 0$$

$$M(x,y) = 3x^2(1+\ln y) \qquad \frac{\partial M}{\partial y} = \frac{3x^2}{y} \qquad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$N(x,y) = -(2y - \frac{x^3}{y}) \qquad \frac{\partial N}{\partial x} = \frac{3x^2}{y}$$

ур-ние в полных диф.

$$I = \int (3x^2 + 3x^2\ln y)\,dx =$$

$$= x^3 + x^3\ln y + C(y)$$

$$\frac{\partial I}{\partial y} = \frac{x^3}{y} + C'(y)$$

$$\cancel{\frac{x^3}{y}} + C'(y) = -2y + \cancel{\frac{x^3}{y}}$$

$$C'(y) = -2y$$

$$C = -\int (2y)\,dy = -y^2 + \tilde{C}$$

$$I = x^3 + x^3\ln y - y^2 + \tilde{C}$$

$$\boxed{\tilde{C} = x^3 + x^3\ln y - y^2}$$

Место для галочки,
если номер решен верно

2. y=(y')^3+y'

$$y=(y')^3+y'$$

$$y'=p \Rightarrow \frac{dy}{dx}=p \Rightarrow \left.\begin{array}{l} dx=\frac{dy}{p} \\ f'(p)=\frac{dy}{dp} \end{array}\right\} \Rightarrow dx=\frac{f'(p)\,dp}{p}$$

$$y=p^3+p$$

$$dx=\frac{3p^2+1}{p}\,dp$$

$$x=\int\left(3p+\frac{1}{p}\right)dp=\frac{3p^2}{2}+\ln|p|+C$$

$$\boxed{\begin{cases} y=p^3+p \\ x=\frac{3p^2}{2}+\ln|p|+C \end{cases}}$$

3. xy''=y'(lny'-lnx)

$$xy''=y'(\ln y'-\ln x)$$

$$y'=p,\quad y''=p'$$

$$x\cdot p'=p\ln\frac{p}{x}$$

$$p'=\frac{p}{x}\ln\frac{p}{x}$$

$$\frac{p}{x}=z,\quad p=zx,\quad p'=z'x+z$$

$$z'x+z=z\ln z$$

$$z'x=z(\ln z-1)$$

$$\frac{dz}{z(\ln z-1)}=\frac{dx}{x}$$

$$\int\frac{dz}{z(\ln z-1)}=\int\frac{1}{\ln z-1}\,d(\ln z-1)=\ln|\ln z-1|$$

$$\ln|\ln z-1|=\ln x+\ln C$$

$$\ln z-1=Cx$$

$$z=e^{Cx+1}\Rightarrow y'=x\cdot e^{C_1x+1}$$

$$y=\int x\cdot e^{C_1x+1}dx=e\int xe^{C_1x}dx=e\,\frac{xe^{C_1x}}{C_1}-e\int\frac{e^{C_1x}}{C_1}dx=$$

$$=\frac{xe^{C_1x+1}}{C_1}-\frac{e}{C_1^2}\int e^{C_1x}d\,C_1x=\frac{xe^{C_1x+1}}{C_1}-\frac{e^{C_1x+1}}{C_1^2}+C_2$$

$$\boxed{y=\frac{(C_1x-1)e^{C_1x+1}}{C_1^2}+C_2}$$

Место для галочки,
если номер решен верно

4. yy’’-2yy’lny=(y’)^2

$$yy'' - 2yy'\ln y = (y')^2$$

$$y' = p \Rightarrow y'' = p'p$$

$$yp'p - 2yp\ln y = p^2$$

$$yp' - 2y\ln y = p \qquad p = 0 \Rightarrow y' = 0 \Rightarrow y = C\ ?$$

$$p' - 2\ln y - \frac{p}{y} = 0$$

$$p' - \frac{p}{y} = 2\ln y$$ — уравнение Бернулли

$$p' - p\frac{1}{y} = 2\ln y$$

$$p = uv$$

$$u'v + v'u + \frac{1}{y}uv = 2\ln y$$

$$\begin{cases} u'v - \frac{1}{y}uv = 0 \\ vu' = 2\ln y \end{cases} \rightarrow \begin{cases} u' = \frac{1}{y}u \\ v'u = 2\ln y \end{cases} \rightarrow \begin{cases} \frac{du}{dy} = \frac{u}{y} \\ v'u = 2\ln y \end{cases}$$

$$\rightarrow \begin{cases} \frac{du}{u} = \frac{dy}{y} \\ v'u = 2\ln y \end{cases} \rightarrow \begin{cases} u = y \\ v'(y) = 2\ln y \end{cases}$$

$$\frac{dv}{dy}y = 2\ln y$$

$$\int dv = \int \frac{2\ln y}{y}dy$$

$$v = 2\int \ln y\, d(\ln y) = \frac{2\ln^2 y}{2} = \ln^2 y + C$$

$$p = uv = y\ln^2 y + Cy$$

$$y' = y(\ln^2 y + C)$$

$$\frac{dy}{dx} = y(\ln^2 y + C)$$

$$dx = \frac{dy}{y(\ln^2 y + C)}$$

$$x = \int \frac{dy}{y(\ln^2 y + C)} = \int \frac{d(\ln y)}{\ln^2 y + C}$$

$$x = \frac{1}{\sqrt{C_1}}\operatorname{arctg}\frac{\ln y}{\sqrt{C_1}} + C_2$$

$$x = -\frac{1}{\sqrt{C_1}}\operatorname{arcctg}\frac{\ln y}{\sqrt{C_1}} + C_2$$

Место для галочки, если номер решен верно

$$x = \frac{1}{\sqrt{C_1}}\operatorname{arctg}\frac{\ln y}{\sqrt{C_1}} + C_2$$

$$x = -\frac{1}{\sqrt{C_1}}\operatorname{arcctg}\frac{\ln y}{\sqrt{C_1}} + C_2$$

$$C_1 = 0 \cdot x = -\frac{1}{\ln y} + C_2$$

2 вариант

Вариант № 2.

1. $xy' - y = x\cos^2\frac{y}{x}$
2. $x = \ln(1 + (y')^2)$
3. $y'' + \frac{y'}{x} - x = 0$
4. $\frac{y''}{y'} = -\frac{y'}{1+y}$

1. xy’-y=xcos(y/x)^2

$$xy' - y = x\cos^2\frac{y}{x}$$

$$x\frac{dy}{dx} - y = x\cdot\cos^2\frac{y}{x}$$

$$\frac{dy}{dx} = \cos^2\frac{y}{x} + \frac{y}{x}$$

$$t = \frac{y}{x} \Rightarrow y = tx$$

$$y' = t'x + t$$

$$t'x + \cancel{t} = \cos^2 t + \cancel{t}$$

$$\frac{dt}{dx}x = \cos^2 t$$

$$\frac{dt}{\cos^2 t} = \frac{dx}{x}$$

$$tg\,t = \ln x + \ln C$$

$$tg\,\frac{y}{x} = \ln Cx$$

$$\boxed{y = x\cdot arctg(\ln Cx)}$$

2. x=ln(1+(y’)^2)

$$x = \ln(1+(y')^2)$$

$$e^x = 1 + (y')^2$$

$$y' = p \Rightarrow \frac{dy}{dx} = p \Rightarrow dx = \frac{dy}{p} \Rightarrow dy = \frac{p}{dx}$$

$$dy = p\,f'(p)\,dp \quad \left(x = f(p) \Rightarrow \frac{dx}{dp} = f'(p)\right)$$

$$y = \int\frac{2p^2}{1+p^2}dp = 2\int\left(1 - \frac{1}{1+p^2}\right)dp = 2\int dp - 2\int\frac{dp}{1+p^2} =$$

$$= 2p - 2arctg\,p + C$$

$$\boxed{\begin{cases} x = \ln(1+p^2) \\ y = 2p - 2arctg\,p + C \end{cases}}$$

3. y’’+y’/x-x=0

$$y'' + \frac{y'}{x} - x = 0$$

$$y' = p, \quad y'' = p'$$

$$p' + \frac{p}{x} - x = 0$$ – уравнение Бернулли

Место для галочки, если номер решен верно

$$p = u \cdot v \Rightarrow u'v + uv' + \frac{1}{x} u \cdot v = x$$

$$\begin{cases} u'\cancel{v} + \frac{1}{x} u \cdot \cancel{v} = 0 \\ uv' = x \end{cases} \Rightarrow \begin{cases} \frac{du}{dx} = -\frac{u}{x} \\ u \cdot \frac{dv}{dx} = x \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \frac{du}{u} = -\frac{dx}{x} \\ dv = \frac{dx \cdot x}{u} \end{cases} \Rightarrow \begin{cases} u = \frac{1}{x} \\ dv = x^2 dx \end{cases} = \begin{cases} u = \frac{1}{x} \\ V = \frac{x^3}{3} + C \end{cases}$$

$$p = u \cdot V = \frac{1}{x} \cdot \left(\frac{x^3}{3} + C \right) = \frac{x^2}{3} + \frac{C}{x}$$

$$y' = \frac{dy}{dx} = \frac{x^2}{3} + \frac{C}{x} \Rightarrow dy = \left(\frac{x^2}{3} + \frac{C}{x} \right) dx$$

$$y = \int \frac{x^2}{3} dx + \int \frac{C}{x} dx = \frac{1}{3} \cdot \frac{x^3}{3} + C_1 \ln|x| + C_2$$

$$\boxed{y = \frac{x^3}{9} + C_1 \ln|x| + C_2}$$

4. y’’/y’=-(y’/(1+y))

$$\frac{y''}{y'} = -\frac{y'}{1+y}$$

Место для галочки, если номер решен верно

$$y' = p \Rightarrow y'' = p'_y \cdot p$$

$$\frac{p'_y \cdot \cancel{p}}{\cancel{p}} = -\frac{p}{1+y}, \quad p \neq 0$$

$$\frac{dp}{dy} = -\frac{p}{1+y} \Rightarrow \frac{dp}{p} = -\frac{dy}{1+y} \Rightarrow p = \frac{C}{1+y}$$

$$y' = \frac{dy}{dx} = \frac{C}{1+y} \Rightarrow dx = \frac{1+y}{C} dy$$

$$x = \frac{1}{C} \int (1+y) dy = \frac{1}{C_1} \left(y + \frac{y^2}{2} \right) + C_2$$

$$\boxed{x = \frac{y}{C_1} + \frac{y^2}{2C_1} + C_2}$$

3 вариант

Вариант № 3.

1. $xy' - y - x + 2 = 0$
2. $y = (y')^2 + 2(y')^3$
3. $xy'' = y' \ln \frac{y'}{x}$
4. $1 + (y')^2 - 2yy'' = 0$

1. xy’-y-x+2=0

$$xy' - y - x + 2 = 0$$

3 вар

$$\frac{dy}{dx}\cdot x = y + x - 2$$

$$y' = \frac{x + y - 2}{x} \qquad y' = f\left(\frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}\right)$$

$$\begin{cases} x + y - 2 = 0 \\ x = 0 \end{cases} \Rightarrow \begin{cases} y_0 = 2 \\ x_0 = 0 \end{cases} \Rightarrow \begin{cases} X = x \\ Y = y - 2 \end{cases} \Rightarrow z = \frac{y-2}{x}$$

$$y' = 1 + z, \quad y = zx + 2 \Rightarrow y' = z'x + z$$

$$z'x + \cancel{z} = 1 + \cancel{z}$$

$$\frac{dz}{dx}x = 1 \Rightarrow dz = \frac{dx}{x} \Rightarrow z = \ln Cx$$

$$\frac{y-2}{x} = \ln Cx$$

$$\boxed{y = x\cdot \ln Cx + 2}$$

2. y=(y’)^2+2*(y’)^3

$$y = (y')^2 + 2(y')^3 \qquad \frac{dy}{dx} = p \Rightarrow dx = \frac{dy}{p}$$

$$y' = p \qquad f'(p) = \frac{dy}{dp} \Rightarrow dy = f'(p)\,dp$$

$$y = p^2 + 2p^3$$

$$dx = \frac{2p + 6p^2}{p} = 2 + 6p$$

$$x = \int (2 + 6p)\,dp = 2p + 3p^2 + C$$

$$\boxed{\begin{cases} y = p^2 + 2p^3 \\ x = 2p + 3p^2 + C \end{cases}}$$

$$y = f(p)$$

$$f(p) = p^2 + 2p^3$$

$$y' = f'(p) = \frac{dy}{dp}$$

3. xy''=y'ln(y'/x)

$$xy'' = y'\ln\frac{y'}{x}$$

$$y' = p,\quad y'' = p'$$

$$x\cdot p' = p\ln\frac{p}{x}$$

$$p' = \frac{p}{x}\ln\frac{p}{x}$$

$$\frac{p}{x} = z,\ p = zx,\ p' = z'x + z$$

$$z'x + z = z\ln z$$

$$z'x = z(\ln z - 1)$$

$$\frac{dz}{z(\ln z - 1)} = \frac{dx}{x}$$

$$\int\frac{dz}{z(\ln z - 1)} = \int\frac{1}{\ln z - 1}\,d(\ln z - 1) = \ln|\ln z - 1|$$

$$\ln|\ln z - 1| = \ln x + \ln C$$

$$\ln z - 1 = Cx$$

$$z = e^{Cx+1} \Rightarrow y' = x\cdot e^{C_1x+1}$$

$$y = \int x\cdot e^{C_1x+1}dx = e\int xe^{C_1x}dx = e\,\frac{xe^{C_1x}}{C_1} - e\int\frac{e^{C_1x}}{C_1}dx =$$

$$= \frac{xe^{C_1x+1}}{C_1} - \frac{e}{C_1^2}\int e^{C_1x}\,dC_1x = \frac{xe^{C_1x+1}}{C_1} - \frac{e^{C_1x+1}}{C_1^2} + C_2$$

$$\boxed{y = \frac{(C_1x-1)e^{C_1x+1}}{C^2} + C_2}$$

Место для галочки,
если номер решен верно

4. 1+(y’)^2-2yy’’=0

4) $1+(y')^2-2yy''=0$

$y'=p \Rightarrow y''=p'_y\cdot p$

$1+p^2-2pyp'_y=0$

$\frac{dp}{dy}=\frac{1+p^2}{2p}\cdot\frac{1}{y} \Rightarrow \frac{dp\cdot 2p}{1+p^2}=\frac{dy}{y}$

$\int\frac{d(p^2+1)}{p^2+1}=\ln|y|+\ln|C|$

$p^2+1=Cy \Rightarrow p=\sqrt{Cy-1}=\frac{dy}{dx}$

$dx=\frac{dy}{\sqrt{Cy-1}}$

$x=\frac{1}{C}\int\frac{d(Cy-1)}{\sqrt{Cy-1}}=\frac{1}{C_1}\cdot 2(C_1y-1)^{\frac{1}{2}}+C_2$

$$\boxed{x=\frac{2\sqrt{C_1y-1}}{C_1}+C_2}$$

Место для галочки,
если номер решен верно

4 вариант

Вариант № 4.

1. $2x\cos^2y\,dx+(2y-x^2\sin2y)dy=0$
2. $(y')^3+6(y')^4=y+3$
3. $x^4y''+2x^3y'=1$
4. $yy''-y'(1+y')=0$

1. 2xcos(y)^2*dx+(2y-x^2*sin(2y))dy=0

$$2x\cos^2 y\,dx + (2y - x^2\sin 2y)\,dy = 0$$

4 вар

$$M(x,y) = 2x\cos^2 y \qquad \frac{\partial M}{\partial y} = 2x(-2\cos y\cdot\sin y) = -2x\sin 2y$$

$$N(x,y) = 2y - x^2\sin 2y \qquad \frac{\partial N}{\partial x} = -2x\cdot\sin 2y$$

ур-ние в полных диф.

$$I = \int 2x\cos^2 y\,dx = x^2\cos^2 y + C(y)$$

$$\frac{\partial I}{\partial y} = -x^2\cdot 2\cdot\cos y\cdot\sin y + C'(y)$$

$$\cancel{-x^2\sin 2y} + C'(y) = 2y - \cancel{x^2\sin 2y}$$

$$C'(y) = 2y \Rightarrow C = y^2 + \tilde{C}$$

$$I = x^2\cos^2 y + y^2 + \tilde{C}$$

$$\boxed{\tilde{C} = x^2\cos^2 y + y^2}$$

2. (y’)^3+6(y’)^4=y+3

№2.

$$(y')^3 + 6(y')^4 = y + 3.$$

Замена: $y' = p \Rightarrow \frac{dy}{dx} = p$.

$$p^3 + 6p^4 = y + 3.$$

$$\frac{d(..p..)}{dx} = \frac{d(..p..)}{dp}\cdot\frac{dp}{dx}$$

$$y = p^3 + 6p^4 - 3.$$

$$\frac{dy}{dx} = \frac{d(p^3+6p^4-3)}{dp}\cdot\frac{dp}{dx} = (p^3+6p^4-3)'_p\cdot\frac{dp}{dx}.$$

$$\frac{dy}{dx} = (3p^2 + 24p^3)\frac{dp}{dx}.$$

$$p = (3p^2 + 24p^3)\frac{dp}{dx}.$$

$$\frac{dx}{dp} = \frac{3p^2 + 24p^3}{p} = 3p + 24p^2.$$

$$dx = (3p + 24p^2)dp$$

$$\int dx = \int(3p + 24p^2)dp.$$

$$x = \frac{3}{2}p^2 + 8p^3 + C$$

$$\begin{cases} x(p) = \frac{3}{2}p^2 + 8p^3 + C \\ y(p) = 6p^4 + p^3 - 3 \end{cases},\ C\in\mathbb{R}.$$

3. x^4*y''+2x^3*y'=1

№3.

$x^4y''+2x^3y'=1.$

Замена: $y'=p \Rightarrow \frac{dy}{dx}=p.$

$y''=(y')'=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{dp}{dx}=p'.$

$\cancel{x^4p'+2x^3p=1.} \quad |:x^4$

$\cancel{p'+\frac{2}{x}p=1}$ – линейное ур-ие.

$x^4p'+2x^3p=1. \quad |:x^4$

$p'+\frac{2}{x}p=\frac{1}{x^4}.$

$\cancel{p'+\frac{2}{x}p=0.}$

$p=u(x)v(x) \Rightarrow p'=u'v+uv'.$

$u'v+uv'+\frac{2}{x}uv=\frac{1}{x^4}.$

$u'v+\frac{2}{x}uv=0. \quad |:v$

$u'+\frac{2}{x}u=0.$

$\frac{du}{dx}=-\frac{2}{x}u.$

$\frac{du}{u}=-\frac{2dx}{x}.$

$\ln|u|=-2\ln|x|$

$e^{\ln u}=e^{\ln x^{-2}}$

$u=x^{-2}.$

$u=\frac{1}{x^2}.$

$uv'=\frac{1}{x^4}.$

$\frac{1}{x^2}v'=\frac{1}{x^4} \quad |\cdot x^2$

$v'=\frac{1}{x^2}$

$\frac{dv}{dx}=\frac{1}{x^2}.$

$dv=\frac{dx}{x^2}.$

$v=-\frac{1}{x}+C$

$p=uv=\frac{1}{x^2}\left(-\frac{1}{x}+C\right)=-\frac{1}{x^3}+\frac{1}{x^2}C.$

$y'=p.$

$\int dy=\int p\,dx=\int\left(-\frac{1}{x^3}+\frac{1}{x^2}C\right)dx=\frac{1}{2x^2}-\frac{1}{x}C+\tilde{C}$

$\boxed{y=\frac{1}{2x^2}-\frac{1}{x}C+\tilde{C}}$

x

4. yy’’-y’(1+y’)=0

4) $yy'' - y'(1+y') = 0$

$y' = p,\quad y'' = p'_y \cdot p$

$y \cdot p'_y \cdot p - p(1+p) = 0 \quad p = 0$

$y' = 0$

$y = C$ – решение

$\frac{dp}{dy} = \frac{1+p}{y} \Rightarrow \frac{dp}{1+p} = \frac{dy}{y}$

$\ln|p+1| = \ln|y| + \ln|C|$

$p + 1 = Cy \Rightarrow \frac{dy}{dx} = Cy - 1$

$dx = \frac{dy}{Cy-1} =$

$x = \frac{1}{C}\int \frac{d(Cy-1)}{Cy-1} = \frac{1}{C_1} \cdot \ln|C_1y-1| + C_2$

$\boxed{x = \frac{\ln|C_1y-1|}{C_1} + C_2} + y = C$

Место для галочки, если номер решен верно

5 вариант

Вариант № 5.

1. $xy' = \sqrt{x^2 - y^2} + y$
2. $x = y' + \operatorname{arctg} y'$
3. $xy'' - y' = x^2 \sin^2 x$
4. $yy'' + (y')^2 = 2yy'$

1. xy’=sqrt(x^2-y^2)+y

1) $xy' = \sqrt{x^2-y^2}+y$

$x\frac{dy}{dx} = \sqrt{x^2-y^2}+y;$ Замена: $y(x) = xg(x)$

Тогда: $x^2\frac{dg}{dx} - \sqrt{-x^2g^2+x^2} = 0;$

$x^2\frac{dg}{dx} - x\sqrt{1-g^2} = 0 \quad / : x$

$x\frac{dg}{dx} - \sqrt{1-g^2} = 0$

$\frac{dg}{\sqrt{1-g^2}} = \frac{dx}{x}$

$\int\frac{1}{\sqrt{1-g^2}}\,dg = \int\frac{1}{x}\,dx;$

$\arcsin\frac{g}{1} = \log|x| + C_1;$

$g = \sin(\log|x|+C_1);$

Обратная замена:

$y(x) = g(x)x$

$y = x\sin(\log|x|+C_1)$

2. x=y’+arctg(y’)

2) $x = y' + \operatorname{arctg} y'$

$y' = p \Rightarrow \frac{dy}{dx} = p \Rightarrow dy = p\cdot dx$

$x = p + \operatorname{arctg} p$

$dx = 1 + \frac{1}{1+p^2}$

$dy = p\left(1 + \frac{1}{1+p^2}\right)dp$

$y = \frac{p^2}{2} + \frac{1}{2}\int\frac{d(p^2+1)}{p^2+1} =$

$$\begin{cases} y = \frac{p^2}{2} + \frac{1}{2}\ln|p^2+1| + C \\ x = p + \operatorname{arctg} p \end{cases}$$ ← ответ

3. xy’’-y’=x^2*sin(x)^2

$$xy'' - y' = x^2 \cdot \sin^2 x$$ 3

Замена: $y' = t$ $\boxed{t = uv}$

$$xt' - t = x^2 \sin^2 x \quad | : x$$

$$\frac{xt' - t}{x} = x \sin^2 x,$$

$$u'v + v'u - \frac{1}{x} uv = x \sin^2 x$$

$$\begin{cases} u'\cancel{v} - \frac{1}{x} u\cancel{v} = 0 \\ v'u = x\sin^2 x \end{cases} \Rightarrow \begin{cases} u' - \frac{1}{x} u = 0 \\ v'u = x \sin^2 x \end{cases} \Rightarrow \begin{cases} \frac{du}{u} = \frac{dx}{x} \\ dv = dx \cdot \frac{x \sin^2 x}{u} \end{cases}$$

$$\Rightarrow \begin{cases} u = x \\ dv = \sin^2 dx \end{cases} \Rightarrow \begin{cases} u = x \\ v = \frac{x}{2} - \frac{\sin(2x)}{4} + C_1 \end{cases}$$

$$t = x\left(\frac{x}{2} - \frac{\sin(2x)}{4} + C_1\right)$$

$$y' = x\left(\frac{x}{2} - \frac{\sin(2x)}{4} + C_1\right)$$

$$\int y' dy = \int \left(\frac{x^2}{2} - \frac{(\sin x \cos x) x}{2} + C_1 x\right) dx$$

$$y = \frac{x^3}{6} + \frac{1}{8} x \cos(2x) - \frac{1}{16} \sin(2x) + \frac{C_1 x^2}{2} + C_2.$$

4. yy''+(y')^2=2yy'

④ $yy'' + (y')^2 = 2yy'$

$y' = p \Rightarrow y'' = p'_y p$

$y p'_y p + p^2 = 2yp$

$p(y p'_y + p - 2y) = 0$ $\quad p = 0$ $\quad \boxed{y(x) = \tilde{C}}$

$y p'_y + p = 2y$

$\boxed{p = uv}$

$y(u'v + v'u) + uv = 2y$

$$\begin{cases} yu'\cancel{v} + u\cancel{v} = 0 \\ yv'u = 2y \end{cases} \Rightarrow \begin{cases} \int \frac{du}{u} = -\frac{dy}{y} \\ dv = \frac{2dy}{u} \end{cases} \Rightarrow \begin{cases} u = \frac{1}{y} \\ v = y^2 + C_1 \end{cases}$$

$p = y + \frac{C_1}{y}$;

$y' = y + \frac{C_1}{y} \Rightarrow \int dx = \int \frac{dy}{y + \frac{C_1}{y}}$;

$x = \frac{1}{2} \ln|y^2 + C_1| + C_2$

Ответ: $y = \pm\sqrt{C_1 + e^{2x} C_3}$ OR $y = \tilde{C}$.

6 вариант

Вариант № 6.

1. $xy' = x e^{\frac{y}{x}} + y$
2. $\sqrt[3]{(y')^2} - 8(y')^2 = x - y'$
3. $(x-1)y'' + 2y' = x + 1$
4. $y''(1+y) = (y')^2 + y'$

1. xy’=x*e^(y/x)+y

1) $xy' = xe^{\frac{y}{x}} + y$

$y' = e^{\frac{y}{x}} + \frac{y}{x}$

$\frac{y}{x} = t \Rightarrow y = tx \Rightarrow y' = t + t'x$

$\cancel{t} + t'x = e^{t} + \cancel{t}$

$\frac{dt}{dx} x = e^{t} \Rightarrow \frac{dt}{e^{t}} = \frac{dx}{x} \Rightarrow -e^{-t} = \ln|x| + \ln|C|$

$e^{-t} = -\ln|Cx| \Rightarrow \ln(e^{-t}) = \ln(\ln \frac{1}{|Cx|})$

$-t = \ln(\ln \frac{1}{|Cx|})$

$y = -x \cdot \ln(\ln \frac{1}{|Cx|})$

2. (y’)^(⅔)-8(y’)^2=x-y’

2) $\sqrt[3]{(y')^2} - 8(y')^2 = x - y'$

$y' = p \Rightarrow \frac{dy}{dx} = p \Rightarrow dy = p\,dx$

$p^{\frac{2}{3}} - 8p^2 = x - p$

$x = p^{\frac{2}{3}} - 8p^2 + p \Rightarrow x' = \frac{2}{3}p^{-\frac{1}{3}} - 16p + 1$

$dy = \left(\frac{2}{3}p^{\frac{2}{3}} - 16p^2 + p\right)dp$

$y = \frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{5} p^{\frac{5}{3}} - \frac{16p^3}{3} + \frac{p^2}{2} + C$

$$\begin{cases} y = \frac{2}{5}p^{\frac{5}{3}} - \frac{16p^3}{3} + \frac{p^2}{2} + C \\ x = \frac{2}{3}p^{-\frac{1}{3}} - 16p + 1 \end{cases}$$

3. (x-1)y''+2y'=x+1

3) $(x-1)y''+2y'=x+1$

$y'=p,\ y''=p'$

$(x-1)p'+2p=x+1 \quad (x\neq 1)$

$p'+p\cdot\frac{2}{(x-1)}=\frac{x+1}{x-1}$ – Бернулли

$p=u\cdot v \Rightarrow p'=u'v+uv'$

$u'v+uv'+u\cdot v\cdot\frac{2}{x-1}=\frac{x+1}{x-1}$

$$\begin{cases} u'\cancel{v}+u\cancel{v}\cdot\frac{2}{x-1}=0 \Rightarrow \frac{du}{dx}=-\frac{2u}{x-1} \\ uv'=\frac{x+1}{x-1} \end{cases}$$

$\frac{1}{2}\frac{du}{u}=-\frac{d(x-1)}{x-1}$

$\frac{1}{2}\ln|u|=-\ln|x-1|$

$\sqrt{u}=\frac{1}{x-1}$

$u=\frac{1}{(x-1)^2}$

$\frac{1}{(x-1)^2}\frac{dv}{dx}=\frac{x+1}{\cancel{x-1}}$

$dv=(x^2-1)dx$

$v=\frac{x^3}{3}-x+C$

$\frac{dy}{dx}=\frac{1}{(x-1)^2}\left(\frac{x^3}{3}-x+C\right)=\frac{x^3-3x+3C}{3(x-1)^2}$

$y=\int\frac{x^3-3x+3C}{3(x-1)^2}dx$ = сложно

4. y''(1+y)=(y')^2+y'

4) $y''(1+y)=(y')^2+y'$

$y'=p;\ y''=p'_y\cdot p$

$\boxed{y=-1}$ решение

$p'_y p(1+y)=p^2+p \to p=0;\ \boxed{y=C}$ решение

$p'_y=\frac{p+1}{(1+y)} \Rightarrow \frac{dp}{dy}=\frac{p+1}{y+1}$

$\frac{dp}{p+1}=\frac{dy}{y+1} \Rightarrow \ln|p+1|=\ln|y+1|+\overset{?}{\ln|C|}$

$p+1=C|y+1|$

$y'=C|y+1|-1$

$\frac{dy}{dx}=C|y+1|-1$

$\int dx=\int\frac{dy}{C|y+1|-1}=\frac{1}{C}\int\frac{dk}{k}=\frac{\ln|k|}{C}$

$k=C|y+1|-1 \Rightarrow dk=\frac{dy}{C}$

$x=\frac{\ln|C|y+1|-1|}{C}+\tilde{C}$

7 вариант

Вариант № 7.

1. $\left(\frac{\sin 2x}{y}+x\right)dx+\left(y-\frac{\sin^2 x}{y^2}\right)dy=0$
2. $yy'-(y')^3-1=0$
3. $(1+x)y''-y'=\sqrt{(1+x)^3}$
4. $3yy'y''=1+(y')^3$

1. (sin(2x)/y+x)dx+(y-sin(x)^2/y^2)dy=0

7 вариант.

$$1\left(\frac{\sin 2x}{y}+x\right)dx+\left(y-\frac{\sin^2 x}{y^2}\right)dy=0$$

$$\frac{\partial M}{\partial y}=\frac{\partial\left(\frac{\sin 2x}{y}+x\right)}{\partial y}=-\frac{\sin 2x}{y^2}$$

$$\frac{\partial N}{\partial x}=\frac{\partial\left(y-\frac{\sin^2 x}{y^2}\right)}{\partial x}=-\frac{\sin 2x}{y^2}$$

$$U(x,y)=\int M(x,y)dx+\varphi(y)=$$

$$=\int\left(\frac{\sin 2x}{y}+x\right)dx+\varphi(y)=$$

$$=-\frac{\cos 2x}{2y}+\frac{x^2}{2}+\varphi(y)$$

$$\frac{\partial U}{\partial y}=\frac{\partial\left(-\frac{\cos 2x}{2y}+\frac{x^2}{2}+\varphi(y)\right)}{\partial y}=y-\frac{\sin^2 x}{y^2}$$

$$\frac{\cos 2x}{2y^2}+\frac{d\varphi(y)}{dy}=y-\frac{\sin^2 x}{y^2}$$

$$\frac{d\varphi(y)}{dy}=y-\frac{2\sin^2 x-\cos 2x}{2y^2}=y-\frac{1}{2y^2}=\frac{2y^3-1}{2y^2}$$

$$d\varphi(y)=\frac{2y^3-1}{2y^2}dy$$

$$\varphi(y)=\frac{1}{2}x^2+\frac{1}{2x}+C$$

$$U(x,y)=-\frac{\cos 2x}{2y}+\frac{x^2}{2}+\frac{1}{2}x^2+\frac{1}{2x}+C$$

$$C=-\frac{\cos 2x}{2y}+\frac{x^2}{2}+\frac{1}{2}x^2+\frac{1}{2x}$$

2. yy’-(y’)^3-1=0

2. $yy' - (y')^3 - 1 = 0$

$y = \frac{1+(y')^3}{y'}$ $\qquad y' = p(x)$

$y = \frac{1+p^3}{p}$

$\frac{dy}{dx} = \frac{d\left(\frac{1+p^3}{p}\right)}{dp} \cdot \frac{dp}{dx}$

$p = \left(\frac{1+p^3}{p}\right)'_p \cdot \frac{dp}{dx} =$

$= \left(\frac{1}{p} + p^2\right)'_p \cdot \frac{dp}{dx}$

$\frac{dx}{dp} = \frac{1}{p}\left(p^{-1} + p^2\right)' = \frac{1}{p}\left(-p^{-2} + 2p\right) =$

$= -p^{-3} + 2$

$x(p) = \int(-p^{-3} + 2)\,dp = \frac{p^{-2}}{2} + 2p + C$

Ответ: $\begin{cases} y(p) = \frac{1}{p} + p^2 \\ x(p) = \frac{p^{-2}}{2} + 2p + C \end{cases}$

3. (1+x)y''-y'=sqrt((1+x)^3)

$$(1+x)y''-y'=\sqrt{(1+x)^3}$$

$$y'=p, \; y''=p'$$

$$(1+x)p'-p=\sqrt{(1+x)^3} \qquad x\neq -1$$

$$p'-\frac{1}{(1+x)}p=\sqrt{1+x} \text{ – Бернулли}$$

$$p=uv, \; p'=u'v+uv'$$

$$u'v+uv'-\frac{1}{1+x}uv=\sqrt{1+x}$$

$$\begin{cases} u'\cancel{v}-\frac{1}{1+x}u\cancel{v}=0 \\ uv'=\sqrt{1+x} \end{cases} \Rightarrow \begin{cases} \frac{du}{dx}=\frac{u}{1+x} \\ v'=\frac{\sqrt{1+x}}{u} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \frac{du}{u}=\frac{d(x+1)}{x+1} \\ dv=\frac{\sqrt{1+x}}{u}dx \end{cases} \Rightarrow \begin{cases} \ln|u|=\ln|x+1| \\ dv=\frac{\sqrt{1+x}}{(x+1)}dx \end{cases}$$

$$u=(x+1)$$

$$v=\int (x+1)^{-\frac{1}{2}}d(x+1)=2\sqrt{x+1}+C$$

$$y'=\frac{dy}{dx}=2(x+1)^{\frac{3}{2}}+C(x+1)$$

$$y=2\int (x+1)^{\frac{3}{2}}d(x+1)+C\int (x+1)\,d(x+1)=$$

$$=2\,\frac{2}{5}(x+1)^{\frac{5}{2}}+C\,\frac{(x+1)^2}{2}+\tilde{C}$$

$$y=\frac{4}{5}(x+1)^{\frac{5}{2}}+\frac{C}{2}(x+1)^2+\tilde{C}$$

4. 3yy’y’’=1+(y’)^3

$$3yy'y''=1+(y')^3$$

$$y'=p,\ y''=p'\cdot p$$

$$3yp^2\cdot p'=1+p^3$$

$$p'-\frac{p}{3y}=\frac{1}{3y}\cdot p^{-2}$$

$$p=u\cdot v,\ p'=u'v+uv'$$

$$u'v+uv'-\frac{1}{3y}uv=\frac{1}{3y}\cdot u^{-2}v^{-2}$$

$$\begin{cases} u'v=\frac{1}{3y}\cdot uv \\ uv'=\frac{1}{3y}\cdot\frac{1}{u^2v^2}\end{cases}$$

$$v=0\Rightarrow p=0\Rightarrow \cancel{y=C}$$

$$\frac{du}{dy}=\frac{u}{3y}\Rightarrow \ln u=\frac{1}{3}\ln y\Rightarrow u=y^{\frac{1}{3}}$$

$$\frac{dv}{dy}=\frac{1}{3y^2v^2}\Rightarrow v^2dv=\frac{1}{3}\frac{dy}{y^2}\Rightarrow \frac{v^3}{3}=-\frac{1}{3y}+C\Rightarrow v=\left(C-\frac{1}{y}\right)^{\frac{1}{3}}$$

$$p=uv=y'=y^{\frac{1}{3}}\left(C-\frac{1}{y}\right)^{\frac{1}{3}}=(Cy-1)^{\frac{1}{3}}$$

$$\frac{dy}{dx}=(Cy-1)^{\frac{1}{3}}\Rightarrow dx=(Cy-1)^{-\frac{1}{3}}dy$$

$$x=\int\frac{dy}{(Cy-1)^{\frac{1}{3}}}=\frac{1}{C}\int t^{-\frac{1}{3}}dt=\frac{1}{C_1}\cdot\frac{3}{2}\cdot t^{2/3}+C_2=\frac{3}{2}\frac{t^{2/3}}{C_1}+C_2$$

$$t=Cy-1,\ t'=C$$

$$\boxed{x=\frac{3(C_1y-1)^{2/3}}{2C_1}+C_2}$$

8 вариант

Вариант № 8.

1. $(xy+y^3)y'=1$
2. $2(y')^3-e^{-y'}-x=0$
3. $2xy'y''=(y')^2+1$
4. $yy''-2(y')^2=0$

1. (xy+y^3)y'=1

$$1)\ (xy+y^3)y'=1$$

8 вар

Место для галочки, если номер решен верно

$$(xy+y^3)\frac{dy}{dx}=1$$

$$xy+y^3=\frac{dx}{dy}$$

$x' - x\cdot y = +y^3$ — ур-ние бернулли

$$x=u\cdot v$$

$$u'v+uv'-y\cdot u\cdot v=+y^3$$

$$\begin{cases} u'v-y\cdot u\cdot v=0 \\ uv'=+y^3 \end{cases} \rightarrow \begin{cases} \frac{du}{dy}=y\cdot u \\ u\cdot v'=+y^3 \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} \frac{du}{u}=y\,dy \\ u\cdot v'=+y^3 \end{cases} \rightarrow \begin{cases} \ln u=\frac{y^2}{2} \\ u\cdot\frac{dv}{dy}=+y^3 \end{cases} \rightarrow \begin{cases} u=e^{\frac{y^2}{2}} \\ e^{\frac{y^2}{2}}\frac{dv}{dy}=+y^3 \end{cases} \rightarrow$$

$$\xrightarrow[t=\frac{y^2}{2}]{} v=\int\frac{y^3}{e^{\frac{y^2}{2}}}dy=\int\frac{y^2}{2e^{\frac{y^2}{2}}}d(y^2)=2\int\frac{y^2}{2}\cdot\frac{1}{e^{\frac{y^2}{2}}}d\left(\frac{y^2}{2}\right)=$$

$$\overset{1}{=}2\int t\cdot\frac{1}{e^t}dt=2\left(-t\cdot e^{-t}-\int -e^{-t}dt\right)=2\left(-t\cdot e^{-t}-e^{-t}\right)=$$

$$=2\cdot -e^{-t}(t+1)+C=-2e^{-\frac{y^2}{2}}\left(\frac{y^2}{2}+1\right)+C=$$

$$=-e^{-\frac{y^2}{2}}(y^2+1)+C$$

$$x=V\cdot u=e^{\frac{y^2}{2}}\cdot\left(-e^{-\frac{y^2}{2}}(y^2+1)+C\right)=\boxed{-y^2-1+e^{\frac{y^2}{2}}C}$$

2. 2(y')^3-e^(-y')-x=0

$$2)\ 2(y')-e^{-y}-x=0$$

Место для галочки, если номер решен верно

$$\boxed{y'=p \Rightarrow \frac{dy}{dx}=p \Rightarrow dy=p\cdot dx}$$

$$x=2p^3-e^{-p}$$

$$x'=6p^2+e^{-p}=6p^2+e^{-p}\quad dx$$

$$dy=p\underbrace{(6p^2+e^{-p})dp}_{dx}=(6p^3+pe^{-p})dp$$

$\int u\,dv=uv-\int v\,du$

$$\begin{cases} u=p \\ dv=e^{-p} \end{cases} \rightarrow \begin{cases} du=1 \\ v=-e^{-p} \end{cases}$$

$$y=\int 6p^3dp+\int pe^{-p}dp=\frac{6p^4}{4}+\int pe^{-p}dp=$$

$$=\frac{3p^4}{2}-pe^{-p}+\int e^{-p}dp=\frac{3p^4}{2}-pe^{-p}-e^{-p}+C$$

$$\boxed{y=\frac{3p^4}{2}-e^{-p}(p+1)+C}$$

3. 2xy'y''=(y')^2+1

$$3)\ 2xy'y'' = (y')^2 + 1$$

$$y' = p \Rightarrow y'' = p'$$

$$2xpp' = p^2 + 1$$

$$2xp\frac{dp}{dx} = p^2 + 1$$

$$\frac{2p}{p^2+1}\,dp = \frac{dx}{x}$$

$$\frac{d(p^2+1)}{p^2+1} = \frac{dx}{x} \Rightarrow \ln|p^2+1| = \ln|x| + \ln|C|$$

$$p^2 + 1 = x \cdot C \Rightarrow p = \sqrt{x \cdot C - 1} \Rightarrow y' = \sqrt{x \cdot C - 1}$$

$$y = \int \sqrt{xC - 1}\,dx = \frac{1}{C}\int (Cx-1)^{\frac{1}{2}}\,d(Cx-1) =$$

$$= \frac{1}{C_1} \cdot \frac{2}{3}(C_1x - 1)^{\frac{3}{2}} + C_2$$

$$\boxed{y = \frac{2(C_1x-1)^{\frac{3}{2}}}{3C_1} + C_2}$$

Место для галочки,
если номер решен верно

4. yy’’-2(y’)^2=0

4) $yy'' - 2(y')^2 = 0$

$y' = p = p(y) \Rightarrow y'' = y''_{xx} = (y'_x)'_x = \frac{d}{dx}(y'_x) =$

$= \frac{d}{dy}(\underbrace{y'_x}_{p}) \cdot \underbrace{\frac{dy}{dx}}_{p} = p'_y \cdot p$

Место для галочки,
если номер решен верно

$y\frac{dp}{dy} \cdot p - 2p^2 = 0$

$y\frac{dp}{dy} = 2p$ и $p = 0 \Rightarrow y' = 0 \Rightarrow y = C$

$\frac{dp}{2p} = \frac{dy}{y} \Rightarrow \ln|p| = 2\ln|y| + \ln|C| \Rightarrow$

$\Rightarrow p = C \cdot y^2 \Rightarrow y' = Cy^2$

$\frac{dy}{dx} = Cy^2 \Rightarrow dx = \frac{dy}{Cy^2}$

$x = \int \frac{dy}{Cy^2} = \frac{1}{C_1} \cdot \frac{-1}{y} + C_2$

$$\boxed{x = -\frac{1}{C_1 y} + C_2}$$

9 вариант

Вариант № 9.

1. $xy' = y + x\sin^2\frac{y}{x}$
2. $y = \sqrt[3]{y'} + \ln y'$
3. $x^2y'' - 2xy' - 3 = 0$
4. $yy'' - (y')^2 - 1 = 0$

1. xy’=y+xsin(y/x)^2

$$xy' = y + x\sin^2\frac{y}{x}$$

9 вар

$$y' = \frac{y}{x} + \sin^2\frac{y}{x} \qquad u = \frac{y}{x}$$

$$u'x + u = u + \sin^2 u$$

$$\frac{du}{dx}x = \sin^2 u$$

$$\int\frac{dx}{x} = \int\frac{du}{\sin^2 u}$$

$$\ln x + \ln C = -\mathrm{ctg}\, u$$

$$u = \mathrm{arcctg}\,(-\ln Cx)$$

$$y = -x\,\mathrm{arcctg}\,(\ln Cx)$$

Верно/нет?

2. y=(y’)^(⅓)+ln(y’)

2) $y = \sqrt[3]{y'} + \ln y'$

$$y' = p \Rightarrow y = p^{\frac{1}{3}} + \ln p \Rightarrow y' = \frac{1}{3}p^{-\frac{2}{3}} + \frac{1}{p}$$

$$\frac{dy}{dx} = p \Rightarrow dx = \frac{dy}{p} = \frac{1}{3p^{\frac{5}{3}}} + \frac{1}{p^2}$$

$$x = \int\frac{1}{3p^{\frac{5}{3}}}dp + \int\frac{1}{p^2}dp =$$

$$= \frac{1}{3}\left(-\frac{3}{2}\right)p^{-\frac{2}{3}} - p^{-1} + C$$

$$\boxed{x = C - \frac{1}{p} - \frac{1}{2p^{\frac{2}{3}}}}$$

3. x^2y''-2xy'-3=0

3) $x^2y'' - 2xy' - 3 = 0$

$y' = p, \quad y'' = p'$

$x^2 p' - 2xp - 3 = 0$

$p' - \frac{2}{x} \cdot p = \frac{3}{x^2}, \quad x^2 \neq 0$ – уравнение Бернулли

$p = u \cdot v$

$u'v + uv' - \frac{2}{x} u \cdot v = \frac{3}{x^2}$

$$\begin{cases} u'\cancel{v} - \frac{2}{x} u \cdot \cancel{v} = 0 \\ uv' = \frac{3}{x^2} \end{cases} \Rightarrow \begin{cases} \frac{du}{dx} = \frac{2u}{x} \\ u \cdot \frac{dv}{dx} = \frac{3}{x^2} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \frac{du}{u} = 2\frac{dx}{x} \\ dv = \frac{3}{x^2 \cdot u} dx \end{cases} \Rightarrow \begin{cases} u = x^2 \\ dv = \frac{3}{x^4} dx \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} u = x^2 \\ v = 3\left(-\frac{1}{3}\right) x^{-3} + C \end{cases} \Rightarrow \begin{cases} u = x^2 \\ v = -x^{-3} + C \end{cases}$$

$p = y' = \frac{dy}{dx} = u \cdot v = -x + x^2 \cdot C$

$dy = (-x + x^2 \cdot C)dx$

$$\boxed{y = -\frac{x^2}{2} + \frac{x^3}{3} \cdot C_1 + C_2}$$

Место для гало
если номер ре

4. yy’’-(y’)^2-1=0

4) $yy'' - (y')^2 - 1 = 0$

$y' = p \Rightarrow y'' = p'_y \cdot p$

$y \cdot \frac{dp}{dy} \cdot p - p^2 - 1 = 0$

$y \cdot p \cdot \frac{dp}{dy} = p^2 - 1 \Rightarrow \frac{dp}{dy} = \frac{p^2-1}{p \cdot y}$

$dp \cdot \frac{p}{p^2-1} = \frac{dy}{y} \Rightarrow \int \frac{1}{2} \cdot \frac{dp^2}{p^2-1} = \int \frac{dy}{y} \Rightarrow$

$\Rightarrow \frac{1}{2} \cdot \ln|p^2-1| = \ln|y| + \ln|C|$

$\sqrt{p^2-1} = Cy \Rightarrow p = \sqrt{C^2y^2+1}$

$\frac{dy}{dx} = \sqrt{C^2y^2+1} \Rightarrow dx = \frac{dy}{\sqrt{C^2y^2+1}}$

$x = \frac{1}{C} \int \frac{dCy}{\sqrt{(Cy)^2+1}} = \frac{1}{C_1} \cdot \ln|C_1y + \sqrt{C_1^2y^2+1}| + C_2$

$\downarrow$

$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln|x + \sqrt{x^2 \pm a^2}| + C$

$$\boxed{x = \frac{\ln|C_1y + \sqrt{C_1^2y^2+1}|}{C_1} + C_2}$$

Место для галочки, если номер решен верно

10 вариант

Вариант № 10.

1. $e^{-y}dx + (1 - xe^{-y})dy = 0$
2. $x = (y')^3 - 2\operatorname{arctg} y'$
3. $(1 + x^2)y'' - 2xy' = 0$
4. $yy'' + (y')^2 - yy' = 0$

1. e^(-y)dx+(1-xe^(-y))dy=0

1) Вар 10

$$e^{-y}dx + (1 - xe^{-y})dy = 0$$

$$M(x,y) = e^{-y} \qquad \frac{\partial M}{\partial y} = -e^{-y}$$

$$N(x,y) = (1 - xe^{-y}) \qquad \frac{\partial N}{\partial x} = -e^{-y} \Rightarrow$$ ур-ние в полных дифференциалах

$$I = \int e^{-y}dx = xe^{-y} + C(y)$$

$$\frac{\partial I}{\partial y} = -\cancel{xe^{-y}} + C'(y) = 1 - \cancel{xe^{-y}} \Rightarrow C'(y) = 1 \Rightarrow C(y) = y + \tilde{C}$$

$$I = xe^{-y} + y + \tilde{C}$$

$$\boxed{C = xe^{-y} + y}$$

2. x=(y’)^3-2arctg(y’)

2) $x = (y')^3 - 2\operatorname{arctg} y'$

$$y' = p \Rightarrow \frac{dy}{dx} = p \Rightarrow dy = p \cdot dx$$

$$x = p^3 - 2\operatorname{arctg} p$$

$$x' = 3p^2 - \frac{2 \cdot 1}{1+p^2}$$

$$dy = p \cdot \left(3p^2 - \frac{1 \cdot 2}{1+p^2}\right)$$

$$y = \int 3p^3\,dp - 2\int \frac{p\,dp}{1+p^2} = 3\frac{p^4}{4} -$$

$$- \int \frac{d(p^2+1)}{p^2+1} = \frac{3p^4}{4} - \ln|p^2+1| + C$$

Место для галочки, если номер решен верн

$$\boxed{\begin{aligned} x &= p^3 - \operatorname{arctg} p \\ y &= \frac{3p^4}{4} - \ln|p^2+1| + C \end{aligned}}$$

3. (1+x^2)y''-2xy'=0

$$3)\ (1+x^2)y''-2xy'=0$$

$$y'=p \Rightarrow y''=p'$$

$$(1+x^2)p'-2xp=0$$

$$\frac{dp}{dx}=\frac{2xp}{1+x^2} \Rightarrow \frac{dp}{p}=\frac{2x\,dx}{1+x^2}$$

$$\ln|p|=\int\frac{d(x^2+1)}{x^2+1}=\ln|x^2+1|+\ln|C|$$

$$p=\frac{dy}{dx}=C(x^2+1) \Rightarrow dy=(Cx^2+C)dx$$

$$y=C_1\frac{x^3}{3}+C_1x+C_2$$

$$\boxed{y=\frac{C_1\cdot x^3}{3}+C_1x+C_2}$$

Место для галочки,
если номер решен верно

4. yy’’+(y’)^2-yy’=0

4) $yy'' + (y')^2 - yy' = 0$

$y' = p \Rightarrow y'' = p'_y \cdot p$

$y \cdot \frac{dp}{dy} \cdot p + p^2 - y \cdot p = 0$

$y \cdot \frac{dp}{dy} + p - y = 0$ и $p = 0$

$y \cdot p'_y + p - y = 0$ — ур-ние Бернулли ?

$p'_y + \frac{1}{y} \cdot p - 1 = 0$ $\quad y \neq 0$

$p = u \cdot v$

$u'v + uv' + \frac{1}{y} u \cdot v - 1 = 0$

$$\begin{cases} u'\cancel{v} + \frac{1}{y} \cdot u \cdot \cancel{v} = 0 \\ uv' - 1 = 0 \end{cases} \Rightarrow \begin{cases} \frac{du}{dy} = -\frac{u}{y} \\ u \cdot \frac{dv}{dy} = 1 \end{cases} \Rightarrow \begin{cases} \frac{du}{u} = -\frac{dy}{y} \\ dv = dy \cdot \frac{1}{u} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} u = \frac{1}{y} \\ v = \frac{y^2}{2} + C \end{cases} \Rightarrow p = \frac{dy}{dx} = \frac{y}{2} + \frac{C}{y} \Rightarrow$$

$$\Rightarrow dx = \frac{dy}{\frac{y}{2} + \frac{C}{y}} = \frac{dy \cdot 2y}{y^2 + 2C} = \frac{d(y^2)}{y^2 + 2C} = \frac{d(y^2 + 2C)}{y^2 + 2C}$$

$$x = \ln|y^2 + \underbrace{2C_1}_{= \tilde{C}_1\,?}| + C_2$$

так можно решать вообще?

11 вариант

Вариант № 11.

1. $ydx + (2\sqrt{xy} - x)dy = 0$
2. $2y\sqrt{y'} = y' + 1$
3. $y'' + y'\mathrm{tg}x = \sin 2x$
4. $4y(y')^2y'' = 1 + (y')^4$

1. ydx+(2*sqrt(xy)-x)dy=0

1) $y\,dx + (2\sqrt{xy} - x)dy = 0$

$y\,dx = -(2\sqrt{xy} - x)dy$

$\frac{dx}{dy} = -\left(2\sqrt{\frac{x}{y}} - \frac{x}{y}\right),\ y \neq 0$

$t = \frac{x}{y} \Rightarrow x = ty \Rightarrow x' = t'y + t$

$t'y + t = -(2\sqrt{t} - t)$

$t'y = -2\sqrt{t}$

$\frac{dt}{dy} = -\frac{2\sqrt{t}}{y} \Rightarrow -\frac{dt}{2\sqrt{t}} = \frac{dy}{y}$

$\int -\frac{dt}{2\sqrt{t}} = \int \frac{dy}{y} \Rightarrow -\sqrt{t} = \ln|y| + \ln C$

$-\sqrt{t} = \ln|y| + C$

$\ln|Cy| = -\sqrt{\frac{x}{y}}$

$C - \sqrt{\frac{x}{y}} = \ln|y|$

$\boxed{x = -y \cdot \ln^2|Cy|}$

$+\ y = 0$

ВАР – 11

Место для галочки, если номер решен верно

2. 2y*sqtr(y’)=y’+1

$$2y\sqrt{y'} = y'+1$$

$$y'=p \Rightarrow \frac{dy}{dx}=p \Rightarrow dx=\frac{dy}{p}$$

$$2y\sqrt{p}=p+1$$

$$y=\frac{p+1}{2\sqrt{p}}$$

$$y'=\frac{(p+1)'\cdot 2\sqrt{p}-(2\sqrt{p})'(p+1)}{4p}=$$

$$=\frac{2\sqrt{p}-\frac{1}{\sqrt{p}}\cdot(p+1)}{4p}=\frac{2p-p-1}{4p\sqrt{p}}=$$

$$=\frac{p-1}{4p\sqrt{p}}$$

$$dx=\frac{p-1}{4p^2\sqrt{p}} \Rightarrow x=\int\frac{p-1}{4p^2\sqrt{p}}\,dp=$$

$$=\int\frac{1}{4p\sqrt{p}}\,dp-\int\frac{1}{4p^2\sqrt{p}}\,dp=$$

$$=\frac{1}{4}\int\frac{1}{p^{\frac{3}{2}}}\,dp-\frac{1}{4}\int\frac{1}{p^{\frac{5}{2}}}\,dp=$$

$$=\frac{1}{4}\left(-\frac{2}{\sqrt{p}}\right)-\frac{1}{4}\left(-\frac{2}{3p\sqrt{p}}\right)=$$

$$=-\frac{1}{2\sqrt{p}}+\frac{1}{6p\sqrt{p}}=\frac{1-3p}{6p\sqrt{p}}$$

$$\boxed{\begin{aligned} x&=\frac{1-3p}{6p\sqrt{p}} \\ y&=\frac{p+1}{2\sqrt{p}} \end{aligned}}$$

Место
если н

3. y''+y'tgx=sin(2x)

Место для галочки, если номер решен верн

$$y'' + y'\,\mathrm{tg}\,x = \sin 2x$$

$$y' = p,\; y'' = p' \Rightarrow p' = \frac{dp}{dx}$$

$p' + p\,\mathrm{tg}\,x = \sin 2x$ — уравнение Бернулли

$$p = uv \Rightarrow u'v + v'u + u\cdot v\,\mathrm{tg}\,x = \sin 2x$$

$$\begin{cases} u'\cancel{v} + u\cancel{v}\,\mathrm{tg}\,x = 0 \\ v'u = \sin 2x \end{cases} \Rightarrow \begin{cases} \frac{du}{dx} = -u\,\mathrm{tg}\,x \\ \frac{dv}{dx}\,u = \sin 2x \end{cases} \Rightarrow \begin{cases} \frac{du}{u} = -\frac{dx}{\mathrm{tg}\,x} \\ \frac{dv}{dx}\,u = \sin 2x \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \ln|u| = -\int \frac{\cos x}{\sin x}\,dx = -\int \frac{d\sin x}{\sin x} = -\ln|\sin x| \\ \frac{dv}{dx}\,\frac{1}{\sin x} = 2\sin x\cos x \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} u = \frac{1}{\sin x} \\ \frac{dv}{dx} = 2\sin^2 x\cos x \end{cases} \Rightarrow \begin{cases} u = \frac{1}{\sin x} \\ dv = 2\sin^2 x\cos x\,dx \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} u = \frac{1}{\sin x} \\ v = \int 2\sin^2 x\, d(\sin x) = 2\,\frac{\sin^3 x}{3} + C_1 \end{cases}$$

$$p = u\cdot v = \frac{2\sin^2 x}{3} + C_1\,\frac{1}{\sin x}\ ?$$

$$y' = \frac{dy}{dx} = \frac{2\sin^2 x}{3} + \frac{C_1}{\sin x}$$

$$dy = \left(\frac{2\sin^2 x}{3} + \frac{C_1}{\sin x}\right)dx$$

$$y = \frac{2}{3}\int \sin^2 x\,dx + C_1\int\frac{dx}{\sin x} = -\frac{\cos x\sin x}{2} + \frac{1}{2}\int dx +$$

$$+ C_1\int \frac{\sin x}{\sin^3 x}\,dx = \boxed{-\frac{\cos x\sin x}{2} + \frac{1}{2}x + \frac{C_1}{2}\ln\left|\frac{\cos x - 1}{\cos x + 1}\right| + C_2}$$

$$\int \frac{\sin x}{\sin^2 x}\,dx = \int \frac{d\cos x}{\cos^2 x - 1} = \frac{1}{2}\ln\left|\frac{\cos x - 1}{\cos x + 1}\right|$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

4y(y’)^2y’’=1+(y’)^4

4) $4y(y')^2y'' = 1 + (y')^4$

$y' = p,\quad y'' = p'_y \cdot p$

Место для галочки, если номер решен верн

$4yp^2 \cdot \frac{dp}{dy} \cdot p = 1 + p^4$

$\frac{4p^3}{1+p^4}\,dp = \frac{dy}{y} \Rightarrow \int \frac{4p^3}{1+p^4}\,dp = \int \frac{dy}{y} \Rightarrow$

$\int \frac{d(1+p^4)}{1+p^4} = \ln|y| + \ln|C|$

$\ln|p^4+1| = \ln|Cy| \Rightarrow p^4 + 1 = Cy$

$p = \sqrt[4]{Cy-1} \Rightarrow \frac{dy}{dx} = \sqrt[4]{Cy-1}$

$dx = \frac{dy}{\sqrt[4]{Cy-1}}$

$x = \int \frac{dy}{\sqrt[4]{Cy-1}} = \frac{1}{C}\int \frac{d(Cy-1)}{\sqrt[4]{Cy-1}} =$

$= \frac{1}{C_1} \cdot \frac{4}{3}\sqrt[4]{(C_1y-1)^3} + C_2$

$$\boxed{x = \frac{4(C_1y-1)^{\frac{3}{4}}}{3C_1} + C_2}$$

12 вариант

Вариант № 12.

1. $xy' - y - x \cdot \operatorname{tg}\frac{y}{x} = 0$
2. $(y')^2 - 4(y')^3 + 2x - 1 = 0$
3. $x^2y'' + 2xy' = \frac{1}{x^2}$
4. $y''(3-2y) + 5(y')^2 = 0$

1. xy’-y-x*tg(y/x)=0

№ 1

$$xy' - y - x \cdot tg\frac{y}{x} = 0 \quad |:x$$

Вар 12

$$y' - \frac{y}{x} - tg\frac{y}{x} = 0$$ – однородное ур-ие

$$\begin{cases} y = tx \Rightarrow y' = t'_x \cdot x + t \\ x = x \end{cases}$$

$$t'x + t - \frac{tx}{x} - tg\frac{tx}{x} = 0$$

$$t'x + \cancel{t} - \cancel{t} - tg\,t = 0$$

$$t'x = tg\,t$$

$$\frac{dt}{dx} \cdot x = tg\,t$$

$$\frac{dt}{tg\,t} = \frac{dx}{x}$$

$$\int \frac{dt}{tg\,t} = \int \frac{\cos t\,dt}{\sin t} = \int \frac{d\sin t}{\sin t} = \ln|\sin t| + C$$

$$\ln|\sin t| = \ln x + \ln C$$

$$\sin t = Cx$$

$$t = \arcsin Cx$$

$$y = tx \Rightarrow t = \frac{y}{x}$$

$$\frac{y}{x} = \arcsin Cx$$

$$\boxed{y = x \cdot \arcsin Cx}$$

2. (y’)^2-4(y’)^3+2x-1=0

$$2.\ (y')^2 - 4(y')^3 + 2x - 1 = 0$$

$$y' = p(y)$$

$$x = \frac{-p^2 + 4p^3 + 1}{2}$$

$$\frac{dx}{dy} = \frac{d\left(\frac{4p^3 - p^2 + 1}{2}\right)}{dp} \cdot \frac{dp}{dy}$$

$$\frac{1}{\frac{dy}{dx}} = \left(2p^3 - \frac{p^2}{2} + \frac{1}{2}\right)'_p \cdot p'_y$$

$$\frac{1}{p} = (6p^2 - p)\,p'_y$$

$$dy = (6p^2 - p)\,dp$$

$$y = \int (6p^2 - p)\,dp = 2p^3 - \frac{p^2}{2} + C$$

Ответ: $$\begin{cases} x(p) = -\frac{p^2}{2} + 2p^3 + \frac{1}{2} \\ y(p) = 2p^3 - \frac{p^2}{2} + C \end{cases}$$

3. x^2*y''+2xy'=1/x^2

3) $x^2y'' + 2xy' = \frac{1}{x^2}$

$y' = p,\ y'' = p'$

$x^2p' + 2xp = \frac{1}{x^2}$

$p' + \frac{2}{x}\cdot p = \frac{1}{x^4},\ x \neq 0$ Бернулли

$u'v + uv' + \frac{2}{x}\cdot u\cdot v = \frac{1}{x^4}$

$\begin{cases} u'\cancel{v} + \frac{2}{x}\cdot u\cdot \cancel{v} = 0 \\ uv' = \frac{1}{x^4} \end{cases} \Rightarrow$ $\frac{du}{dx} = -\frac{2u}{x}$ $\Downarrow$ $\frac{du}{2u} = -\frac{dx}{x} \Rightarrow \frac{1}{2}\ln|u| = -\ln|x|$ $\Uparrow$ $\sqrt{u} = \frac{1}{x} \Rightarrow u = \frac{1}{x^2}$

$\frac{1}{x^2}\cdot\frac{dv}{dx} = \frac{1}{x^4} \Rightarrow dv = \frac{dx}{x^2} \Rightarrow v = -\frac{1}{x} + C$

$\frac{dy}{dx} = \frac{1}{x^2}\left(-\frac{1}{x} + C\right) = -\frac{1}{x^3} + \frac{C}{x^2} \Rightarrow y = +\frac{1}{2x^2} - \frac{C}{x} + \tilde{C}$

4. y''(3-2y)+5(y')^2=0

4. $y''(3-2y) + 5(y')^2 = 0$ $y' = p,\ y'' = pp'$

$pp'(3-2y) = -5p^2$

$\frac{dp}{dy}(3-2y) = -5p$

$-\int\frac{dp}{5p} = \int\frac{dy}{(3-2y)}$

$-\frac{1}{5}\ln p + \ln C = -\frac{1}{2}\ln|3-2y|$

$p^{\frac{1}{5}} = C(3-2y)^{\frac{1}{2}}$

$p = C^5(3-2y)^{\frac{5}{2}}$

$y' = C_1(3-2y)^{\frac{5}{2}}$

$\frac{dy}{dx} = C_1(3-2y)^{\frac{5}{2}}$

$\int\frac{dy}{C_1(3-2y)^{\frac{5}{2}}} = \int dx$

$x = \frac{1}{3C_1\sqrt{3-2y}\,(3-2y)} + C_2$

13 вариант

Вариант № 13.

1. $ydx + (y^3 - x)dy = 0$
2. $yy' = 1 + (y')^2 e^{y'}$
3. $xy'' = y'(1 + \ln \frac{y'}{x})$
4. $2yy'' - (y')^2 - 1 = 0$

1. ydx+(y^3-x)dy=0

Вариант №13.

№1 $ydx + (y^3 - x)dy = 0$. – линейное относ. x.

$\Rightarrow x = uv$.

$dx = udv + vdu$

$yd(uv) + (y^3 - uv)dy = 0$

$y(udv + vdu) + y^3 dy - uvdy = 0$

$yudv + yvdu + y^3 dy - uvdy = 0$

$$\begin{cases} yudv - uvdy = 0 \to ydv - vdy = 0 \to ydv = vdy \\ yvdu + y^3 dy = 0 \to vdu + y^2 dy = 0 \end{cases}$$

$ydv = vdy$ | $y = 0$ – решение

$\frac{dv}{v} = \frac{dy}{y}$

$\ln|v| = \ln|y|$

$$\begin{cases} v = y \\ vdu + y^2 dy = 0 \end{cases}$$

$ydu + y^2 dy = 0$

$\int du = \int -\frac{y^2}{y} dy$

$\int du = -\int y dy$

$u = -\frac{y^2}{2} + C$

$x = uv \to x = (-\frac{y^2}{2} + C) \cdot y = -\frac{y^3}{2} + yC$.

Ответ: $\begin{cases} 2x = -y^3 + 2yC \\ y = 0. \end{cases}$

y' ... не разрешимое относ

2. yy’=1+(y’)^2*e^(y’)

$$yy' = 1 + (y')^2 e^{y'}$$ – не разр. отн-но y'

$$y' = p \quad dy = p\,dx \quad dx = \frac{dy}{p}$$

$$y = \frac{p^2 \cdot e^p + 1}{p} = p \cdot e^p + \frac{1}{p}$$

$$dy = \left(e^p + p \cdot e^p - \frac{1}{p^2}\right) dp$$

$$dx = \frac{\left(e^p + p \cdot e^p - \frac{1}{p^2}\right)}{p} =$$

$$= \frac{e^p}{p} + e^p - \frac{1}{p^3}$$

$$\boxed{\begin{cases} x = e^p - \dfrac{1}{2p^2} + \displaystyle\int \frac{e^p}{p} \\ y = p \cdot e^p + \dfrac{1}{p} \end{cases}}$$

Второе реш.-ие

№2

$$y y' = 1 + (y')^2 e^{y'}$$

Замена. $y' = p = \frac{dy}{dx}$

$$yp = 1 + p^2 e^p \mid : p$$

$$y = \frac{1}{p} + pe^p$$

$$\frac{dy}{dx} = \frac{d\left(\frac{1}{p} + pe^p\right)}{dp} \frac{dp}{dx}$$

$$\frac{dy}{dx} = \left(-\frac{1}{p^2} + e^p + pe^p\right) \frac{dp}{dx}$$

$$p = \left(-\frac{1}{p^2} + e^p + pe^p\right) \frac{dp}{dx} \quad | : p$$

$$dx = \left(-\frac{1}{p^3} + \frac{e^p}{p} + e^p\right) dp$$

$$x = -\frac{p^{-2}}{-2} + e^p + \int \frac{e^p}{p} dp = \frac{1}{p^2} + e^p + \int \frac{e^p}{p} dp$$

$$\begin{cases} x(p) = \dfrac{1}{2p^2} + e^p + \displaystyle\int \frac{e^p}{p} dp \\ y(p) = \dfrac{1}{p} + pe^p \end{cases}$$

3. xy''=y'(1+ln(y'/x))

$$xy'' = y'\left(1+\ln\frac{y'}{x}\right)$$

ур., допуск-ее понижение порядка, II тип

$y'=p,\ y''=p',\ y'''=p''$

Тип $y,\ p=y',\ y''=pp',\ \dots$

$$y'=p$$

$$xp' = p\left(1+\ln\frac{p}{x}\right)$$

$$xp' - p - p\ln\frac{p}{x} = 0$$

$$1+m-1 = m = m+m-1$$

$$m = m = 2m-1$$

$$m=1$$

$$\begin{cases} p = tx \Rightarrow p' = t'\cdot x + t \\ x = x \end{cases}$$

$$x\cdot(t'\cdot x + t) - tx - tx\cdot\ln\frac{tx}{x} = 0$$

$$t'x^2 + \cancel{tx} - \cancel{tx} - tx\cdot\ln t = 0$$

$$t'x^2 - tx\ln t = 0 \quad |:x$$

$$t'x - t\ln t = 0$$

$$t'\cdot x = t\ln t$$

$$\frac{dt}{dx}\cdot x = t\ln t$$

$$\frac{dt}{t\ln t} = \frac{dx}{x}$$

$$\frac{d(\ln t)}{\ln t} = \frac{dx}{x}$$

$$\ln(\ln(t)) = \ln(x) + \ln C$$

$$\ln(\ln(t)) = \ln(Cx)$$

$$\ln(t) = Cx$$

$$t = e^{Cx}$$

$$p = tx \Rightarrow p = e^{Cx}\cdot x = \frac{dy}{dx}$$

$$dy = e^{Cx}\cdot x\,dx$$

$$y = \underbrace{\frac{1}{C}x\,de^{Cx}}_{} = \frac{1}{C}xe^{Cx} - \frac{1}{C}\int e^{Cx}dx = \frac{1}{C}xe^{Cx} - \frac{1}{C^2}e^{Cx} + \tilde{C}$$

$$\boxed{y = \frac{xe^{Cx}}{C} - \frac{e^{Cx}}{C^2} + \tilde{C}}$$

$$e^{Cx}\cdot x\,dx =$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

4. 2yy’’-(y’)^2-1=0

$$2yy'' - (y')^2 - 1 = 0$$

Замена: $y' = p = \frac{dy}{dx}$, $y'' = \frac{d}{dx}y' = \frac{d}{dy}\cdot y' \cdot \frac{dy}{dx} = p'_y \cdot p$

$$2yp\cdot p'_y - p^2 - 1 = 0 \quad |:p$$

$$2yp'_y - p - \frac{1}{p} = 0 \Rightarrow 2yp'_y = p + \frac{1}{p}$$

$$2yp'_y = \frac{p^2+1}{p}$$

$$2\frac{dp}{dy} = \frac{1}{y}\left(\frac{p^2+1}{p}\right)$$

$$\frac{p\,dp}{p^2+1} = \frac{dy}{y}$$

$$\frac{2}{2}\frac{d(p^2+1)}{p^2+1} = \frac{dy}{y}$$

$$\ln|p^2+1| = \ln|y| + \ln C$$

$$p^2+1 = yC$$

$$p^2 = yC-1$$

$$p = (yC-1)^{\frac{1}{2}}$$

$$p = \frac{dy}{dx} = (yC-1)^{\frac{1}{2}}$$

$$dx = \frac{dy}{(yC-1)^{\frac{1}{2}}} \qquad x = \frac{1}{C}\int\frac{d(yC-1)}{(yC-1)^{\frac{1}{2}}} = \frac{1}{C}\frac{2(yC-1)^{\frac{1}{2}}}{1} + \tilde{C} =$$

$$= \frac{2(yC-1)^{\frac{1}{2}}}{C} + \tilde{C}$$

14 вариант

Вариант № 14.

1. $(2x - \frac{y}{x})dx = (\ln x - 2y)dy$
2. $y(y')^5 - 4(y')^4 + 2 = 0$
3. $y'' - \frac{y'}{x} = \frac{1}{x}\sqrt{(y')^2 - x^2}$
4. $y''(1 + y) - y'(y' - 1) = 0$

1. (2x-(y/x))dx=(lnx-2y)dy

Вариант 14

№1

$$\left(2x - \frac{y}{x}\right)dx = (\ln x - 2y)dy \Rightarrow \left(2x - \frac{y}{x}\right)dx + (2y - \ln x) = 0$$

$$\frac{\partial M}{\partial y} = -\frac{1}{x}, \quad \frac{\partial N}{\partial x} = -\frac{1}{x}$$

ур-ие в полных дифференциалах

$$I = \int \left(2x - \frac{y}{x}\right)dx = 2\cdot\frac{x^2}{2} - y\ln x + C(y) = x^2 - y\ln x + C(y)$$

$$\frac{\partial I}{\partial y} = -\ln x + C'(y)$$

$$-\ln(x) + C'(y) = 2y - \ln x \Rightarrow C'(y) = 2y$$

$$C(y) = 2\cdot\frac{y^2}{2} + C = y^2 + C$$

$$I = x^2 + y^2 - y\ln x + C$$

$$\boxed{C = x^2 + y^2 - y\ln x}$$

На всякий случай для N:

$$I = \int(-\ln x + 2y)dy = 2\cdot\frac{y^2}{2} + C(x) = y^2 + C(x)$$

$$\frac{\partial I}{\partial x} = C'(x)$$

$$C'(x) = 2x - \frac{y}{x} \Rightarrow C(x) = \int\left(2x - \frac{y}{x}\right)dx = 2\cdot\frac{x^2}{2} - y\ln x + C = x^2 - y\ln x + C$$

$$I = y^2 + x^2 - y\ln x + C \Rightarrow \boxed{C = x^2 + y^2 - y\ln x}$$

2. y(y’)^5-4(y’)^4+2=0

№2

$$y(y')^5 - 4(y')^4 + 2 = 0 \quad |:(y')^5$$

$$y - \frac{4}{y'} + \frac{2}{(y')^5} = 0$$

$y = \frac{4}{y'} - \frac{2}{(y')^5}$ – ур-ие вида $y = f(y')$

Замена $y' = p \Rightarrow \frac{dy}{dx} = p$

$$y = \frac{4}{p} - \frac{2}{p^5}$$

$$\frac{dy}{dx} = \frac{d\left(\frac{4}{p} - \frac{2}{p^5}\right)}{dx} = \frac{d\left(\frac{4}{p} - \frac{2}{p^5}\right)}{dp} \cdot \frac{dp}{dx} = \left(-\frac{4}{p^2} - \frac{2\cdot(-5)}{p^6}\right) \cdot \frac{dp}{dx}$$

$$p = \left(-\frac{4}{p^2} + \frac{10}{p^6}\right)\frac{dp}{dx} \Rightarrow dx = \left(-\frac{4}{p^3} + \frac{10}{p^7}\right)dp$$

$$x = \int\left(-\frac{4}{p^3} + \frac{10}{p^7}\right)dp = -4 \cdot \frac{1}{(-2)p^2} + \cancel{10}^{5} \cdot \frac{1}{\underset{-3}{\cancel{(-6)}}\,p^6} = \frac{2}{p^2} - \frac{5}{3p^6} + C$$

Ответ: $\begin{cases} x(p) = \frac{2}{p^2} - \frac{5}{3p^6} + C \\ y(p) = \frac{4}{p} - \frac{2}{p^5} \end{cases}$

3. y''-(y')/x=1/x*sqrt((y')^2-x^2)

№ 3 $\quad y'' - \frac{y'}{x} = \frac{1}{x}\sqrt{(y')^2 - x^2}$

Замена: $y' = \frac{dy}{dx} = p$; $y'' = \frac{d}{dx}\cdot\frac{dy}{dx} = \frac{dp}{dx} = p'$

$p' - \frac{p}{x} = \frac{1}{x}\sqrt{p^2 - x^2}$

$p' - \frac{1}{x}\left(p + \sqrt{p^2 - x^2}\right) = 0$ – линейное однородное ур-ие относительно p.

Замена: $p = tx \Rightarrow p' = t'x + t$

$t'x + \cancel{t} - \cancel{\frac{tx}{x}} - \frac{\sqrt{t^2x^2 - x^2}}{x} = 0 \qquad t = \frac{p}{x}$

$t'x - \frac{x\sqrt{t^2-1}}{x} = 0$

$t'x = \sqrt{t^2-1} \; | : x$

$t' = \frac{\sqrt{t^2-1}}{x} \Rightarrow \frac{dt}{dx} = \frac{\sqrt{t^2-1}}{x}$

$\frac{dt}{\sqrt{t^2-1}} = \frac{dx}{x} \Rightarrow \ln\left|t + \sqrt{t^2-1}\right| = \ln|x| + \ln|C|$

$t + \sqrt{t^2-1} = Cx$

$t + \sqrt{t^2-1} = Cx \Rightarrow \frac{p}{x} + \frac{1}{x}\sqrt{p^2 - x^2} = Cx \; | \cdot x$

$\frac{p}{x} + \sqrt{\frac{p^2}{x^2} - 1}$

$p + \sqrt{p^2 - x^2} = Cx^2$

Пока что не получилось. (бабуля спешит на помощь)

$Cx - t = \sqrt{t^2 - 1} \Rightarrow C^2x^2 - 2Cx\cdot t + \cancel{t^2} = \cancel{t^2} - 1 \Rightarrow C^2x^2 + 1 = 2Cxt \Rightarrow t = \frac{C^2x^2}{2Cx} + \frac{1}{2Cx} = \frac{Cx}{2} + \frac{1}{2Cx}$

$p = t\cdot x = \frac{x}{2Cx} + \frac{Cx^2}{2} = \frac{Cx^2}{2} + \frac{1}{2C},\; x \neq 0$

$\frac{dy}{dx} = \frac{Cx^2}{2} + \frac{1}{2C} \Rightarrow dy = \left(\frac{Cx^2}{2} + \frac{1}{2C}\right)dx$

$\boxed{y = \frac{Cx^3}{6} + \frac{x}{2C} + \tilde{C}}$

4. y''(1+y)-y'(y'-1)=0

№ 4 $\quad y''(1+y) - y'(y'-1) = 0$ – ур-ие, допускающее понижение степени

Замена: $y' = \frac{dy}{dx} = p$; $y'' = \frac{d}{dx}\cdot\frac{dy}{dx} = \frac{d}{dx}\cdot p = \frac{d}{dy}\cdot p\cdot\frac{dy}{dx} = p'_y\cdot p$

$p'_y\cdot p\,(1+y) - p(p-1) = 0 \; | : p$

$p'_y(1+y) - p + 1 = 0$

$p'_y = \frac{p-1}{1+y} \Rightarrow \frac{dp}{dy} = \frac{p-1}{1+y} \Rightarrow \frac{dp}{p-1} = \frac{dy}{y+1} \Rightarrow \ln|p-1| = \ln|y+1| + \ln C$

$p - 1 = C(y+1) \Rightarrow p = C(y+1) + 1$

$p = \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = C(y+1) + 1 \Rightarrow \frac{dy}{C(y+1)+1} = dx \Rightarrow x = \frac{1}{C}\int\frac{d(C(y+1)+1)}{C(y+1)+1} =$

$= \frac{1}{C}\ln|C(y+1)+1| + \tilde{C}$

Если $p = 0 \Rightarrow y' = 0 \Rightarrow y = C_1$ – решение, но оно уже входит в решение

$\boxed{x = \frac{1}{C}\ln|C(y+1)+1| + \tilde{C}}$

15 вариант

Вариант № 15.

1. $x(y' - y) = e^x$
2. $(y')^5 - 2(y')^4 - y' + x = 0$
3. $y'' = 2(y' - 1)\text{ctg}x$
4. $y''y - (y')^2 - 4y' = 0$

1. x(y’-y)=e^x

$$xy' - xy = e^x$$

бернулии

$$y' - y = \frac{e^x}{x}, \quad x \neq 0$$

$$y = u \cdot v, \quad y' = u'v + uv'$$

$$u'v + uv' - uv = \frac{e^x}{x}$$

what?

$$\begin{cases} u'\cancel{v} - u\cancel{v} = 0 \\ uv' = \frac{e^x}{x} \end{cases} \quad \begin{cases} \frac{du}{dx} = u \\ v' = \frac{e^x}{ux} \end{cases} \quad \begin{cases} \frac{du}{u} = dx \\ dv = \frac{e^x}{xu} dx \end{cases}$$

$$\ln|u| = x \Rightarrow u = e^x$$

$$dv = \frac{e^x}{xe^x} dx \Rightarrow v = \int \frac{1}{x} dx = \ln|x| + \ln|C|$$

$$v = \ln|Cx|$$

$$y = uv = e^x \ln(Cx)$$

2. (y’)^5-2(y’)^4-y’+x=0

2. $(y')^5-2(y')^4-y'+x=0$

$x=2(y')^4+y'-(y')^5 \qquad y'=p$

$x=2p^4+p-p^5$

$$\frac{dx}{dy}=\frac{d(2p^4+p-p^5)}{dp}\cdot\frac{dp}{dy}$$

$$\frac{1}{\frac{dy}{dx}}=(2p^4+p-p^5)'_p\,p'_y$$

$$\frac{1}{p}=(8p^3+1-5p^4)\,p'_y$$

$$dy=(8p^4+p-5p^5)\,dp$$

$$y=\int(8p^4+p-5p^5)\,dp=$$

$$=\frac{8}{5}p^5+\frac{p^2}{2}-\frac{5}{6}p^6+C$$

Ответ: $\begin{cases} x(p)=2p^4+p-p^5 \\ y(p)=\frac{8}{5}p^5+\frac{p^2}{2}-\frac{5}{6}p^6+C \end{cases}$

3. y''=2(y'-1)ctgx

$(y')^5-2(y')^4-y'+x=0$

$x=(y')^5+2(y')^4+y'$ – ур-ие вида $x=f(y')$.

Замена: $y'=p$

$x=p^5+2p^4+p$

$\frac{dx}{dy}=\frac{d(p^5+2p^4+p)}{dp}\cdot\frac{dp}{dy}$ $\qquad \frac{dx}{dy}=\frac{d(p^5+2p^4+p)}{dy}=\frac{d(p^5+2p^4+p)}{dp}\cdot\frac{dp}{dy}$

$\frac{dx}{dy}=(5p^4+8p^3+1)\frac{dp}{dy}$

$\frac{1}{\frac{dy}{dx}}=(5p^4+8p^3+1)\frac{dp}{dy}$

$\frac{dy}{dx}=\frac{1}{p}$

$\frac{1}{p}=(5p^4+8p^3+1)\frac{dp}{dy} \quad |\cdot p$

$dy=(5p^5+8p^4+p)\,dp$

$y=5\cdot\frac{p^6}{6}+8\cdot\frac{p^5}{5}+\frac{p^2}{2}+C$

Ответ: $\begin{cases} x(p)=p^2+2p^4+p \\ y(p)=\frac{5}{6}p^6+\frac{8}{5}p^5+\frac{1}{2}p^2+C \end{cases}$, $C\in\mathbb{R}$.

4. y''y-(y')^2-4y'=0

$$y''y - (y')^2 - 4y' = 0$$

$$y' = p, \quad y'' = p'_y p$$

$$p'_y p y - p^2 - 4p = 0 \qquad p = 0 . \ y = C \text{ — реш}$$

$$p'_y y - p - 4 = 0$$

$$p'_y - \frac{p}{y} = \frac{4}{y} \text{ — Бернулли} \qquad y \neq 0 . \quad y = 0 \text{ — реш}$$

$$p = uv, \quad p' = u'v + uv'$$

$$u'v + uv' - uv\frac{1}{y} = \frac{4}{y}$$

$$\begin{cases} u'v - uv \cdot \frac{1}{y} = 0 \\ uv' = \frac{4}{y} \end{cases} \Rightarrow \begin{cases} \frac{du}{dy} = \frac{u}{y} \\ v' = \frac{4}{uy} \end{cases} \Rightarrow \begin{cases} \frac{du}{u} = \frac{dy}{y} \\ dv = \frac{4}{uy}\,dy \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} u = y \\ v = \int \frac{4}{y^2}\,dy \end{cases} \Rightarrow \begin{cases} u = y \\ v = -\frac{4}{y} + C \end{cases}$$

$$p = uv = \frac{dy}{dx} = -4 + Cy$$

$$dx = \frac{dy}{Cy - 4} \Rightarrow x = \frac{1}{C} \int \frac{d(Cy-4)}{Cy-4} = \frac{1}{C} \ln|Cy - 4| + \tilde{C}$$

16 вариант.

Вариант № 16.

1. $x \cdot \sin\frac{y}{x} y' = y \cdot \sin\frac{y}{x} - x$
2. $2y\sqrt{(y')^3} = (y')^2 + 3$
3. $xy'' - 2y' - 2x^4 = 0$
4. $y'' + \frac{2}{1-y}(y')^2 = 0$

1. x*sin(y/x)*y'=y*sin(y/x)-x

В-16

1) $xy' \sin\frac{y}{x} = y \sin\frac{y}{x} - x;$

Замена: $y = gx$

$x\left(x\frac{dg}{dx} + g\right)\sin g = x(g \sin g - 1)$ $x \neq 0$

$\left(x\frac{dg}{dx} + g\right)\sin g = g\sin g - 1;$

$x \sin g \frac{dg}{dx} = -1;$

$\int \sin g \, dg = -\int \frac{dx}{x};$

$-\cos g = -\ln|x| + C;$

$\cos g = \ln|x| + C_1;$

$g = \cos^{-1}(\ln|x| + C_1);$

Обратная замена.

$y = x\cos^{-1}(\ln|x| + C_1).$

2. 2y*(y')^1/3=(y')^2+3

(2) $2y\sqrt{(y')^3} = (y')^2 + 3$

Замена: $y' = p \Rightarrow \frac{dy}{dx} = p$

$$2y\,p^{\frac{3}{2}} = p^2 + 3$$

$$y = \frac{p^2+3}{p^{3/2}\cdot 2}$$

$$\cancel{\int dx = \int \frac{p^2+3}{p^{3/2}\,2p}\,dy}$$

$$\cancel{\begin{cases} x = \frac{p^3+9p}{6p^{\frac{5}{2}}} \\ y = \frac{p^2+3}{2p^{3/2}} \end{cases}}$$

$$f(p) = \frac{1}{2}p^{\frac{1}{2}} + \frac{3}{2}p^{-\frac{5}{2}}$$

$$f'(p) = \frac{1}{4p^{\frac{1}{2}}} - \frac{9}{4p^{\frac{5}{2}}};$$

$$\int dx = \int\left(\frac{1}{4p^{\frac{1}{2}}} - \frac{9}{4p^{\frac{5}{2}}}\right)dp$$

$$\begin{cases} x = \frac{p^{\frac{1}{2}}}{8} + \frac{p}{10p^{\frac{5}{2}}} + C \\ y = \frac{p^2+3}{2p^{3/2}} \end{cases}$$

3. x*y''-2y'-2x^4=0

(3) $xy'' - 2y' - 2x^4 = 0$

Замена: $y' = p$

$xp' - 2p - 2x^4 = 0$

$x(u'v + v'u) - 2uv = 2x^4;$

$\begin{cases} xu'v - 2uv = 0 \\ xv'u = 2x^4 \end{cases}$ $\begin{cases} xu' - 2u = 0 \\ v'u = 2x^3 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x\frac{du}{dx} = 2u \\ v'u = 2x^3 \end{cases} \Rightarrow \begin{cases} \int\frac{du}{2u} = \int\frac{dx}{x} \\ v'v = 2x^3 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} u = x^2 \\ v' = 2x \end{cases} \Rightarrow \begin{cases} u = x^2 \\ v = x^2 + C \end{cases}$

$p = x^2(x^2 + C)$

$y = \frac{x^5}{5} + \frac{x^3 C}{3} + C_1$

4. y''+(2/(1-y))*(y')^2=0

4) $y'' + \frac{2}{1-y}(y')^2 = 0$

$y' = p$

$p p'_y + p^2 \frac{2}{1-y} = 0,$ $\quad p = 0$, $y(x) = \tilde{C}$

$p'_y + p\frac{2}{1-y} = 0;$

$(u'v + v'u)_y + uv\frac{2}{1-y} = 0;$

$$\begin{cases} u'v + uv\frac{2}{1-y} = 0; \\ v'u_y = 0 \end{cases} \Rightarrow \begin{cases} u' = -\frac{2u}{1-y}; \\ v'u_y = 0 \end{cases} \Leftrightarrow$$

$$\Rightarrow \begin{cases} \frac{du}{dx} = -\frac{2u}{1-y} \\ v'u_y = 0 \end{cases} \Leftrightarrow \begin{cases} u = (1-y)^2 \\ v = C \end{cases}$$

$p = C(1-y)^2;$

$\frac{dy}{dx} = C(1-y^2)$

$\int \frac{dy}{C(1-y^2)} = \int dx$

$x = \frac{1}{C(1-y)} + C_1$

17 вариант.

Вариант № 17.

1. $ydx = 2(x+y)dy$
2. $x = (y')^3 + \ln y'$
3. $y'' = \frac{y'}{x} + x$
4. $y''(4y-1) - 3(y')^2 = 0$

1. ydx=2*(x+y)dy

$$\underbrace{y\,dx}_{1} \neq \underbrace{2(x+y)dy}_{2}$$

$$y = 2(x+y)y'$$

$$y = 2x + 2yy' \quad | \, y$$

Solve $y(x) = 2(x + y(x))\dfrac{d\,y(x)}{d\,x}$:

Let $y(x) = x\,v(x)$, which gives $\dfrac{d\,y(x)}{d\,x} = x\dfrac{d\,v(x)}{d\,x} + v(x)$:

$$x\,v(x) = 2(x + x\,v(x))\left(x\frac{d\,v(x)}{d\,x} + v(x)\right)$$

Simplify:

$$x\,v(x) = 2\,x\left(x\frac{d\,v(x)}{d\,x} + v(x)\right)(v(x) + 1)$$

Solve for $\dfrac{d\,v(x)}{d\,x}$:

$$\frac{d\,v(x)}{d\,x} = \frac{-2\,v(x)^2 - v(x)}{2\,x\,(v(x) + 1)}$$

Divide both sides by $\dfrac{-2\,v(x)^2 - v(x)}{2\,(v(x) + 1)}$:

$$\frac{2\,\frac{d\,v(x)}{d\,x}\,(v(x) + 1)}{-2\,v(x)^2 - v(x)} = \frac{1}{x}$$

Integrate both sides with respect to x:

$$\int \frac{2\,\frac{d\,v(x)}{d\,x}\,(v(x) + 1)}{-2\,v(x)^2 - v(x)}\,d\,x = \int \frac{1}{x}\,d\,x$$

Evaluate the integrals:

$\log(2\,v(x) + 1) - 2\log(v(x)) = \log(x) + c_1$, where c_1 is an arbitrary constant.

Solve for $v(x)$:

$$v(x) = -\frac{e^{-c_1}\left(\sqrt{e^{c_1}\,x + 1} - 1\right)}{x} \text{ or } v(x) = \frac{e^{-c_1}\left(\sqrt{e^{c_1}\,x + 1} + 1\right)}{x}$$

Substitute back for $y(x) = x\,v(x)$:

Answer:

$$y(x) = -e^{-c_1}\left(\sqrt{e^{c_1}\,x + 1} - 1\right)$$

$$\text{or } y(x) = e^{-c_1}\left(\sqrt{e^{c_1}\,x + 1} + 1\right)$$

$$2yy' - y = -2x$$

$$y(2y' - 1) = -2x$$

$$uv(2(u'v + v'u) - 1) = -2x$$

$$2uv^2u'$$

$$2y' - 1 + \frac{2x}{y} = 0$$

$$2(u'v + uv') - 1 + \frac{2x}{uv} = 0$$

$$\begin{cases} 2u'v + \dfrac{2x}{uv} = 0 \\ 2uv' = 1 \end{cases}$$

$$y\,dx = 2(x+y)dy$$

$$y = 2(x+y)\frac{dy}{dx}$$

$$y = 2xy' + 2yy'$$

$$m = 1 + m - 1 = m + m - 1$$

$$m = 2m - 1 \Rightarrow m = 1$$

$$\begin{cases} y = tx \\ x = x \end{cases}$$

2. x=(y')^5+lny'

2. $x=(y')^5+\ln y'$ $\quad y'=p$

$x=p^5+\ln p$

$$\frac{dx}{dy}=\frac{d(p^5+\ln p)}{dp}\cdot\frac{dp}{dy}$$

$$\frac{1}{\frac{dy}{dx}}=(p^5+\ln p)'_p\cdot p'_y$$

$$\frac{1}{p}=(5p^4+p^{-1})\cdot p'_y$$

$$dy=(5p^5+1)dp$$

$$y=\int(5p^5+1)\,dp=$$

$$=\frac{5p^6}{6}+p+C$$

Ответ: $\begin{cases} x(p)=p^5+\ln p \\ y(p)=\frac{5p^6}{6}+p+C \end{cases}$

3. y''=y'/x+x

$$y'' = \frac{y'}{x} + x$$

$$p = y'$$

$$p' = \frac{p}{x} + x$$

$$uv' + u'v = \frac{uv}{x} + x$$

$$\begin{cases} u'v - \frac{uv}{x} = 0 \\ uv' = x \end{cases} \Rightarrow \begin{cases} u' = \frac{u}{x} \\ uv' = x \end{cases} \Rightarrow \ln u = \ln x \quad \begin{cases} u = x \\ v = x \end{cases}$$

$$y' = x^2, \quad \underline{y = \frac{x^3}{3} + C}$$

4. y''*(4y-1)-3(y')^2=0

$$y''(4x-1) - 3(y')^2 = 0$$

$$y' = p$$

$$p'(4x-1) = 3p^2$$

$$y' = p$$

$$y'' = p'$$

$$p'(4x-1) - 3p^2 = 0$$

$$p'(4x-1) = 3p^2$$

$$p' = \frac{3p^2}{4x-1}$$

$$\frac{dp}{dx} = \frac{3p^2}{4x-1}$$

$$\frac{dp}{p^2} = \frac{3dx}{4x-1}$$

$$p^{-2}dp = 3\frac{dx}{4x-1}$$

$$-\frac{1}{p} = \underline{3\frac{d(4x-1)}{4x-1} = 3\ln|4x-1|}$$

$$p = -\frac{1}{3} \cdot \frac{1}{\ln|4x-1|}$$

$$\frac{dy}{dx} = -\frac{1}{3} \cdot \frac{1}{\ln|4x-1|}$$

$$dy = -\frac{1}{3} \cdot \frac{1}{\ln|4x-1|} dx$$

$$\boxed{y = -\frac{1}{3} \cdot \int \frac{1}{\ln|4x-1|} dx}$$

18 вариант.

Вариант № 18.

1. $y x^{y-1} dx + x^y \ln x \, dy = 0$
2. $x = (y')^2 + \sin y'$
3. $\sin x \cdot y'' + y' \cdot \cos x = 1$
4. $y''(1 + y) + (y')^2 = 0$

1. y*x^(y-1)dx+x^y*lnxdy=0

1) $\underbrace{y x^{y-1}}_{F(x,y)} dx + \underbrace{x^y \ln x}_{G(x,y)} dy = 0$

$\frac{\partial F(x,y)}{\partial y} = x^{y-1} + y x^{y-1} \ln x$

$\frac{\partial G(x,y)}{\partial x} = \frac{x^y}{x} + y x^{y-1} \ln x = x^{y-1} + y x^{y-1} \ln x$

$F'_y = G'_x$ Метод полных дифференциалов

$dI = F dx$

$I = \int y x^{y-1} dx = \frac{\cancel{y} x^y}{\cancel{y}} + \varphi(y) = x^y + \varphi(y)$

$\frac{dI}{dy} = x^y \ln x + \varphi'(y)$

$x^y \ln x + \varphi'(y) = x^y \ln x$

$\varphi'(y) = 0$

$\varphi(y) = C$

$I = x^y + C$

$\boxed{C = -x^y}$

2. x=(y’)^2+siny’

2)

$X=(y')^2+\sin y' \qquad y'=p \Rightarrow \frac{dy}{dx}=p \Rightarrow dy=p\,dx$

$X=p^2+\sin p$

$\int u\,dV = UV-\int V\,dU$

$X'=2p+\cos p$

$\begin{cases} U=p & dU=dp \\ dV=\sin p & V=\sin p \end{cases}$

$dy=p(2p+\cos p)dp$

$y=\int(2p^2+p\cos p)dp=\frac{2p^3}{3}+\int p\cos p\,dp=\frac{2p^3}{3}+\int p\,d(\sin p) =$

$= \frac{2p^3}{3}+p\sin p-\int \sin p\,dp=\frac{2}{3}p^3+p\sin p+\cos p+C$

$$\boxed{\begin{cases} y=\frac{2}{3}p^3+p\sin p+\cos p+C \\ X=p^2+\sin p \end{cases}}$$

3. sinx*y’’+y’*cosx=1

3)

$\sin x\, y''+y'\cos x=1 \qquad \boxed{y'=p}$

$\sin x\, p'+p\cos x=1$

$p'+p\,\mathrm{ctg}\,x=\frac{1}{\sin x}$

$\sin x=0$
$y'=1$
$y=x+C$

$\boxed{p=UV,\ p'=U'V+UV'}$

$U'V+UV'+UV\mathrm{ctg}\,x=\frac{1}{\sin x}$

$\begin{cases} U'V+UV\mathrm{ctg}\,x=0 \\ V'U=\frac{1}{\sin x} \end{cases}$

$U'=U\mathrm{ctg}\,x$

$\frac{dU}{dx}=U\mathrm{ctg}\,x$

$\frac{dU}{U}=\mathrm{ctg}\,x\,dx$

$\ln U=\int\frac{\cos x}{\sin x}dx=\int\frac{1}{\sin x}d\sin x=\ln|\sin x|$

$U=\sin x$

$V'\sin x=\frac{1}{\sin x}$

$dV=\frac{dx}{\sin^2 x}$

$V=-\mathrm{ctg}\,x+C_1$

$UV=-\cos x+\sin x\, C_1$

$y'=-\cos x+\sin x\, C_1$

$y=\int(-\cos x+\sin x\, C_1)dx=-\sin x-\cos x\, C_1+C_2$

$$\boxed{y=-\sin x-\cos x\, C_1+C_2}$$

4. y’’*(1+y)+(y’)^2=0

(4) $y''(1+y)+(y')^2=0$ $\boxed{y'=p}$

$pp'(1+y)+p^2=0$

$\frac{p'}{p}=-\frac{1}{y+1} \Rightarrow \frac{dp}{p}=-\frac{dy}{y+1}$

$\int\frac{dp}{p}=-\int\frac{d(y+1)}{y+1}$

$\ln p=-\ln(y+1)+\ln C$

$p=\frac{C}{y+1} \Rightarrow y'=\frac{C_1}{y+1}$

$\frac{dy}{dx}=\frac{C_1}{y+1}$

$(y+1)dy=C_1dx$

$\int y+1\,dy=C_1\int dx$

$y^2+y+C_2=C_1x$

$\boxed{x=\frac{y^2+y+C_2}{C_1}}$

19 вариант.

Вариант № 19.

1. $xyy'=x^2+y^2$
2. $y=(y')^2-(y')^2\cdot\cos y'$
3. $x^2y''+xy'+1=0$
4. $5y(y')^3\cdot y''-(y')^5-1=0$

1. xyy'=x^2+y^2

Вариант 19.

№1

$$xyy' = x^2 + y^2 \;\Big|\; : xy \Leftrightarrow x \neq 0;\ y \neq 0$$

$$\cancel{y' = \frac{x}{y} + \left(\frac{x}{y}\right)^{-1}} \qquad y' = \left(\frac{y}{x}\right)^{-1} + \frac{y}{x}$$

Замена: $t = \frac{y}{x}$, $y = tx$; $y' = xt' + t$

$$xt' + t = \frac{1}{t} + t$$

$$x \cdot \frac{dt}{dx} + \cancel{t} = \frac{1}{t} + \cancel{t}; \;\Big|\; : x \Leftrightarrow x \neq 0$$

$$\int t\, dt = \int \frac{1}{x}\, dx$$

$$\frac{t^2}{2} = \ln|x| + C$$

Обр. замена:

$$\left(\frac{y}{x}\right)^2 = \ln x^2 + 2C, \qquad y^2 = x^2\left(\ln x^2 + 2C\right)$$

$$y = x\sqrt{\ln x^2 + 2C}$$

Проверка:

1) $xy = 0 \Leftrightarrow \left[\begin{array}{l} x = 0 \longrightarrow y = 0 \\ y = 0 \longrightarrow x = 0 \end{array}\right\}$ частн. реш.

Ответ: $y = x\sqrt{\ln x^2 + 2C}$; $y = 0$, $x = 0$

2. y=(y’)^3-(y’)^2*cosy’

№2

$y = (y')^3 - (y')^2 \cdot \cos y'$ $\quad y = F(y')$

$y' = p \;;\; \frac{dy}{dx} = p \;;\; dy = p\,dx$

$$\begin{cases} y = p^3 - p^2 \cdot \cos p \\ dy = (3p^2 - 2p\cos p + p^2 \cdot \sin p)\,dp \quad (2) \end{cases}$$

(2) $p\,dx = (3p^2 - 2p\cos p + p^2 \sin p)\,dp \;|:p$

$\int dx = \int (3p - 2\cos p + p\sin p)\,dp$

$x = 3\frac{p^2}{2} - 2\sin p \underbrace{- p\cos p + \sin p}_{*} + C$

$* \int p\sin p\,dp = \begin{vmatrix} u = p & dv = \sin p\,dp \\ du = dp & v = -\cos p \end{vmatrix} =$

$= -p\cos p + \int \cos p\,dp = -p\cos p + \sin p + C$

$$\begin{cases} y = p^3 - p^2 \cos p \\ x = \frac{3}{2}p^2 - p\cos p - \sin p + C \end{cases}$$

Проверка: $p = 0 \;;\; y' = 0 \rightarrow y = 0$

2

Ответ: $\begin{cases} y = p^3 - p^2 \cos p \\ x = \frac{3}{2}p^2 - p\cos p - \sin p + C \end{cases}$; $y = 0$

№3

$x^2 y'' + xy' + 1 = 0$

Замена: $p = y'$; $\cancel{y'' = p p'}$

$x^2 p' + px + 1 = 0$

$\cancel{x^2 \cdot \frac{dp}{dx} + px + 1 = 0 \;|: x^2}$

$\cancel{\frac{dp}{dx} + \frac{p}{x} + \frac{1}{x^2} = 0}$

Решаем однородное ур-е:

$x^2 p' + xp = 0 \quad |: x^2 \neq 0$

$\frac{dp}{dx} = -\frac{p}{x}$;

$p = C \cdot x^{-1}$

$p = \frac{C}{x}$; $p' = \frac{C' \cdot x - C}{x^2}$

Подставим в неоднородн. ур-е:

$C' \cdot x - \cancel{C} + \cancel{C} + 1 = 0$

$C' = -\frac{1}{x}$

$C = -\ln|x| + C_1$

$p = \frac{-\ln|x| + C_1}{x}$

3

3. x^2y''+xy'+1=0

№3

$x^2 y'' + xy' + 1 = 0$

Замена: $p = y'$; $\cancel{y'' = pp'}$

$x^2 p' + px + 1 = 0$

$\cancel{x^2 \cdot \frac{dp}{dx} + px + 1 = 0 \;\; |: x^2}$

Решаем однородное ур-е:

$x^2 p' + xp = 0 \quad |: x^2 \neq 0$

$\frac{dp}{dx} = -\frac{p}{x}$;

$\frac{dp}{dx} + \frac{p}{x} + \frac{1}{x^2} = 0$

$p = C \cdot x^{-1}$

$p = \frac{C}{x}$; $p' = \frac{C' x - C}{x^2}$

Подставим в неоднородн. ур-е:

$C' x - \cancel{C} + \cancel{C} + 1 = 0$

$C' = -\frac{1}{x}$

$C = -\ln|x| + C_1$

$p = \frac{-\ln|x| + C_1}{x}$

3

Обр. замена:

$y' = \frac{-\ln|x| + C_1}{x}$

$\int dy = \int \left(-\frac{\ln|x|}{x} + \frac{C_1}{x} \right) dx$

$y = -\int \ln|x| \, d(\ln|x|) + C_1 \cdot \ln|x| + C_2$

$y = -\frac{1}{2} \ln^2|x| + C_1 \cdot \ln|x| + C_2$

4. 5y*(y’)^3*y’’-(y’)^5-1=0

Обр. замена:

№4

$5y\,(y')^3\cdot y'' - (y')^5 - 1 = 0$

$p = p(y) = y'$; $y'' = p\cdot p'$

$5y\,p^3\cdot(pp') - p^5 = 1$

$5y\,p^4 p' - p^5 = 1$

Решим однородное ур-е:

$5y\,p^4\frac{dp}{dy} - p^5 = 0 \;\; | : p^4 \neq 0\,;\; y \neq 0$

$\frac{dp}{dy} = \frac{1}{5}\cdot\frac{p}{y}$

$p = C\sqrt[5]{y}$ (*) $\quad p' = C\cdot\frac{1}{5}\cdot\frac{1}{\sqrt[5]{y^4}} + C'\cdot\sqrt[5]{y}$

4

Подставим в неоднородное ур-е:

$\cancel{5y(C\sqrt[5]{y})^4\cdot C\cdot\frac{1}{5}\cdot\frac{1}{\sqrt[5]{y^4}} - C^5 y = 1}$

$5y\,C^4\cdot\sqrt[5]{y^4}\left(\frac{C}{5\sqrt[5]{y^4}} + C'\sqrt[5]{y}\right) - C^5 y = 1$

$\cancel{5y}\cdot C^4\cancel{\sqrt[5]{y^4}}\,\frac{C + 5C'y}{\cancel{5\sqrt[5]{y^4}}} - C^5 y = 1$

$\cancel{yC^5} + 5y^2 C' C^4 - \cancel{yC^5} = 1 \;\; | : y^2 \neq 0$

$5\int C^4\,dC = \int\frac{1}{y^2}\,dy$

$C^5 = -\frac{1}{y} + C_1$;

$C = \sqrt[5]{-\frac{1}{y} + C_1}$ – подставим в (*)

$p = \sqrt[5]{-1 + yC_1}$

Обр. замена:

$y' = \sqrt[5]{-1 + yC_1}$

$\frac{1}{C_1}\int\frac{1}{\sqrt[5]{C_1y - 1}}\,d(C_1y - 1) = \int dx$

$\frac{1}{C_1}\cdot\frac{5}{4}\cdot(C_1y - 1)^{\frac{4}{5}} = x + C_2$

5

$\sqrt[5]{(C_1y - 1)^4} = \frac{4C_1}{5}(x + C_2)$

$(C_1y - 1)^4 = \left(\frac{4}{5}C_1x + \frac{4}{5}C_1C_2\right)^5$

$C_1y - 1 = \frac{4}{5}C_1(x + C_2)\sqrt[5]{\frac{4}{5}C_1(x + C_2)}$

$y = \frac{4}{5}(x + C_2)\sqrt[5]{\frac{4}{5}C_1(x + C_2)} + \frac{1}{C_1}$

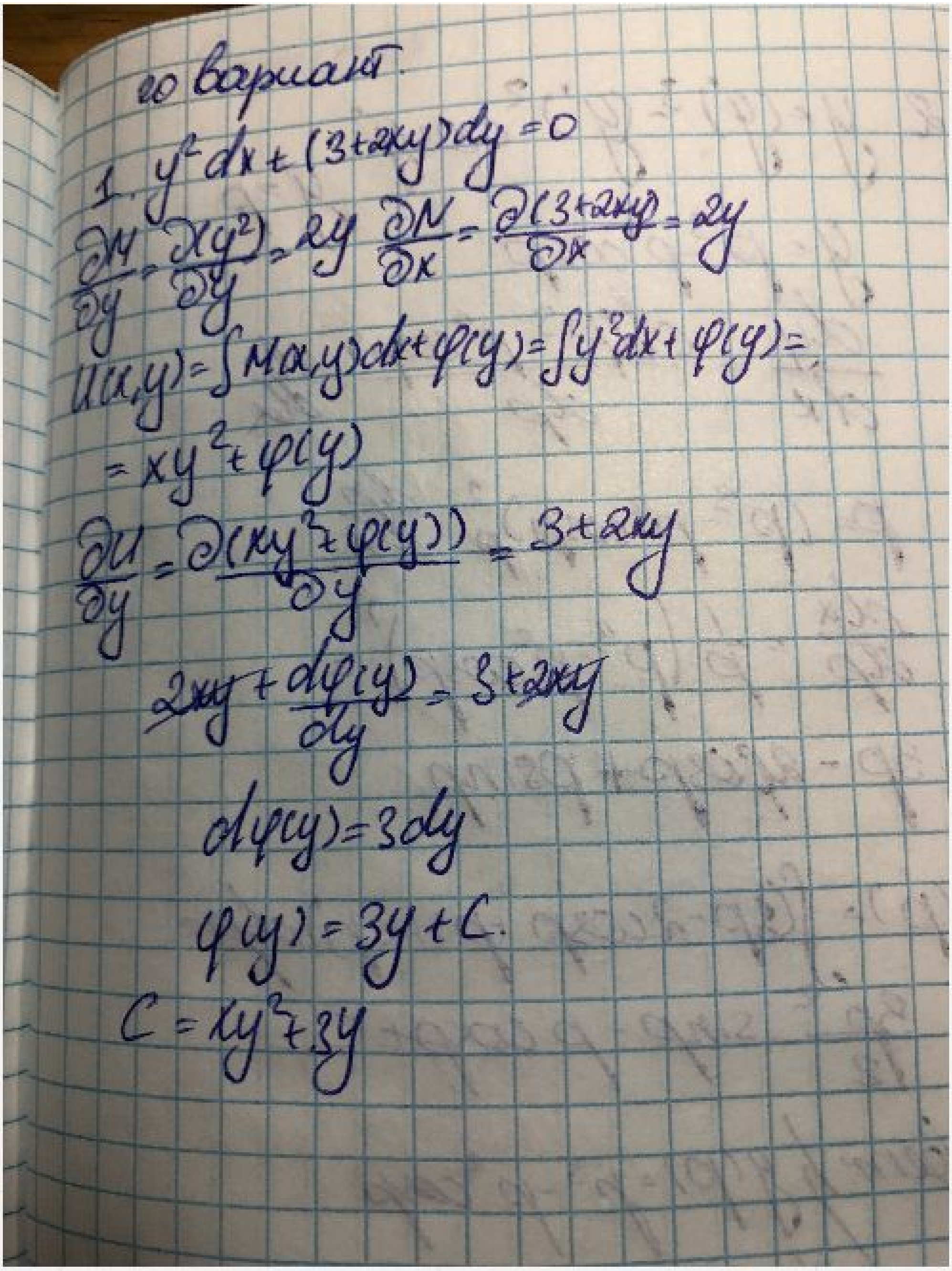
Вор. 19

20 вариант.

Вариант № 20.

1. $y^2dx + (3 + 2xy)dy = 0$
2. $yy' = 1 + (y')^3\cdot e^{y'}$
3. $xy'' = y'(\ln y' - \ln x)$
4. $y''(2y + 3) - 2(y')^2 = 0$

1. y^2dx+(3+2xy)dy=0



20 вариант.

1. $y^2dx+(3+2xy)dy=0$

$$\frac{\partial M}{\partial y}=\frac{\partial(y^2)}{\partial y}=2y \qquad \frac{\partial N}{\partial x}=\frac{\partial(3+2xy)}{\partial x}=2y$$

$$U(x,y)=\int M(x,y)dx+\varphi(y)=\int y^2dx+\varphi(y)=$$

$$=xy^2+\varphi(y)$$

$$\frac{\partial U}{\partial y}=\frac{\partial(xy^2+\varphi(y))}{\partial y}=3+2xy$$

$$\cancel{2xy}+\frac{d\varphi(y)}{dy}=3+\cancel{2xy}$$

$$d\varphi(y)=3dy$$

$$\varphi(y)=3y+C$$

$$C=xy^2+3y$$

2. yy'=1+(y')^3*e^y'

№ 2.

$$yy' = 1 + (y')^2 e^{y'}$$

Замена: $y' = p = \frac{dy}{dx}$

$yp = 1 + p^2 e^p \quad |:p$

$y = \frac{1}{p} + pe^p$

$$\frac{dy}{dx} = \frac{d\left(\frac{1}{p} + pe^p\right)}{dp} \cdot \frac{dp}{dx}$$

$$\frac{dy}{dx} = \left(-\frac{1}{p^2} + e^p + pe^p\right)\frac{dp}{dx}$$

$$p = \left(-\frac{1}{p^2} + e^p + pe^p\right)\frac{dp}{dx} \quad |:p$$

$$dx = \left(-\frac{1}{p^3} + \frac{e^p}{p} + e^p\right)dp$$

$$x = -\frac{p^{-2}}{-2} + e^p + \int \frac{e^p}{p}\,dp = \frac{1}{p^2} + e^p + \int \frac{e^p}{p}\,dp$$

$$\begin{cases} x(p) = \frac{1}{2p^2} + e^p + \int \frac{e^p}{p}\,dp \\ y(p) = \frac{1}{p} + pe^p \end{cases}$$

3. xy''=y'(lny'-lnx)

$$xy'' = y'(\ln y' - \ln x)$$

Замена: $y' = p,\ y'' = p'$

$$xp' = p(\ln p - \ln x)$$

$$xp' = p\ln\frac{p}{x}$$

$$xp' - p\ln\frac{p}{x} = 0$$

$$\underline{1 + m - 1 = m + m - 1}$$

$$m = 2m - 1 \Rightarrow m = 1$$

$$\begin{cases} p = tx^m \Rightarrow p' = t'\cdot x + t \\ x = x \end{cases}$$

$$x(t'\cdot x + t) - tx\ln\frac{tx}{x} = 0$$

$$t'x^2 + tx - tx\ln t = 0 \quad |:x$$

$$t'x + t - t\ln t = 0$$

$$t'x + t(1 - \ln t) = 0$$

$$\frac{dt}{dx}\cdot x = t(\ln t - 1)$$

$$\frac{dt}{dx} = \frac{t(\ln t - 1)}{x}$$

$$\frac{dt}{t(\ln t - 1)} = \frac{dx}{x}$$

$$\frac{d(\ln t - 1)}{(\ln t - 1)} = \frac{dx}{x}$$

$$\ln|\ln t - 1| = \ln x + \ln C$$

$$\ln t - 1 = Cx$$

$$\ln t = Cx + 1$$

$$t = e^{Cx+1}$$

$$t = \frac{p}{x}$$

$$\frac{p}{x} = e^{Cx+1}$$

$$p = e^{Cx+1}\cdot x$$

$$p = \frac{dy}{dx} = e^{Cx+1}\cdot x$$

$$y = \int xe^{Cx+1}dx = \frac{1}{C}\int x\cdot d(e^{Cx+1}) = \frac{1}{C}xe^{Cx+1} - \frac{1}{C}\int e^{Cx+1}dx = \frac{1}{C}xe^{Cx+1} - \frac{1}{C^2}e^{Cx+1} + \tilde{C}$$

$$\boxed{y = \frac{xe^{Cx+1}}{C} - \frac{e^{Cx+1}}{C^2} + \tilde{C}}$$

4. y''(2y+3)-2(y')^2=0

$$y''(2x+3)-2(y')^2=0$$

$$\boxed{p=y'}$$

$$p'(2x+3)-2p^2=0$$

$$\frac{du}{dx}=k\frac{y}{x} \Rightarrow y=Cx^k$$

$$(uv'+u'v)(2x+3)-2uv=0$$

$$\begin{cases} u'\cancel{v}(2x+3)-2u\cancel{v}=0 \\ uv'(2x+3)=0 \end{cases} \Rightarrow \begin{cases} (2x+3)u'=2u \\ \dots \end{cases} \Rightarrow \begin{cases} \frac{du}{dx}=\frac{2u}{2x+3} \\ \dots \end{cases} \Rightarrow \begin{cases} u=(2x+3)^2 \\ v=const \end{cases}$$

$$p=C(2x+3)^2$$

$$y=\int C(2x+3)^2\,dx$$

$$y=C\left(\frac{4x^3}{3}+6x^2+9x\right)+C_1$$

21 вариант.

Вариант № 21.

1. $(2x-\frac{y^2}{x})dx=3y^2(1+\ln x)dy$
2. $(y')^2-6(y')^4-3-x=0$
3. $(1+x^2)y''+2xy'-x^2=0$
4. $yy''-(y')^2-1=0$

1. (2x-y^3/x)dx=3y^2(1+lnx)dy

1)

$$\left(2x - \frac{y^3}{x}\right)dx = 3y^2(1+\ln x)dy$$

$$\underbrace{\left(2x - \frac{y^3}{x}\right)}_{F(x,y)}dx + \underbrace{\left(-3y^2 - 3y^2\ln x\right)}_{G(x,y)}dy = 0$$

$$F'_y = -\frac{3y^2}{x}; \quad G'_x = -\frac{3y^2}{x}$$

$$dI = F dx$$

$$dI = \left(2x - \frac{y^3}{x}\right)dx$$

$$\int dI = \int 2x - \frac{y^3}{x}\, dx$$

$$I = x^2 - y^3 \ln x + \varphi(y)$$

$$I'_y = -3y^2 \ln x + \varphi'(y)$$

$$-3y^2 - 3y^2 \ln x = -3y^2 \ln x + \varphi'(y)$$

$$\varphi'(y) = -3y^2$$

$$\frac{d\varphi}{dy} = -3y^2$$

$$d\varphi = -3y^2 dy$$

$$\int d\varphi = \int -3y^2 dy$$

$$\varphi = -y^3 + C$$

$$I = x^2 - y^3 \ln x - y^3 + C$$

$$\boxed{C = y^3 + y^3 \ln x - x^2}$$

2. (y’)^2-6(y’)^4-3-x=0

$$2)\quad (y')^2-6(y')^4-3-x=0 \qquad y'=p \qquad \frac{dy}{dx}=p$$

$$p^2-6p^4-3=x$$

$$x'=2p-24p^3$$

$$dy=p(2p-24p^3)dp$$

$$\int dy=\int 2p^2-24p^4\,dp$$

$$\boxed{\begin{cases} y=\frac{2}{3}p^3-\frac{24}{5}p^5+C \\ x=p^2-6p^4-3 \end{cases}}$$

3. (1+x^2)y’’+2xy’-x^2=0

$$3)\quad (1+x^2)y''+2xy'-x^2=0 \qquad y'=p$$

$$(1+x^2)p'+2xp-x^2=0 \qquad p=uv,\ p'=u'v+uv'$$

$$p'+\frac{2xp}{(1+x^2)}=\frac{1}{(1+x^2)}$$

$$uv'+u'v+\frac{2xuv}{(1+x^2)}=\frac{1}{(1+x^2)}$$

$$\begin{cases} uv'+\frac{2xuv}{(1+x^2)}=0 \\ u'v=\frac{1}{(1+x^2)} \end{cases}$$

$$v'+\frac{2xv}{(1+x^2)}=0$$

$$\frac{dv}{v}=-\frac{2x\,dx}{(1+x^2)}$$

$$\frac{dv}{v}=-\frac{d(1+x^2)}{(1+x^2)}$$

$$\ln v=-\ln(1+x^2)$$

$$v=\frac{1}{1+x^2}$$

$$\frac{u'}{1+x^2}=\frac{1}{1+x^2}$$

$$du=dx$$

$$u=x+C$$

$$p=\frac{x+C}{1+x^2}$$

$$y'=\frac{x+C_1}{1+x^2}$$

$$\frac{dy}{dx}=\frac{x+C_1}{1+x^2}$$

$$dy=\frac{x+C_1}{1+x^2}dx$$

$$\int dy=\int\frac{x}{1+x^2}dx+C_1\int\frac{1}{1+x^2}dx$$

$$\boxed{y=\frac{1}{2}\ln|1+x^2|+C_1\operatorname{arctg}(x)+C_2}$$

4. yy’’-(y’)^2-1=0

4)

$$yy'' - (y')^2 - 1 = 0 \qquad y' = p$$

$$ypp' - p^2 - 1 = 0$$

$$ypp' = p^2 + 1$$

$$\frac{pp'}{p^2+1} = \frac{1}{y}$$

$$\frac{p\,dp}{p^2+1} = \frac{dy}{y}$$

$$\ln|p^2+1| = \ln|y| + \ln C$$

$$p^2 + 1 = yC$$

$$p = \sqrt{yC - 1}$$

$$y' = \sqrt{yC - 1}$$

$$\frac{dy}{\sqrt{yC_1 - 1}} = dx$$

$$\int dx = \frac{1}{C_1}\int \frac{d(C_1 y - 1)}{\sqrt{yC_1 - 1}}$$

$$\boxed{x = \frac{2\sqrt{yC_1 - 1}}{C_1} + C_2}$$

22 вариант.

Вариант № 22.

1. $xy' - y + x\,\mathrm{ctg}\,\frac{y}{x} = 0$
2. $y = \frac{1}{2}(y')^2 + \ln y'$
3. $2xy'y'' = 1 + (y')^2$
4. $yy'' + (y')^2 = \frac{y'}{y^2}$

1. xy’-y+x*ctg(y/x)=0

22 вариант.

1. $xy' - y + x\,ctg\frac{y}{x} = 0$

$xy' = y + x\,ctg\frac{y}{x}$

$y' = \frac{y}{x} + ctg\frac{y}{x}$ $\quad \frac{y}{x} = u$

$u'x + u = u + ctg\,u$

$\frac{du}{dx}\cdot x = ctg\,u$

$\int\frac{du}{ctg\,u} = \int\frac{dx}{x}$

$-\ln|\cos u| = \ln x + \ln C$

$\ln|\cos u| = -\ln Cx$

$\cos u = -Cx$

$u = \arccos(-Cx)$

$y = x\arccos(-Cx)$

2. y=1/2(y')^2+lny'

$$2.\ y=\frac{1}{2}(y')^2+\ln y' \qquad y'=p$$

$$y=\frac{1}{2}p^2+\ln p$$

$$\frac{dy}{dx}=\frac{d\left(\frac{1}{2}p^2+\ln p\right)}{dp}\cdot\frac{dp}{dx}$$

$$p=\left(\frac{1}{2}p^2+\ln p\right)'\frac{dp}{dx}$$

$$\frac{dx}{dp}=\frac{1}{p}\left(\frac{1}{2}p^2+\ln p\right)'=$$

$$=1+\frac{1}{p^2}$$

$$x(p)=\int\left(1+\frac{1}{p^2}\right)dp=$$

$$=p-\frac{1}{p}+C$$

Ответ: $\begin{cases} y(p)=\frac{1}{2}p^2+\ln p \\ x(p)=p-\frac{1}{p}+C \end{cases}$

3. 2xy’y’’=1+(y’)^2

$$2xy'y'' = 1 + (y')^2$$

$$y' = p, \quad y'' = p'$$

$$2xpp' = 1 + p^2$$

$$p'x = \frac{1+p^2}{2p}$$

$$\frac{dp}{dx}x = \frac{1+p^2}{2p} \Rightarrow \frac{2p\,dp}{1+p^2} = \frac{dx}{x}$$

$$\frac{d(p^2+1)}{p^2+1} = \frac{dx}{x}$$

$$\ln|p^2+1| = \ln|x| + \ln|C|$$

$$p^2+1 = Cx$$

$$p = \sqrt{Cx-1}$$

$$\frac{dy}{dx} = \sqrt{Cx-1}$$

$$dy = \sqrt{Cx-1}\,dx$$

$$y = \frac{2(Cx-1)^{\frac{3}{2}}}{3C} + \tilde{C}$$

4. yy’’+(y’)^2=y’/y^2

$$yy'' + (y')^2 = \frac{y'}{y^2}$$

$$y' = p,\quad y'' = p'_y\, p$$

$$y \cdot p'_y \cdot p + p^2 = \frac{p}{y^2} \qquad p = 0.\ y = C \text{ - реш}$$

$$y \cdot p'_y + p = \frac{1}{y^2} \text{ — вернуть}$$

$$p'_y + \frac{p}{y} = \frac{1}{y^3} \longrightarrow y \neq 0 \qquad y = 0 \text{ реш.}$$

$$p = u \cdot v \Rightarrow p' = u'v + uv'$$

$$u'v + uv' + \frac{1}{y}uv = \frac{1}{y^3}$$

$$\begin{cases} u'\cancel{v} + \frac{1}{y}u\cancel{v} = 0 \\ uv' = \frac{1}{y^3} \end{cases} \Rightarrow \begin{cases} \frac{du}{dy} = -\frac{1}{y} \\ v' = \frac{1}{u \cdot y^3} \end{cases} \Rightarrow \begin{cases} du = -\frac{dy}{y} \\ \frac{dv}{dy} = \frac{1}{u \cdot y^3} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} u = \frac{1}{y} \\ dv = \frac{1}{y^2} \cdot dy \end{cases} \Rightarrow \begin{cases} u = \frac{1}{y} \\ v = -\frac{1}{y} + C \end{cases}$$

Что за какашка?

$$y' = uv = -\frac{1}{y^2} + \frac{C}{y}$$

$$\frac{dy}{dx} = -\frac{1}{y^2} + \frac{C}{y} \Rightarrow dx = \frac{dy}{-\frac{1}{y^2} + \frac{C}{y}} = \frac{y^2 dy}{-1 + Cy}$$

23 вариант.

Вариант № 23.

1. $y' + y \cdot \mathrm{tg}x = \sin 2x$
2. $x = (y')^2 + \arcsin y'$
3. $xy'' - y' - \sqrt{(y')^2 - x^2} = 0$
4. $y''(5y + 2) - 4(y')^2 = 0$

1. y’+y*tgx=sin2x

$y' + y\,tgx = \sin 2x$ - неоднор. ур

1-го порядка

$$\begin{cases} y = u(x)v(x) \\ x = x \end{cases}$$

$y' + uv\,tgx = \sin 2x$

$u'v + v'u + uv\,tgx = \sin 2x$

$v(u' + u\,tgx) + v'u = \sin 2x$

$u' + u\,tgx = 0$

$\frac{du}{dx} = -u\,tgx \Rightarrow -\frac{1}{u}du = tgx\,dx$

$-\ln|u| = \int \frac{d\cos x}{\cos x}$

$-\ln|u| = -\ln|\cos x|$

$u = \cos x$ *

$v' \cos x = \sin 2x$

$\frac{dv}{dx} \cos x = \sin 2x$

$\frac{dv}{dx} = \frac{2\sin x \cancel{\cos x}}{\cancel{\cos x}}$ $\quad dv = 2\sin x\,dx$

$v = -2\cos x + C$

$$\boxed{y = (-2\cos x + C) \cdot \cos x}$$

2. x=(y’)^2+arcsiny’

$$2.\ x=(y')^2+\arcsin y' \quad y'=p$$

$$x=p^2+\arcsin p$$

$$\frac{dx}{dy}=\frac{d(p^2+\arcsin p)}{dp}\cdot\frac{dp}{dy}$$

$$\frac{1}{\frac{dy}{dx}}=(p^2+\arcsin p)'_p\cdot p'_y$$

$$\frac{1}{p}=(p^2+\arcsin p)'_p\, p'_y$$

$$\frac{1}{p}=\left(2p+\frac{1}{\sqrt{1-p^2}}\right)p'_y$$

$$dy=\left(2p^2+\frac{p}{\sqrt{1-p^2}}\right)dp$$

$$y=\int\left(2p^2+\frac{p}{\sqrt{1-p^2}}\right)dp=$$

$$=\frac{2p^3}{3}-\sqrt{1-p^2}+C$$

Ответ: $\begin{cases} x(p)=p^2+\arcsin p \\ y(p)=\frac{2p^3}{3}-\sqrt{1-p^2}+C \end{cases}$

3. xy’’-y’-((y’)^2-x^2)^1/2=0 (задание вбито как xy’-y’(y^2-x^2)^(1/2)=0, то есть уже с заменой y’=p)

Solve $x\frac{d\,y(x)}{d\,x}-\sqrt{\frac{d\,y(x)}{d\,x}\left(-x^2+y(x)^2\right)}=0$:

Let $y(x)=x\,v(x)$, which gives $\frac{d\,y(x)}{d\,x}=x\frac{d\,v(x)}{d\,x}+v(x)$:

$$x\left(x\frac{d\,v(x)}{d\,x}+v(x)\right)-\sqrt{\left(-x^2+x^2\,v(x)^2\right)\left(x\frac{d\,v(x)}{d\,x}+v(x)\right)}=0$$

Simplify:

$$x\left(x\frac{d\,v(x)}{d\,x}-\sqrt{\left(x\frac{d\,v(x)}{d\,x}+v(x)\right)\left(v(x)^2-1\right)}+v(x)\right)=0$$

Solve for $\frac{d\,v(x)}{d\,x}$:

$$\frac{d\,v(x)}{d\,x}=-\frac{v(x)}{x}\ \text{or}\ \frac{d\,v(x)}{d\,x}=\frac{v(x)^2-v(x)-1}{x}$$

For $\frac{d\,v(x)}{d\,x}=-\frac{v(x)}{x}$:

Divide both sides by $v(x)$:

$$\frac{\frac{d\,v(x)}{d\,x}}{v(x)}=-\frac{1}{x}$$

Integrate both sides with respect to x:

$$\int\frac{\frac{d\,v(x)}{d\,x}}{v(x)}\,d\,x=\int-\frac{1}{x}\,d\,x$$

Evaluate the integrals:

$\log(v(x))=-\log(x)+c_1$, where c_1 is an arbitrary constant.

Solve for $v(x)$:

$$v(x)=\frac{e^{c_1}}{x}$$

For $\frac{d\,v(x)}{d\,x}=\frac{v(x)^2-v(x)-1}{x}$:

Divide both sides by $v(x)^2-v(x)-1$:

$$\frac{\frac{d\,v(x)}{d\,x}}{v(x)^2-v(x)-1}=\frac{1}{x}$$

Integrate both sides with respect to x:

$$\int\frac{\frac{d\,v(x)}{d\,x}}{v(x)^2-v(x)-1}\,d\,x=\int\frac{1}{x}\,d\,x$$

Evaluate the integrals:

$$\frac{\log\left(-2\,v(x)+1+\sqrt{5}\right)-\log\left(2\,v(x)-1+\sqrt{5}\right)}{\sqrt{5}}$$

$=\log(x)+c_1$, where c_1 is an arbitrary constant.

Solve for $v(x)$:

$$v(x)=\frac{-e^{\sqrt{5}\,c_1}\left(-1+\sqrt{5}\right)x^{\sqrt{5}}+1+\sqrt{5}}{2\,e^{\sqrt{5}\,c_1}\,x^{\sqrt{5}}+2}$$

Collect solutions and simplify the arbitrary constants:

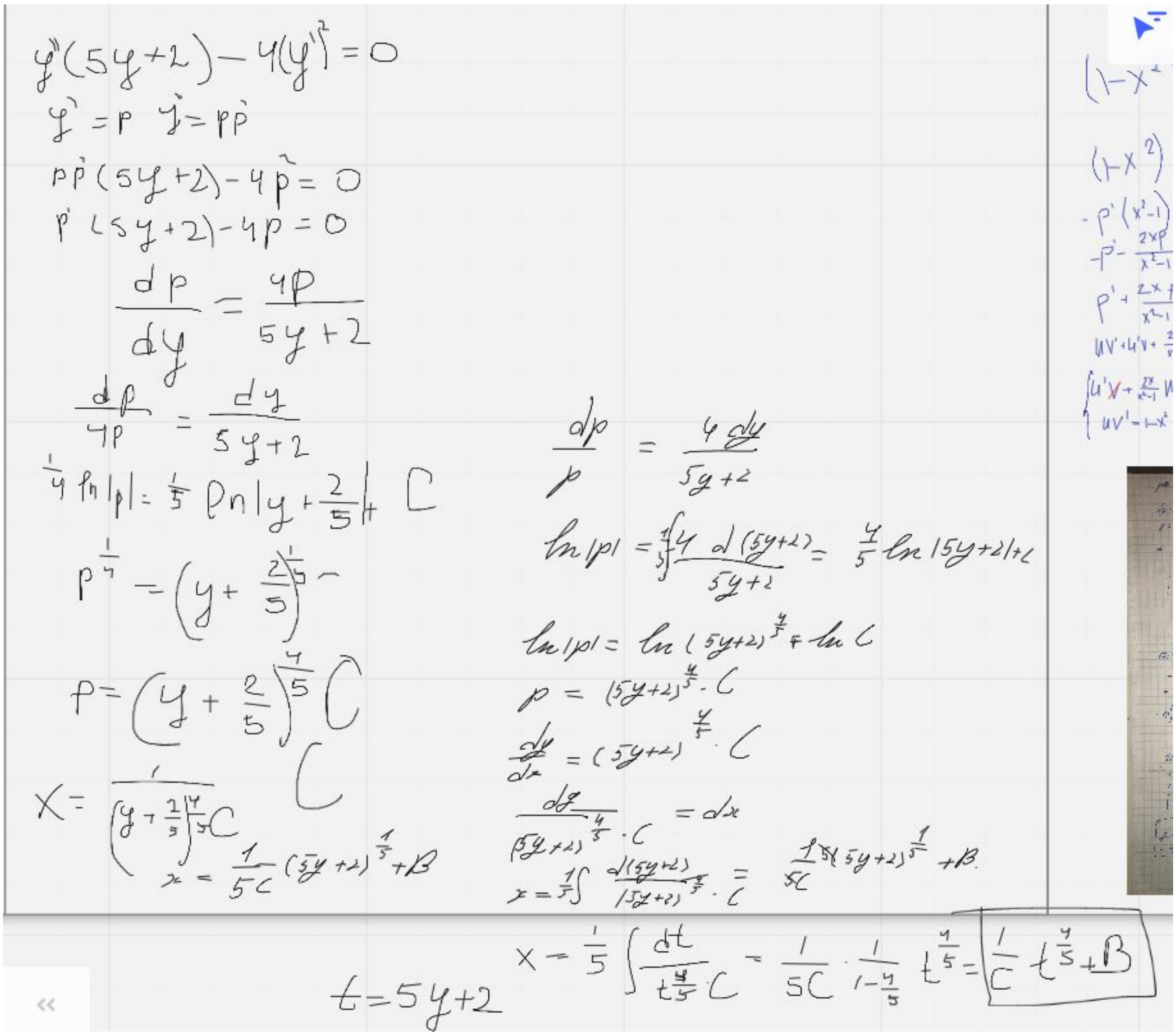
$$v(x)=\frac{c_1}{x}\ \text{or}\ v(x)=\frac{-\left(-1+\sqrt{5}\right)c_1\,x^{\sqrt{5}}+1+\sqrt{5}}{2\,c_1\,x^{\sqrt{5}}+2}$$

Substitute back for $y(x)=x\,v(x)$:

Answer:

$$y(x)=c_1\ \text{or}\ y(x)=\frac{x\left(-\left(-1+\sqrt{5}\right)c_1\,x^{\sqrt{5}}+1+\sqrt{5}\right)}{2\,c_1\,x^{\sqrt{5}}+2}$$

4. y’’(5y+2)-4(y’)^2=0

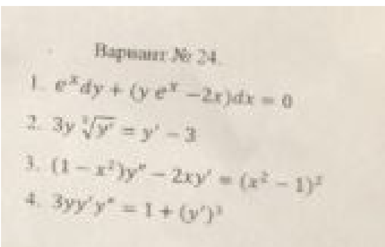


$$y''(5y+2) - 4(y')^2 = 0$$
$$y' = p \quad y'' = pp'$$
$$pp'(5y+2) - 4p^2 = 0$$
$$p'(5y+2) - 4p = 0$$
$$\frac{dp}{dy} = \frac{4p}{5y+2}$$
$$\frac{dp}{4p} = \frac{dy}{5y+2}$$
$$\frac{1}{4}\ln|p| = \frac{1}{5}\ln\left|y + \frac{2}{5}\right| + C$$
$$p^{\frac{1}{4}} = \left(y + \frac{2}{5}\right)^{\frac{1}{5}} C$$
$$p = \left(y + \frac{2}{5}\right)^{\frac{4}{5}} C$$
$$x = \frac{1}{\left(y + \frac{2}{5}\right)^{\frac{4}{5}} C}$$
$$x = \frac{1}{5C}(5y+2)^{\frac{1}{5}} + B$$

$$\frac{dp}{p} = \frac{4\,dy}{5y+2}$$
$$\ln|p| = \frac{1}{5}\int \frac{4\,d(5y+2)}{5y+2} = \frac{4}{5}\ln|5y+2| + c$$
$$\ln|p| = \ln(5y+2)^{\frac{4}{5}} + \ln C$$
$$p = (5y+2)^{\frac{4}{5}} \cdot C$$
$$\frac{dy}{dx} = (5y+2)^{\frac{4}{5}} \cdot C$$
$$\frac{dy}{(5y+2)^{\frac{4}{5}} \cdot C} = dx$$
$$x = \frac{1}{5}\int \frac{d(5y+2)}{(5y+2)^{\frac{4}{5}} \cdot C} = \frac{1}{5C}(5y+2)^{\frac{1}{5}} + B$$

$$t = 5y+2 \qquad x = \frac{1}{5}\int \frac{dt}{t^{\frac{4}{5}} C} = \frac{1}{5C} \cdot \frac{1}{1-\frac{4}{5}}\, t^{\frac{4}{5}} = \boxed{\frac{1}{C}\, t^{\frac{4}{5}} + B}$$

24 вариант.



Вариант № 24

1. $e^x dy + (y e^x - 2x)dx = 0$
2. $3y\sqrt[3]{y'} = y' - 3$
3. $(1 - x^2)y'' - 2xy' = (x^2 - 1)^2$
4. $3yy'y'' = 1 + (y')^3$

1. e^xdy+(y*e^x-2x)dx=0

24 вариант

1. $e^x dy + (ye^x - 2x)dx = 0$

$\frac{\partial M}{\partial y} = \frac{\partial(ye^x-2x)}{\partial y} = e^x$ $\frac{\partial N}{\partial x} = \frac{\partial(e^x)}{dx} = e^x$

$u(x,y) = \int M(x,y)dx + \varphi(y) =$

$= \int (ye^x - 2x)dx + \varphi(y) =$

$= ye^x - x^2 + \varphi(y)$

$\frac{\partial u}{\partial y} = \frac{\partial(ye^x - x^2 + \varphi(y))}{\partial y} = e^x$

$e^x + \frac{d\varphi(y)}{dy} = e^x$

$d\varphi(y) = dy$

$\varphi(y) = y + C$

$C = ye^x - x^2 + y$

2. 3y*(y’)^1/3=y’-3

2. $3y\sqrt[3]{y'} = y' - 3.$

$$y = \frac{y'}{3\sqrt[3]{y'}} - \frac{1}{\sqrt[3]{y'}} \qquad y' = p$$

$$y = \frac{p-3}{3\sqrt[3]{p}}$$

$$\frac{dy}{dx} = \frac{d\left(\frac{p-3}{3\sqrt[3]{p}}\right)}{dp} \cdot \frac{dp}{dx}$$

$$p = \left(\frac{p}{3\sqrt[3]{p}} - \frac{1}{\sqrt[3]{p}}\right)'_p \frac{dp}{dx}$$

$$\frac{dx}{dp} = \frac{1}{p}\left(\frac{p}{3\sqrt[3]{p}} - \frac{1}{\sqrt[3]{p}}\right)' =$$

$$= \frac{2p+3}{9p^2\sqrt[3]{p}}$$

$$x(p) = \int \frac{2p+3}{9p^2\sqrt[3]{p}}\,dp =$$

$$= -\frac{2}{3\sqrt[3]{p}} - \frac{1}{4p\sqrt[3]{p}} + C$$

Ответ: $\begin{cases} y(p) = \frac{p-3}{3\sqrt[3]{p}} \\ x(p) = -\frac{2}{3\sqrt[3]{p}} - \frac{1}{4p\sqrt[3]{p}} + C \end{cases}$

3. (1-x^2)y''-2xy'=(x^2-1)^2

$$(1-x^2)y'' - 2xy' = (x^2-1)^2.$$

Замена: $y' = p$, $y'' = p'$.

$$(1-x^2)p' - 2xp = (x^2-1)^2.$$

$$-(x^2-1)p' - 2xp = (x^2-1)^2 \quad | : (x^2-1).$$

$$-p' - \frac{2x}{x^2-1}p = x^2-1 \quad | \cdot (-1).$$

$$p' + \frac{2x}{x^2-1}p = 1-x^2$$ – лин. ур-ие отн. p

Замена: $p = uV$.

$$\underline{u'V} + uV' + \underline{\frac{2x}{x^2-1}uV} = 1-x^2.$$

$$u'V + \frac{2x}{x^2-1}uV = 0 \quad | : V$$

$$u' + \frac{2x}{x^2-1}u = 0$$

$$u' = \frac{2x}{1-x^2}u.$$

$$\frac{du}{dx} = \frac{2x}{1-x^2}u.$$

$$\frac{du}{u} = \frac{2x\,dx}{1-x^2} = \frac{dx^2}{1-x^2} = \frac{-d(1-x^2)}{1-x^2}$$

$$\ln|u| = -\ln|1-x^2|.$$

$$u = \frac{1}{1-x^2}.$$

$$uV' = 1-x^2.$$

$$\frac{1}{1-x^2}V' = 1-x^2 \quad | \cdot (1-x^2)$$

$$\frac{dt}{dx} = t' \Rightarrow dx = \frac{1}{t'}dt.$$

$$\frac{dV}{dx} = (1-x^2)^2.$$

$$dV = (1-x^2)^2 dx.$$

$$t = 1-x^2 \Rightarrow \frac{dt}{dx} = -2x.$$

$$dx = \frac{1}{t'}dt = -\frac{1}{2x}dt$$

$$\int (1-x^2)^2 dx = \int (x^4 - 2x^2 + 1)\,dx = \frac{x^5}{5} - \frac{2}{3}x^3 + x + C.$$

$$V = \frac{x^5}{5} - \frac{2}{3}x^3 + x + C.$$

$$p = uV = \frac{1}{1-x^2}\left(\frac{3x^5}{15} - \frac{10x^3}{15} + \frac{15x}{15} + \frac{15C}{15}\right) = \frac{3x^5 - 10x^3 + 15x + 15C}{15(1-x^2)} =$$

$$= \frac{1}{15}\left(\frac{3x^5}{1-x^2} - \frac{10x^3}{1-x^2} + \frac{15x}{1-x^2} + \frac{15C}{1-x^2}\right)$$

where $\frac{dy}{dx} = p$.

$dy = p\,dx$

$y = \int p\,dx = \frac{1}{15}\int\left(\frac{3x^5}{1-x^2} - \frac{10x^3}{1-x^2} + \frac{15x}{1-x^2} + \frac{15C}{1-x^2}\right)dx =$

$=$

$\int \frac{3x^5}{1-x^2}dx = 3\int\frac{x^5}{1-x^2}dx = 3\int\frac{x^{\cancel{5}\,4}}{\cancel{t}\cdot -2x\cdot(1-x^2)}dt \ominus$

$t = 1-x^2 \Rightarrow dt = -2x.$

$dx = d\frac{1}{t}dx = -\frac{1}{2x}dt$

$1-x^2 = t \Rightarrow x^2 = 1-t \Rightarrow x^5 = (1-t)^{5}$

$dx =$

$x^4 = (1-t)^2$

$\ominus -\frac{3}{2}\cdot\int\frac{(1-t)^2}{t}dt = -\frac{3}{2}\int\left(\frac{t^2}{t} - \frac{2t}{t} + \frac{1}{t}\right)dt =$

$= -\frac{3}{2}\frac{t^2}{2} + 3t - \frac{3}{2}\ln|t| + C = -\frac{3}{4}t^2 + 3t - \frac{3}{2}\ln|t| + C =$

$= -\frac{3}{4}(1-x^2)^2 + 3(1-x) - \frac{3}{2}\ln|1-x| + C.$

$-10\int\frac{x^3}{1-x^2}dx = -10\int\frac{x^{\cancel{3}\,2}}{-2x\cdot(1-x^2)}dt = \frac{\cancel{10}}{\cancel{2}}5\int\frac{x^2}{1-x^2}dt = 5\int\frac{1-t}{t} \ominus$

$t = 1-x^2 \Rightarrow dt = -2x.$

$\Rightarrow x^2 = 1-t$

$\ominus 5\int\left(\frac{1}{t} - 1\right)dt = 5\ln|t| - 5t + C = 5\ln|1-x^2| - 5(1-x^2) + C$

$\int\frac{15x}{1-x^2}dx = \frac{15}{2}\int\frac{dx^2}{1-x^2} = -\frac{15}{2}\int\frac{d(1-x^2)}{1-x^2} = -\frac{15}{2}\ln|1-x^2| + C$

$\int\frac{15C}{1-x^2} = 15C\cdot\ln|1-x^2|$

$$\boxed{y = \frac{1}{15}\left(-\frac{3}{4}(1-x^2)^2 + 3(1-x) - \frac{3}{2}\ln|1-x| + 5\ln|1-x^2| - 5(1-x^2) - \frac{15}{2}\ln|1-x^2| + 15C\ln|1-x^2| + \tilde{C}\right)}$$

4. 3yy’y’’=1+(y’)^3

$$3yy'y'' = 1+(y')^3$$

$$pp'_y = y''$$

$$3yp^2p' = 1+p^3$$

$$3yp' = \frac{1}{p^2} + p; \quad \cancel{|:p^3}$$

$$3y(uv'+u'v) = \frac{1}{uv} + 1$$

$$3yu'v = \frac{1}{uv} + 1$$

$$3y\cdot p\cdot p\cdot p'_y = 1+p^3$$

$$3yp^2\cdot p'_y = 1+p^3 \quad |:p^2$$

$$3yp'_y = \frac{1}{p^2} + p$$

$$p'_y = \frac{1}{3}\left(\frac{1}{p^2} + p\right)\cdot\frac{1}{y}$$

$$dp = \frac{1}{3}\left(\frac{1}{p^2} + p\right)\cdot\frac{1}{y}\,dy$$

$$\frac{p^2\,dp}{1+p^3} = \frac{1}{3y}\,dy$$

$$\cancel{\frac{1}{3}}\frac{d(1+p^3)}{1+p^3} = \cancel{\frac{1}{3}}\cdot\frac{1}{y}\,dy$$

$$\ln|1+p^3| = \ln|y| + \ln C$$

$$1+p^3 = yC$$

$$p^3 = yC-1$$

$$p = (yC-1)^{\frac{1}{3}}$$

$$\boxed{x = \frac{1}{C}\cdot\frac{3(yC-1)^{\frac{2}{3}}}{2} + \tilde{C}}$$

$$p = \frac{dy}{dx}$$

$$\frac{dy}{dx} = (yC-1)^{\frac{1}{3}} \Rightarrow dx = \frac{dy}{(yC-1)^{\frac{1}{3}}} = \frac{1}{C}\,\frac{d(yC-1)}{(yC-1)^{\frac{1}{3}}}$$

25 вариант.

Вариант № 25.

1. $x\cos(xy)dy + (y\cos(xy) - x)dx = 0$
2. $y = (y')^3 - \ln(1+(y')^2)^2$
3. $xy'' + y' = 16x^3$
4. $y(1-\ln y)y'' + (1+\ln y)(y')^2 = 0$

1. xcos(xy)dy+(ycos(xy)-x)dx=0

Б-25

1) $\underbrace{x\cos(xy)\,dy}_{F} + \underbrace{(y\cos(xy)-x)\,dx}_{G} = 0$

$\cos(xy) - \sin(xy)yx \qquad \cos(xy) - xy\sin(xy)$

$I = \int x\cos(xy)\,dy = \int \cos(xy)\,d(xy) = \sin(xy) + C(y)$

$\frac{\partial F}{\partial x} = y\cos(xy) + C'(y)$

$y\cos(xy) + C'(y) = y\cos(xy) - x;$

$C'(y) = -x$

$C = -\frac{x^2}{2} + C_1$

$I = \sin(xy) - \frac{x^2}{2} + C_1$

$C_1 = \frac{x^2}{2} - \sin(xy)$

2. y=(y’)^3-ln(1+(y’)^2)^2

(2) $y = (y')^3 - \ln(1+(y')^2)^2$

Замена: $p = y'$

$y = p^3 - \ln(1+p^2)^2;$

$f(p) = p^3 - \ln(1+p^2)^2$

$f'(p) = 3p^2 - \frac{4p\ln(1+p^2)}{1+p^2}$

$\int dx = \int\left(\frac{3p^2}{p} - \frac{4p\ln(1+p^2)}{p(1+p^2)}\right)dp;$

$$\begin{cases} x = \frac{3p^2}{2} - 4\int\frac{\ln(1+p^2)}{1+p^2} \\ y = p^3 - \ln(1+p^2)^2 \end{cases}$$

$y = (y')^3 - \ln(1+(y')^2)^2$

$y = (y')^3 - 2\ln(1+(y')^2)$

Замена: $y = p^3 - 2\ln(1+p^2)$

$\frac{dy}{dx} = 3p^2 - 2\cdot 2p\cdot\frac{1}{1+p^2}.$

3. xy''+y'=16x^3

3) $xy'' + y' = 16x^3$

Замена: $p = y'$

$xp' + p = 16x^3$ $\qquad p = uv$

$x(uv' + u'v) + uv = 16x^3$

$$\begin{cases} xu'v + uv = 0 \\ \cancel{x}uv' = 16x^3 \end{cases} \Rightarrow \begin{cases} xu' = u \\ uv' = 16x^2 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} u = \frac{1}{x} \\ v' = 16x^3 \end{cases} \Rightarrow \begin{cases} u = \frac{1}{x} \\ v = 4x^4 + C \end{cases}$$

$p = (4x^4 + C)\frac{1}{x}$

$y' = 4x^3 + \frac{C}{x}$

$y = \int \left(4x^3 + \frac{C}{x}\right) dx$

$y = x^4 + C \ln|x| + C_1$

4. y(1-lny)y''+(1+lny)(y')^2=0

$$y(1-\ln y)y'' + (1+\ln y)(y')^2 = 0$$

$$y' = p, \quad y'' = p'_y\, p$$

$$y(1-\ln y)\, p'_y\, p + (1+\ln y)\, p^2 = 0 \qquad p=0;\ \overset{\text{реш}}{y=C}$$

$$y(1-\ln y)\, p'_y + (1+\ln y)\, p = 0$$

$$\frac{dp}{dy} = -\frac{p(1+\ln y)}{y(1-\ln y)} \Rightarrow \frac{dp}{p} = \frac{-(1+\ln y)dy}{y(1-\ln y)}$$

$$\ln|p| = -\int \frac{(1+\ln y)}{(1-\ln y)}\, d(\ln y) =$$

$$= \ln y + 2\ln(\ln y - 1) + \ln C$$

$$p = Cy(\ln y - 1)^2$$

$$\frac{dy}{dx} = Cy(\ln y - 1)^2$$

$$dx = \frac{dy}{Cy(\ln y - 1)^2}$$

?

26 вариант.

Вариант № 26.

1. $ydx = (e^y + x)dy$
2. $x = (y')^4 - 2(y')^2 + 2$
3. $y'' = \frac{y'}{x}(1 + \ln\frac{y'}{x})$
4. $yy'' - y'(1 + y') = 0$

1. ydx=(e^y+x)dy

$$y\,dx = (e^y + x)\,dy$$

$$M(x,y) = y$$

$$N(x,y) = e^y + x$$

$$\frac{\partial M}{\partial y} = 1, \quad \frac{\partial N}{\partial x} = 1 \quad =$$

ур-ние в полных диф

$$I = \int y\,dx = y\int dx = yx + C(y)$$

$$\frac{\partial I}{\partial y} = x + C'(y) = e^y + x$$

$$C'(y) = e^y \Rightarrow C(y) = e^y + \tilde{C} \; ?$$

$$I = xy + e^y + C$$

2. x=(y’)^4-2(y’)^2+2

$$x = (y')^4 - 2(y')^2 + 2$$

$$y' = p \Rightarrow \frac{dy}{dx} = p \Rightarrow dy = dx\,p$$

$$x = p^4 - 2p^2 + 2 \Rightarrow x' = 4p^3 - 4p$$

$$dy = p(4p^3 - 4p) = 4p^4 - 4p^2$$

$$y = (4p^4 - 4p^2)\,dp = \frac{4p^5}{5} - \frac{4p^3}{3} + C$$

что-то слишком изи

3. y''=(y'/x)*(1+ln(y/x))

$$y'' = \frac{y'\left(1+\ln\frac{y'}{x}\right)}{x}$$

ур. допуск-ее понижение порядка, II тип

$y'=p,\ y''=p',\ y'''=p'',$

тип $y,\ p=y',\ y''=pp',\ \dots$

$y'=p$

$xp' = p\left(1+\ln\frac{p}{x}\right)$

$xp' - p - p\ln\frac{p}{x} = 0$

$1+m-1 = m = m+m-1$

$m = m = 2m-1$

$m=1$

$$\begin{cases} p = tx \Rightarrow p' = t'\cdot x + t \\ x = x \end{cases}$$

$x\cdot(t'\cdot x + t) - tx - tx\cdot\ln\frac{tx}{x} = 0$

$t'x^2 + \cancel{tx} - \cancel{tx} - tx\cdot\ln t = 0$

$t'x^2 - tx\ln t = 0 \quad |:x$

$t'x - t\ln t = 0$

$t'\cdot x = t\ln t$

$\frac{dt}{dx}\cdot x = t\ln t$

$\frac{dt}{t\ln t} = \frac{dx}{x}$

$\frac{d(\ln t)}{\ln t} = \frac{dx}{x}$

$\ln(\ln(t)) = \ln(x) + \ln C$

$\ln(\ln(t)) = \ln(Cx)$

$\ln(t) = Cx$

$t = e^{Cx}$

$p = tx \Rightarrow p = e^{Cx}\cdot x = \frac{dy}{dx}$

$dy = e^{Cx}\cdot x\,dx$

$y = \underbrace{\frac{1}{C}x\,de^{Cx}}$ $= \frac{1}{C}xe^{Cx} - \frac{1}{C}\int e^{Cx}dx = \frac{1}{C}xe^{Cx} - \frac{1}{C^2}e^{Cx} + \tilde{C}$

$e^{Cx}\cdot x\,dx =$

$$\boxed{y = \frac{xe^{Cx}}{C} - \frac{e^{Cx}}{C^2} + \tilde{C}}$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

4. yy''-y'(1+y')=0

$$yy'' - y'(1+y') = 0$$

$$y' = p,\quad y'' = p'_y \cdot p$$

$$y \cdot p'_y\, p - p(1+p) = 0 \longrightarrow p = 0,\ y = C \text{ - реш}$$

$$y\frac{dp}{dy} = (1+p)$$

$$\frac{d(p+1)}{p+1} = \frac{dy}{y} \Rightarrow \ln|p+1| = \ln|y| + \ln|C|$$

$$p+1 = Cy$$

$$p = Cy - 1$$

$$\frac{dy}{dx} = Cy - 1 \Rightarrow dx = \frac{dy}{Cy-1}$$

$$x = \frac{1}{C}\int \frac{d(Cy-1)}{Cy-1} = \frac{\ln|Cy-1|}{C} + \tilde{C}$$

27 вариант.

Вариант № 27.

1. $(\sin x + y)dy + (y\cos x - x^2)dx = 0$
2. $y(y')^3 - 2(y')^2 - 3 = 0$
3. $x^2 y'' = (y')^2$
4. $yy'' = (y')^2 + yy'$

1. (sinx+y)dy+(ycosx-x^2)dx=0

№1

$$\underbrace{(\sin x + y)}_{N(x,y)}\,dy + \underbrace{(y\cos x - x^2)}_{M(x,y)}\,dx = 0$$

Проверим явл. ли ур-ие в полн. диф-ах

$$\frac{\partial M}{\partial y} = \cos x = \frac{\partial N}{\partial x} = \cos x$$

$\downarrow$

да

$$I = \int M(x,y)\,dx = \int (y\cos x - x^2)\,dx = y\sin x - \frac{x^3}{3} + C(y)$$

$$\frac{\partial I}{\partial y} = \sin x + C'_y$$

$$\cancel{\sin x} + C'_y = \cancel{\sin x} + y \; ; \quad C = \frac{y^2}{2} + C_1$$

$$I = y\sin x - \frac{x^3}{3} + \frac{y^2}{2} + C_1$$

1

Сделаем проверку:

$$\frac{\partial I}{\partial x} = y\cos x - x^2 \; ; \quad \frac{\partial I}{\partial y} = \sin x + y$$

№2

2

2. y(y')^3-2(y')^2-3=0

№2

$y(y')^3 - 2(y')^2 - 3 = 0.$ Ур-ие 1-го порядка, приводящееся к виду $y = f(y')$.

$$y = \frac{3 + 2(y')^2}{(y')^3}.$$

Замена: $y' = p(x)$; $y = \dfrac{2p^2+3}{p^3}$. $(*)$

Дифференцируем $(*)$ по x.

$$\frac{d(\ldots p \ldots)}{dx} = \frac{d(\ldots p \ldots)}{dp} \cdot \frac{dp}{dx}.$$

$$\frac{dy}{dx} = \frac{d\left(\frac{2p^2+3}{p^3}\right)}{dx} = \frac{d\left(\frac{2p^2+3}{p^3}\right)}{dp} \cdot \frac{dp}{dx}.$$

Замена: $\dfrac{dy}{dx} = p$.

$$p = \left(\frac{2p^2+3}{p^3}\right)'_p \cdot \frac{dp}{dx}$$

$$p = \left(\frac{4p \cdot p^3 - 3p^2(2p^2+3)}{p^6}\right)\frac{dp}{dx} = \left(\frac{4p^4 - 6p^4 - 9p^2}{p^6}\right)\frac{dp}{dx} = \left(-\frac{2p^4 - 9p^2}{p^6}\right)\frac{dp}{dx} =$$

$$= \frac{-2p^2 - 9}{p^4}\,\frac{dp}{dx}$$

Выразить $\dfrac{dx}{dp}$:

$$\frac{dx}{dp} = -\frac{1}{p} \cdot \frac{2p^2+9}{p^4}$$

Найти x.

$$x = -\int \frac{2p^2+9}{p^5}\,dp = -\int \frac{2}{p^3}\,dp - \int \frac{9}{p^5}\,dp = \frac{1}{p^2} + \frac{9}{4p^4} + C$$

Ответ: $\begin{cases} x(p) = \dfrac{1}{p^2} + \dfrac{9}{4p^4} + C \\ y(p) = \dfrac{2}{p} + \dfrac{3}{p^3} \end{cases}$, $C = const.$

3. x^2y''=(y')^2

3. Разделение переменных

$$x^2 y'' = (y')^2$$

$$x^2 y'' = \left(\frac{dy}{dx}\right)^2$$

$$y' = p(x)$$

$$x^2 p' = p^2 ; \quad x^2 \frac{dp}{dx} = p^2 ; \quad \int \frac{1}{x^2} dx = \int \frac{1}{p^2} dp ;$$

$$-\frac{1}{x} = -\frac{1}{p} + C ; \quad -\left(\frac{1+Cx}{x}\right) = -\frac{1}{p} ;$$

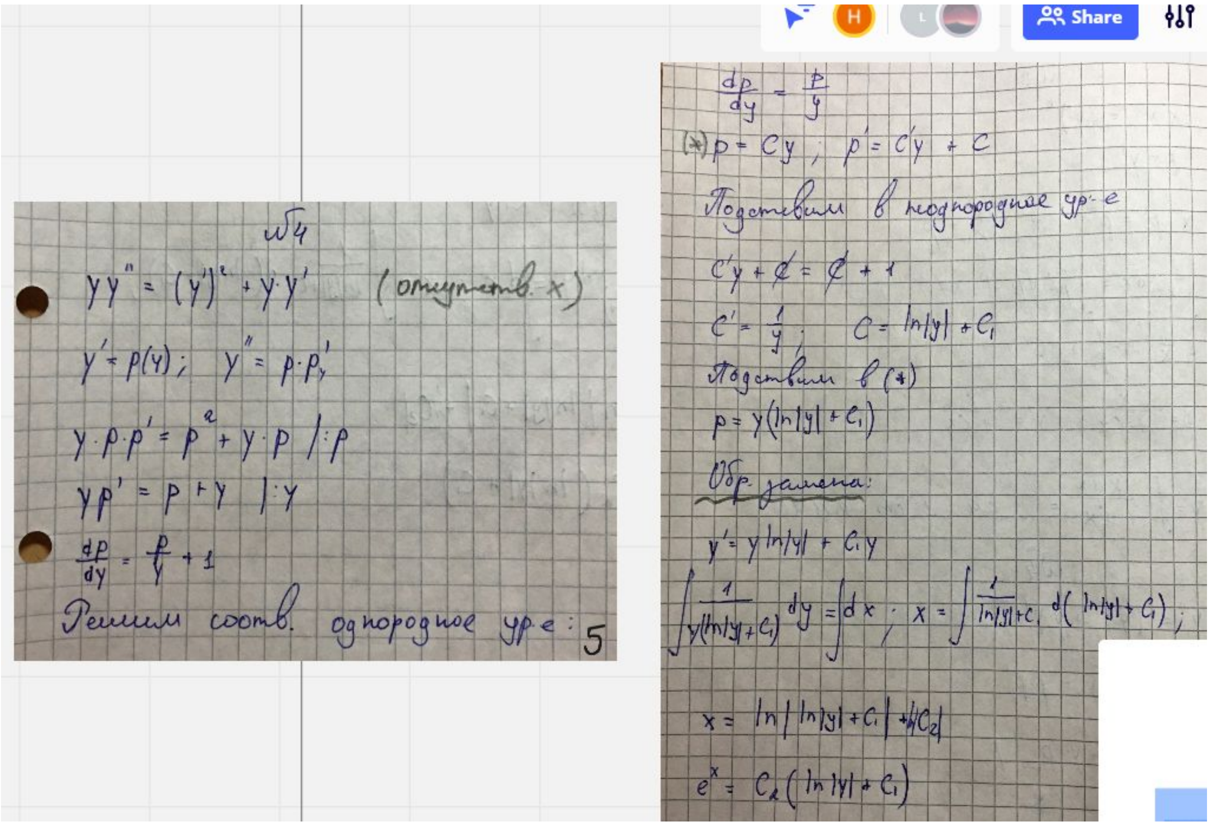
$$p = \frac{x}{1+Cx} ; \quad \frac{dy}{dx} = \frac{x}{1+Cx} ; \quad \int dy = \int \frac{x}{1+Cx} dx ;$$

$$y = \frac{1}{C}\left(\int \frac{Cx+1}{Cx+1} dx - \frac{1}{C}\int \frac{1}{Cx+1} dx\right) = \frac{1}{C}\left(x - \frac{\ln|1+Cx|}{C}\right) =$$

$$= \frac{x}{C} - \frac{\ln|1+Cx|}{C^2} + C_1$$

4

4. yy’’=(y’)^2+yy’



№4

$yy'' = (y')^2 + y \cdot y'$ (отсутств. x)

$y' = p(y)$; $y'' = p \cdot p'_y$

$y \cdot p \cdot p' = p^2 + y \cdot p \;/ : p$

$yp' = p + y \;/ : y$

$\frac{dp}{dy} = \frac{p}{y} + 1$

Решим соотв. однородное ур-е: 5

$\frac{dp}{dy} = \frac{p}{y}$

(*) $p = Cy$; $p' = C'y + C$

Подставим в неоднородное ур-е

$C'y + \cancel{C} = \cancel{C} + 1$

$C' = \frac{1}{y}$; $C = \ln|y| + C_1$

Подставим в (*)

$p = y(\ln|y| + C_1)$

Обр. замена:

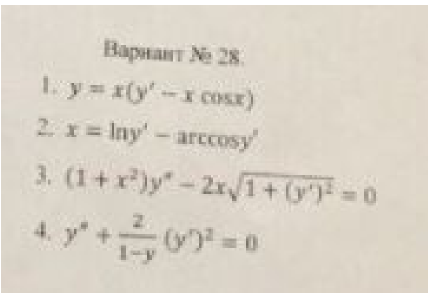
$y' = y\ln|y| + C_1 y$

$\int \frac{1}{y(\ln|y| + C_1)} dy = \int dx$; $x = \int \frac{1}{\ln|y| + C_1} d(\ln|y| + C_1)$;

$x = \ln|\ln|y| + C_1| + \ln|C_2|$

$e^x = C_2(\ln|y| + C_1)$

28 вариант.



Вариант № 28.

1. $y = x(y' - x\cos x)$
2. $x = \ln y' - \arccos y'$
3. $(1 + x^2)y'' - 2x\sqrt{1 + (y')^2} = 0$
4. $y'' + \frac{2}{1-y}(y')^2 = 0$

1. y=x(y'-xcosx)

28 Вар

$$y = x(y' - x\cos x)$$

$$y = xy' - x^2\cos x$$

Вариация производной постоянной Лагранжа

$$y = x\,y'$$

$$y = \frac{x\,dx}{dy} \Rightarrow \int\frac{dy}{y} = \int\frac{dx}{x} \Rightarrow \ln y = \ln x + \ln C \Rightarrow y = Cx$$

$$y = C(x)\cdot x$$

$$\cancel{C(x)\cdot x} = x^2C'(x) + \cancel{xC(x)} - x^2\cos x$$

$$\cancel{x^2}C'(x) = \cancel{x^2}\cos x$$

$$C(x) = \int\cos x\,dx = \sin x + C_1$$

√

$$y = x\sin x + C_1x$$

Верно/нет?

2. x=lny’-arccosy’

№8.

Ур-ия 1-го порядка, приводящиеся к виду $x = f(y')$.

$x = \ln y' - \arccos y'$ – приводящееся к виду $x = f(y')$.

~~Дифференцируем по y~~

Замена: $y' = p$.

$x = \ln p - \arccos p$. (*)

Дифференцируем (*) по y

$$\frac{d(..p..)}{dy} = \frac{d(..p..)}{dp} \cdot \frac{dp}{dy}$$

$$\cancel{\frac{dx}{dy} = \frac{d(\ln p - \arccos p)}{dp} \cdot \frac{dp}{dy}}$$

$$\frac{dx}{dy} = \frac{d(\ln p - \arccos p)}{dy} = \frac{d(\ln p - \arccos p)}{dp} \cdot \frac{dp}{dy}.$$

$$\frac{1}{\frac{dy}{dx}} = (\ln p - \arccos p)'_p \cdot p'_y.$$

Замена: $\frac{dy}{dx} = p$.

$$\frac{1}{p} = (\ln p - \arccos p)'_p \cdot p'_y.$$

$$\frac{1}{p} = \left(\frac{1}{p} + \frac{1}{\sqrt{1-p^2}}\right) \cdot p'_y.$$

Разделяем переменные.

$$\frac{1}{p} = \left(\frac{1}{p} + \frac{1}{\sqrt{1-p^2}}\right) \cdot \frac{dp}{dy}$$

$$dy = p\left(\frac{1}{p} + \frac{1}{\sqrt{1-p^2}}\right) dp.$$

$$dy = \left(1 + \frac{p}{\sqrt{1-p^2}}\right) dp.$$

Интегрируем.

$$y = \int \left(1 + \frac{p}{\sqrt{1-p^2}}\right) dp = p - \frac{1}{2}\int \frac{p}{p\sqrt{1-p^2}}\, d(1-p^2) = p - \frac{1}{2}\int (1-p^2)^{-\frac{1}{2}} d(1-p^2)$$

$$= p - \frac{1}{2} \cdot \frac{2\sqrt{1-p^2}}{1} + C = p - \sqrt{1-p^2} + C.$$

Ответ: $\begin{cases} x(p) = \ln p - \arccos p \\ y(p) = p - \sqrt{1-p^2} + C, \quad C = const. \end{cases}$

3. (1+x^2)y''-2x(1+(y')^2)^1/2=0

3) $(1+x^2)y''-2x\sqrt{1+(y')^2}=0$

$y'=p,\ y''=p'$

$(1+x^2)p'-2x\sqrt{1+(p)^2}=0$

$p'=\frac{2x}{(1+x^2)}\sqrt{1+(p)^2}$

$\frac{dp}{dx}=\frac{2x}{(1+x^2)}\sqrt{1+(p)^2}$

$\frac{dp}{\sqrt{1+p^2}}=\frac{2x\,dx}{(1+x^2)}$

$\ln\left|\sqrt{1+p^2}+p\right|=\int\frac{d(x^2+1)}{x^2+1}=\ln|x^2+1|+\ln|C|$

$\sqrt{1+p^2}+p=C(x^2+1)$

$\sqrt{p^2+1}=C(x^2+1)-p$

$\cancel{p^2}+1=C^2(x^2+1)^2-2pC(x^2+1)+\cancel{p^2}$

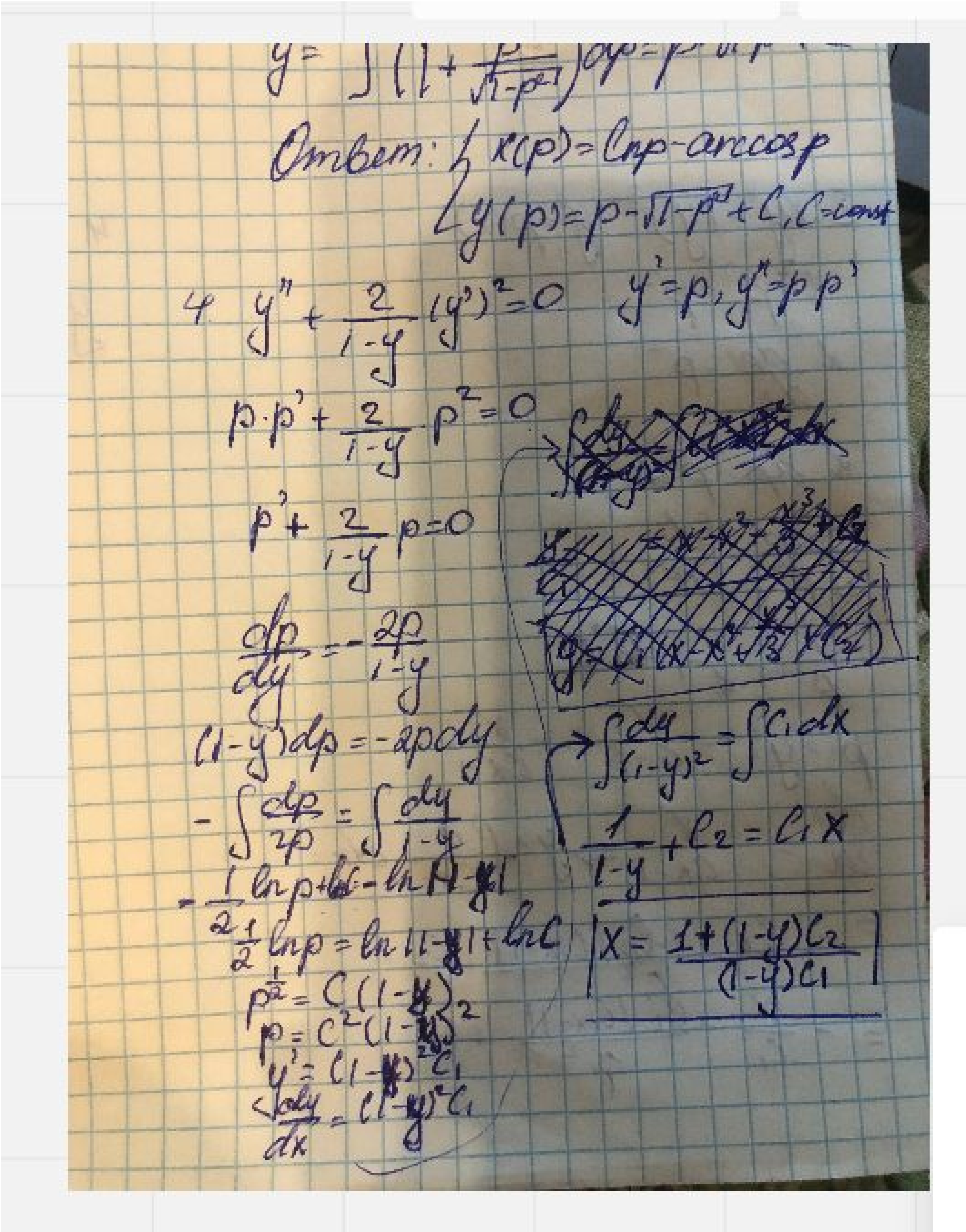
$p=\frac{C^2(x^2+1)^2-1}{2C(x^2+1)}=\frac{C(x^2+1)}{2}-\frac{1}{2C(x^2+1)}$

$\int dy=\int\frac{C(x^2+1)}{2}dx-\int\frac{dx}{2C(x^2+1)}=\frac{C}{2}\left(\frac{x^3}{3}+x\right)-$

$-\frac{1}{2C}\cdot\operatorname{arctg}(x)+\tilde{C}$

$y=\frac{Cx^3}{6}+\frac{Cx}{2}-\frac{\operatorname{arctg}x}{2C}+\tilde{C}$

4. y’’+(2/(1-y))*(y’)^2=0



$$y=\int\left(1+\frac{p}{\sqrt{1-p^2}}\right)dp=p \ldots$$

Ответ: $\begin{cases} x(p)=\ln p-\arccos p \\ y(p)=p-\sqrt{1-p^2}+C,\ C\text{-const}\end{cases}$

4. $y''+\frac{2}{1-y}(y')^2=0 \qquad y'=p,\ y''=pp'$

$$p\cdot p'+\frac{2}{1-y}p^2=0$$

$$p'+\frac{2}{1-y}p=0$$

$$\frac{dp}{dy}=-\frac{2p}{1-y}$$

$$(1-y)dp=-2p\,dy$$

$$-\int\frac{dp}{2p}=\int\frac{dy}{1-y}$$

$$-\frac{1}{2}\ln p+\ln C=-\ln|1-y|$$

$$\frac{1}{2}\ln p=\ln|1-y|+\ln C$$

$$p^{\frac{1}{2}}=C(1-y)$$

$$p=C^2(1-y)^2$$

$$y'=(1-y)^2C_1$$

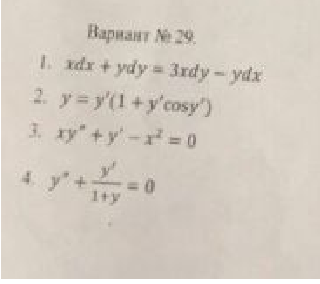
$$\frac{dy}{dx}=(1-y)^2C_1$$

$$\int\frac{dy}{(1-y)^2}=\int C_1dx$$

$$\frac{1}{1-y}+C_2=C_1x$$

$$\boxed{x=\frac{1+(1-y)C_2}{(1-y)C_1}}$$

29 вариант.



Вариант № 29.

1. $xdx+ydy=3xdy-ydx$
2. $y=y'(1+y'\cos y')$
3. $xy''+y'-x^2=0$
4. $y''+\frac{y'}{1+y}=0$

1. xdx+ydy=3xdy-ydx

$$x\,dx + y\,dy = 3x\,dy - y\,dx$$ вар 29

$$\frac{dy}{dx} = \frac{x+y}{3x-y} \Rightarrow \begin{cases} x+y=0 \\ 3x-y=0 \end{cases} \Rightarrow \begin{cases} x=-y \\ y=0 \end{cases} \Rightarrow \begin{cases} x_0=0 \\ y_0=0 \end{cases}$$

$$z = \frac{y-y_0}{x-x_0} = \frac{y}{x} \Rightarrow y = zx \Rightarrow y' = z'x + z$$

$$z'x + z = \frac{x+zx}{3x-zx} = \frac{1+z}{3-z} \Rightarrow z'x = \frac{1+z-3z+z^2}{3-z} = \frac{(1-z)^2}{3-z}$$

$$dz \cdot \frac{3-z}{(1-z^2)} = \frac{dx}{x}$$

$$\int \frac{3-z}{(1-z^2)}\,dz = -\int \frac{z-3}{(z-1)^2}\,dz = -\int \frac{z-1}{(z-1)^2}\,dz + \int \frac{2}{(z-1)^2}\,dz =$$

$$= -\int \frac{1}{z-1}\,dz + 2\int \frac{1}{(z-1)^2}\,dz = -\ln|z-1| + 2\int \frac{d(z-1)}{(z-1)^2} =$$

$$= -\ln|z-1| - \frac{2}{z-1} + \ln C$$

$$\ln x = \ln \frac{C}{z-1} - \frac{2}{z-1}$$ √1

$$x = e^{\frac{2}{1-z}} \cdot \frac{C}{z-1}$$

$$x = e^{\frac{2}{1-\frac{y}{x}}} \cdot \frac{C}{\frac{y}{x}-1}$$

+ особе. реш.: $y=x$;

2. y=y’(1+y’cosy’)

№2

$$y = y'(1 + y'\cos y') \quad ; \quad y = F(y')$$

$$y' = p(x) \; ;$$

$$y = p(1 + p\cos p)$$

$$\frac{dy}{dx} = \frac{d(p(1+p\cos p))}{dx} = \frac{d(p + p^2\cos p)}{dp} \cdot \frac{dp}{dx}$$

$$p = (1 + 2p\cos p - p^2\sin p)\frac{dp}{dx} \quad \Big| : p \neq 0 \Rightarrow y' = 0 \Rightarrow$$

$$\Rightarrow y = 0$$

$$\frac{dx}{dp} = \frac{1}{p} + 2\cos p - p\sin p$$

$$x = \int\left(\frac{1}{p} + 2\cos p - p\sin p\right)dp =$$

$$= \ln|p| + 2\sin p - \int p\sin p \; \circeq$$

$$\int p\sin p = -p\cos p + \sin p + C$$

$$\circeq \ln|p| + 2\sin p + p\cos p - \sin p + C =$$

$$= \ln|p| + \sin p + p\cos p + C$$

$$\begin{cases} x = \ln|p| + \sin p + p\cos p + C \\ y = p(1 + p\cos p) \end{cases}, \; y = 0$$

3. xy''+y'-x^2=0

№3

$xy'' + y' - x^2 = 0$

$y' = p(x)$

$xp' + p - x^2 = 0$

$\cancel{x\frac{dp}{dx} = x^2 - p}$, $\cancel{\frac{dp}{dx} = x - \frac{p}{x}}$, $\cancel{p' = x - \frac{p}{x}}$

$\cancel{z = \frac{p}{x}}$; $\cancel{p' = z + xz'}$

$\cancel{z + xz' = x - z}$

$xp' + p = x^2$ - неоднородное ур-е

Решим соотв. одн-е ур-е

$x\frac{dp}{dx} + p = 0$

$\int \frac{1}{x}dx = -\int \frac{1}{p}dp$; $\ln|x| = -\ln|p| + \ln|C(x)|$

$x = \frac{C(x)}{p}$; $p \overset{(*)}{=} \frac{C(x)}{x}$; $p' = \frac{C'\cdot x - C}{x^2}$

№3

$\frac{C'x - C(x)}{x} + \frac{C(x)}{x} = x^2$

$C'x - \cancel{C} + \cancel{C} = x^3$; $C' = x^2$; $\underbrace{C = \frac{x^3}{3} + C_1}_{(*)}$

$p = \frac{1}{x}\left(\frac{x^3}{3} + C_1\right)$

Обр. замена:

$\frac{dy}{dx} = \frac{x^2}{3} + \frac{C_1}{x}$

$\int dy = \int\left(\frac{x^2}{3} + \frac{C_1}{x}\right)dx$

$y = \frac{x^3}{9} + C_1 \cdot \ln|x| + C_2$

? ? ?

4. y''+y'/(1+y)=0

№4

$$y'' + \frac{y'}{1+y} = 0 \quad (\text{нет } x)$$

$$y' = p(y) = p; \quad y'' = p \cdot p'$$

$$p \cdot p' + \frac{p}{1+y} = 0 \quad | : p \Rightarrow p = 0 \Rightarrow y' = 0 \text{ – реш-е}$$

$$\frac{dp}{dy} = -\frac{1}{y+1}$$

$$p = -\ln|y+1| + \ln|C|;$$

$$y' = \ln\left|\frac{C}{y+1}\right|$$

$$\int dx = \int \frac{1}{\ln\left|\frac{C}{y+1}\right|}\,dy \qquad y = \frac{C}{e^t} - 1$$

$$x = \int \frac{1}{\ln\left|\frac{C}{y+1}\right|}\,dy = \left|\begin{array}{l} t = \ln\left|\frac{C}{y+1}\right| \quad e^t = \frac{C}{y+1} \\ dt = \frac{1}{\frac{C}{y+1}} \cdot C \cdot \left(-\frac{1}{(y+1)^2}\right) dy \\ dt = -\frac{1}{y+1}\,dy \\ dy = -(y+1)\,dt; \quad dy = -\frac{C}{e^t}\,dt \end{array}\right| =$$

$$= \int \frac{1}{t} \cdot \left(-\frac{C}{e^t}\right) dt =$$

$= ???$

30 вариант.

Вариант № 30.

1. $(1 + y^2 \sin 2x)dx = 2y \cdot \cos^2 x\, dy$
2. $x(y')^3 = 1 + y'$
3. $y'' \sin x - 2y' \cos x = \sin^3 x$
4. $y''(3 - 2y) + 5(y')^2 = 0$

1. (1+y^2*sin2x)dx=2y*(cosx)^2dy

1)

$$(1+y^2\sin 2x)dx = 2y\cdot\cos^2 x\,dy$$

$$\underbrace{(1+y^2\sin 2x)}_{F}dx + \underbrace{(-2y\cos^2 x)}_{G}dy = 0$$

$$F'_y = 2y\sin 2x$$

$$G'_x = 4y\cos x\sin x = 2y\sin x$$

$$dI = F\,dx$$

$$dI = (1+y^2\sin 2x)dx$$

$$\int dI = \int dx + \frac{y^2}{2}\int \sin 2x\,d2x$$

$$I = x - \frac{y^2}{2}\cos 2x + \varphi(y)$$

$$I'_y = -y\cos 2x + \varphi'(y) = -2y\cos^2 x - y + \varphi'(y)$$

$$-2y\cos^2 x = -2y\cos^2 x - y + \varphi'(y)$$

$$\varphi'(y) = y$$

$$\frac{d\varphi}{dy} = y$$

$$\int d\varphi = \int y\,dy$$

$$\varphi = \frac{1}{2}y^2 + C$$

$$I = x - \frac{y^2}{2}\cos 2x + \frac{1}{2}y^2 + C$$

$$\boxed{C = \frac{1}{2}y^2(\cos 2x - 1) - x}$$

2. x(y')^3=1+y'

2)

$$x(y')^3 = 1+y'$$

$$xp^3 = 1+p$$

$$x = \frac{1}{p^3} + \frac{1}{p^2}$$

$$x' = -\frac{3}{p^4} - \frac{2}{p^3}$$

$$dy = p\left(-\frac{3}{p^4} - \frac{2}{p^3}\right)dp$$

$$\int dy = \int -\frac{3}{p^3} - \frac{2}{p^2}\, dp$$

$$\boxed{\begin{cases} y = \frac{3}{2p^2} + \frac{2}{p} \\ x = \frac{1}{p^3} + \frac{1}{p^2} \end{cases}}$$

$$y' = p \Rightarrow \frac{dy}{dx} = p \Rightarrow dy = dx\,p$$

$$x'_p = f'(p) = \frac{dx}{dp} \Rightarrow dx = f'(p)dp$$

$$dy = p\,f'(p)dp$$

3. y''sinx-2y'cosx=(sinx)^3

3) $y'' \sin x - 2y' \cos x = \sin^3 x \qquad y' = p$

$p' \sin x - 2p \cos x = \sin^3 x$

$p' - 2p \frac{\cos x}{\sin x} = \sin^2 x \quad (! \; / \sin x) \qquad p = uv, \; p' = u'v + uv'$

$u'v + uv' - 2uv \frac{\cos x}{\sin x} = \sin^2 x$

$\begin{cases} u'v - 2uv \frac{\cos x}{\sin x} = 0 \\ uv' = \sin^2 x \end{cases} \rightarrow \quad u'v - 2uv \frac{\cos x}{\sin x} = 0$

$\frac{u'}{u} = 2 \frac{\cos x}{\sin x}$

$\frac{du}{u} = 2 \frac{\cos x}{\sin x} dx$

$\int \frac{du}{u} = 2 \int \frac{d(\sin x)}{\sin x}$

$\ln u = 2 \ln(\sin x)$

$u = \sin^2 x$

$\sin^2 x \, v' = \sin^2 x$

$v' = 1$

$dv = dx$

$\int dv = \int dx$

$v = x + C$

$\cos 2x = 1 - 2\sin^2 x$

$\sin^2 x = \frac{1}{2} - \frac{\cos 2x}{2}$

$\int \sin^2 x \, dx =$

$= \int \frac{1}{2} dx - \frac{1}{2} \int \cos 2x \, dx =$

$= \frac{1}{2} x - \frac{1}{4} \sin 2x$

$\int x \sin^2 x \, dx$

$y' = x \sin^2 x + C_1 \sin^2 x$

$dy = (x \sin^2 x + C_1 \sin^2 x) dx$

$\int dy = \int x \sin^2 x \, dx + C_1 \int \sin^2 x \, dx$

$y = \frac{1}{4} x (x + 2C_1) - \frac{1}{4} \sin 2x (x + C_1) - \frac{1}{8} \cos 2x + C_2 \; ?$

$\int x \sin^2 x \, dx = \frac{1}{2} \int x \, dx - \frac{1}{2} \int x \cos 2x \, dx \; (=)$

$\frac{1}{2}\int x\,dx = \frac{1}{4} x^2$

$\int x \cos 2x \, dx = \frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} dx =$

$\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x$

$(=) \; \frac{1}{4} x^2 - \frac{1}{4} x \sin 2x - \frac{1}{8} \cos 2x$

4. y''(3-2y)+5(y')^2=0

4) $y''(3-2y)+5(y')^2=0 \qquad y'=p,\ y''=pp'$

$pp'(3-2y)+5p^2=0$

$p'(3-2y)=-5p$

$\frac{p'}{p}=-\frac{5}{3-2y}$

$\frac{dp}{p}=-\frac{5dy}{3-2y}$

$\int\frac{dp}{p}=-5\int\frac{dy}{3-2y}$

$\ln p=\frac{5}{2}\ln(3-2y)+\ln C$

$p=(3-2y)^{\frac{5}{2}}C$

$y'=(3-2y)^{\frac{5}{2}}C_1$

$\int\frac{dy}{(3-2y)^{\frac{5}{2}}}=\int C_1dx$

$x=\frac{1}{3C_1(3-2y)^{\frac{3}{2}}}+C_2$

$\int\frac{dy}{3-2y}=-\frac{1}{2}\int\frac{d3-2y}{3-2y}=-\frac{1}{2}\ln|3-2y|$

$\int\frac{dy}{(3-2y)^{\frac{5}{2}}}=-\frac{1}{2}\int\frac{d(3-2y)}{(3-2y)^{\frac{5}{2}}}=$

$=\frac{1}{3(3-2y)^{3/2}}$