Понедельник нитений 13:00

Вариант - 29

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1) $y''-2y'+y=\frac{e^x}{x^2}$

2) $\begin{cases}\dot{x}=-2x-y & x(0)=2\\ \dot{y}=2x-4y & y(0)=1\end{cases}$

3) $y''+2y'-3y=e^{-3x}$

4) $y^{(5)}-21y^{(3)}-100y'=e^{5x}\cos 2x+xe^{-5x}-5x$

№1

$y''-2y'+y=\frac{e^x}{x^2}$

$\lambda^2-2\lambda+1=0$

$(\lambda-1)^2=0$

$\lambda_1=1\quad k_1=2\longrightarrow e^x,\ xe^x$

$y_{o.o}=C_1e^x+C_2xe^x$

$\varphi_1=e^x\qquad \varphi_1'=e^x$

$\varphi_2=xe^x\qquad \varphi_2'=e^x+xe^x$

$\Delta=\begin{vmatrix}e^x & xe^x\\ e^x & e^x(1+x)\end{vmatrix}=e^{2x}\begin{vmatrix}1 & x\\ 1 & 1+x\end{vmatrix}=e^{2x}(x+1-x)=e^{2x}$

$\Delta_1=\begin{vmatrix}0 & xe^x\\ \frac{e^x}{x^2} & e^x(1+x)\end{vmatrix}=-\frac{e^{2x}}{x}\qquad \Delta_2=\begin{vmatrix}e^x & 0\\ e^x & \frac{e^x}{x^2}\end{vmatrix}=\frac{e^{2x}}{x^2}$

$C_1'(x)=\frac{\Delta_1}{\Delta}=\frac{-\frac{e^{2x}}{x}}{e^{2x}}=-\frac{1}{x}\qquad C_1(x)=-\ln|x|+A$

$C_2'(x)=\frac{\Delta_2}{\Delta}=\frac{\frac{e^{2x}}{x^2}}{e^{2x}}=\frac{1}{x^2}\qquad C_2(x)=-\frac{1}{x}+B$

$y=(-\ln|x|+A)e^x+(-\frac{1}{x}+B)xe^x=\boxed{Ae^x+Bxe^x-e^x(\ln|x|+1)}$

№2

$\begin{cases}\dot{x}=-2x-y\\ \dot{y}=2x-4y\end{cases}\qquad A=\begin{pmatrix}-2 & -1\\ 2 & -4\end{pmatrix}$

$\begin{vmatrix}-2-\lambda & -1\\ 2 & -4-\lambda\end{vmatrix}=(\lambda+2)(\lambda+4)+2=\lambda^2+6\lambda+10=0$

$D=36-40=-4$

$\lambda_{1,2}=\frac{-6\pm 2i}{2}=-3\pm i$

$\boxed{\lambda=-3+i}$

$\left(\begin{array}{cc|c}1-i & -1 & 0\\ 2 & -1-i & 0\end{array}\right)\Rightarrow \vec{s}=\begin{pmatrix}1+i\\ 2\end{pmatrix}$

$\vec{\jmath}\cdot e^{\lambda t} = \begin{pmatrix}1+i\\2\end{pmatrix} e^{(-3+i)t} = \left[\begin{pmatrix}1\\2\end{pmatrix} + i\begin{pmatrix}1\\0\end{pmatrix}\right](\cos t + i\sin t)e^{-3t} =$

$= e^{-3t}\left(\begin{pmatrix}1\\2\end{pmatrix}\cos t + i\begin{pmatrix}1\\2\end{pmatrix}\sin t + i\begin{pmatrix}1\\0\end{pmatrix}\cos t - \begin{pmatrix}1\\0\end{pmatrix}\sin t\right) =$

$= e^{-3t}\begin{pmatrix}\cos t - \sin t\\2\cos t\end{pmatrix} + e^{-3t}\cdot i\begin{pmatrix}\sin t + \cos t\\2\sin t\end{pmatrix}$

$\begin{pmatrix}x\\y\end{pmatrix} = C_1\begin{pmatrix}\cos t - \sin t\\2\cos t\end{pmatrix}e^{-3t} + C_2\begin{pmatrix}\sin t + \cos t\\2\sin t\end{pmatrix}e^{-3t}$

$\begin{cases} x(t) = C_1(\cos t - \sin t)e^{-3t} + C_2(\sin t + \cos t)e^{-3t} \\ y(t) = C_1\cdot 2\cos t\cdot e^{-3t} + C_2\cdot 2\sin t\, e^{-3t} \end{cases}$

$x(0) = C_1 e^{-3\cdot 0} + C_2 e^{-3\cdot 0} = C_1 + C_2 = 2$

$y(0) = C_1\cdot 2 = 1$

$C_1 = \frac{1}{2}$

$C_2 = \frac{3}{2}$

$\boxed{\begin{aligned} x(t) &= \tfrac{1}{2}(\cos t - \sin t)e^{-3t} + \tfrac{3}{2}(\sin t + \cos t)e^{-3t} \\ y(t) &= \tfrac{1}{2}\cdot 2\cos t\, e^{-3t} + \tfrac{3}{2}\cdot 2\sin t\, e^{-3t} \end{aligned}}$

№3

$y'' + 2y' - 3y = e^{-3x}$

$\lambda^2 + 2\lambda - 3 = 0$

$\begin{array}{r|l} \lambda^2 + 2\lambda - 3 & \lambda - 1 \\ -\ \lambda^2 - \lambda & \lambda + 3 \\ \hline 3\lambda - 3 & \\ -\ 3\lambda - 3 & \\ \hline 0 & \end{array}$

$(\lambda - 1)(\lambda + 3) = 0$

$\lambda_1 = 1 \quad k_1 = 1 \longrightarrow e^x$

$\lambda_2 = -3 \quad k_2 = 1 \longrightarrow e^{-3x}$

$y_{о.о} = C_1 e^x + C_2 e^{-3x}$

$e^{-3x} = e^{-3\cdot x}[1\cdot\cos 0 + 0\cdot\sin 0]$

$\underbrace{\alpha = -3,\ \beta = 0}_{\lambda = -3} \qquad \underbrace{m = 0,\ n = -\infty}_{N = 0} \qquad r = 1$

$y_{ч.н.} = x e^{-3x}[A_1\cdot\cos 0 + A_2\cdot\sin 0] = x e^{-3x} A_1$

$y'_{чн} = A_1 e^{-3x} - 3A_1 x e^{-3x}$

$y''_{чн} = -3A_1 e^{-3x} - 3(A_1 e^{-3x} - 3A_1 x e^{-3x}) = -6A_1 e^{-3x} + 9A_1 x e^{-3x}$

$(-6A_1 e^{-3x} + 9A_1 x e^{-3x}) + 2(A_1 e^{-3x} - 3A_1 x e^{-3x}) - 3(A_1 x e^{-3x}) = e^{-3x}$

$-4A_1 e^{-3x} = e^{-3x}$

$-4A_1 = 1 \Rightarrow A_1 = -\frac{1}{4}$

$y_{ч.н.} = -\frac{1}{4} x e^{-3x}$

$\boxed{y_{о.н.} = C_1 e^x + C_2 e^{-3x} - \frac{1}{4} x e^{-3x}}$

№4

$y^{(5)} - 21y^{(3)} - 100y' = \underbrace{e^{5x}\cos 2x}_{f_1} + \underbrace{xe^{-5x}}_{f_2} \underbrace{- 5x}_{f_3}$

$\lambda^5 - 21\lambda^3 - 100\lambda = 0$

$\lambda(\lambda^4 - 21\lambda^2 - 100) = 0$

$\lambda_1 = 0 \quad k_1 = 1 \longrightarrow 1$

$\lambda_2 = 5 \quad k_2 = 1 \longrightarrow e^{5x}$

$\lambda_3 = -5 \quad k_3 = 1 \longrightarrow e^{-5x}$

$\lambda_{4,5} = \pm 2i \quad k_{4,5} = 1 \longrightarrow \sin 2x, \cos 2x$

$\lambda^4 - 21\lambda^2 - 100 = 0$

$D = 441 + 400 = 841$

$\lambda^2_{1,2} = \frac{21 \pm 29}{2}$

$\lambda_1^2 = 25 \to \lambda_1 = 5$, $\lambda_2 = -5$

$\lambda_2^2 = -4 \to \lambda_3 = 2i$, $\lambda_4 = -2i$

$D = b^2 - 4ac$

$21 \times 21 - 4(-100) =$

$= 441 + 400 = 841$

$29 \times 29 = 841$

$y_{o.o} = C_1 + C_2 e^{5x} + C_3 e^{-5x} + C_4 \sin 2x + C_5 \cos 2x$

$f_1 = e^{5x} \cos 2x = e^{5 \cdot x} [1 \cdot \cos 2x + 0 \cdot \sin 2x]$

$\underbrace{\alpha = 5,\ \beta = 2}_{\lambda = 5 + 2i} \quad \underbrace{m = 0,\ n = -\infty}_{N = 0} \quad r = 0$

$y_{ч.н.1} = e^{5x} [A_1 \cos 2x + A_2 \sin 2x]$

$f_2 = xe^{-5x} = e^{-5 \cdot x} [x \cdot \cos 0 + 0 \cdot \sin 0]$

$\underbrace{\alpha = -5,\ \beta = 0}_{\lambda = -5} \quad \underbrace{m = 1,\ n = -\infty}_{N = 1} \quad r = 1$

$y_{ч.н.2} = xe^{-5x} [(B_1 x + C_1)\cos 0 + (B_2 x + C_2)\sin 0] = (B_1 x + C_1) e^{-5x} \cdot x = (B_1 x^2 + C_1 x) e^{-5x}$

$f_3 = -5x = e^{0x} [-5x \cdot \cos 0 + 0 \cdot \sin 0]$

$\underbrace{\alpha = 0,\ \beta = 0}_{\lambda = 0} \quad \underbrace{m = 1,\ n = -\infty}_{N = 1} \quad r = 1$

$y_{ч.н.3} = x [(D_1 x + E_1)\cos 0 + (D_2 x + E_2)\sin 0] = D_1 x^2 + E_1 x$

$$\boxed{\begin{aligned} y_{о.н.} = {} & C_1 + C_2 e^{5x} + C_3 e^{-5x} + C_4 \sin 2x + C_5 \cos 2x + e^{5x}(A_1 \cos 2x + A_2 \sin 2x) + \\ & + (B_1 x^2 + C_1^* x) e^{-5x} + (D_1 x^2 + E_1 x) \end{aligned}}$$