Solving Linear Programming Problems

The Simplex Method (PART 2)

Mohammed Brahimi

ENSIA/Intelligent Systems Enginnering

March 11, 2023

Outline

Key Takeaways from Last Lecture

- Simplex is an efficient algorithm for finding optimal solutions to LP problems by navigating through the corner points of the feasible region.

- It iteratively moves from one Basic Feasible Solution (BFS) to a better neighborhood BFS until the optimal BFS is reached.

- By detecting the optimal BFS, the simplex method provides the optimal values of the decision variables and the objective function.

Questions Regarding Simplex

- How can we choose an appropriate initial BFS if the origin is not a basic feasible solution (BFS)?

ensia

Questions Regarding Simplex

- How can we choose an appropriate initial BFS if the origin is not a basic feasible solution (BFS)?

- What are the special cases that may arise when using Simplex?

Questions Regarding Simplex

- How can we choose an appropriate initial BFS if the origin is not a basic feasible solution (BFS)?

- What are the special cases that may arise when using Simplex?

- Does Simplex terminate in every LP?

Artificial starting solution

- LPs where all constraints are of the form "$\leq$" with nonnegative right-hand sides can be conveniently started with an all-slack basic feasible solution.
- Select "all-slack" variables as basic variables to create an initial basic feasible solution (BFS).

$$S_1 = b_1 + \sum_{i=m+1}^{n} a_{1i} x_i$$

$$\vdots$$

$$S_m = b_m + \sum_{i=m+1}^{n} a_{mi} x_i$$

ensia

Artificial starting solution

- If LPs involves constraints of the form "$\geq$" or "$=$" do not have this convenient starting solution.
- This "ill-behaved" LPs, artificial variables should be used to find an initial BFS that we can start with.

$$
\begin{array}{lr}
\text{Maximize} & 3x + 9y \\
\text{Subject to} & x + y \leq 3 \\
 & 5x - y \geq 3 \\
 & y \geq 1
\end{array}
$$

Two-Phase method

$$
\begin{aligned}
\text{Maximize} \quad & c_1x_1 + c_2x_2 + \cdots + c_nx_n \\
\text{Subject to} \quad & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\
& a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\
& \vdots \\
& a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \\
& x_1, x_2, \ldots, x_n \geq 0
\end{aligned}
$$

Two-Phase method

$$
\begin{aligned}
\text{Minimize} \quad & R_1 + R_2 + \cdots + R_m \\
\text{Subject to} \quad & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + \boldsymbol{R_1} = b_1 \\
& a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + \boldsymbol{R_2} = b_2 \\
& \vdots \\
& a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + \boldsymbol{R_m} = b_m
\end{aligned}
$$

- Feasible if the objective value reaches 0.
- All the $\boldsymbol{R_i}$ are zeros.
- I have a BFS without the R_i.

Example: The Standard form (Phase 1)

$$
\begin{array}{llrrrrrl}
\text{Maximize} & z = & 3 & x & + & 9y & & \\
\text{Subject to} & & & x & + & y & \leq & 3 \\
& & 5 & x & - & y & \geq & 3 \\
& & & & & y & \geq & 1
\end{array}
$$

Example: The Standard form (Phase 1)

$$
\begin{array}{lrcrcrcrcr}
\text{Maximize} & z = & 3 & x & + & 9y & & & & \\
\text{Subject to} & & & x & + & y & + & s_1 & = & 3 \\
 & & 5 & x & - & y & - & s_2 & = & 3 \\
 & & & & & y & - & s_3 & = & 1
\end{array}
$$

Example: Two-Phase method (Phase 1)

$$
\begin{array}{llcccccccccc}
\text{Minimize} & z = & R_1 & + & R_2 & + & R_3 & & & & & \\
\text{Subject to} & & x & + & y & + & S_1 & + & R_1 & = & 3 \\
 & & 5x & - & y & - & S_2 & + & R_2 & = & 3 \\
 & & & & y & - & S_3 & + & R_3 & = & 1
\end{array}
$$

ensia

Example: Two-Phase method (Phase 1)

Basic	x	y	S_1	S_2	S_3	R_1	R_2	R_3	RHS	Ratio
z	0	0	0	0	0	-1	-1	-1	0	
R_1	1	1	1	0	0	1	0	0	3	
R_2	5	-1	0	-1	0	0	1	0	3	
R_3	0	1	0	0	-1	0	0	1	1	

- Basic variables = $\{R_1, R_2, R_3\}$
- We should remove basic variables from the objective function to start Simplex.
- $\boldsymbol{Row(z) = Row(z) + Row(R_1) + Row(R_2) + Row(R_3)}$

Example: Two-Phase method (Phase 1)

Basic	x	y	S_1	S_2	S_3	R_1	R_2	R_3	RHS	Ratio
z	6	1	1	-1	-1	0	0	0	7	
R_1	1	1	**1**	0	0	1	0	0	3	
R_2	5	-1	0	-1	0	0	1	0	3	
R_3	0	1	0	0	-1	0	0	1	1	

- Entering variable: S_1 Leaving Variable: R_1
- Basic variables = $\{S_1, R_2, R_3\}$
- We should remove basic variables from the objective function to start Simplex.
- $\boldsymbol{Row(z) = Row(z) -}$ **Pivot row**

Example: Two-Phase method (Phase 1)

Basic	x	y	S_1	S_2	S_3	R_1	R_2	R_3	RHS	Ratio
z	5	0	0	-1	-1	-1	0	0	4	
S_1	1	1	1	0	0	1	0	0	3	3
R_2	**5**	-1	0	-1	0	0	1	0	3	3/5
R_3	0	1	0	0	-1	0	0	1	1	

- Entering variable: x, Leaving Variable: R_2
- Basic variables = $\{S_1, x, R_3\}$
- $\boldsymbol{Row(z) = Row(z) -}$ **5*Pivot row**
- $\boldsymbol{Row(S_1) = Row(S_1) -}$ **Pivot row**

Example: Two-Phase method (Phase 1)

Basic	x	y	S_1	S_2	S_3	R_1	R_2	R_3	RHS	Ratio
z	0	1	0	0	-1	-1	-1	0	1	
S_1	0	6/5	1	1/5	0	1	-1/5	0	12/5	2
x	1	-1/5	0	-1/5	0	0	1/5	0	3/5	
R_3	0	**1**	0	0	-1	0	0	1	1	1

- Entering variable: y, Leaving Variable: R_3
- Basic variables = $\{S_1, x, y\}$
- $\boldsymbol{Row(z) = Row(z) -}$ **Pivot row**
- $\boldsymbol{Row(S_1) = Row(S_1) - \frac{6}{5}}$**Pivot row**
- $\boldsymbol{Row(x) = Row(x) + \frac{1}{5}}$**Pivot row**

ensia

Example: Two-Phase method (Phase 1)

Basic	x	y	S_1	S_2	S_3	R_1	R_2	R_3	RHS	Ratio
z	0	0	0	0	0	-1	-1	-1	0	
S_1	0	0	1	1/5	6/5	1	- 1/5	-6/5	6/5	
x	1	0	0	-1/5	-1/5	0	1/5	1/5	4/5	
y	0	1	0	0	-1	0	0	1	1	

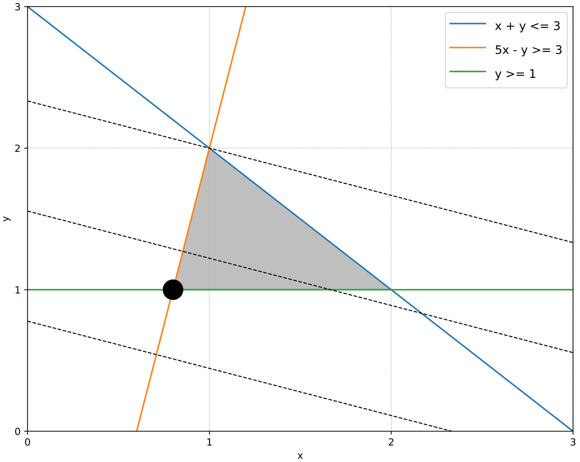
- Optimally detected because ($\boldsymbol{c_i} \leq \mathbf{0}, \forall \boldsymbol{i}$).
- $\boldsymbol{R_1 = R_2 = R_3 = 0}$
- Basic variables = $\{S_1, x, y\}$
- Non-basic variables = $\{S_2, S_3\}$

ensia

Example: Two-Phase method (Phase 2)

$$
\begin{array}{llcl}
\text{Maximize} & z & = & 3x + 9y \\
\text{Subject to} & S_1 & = & \frac{6}{5} + \frac{1}{5} S_2 + \frac{6}{5} S_3 \\
 & x & = & \frac{4}{5} - \frac{1}{5} S_2 - \frac{1}{5} S_3 \\
 & y & = & 1 - S_3
\end{array}
$$

ensia


x + y <= 3
5x - y >= 3
y >= 1
x
y
0
1
2
3

Example: Two-Phase method (Phase 2)

Basic	x	y	S_1	S_2	S_3	RHS	Ratio
z	-3	-9	0	0	0	0	
S_1	0	0	1	1/5	6/5	6/5	
x	1	0	0	-1/5	-1/5	4/5	
y	0	1	0	0	-1	1	

- Basic variables = $\{S_1, x, y\}$
- We should remove basic variables from the objective function to start Simplex.
- $\boldsymbol{Row(z) = Row(z) + 3 \times Row(x) + 9 \times Row(y)}$

Example: Two-Phase method (Phase 2)

Basic	x	y	S_1	S_2	S_3	**RHS**	**Ratio**
z	0.00	0.00	0.00	-3/5	-48/5	57/5	
S_1	0.00	0.00	1.00	1/5	6/5	6/5	
x	1	0	0	-1/5	-1/5	4/5	
y	0	1	0	0	-1	1	

- Entering variable: S_3, Leaving Variable: S_1
- **Pivot row** $= \frac{5}{6} \times$ **Pivot row**
- $\textbf{Row}(z) = \textbf{Row}(z) + \frac{48}{5} \times$ **Pivot row**
- $\textbf{Row}(x) = \textbf{Row}(x) + \frac{1}{5} \times$ **Pivot row**
- $\textbf{Row}(y) = \textbf{Row}(y) +$ **Pivot row**

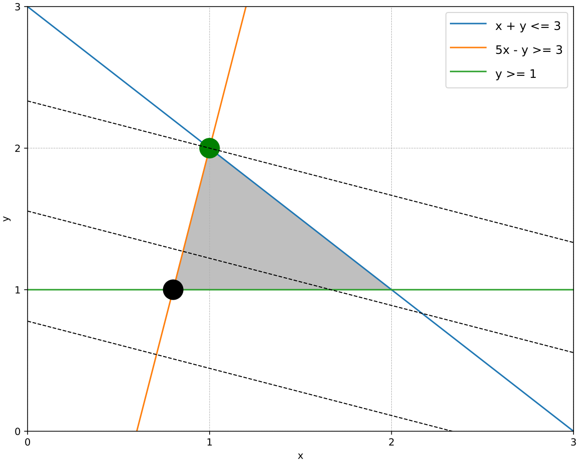
ensia

Example: Two-Phase method (Phase 2)

Basic	x	y	S_1	S_2	S_3	RHS	Ratio
z	0.00	0.00	8.00	1.00	0.00	21.00	
S_3	0	0	5/6	1/6	1	1	
x	1	0	1/6	-1/6	0	1	
y	0	1	5/6	1/6	0	2	

- Optimally detected because ($\boldsymbol{c_i} \geq \mathbf{0}, \forall \boldsymbol{i}$).
- Basic variables = $\{S_3, x, y\}$
- Non-basic variables = $\{S_1, S_2\}$
- Optimum $(\boldsymbol{x} = \mathbf{1}, \boldsymbol{y} = \mathbf{2})$ and $\boldsymbol{z} = \mathbf{21}$.

ensia


x + y <= 3
5x - y >= 3
y >= 1
x
y
0
1
2
3

Simplex: Special cases

- **Infeasibility:** occurs when there is no feasible solution that satisfies all of the constraints.
- **Unboundedness:** occurs when the objective function can be increased indefinitely without violating any of the constraints.
- **Alternative Optima:** occurs when several global optima with same objective value exists.
- **Degeneracy:** occurs when one or more basic variables become zero during the iteration process.

Special cases:Infeasibility

- Empty feasible region.
- Can be detected using Two-Phase method.
- The objective function ($\sum \boldsymbol{R_i}$) in Phase 1 cannot be 0.

$$
\begin{array}{lrrrrl}
\textbf{Maximize} & z = & 2x_1 & - & 3x_2 & \\
\textbf{S.t.} & & x_1 & + & x_2 \leq & 2 \\
 & & 2x_1 & - & 2x_2 \geq & 5 \\
 & & x_1 & , & x_2 \geq & 0
\end{array}
$$

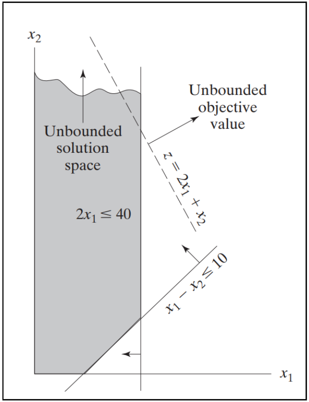
Special cases: Unboundedness

- Unbounded solutions allow for arbitrary increases in variables without violating any constraints.
- Unboundedness may indicate a poorly constructed model.

$$
\begin{array}{llcccccc}
\textbf{Maximize} & z = & 2 & x_1 & + & x_2 & & \\
\textbf{S.t} & & & x_1 & - & x_2 & \leq & 10 \\
 & & 2 & x_1 & & & \leq & 40 \\
 & & & x_1 & , & x_2 & \geq & 0
\end{array}
$$

Special cases: Unboundedness

$$
\begin{array}{llrlcll}
\textbf{Maximize} & z = & 2 & x_1 & + & x_2 & \\
\textbf{S.t} & & & x_1 & - & x_2 & \leq 10 \\
 & & 2 & x_1 & & & \leq 40 \\
 & & & x_1 & , & x_2 & \geq 0
\end{array}
$$



How to detect Unboundedness

Simplex Method indicates unbounded solutions when all Ratios values are either infinite or negative, resulting in no leaving variable.

Basic	x_1	x_2	S_1	S_2	**RHS**	**Ratio**
z	-2	-1	0	0	0	
S_1	1	-1	1	0	10	Negative
S_2	2	0	0	1	40	Infinite

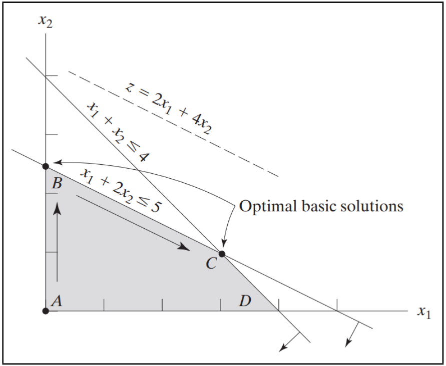
Special cases: Alternative Optima

- An LP problem may have infinite alternative optima when the objective function is parallel to a constraint.
- Any point on that constraint line is also optimal.
- Alternative optima provide different variable combinations with the same optimal objective value.

$$
\begin{array}{llcccccc}
\textbf{Maximize} & z = & 2 & x_1 & + & 4x_2 & & \\
\textbf{S.t} & & & x_1 & + & 2x_2 & \leq & 5 \\
 & & & x_1 & + & x_2 & \leq & 4 \\
 & & & x_1 & , & x_2 & \geq & 0
\end{array}
$$

Special cases: Alternative Optima

$$\begin{array}{llrrl}
\textbf{Max} & z = 2 & x_1 + & 4x_2 & \\
\textbf{S.t} & & x_1 + & 2x_2 & \leq 5 \\
 & & x_1 + & x_2 & \leq 4 \\
 & & x_1, & x_2 & \geq 0
\end{array}$$



Special cases: Alternative Optima

Basic	x_1	x_2	S_1	S_2	**RHS**
z	-2	-4	0	0	0
S_1	1	2	1	0	5
S_2	1	1	0	1	4

- Entering variable: x_2, Leaving Variable: S_1
- $\textbf{Pivot row} = \frac{1}{2} \times \textbf{Pivot row}$
- $\textbf{\textit{Row}}(z) = \textbf{\textit{Row}}(z) + 4 \times \textbf{Pivot row}$
- $\textbf{\textit{Row}}(S_2) = \textbf{\textit{Row}}(S_2) - \textbf{Pivot row}$

ensia

Special cases: Alternative Optima

- If a non-basic variable **has a zero coefficient**, it can be replaced with a basic variable whose right-hand side value is strictly positive without changing the objective function's right-hand side value.
- If we swap x_1 with x_2, where x_2 has a right-hand side value of 2.5, then the objective function's right-hand side value remains at 10. After the swap, x_1 becomes the new basic variable with a value of 5, and x_1 becomes a non-basic variable with a value of 0.

Basic	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{S_1}$	$\boldsymbol{S_2}$	**RHS**
z	0	0	2	0	10
x_2	0.5	1	0.5	0	2.5
S_2	0.5	0	-0.5	1	1.5

Special cases: Degeneracy

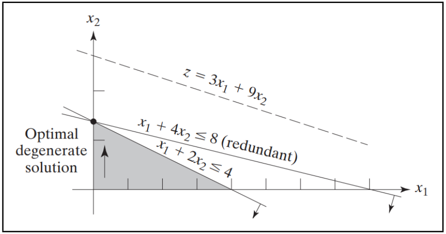
- Feasibility condition of simplex method can have ties for minimum ratio.
- Ties can be broken arbitrarily but will result in a degenerate solution in the next iteration
- Degeneracy can cause the algorithm to cycle indefinitely and not terminate

$$
\begin{array}{llcrlcrlcr}
\textbf{Maximize} & z = & 3 & x_1 & + & 9 & x_2 & & \\
\textbf{S.t} & & & x_1 & + & 4 & x_2 & \leq & 8 \\
 & & & x_1 & + & 2 & x_2 & \leq & 4 \\
 & & & x_1 & , & & x_2 & \geq & 0
\end{array}
$$

ensia

Special cases: Degeneracy

$$\begin{array}{llllll}
\textbf{Max} & z = 3 & x_1 + & 9x_2 & \\
\textbf{S.t} & & x_1 + & 4x_2 & \leq 8 \\
 & & x_1 + & 2x_2 & \leq 4 \\
 & & x_1, & x_2 & \geq 0
\end{array}$$



Special cases: Degeneracy

- Ties in minimum ratio.
- Some basic variable are equal to zero.

Basic	x_1	x_2	S_1	S_2	RHS	Ratio
z	-3	-9	0	0	0	
S_1	1	4	1	0	8	2
S_2	1	2	0	1	4	2

Special cases: Degeneracy

- Ties in minimum ratio.
- Some basic variable are equal to zero.

Basic	x_1	x_2	S_1	S_2	RHS	Ratio
z	-0.75	0	2.25	0	18	
x_2	0.25	1	0.25	0	2	8
S_2	0.5	0	-0.5	1	0	0

Special cases: Degeneracy

- Ties in minimum ratio.
- Some basic variable are equal to zero.

Basic	x_1	x_2	S_1	S_2	RHS	Ratio
z	0	0	1.5	1.5	18	
x_2	0	1	0.5	-0.5	2	
x_1	1	0	-1	2	0	

Degeneracy interpretation

- The presence of degeneracy in an LP suggests the potential existence of a superfluous constraint.
- Shuffling around the basic variables without departing from a corner.
- Dealing with degeneracy in an LP can create the impression that we are moving from one corner to another, while keeping the objective value constant.

Degeneracy can cause cycling

- Cycling happens when the simplex algorithm loops between multiple solutions without reaching the optimal solution due to degeneracy.
- This can cause the simplex algorithm to loop indefinitely.
- To prevent cycling, anti-cycling rules, such as Bland's rule, can be applied to stop revisiting the same solution and improve the efficiency of the simplex algorithm.
- If there are multiple ratios that are minimal, choose the variable x_j with the smallest index as the entering variable.

Example of cycling

$$
\begin{aligned}
\textbf{Max} \quad z = & \; 2.3 x_1 + 2.15 x_2 - 13.55 x_3 - 0.4 x_4 \\
\textbf{S.t} \quad & \; 0.4 x_1 + 0.2 x_2 - 1.4 x_3 - 0.2 x_4 \leq 0 \\
& -7.8 x_1 - 1.4 x_2 + 7.8 x_3 + 0.4 x_4 \leq 0 \\
& \; x_1, \; x_2, \; x_3, \; x_4 \geq 0
\end{aligned}
$$

Example of cycling

Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	-2.3	-2.15	13.55	0.4	0	0	0
x_5	0.4	0.2	-1.4	-0.2	1	0	0
x_6	-7.8	-1.4	7.8	0.4	0	1	0

We are at the origin $(0, 0, ...0)$

Example of cycling

Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	0	-1	5.5	-0.75	5.75	0	0
x_1	1	0.5	-3.5	-0.5	2.5	0	0
x_6	0	2.5	-19.5	-3.5	19.5	1	0

We are at the origin $(0, 0, \ldots 0)$

Example of cycling

Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	0	0	-2.3	-2.15	13.55	0.4	0
x_1	1	0	0.4	0.2	-1.4	-0.2	0
x_2	0	1	-7.8	-1.4	7.8	0.4	0

We are at the origin $(0, 0, ...0)$.

Example of cycling

Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	5.75	0	0	-1	5.5	-0.75	0
x_3	2.5	0	1	0.5	-3.5	-0.5	0
x_6	19.5	1	0	2.5	-19.5	-3.5	0

We are at the origin $(0, 0, ...0)$.

ensia

Example of cycling

Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	13.55	0.4	0	0	-2.3	-2.15	0
x_3	-1.4	-0.2	1	0	0.4	0.2	0
x_4	7.8	0.4	0	1	-7.8	-1.4	0

We are at the origin $(0, 0, ...0)$.

Example of cycling

Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	5.5	-0.75	5.75	0	0	-1	0
x_5	-3.5	-0.5	2.5	0	1	0.5	0
x_4	-19.5	-3.5	19.5	1	0	2.5	0

We are at the origin $(0, 0, ...0)$.

Example of cycling

Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	-2.3	-2.15	13.55	0.4	0	0	0
x_5	0.4	0.2	-1.4	-0.2	1	0	0
x_4	-7.8	-1.4	7.8	0.4	0	1	0

The Simplex has returned to its original state.

Example of cycling

Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	-2.3	-2.15	13.55	0.4	0	0	0
x_5	0.4	0.2	-1.4	-0.2	1	0	0
x_6	-7.8	-1.4	7.8	0.4	0	1	0

Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	-2.3	-2.15	13.55	0.4	0	0	0
x_5	0.4	0.2	-1.4	-0.2	1	0	0
x_4	-7.8	-1.4	7.8	0.4	0	1	0

The Simplex will continuously cycle through these states.

Sensitivity Analysis

- **Sensitivity analysis** (or post-optimality analysis) determines how optimal solutions are affected by changes within specified ranges.
  - Changes in **right-hand side (RHS) values**.
  - Changes in **objective function coefficients**.

Sensitivity Analysis

- **Sensitivity analysis** (or post-optimality analysis) determines how optimal solutions are affected by changes within specified ranges.
  - Changes in **right-hand side (RHS) values**.
  - Changes in **objective function coefficients**.

- Managers must operate in dynamic environments with **imprecise estimates** of coefficients.

Sensitivity Analysis

- **Sensitivity analysis** (or post-optimality analysis) determines how optimal solutions are affected by changes within specified ranges.
  - Changes in **right-hand side (RHS) values**.
  - Changes in **objective function coefficients**.

- Managers must operate in dynamic environments with **imprecise estimates** of coefficients.

- Sensitivity analysis is important for managers to ask **"what-if"** questions about the problem.

Graphical sensitivity Analysis

- We consider two cases:

1. Sensitivity of the optimum solution to changes in the availability of the resources (right-hand side of the constraints)

2. Sensitivity of the optimum solution to changes in unit profit or unit cost (coefficients of the objective function)

Changes in the Right-Hand side

- JOBCO manufactures two products on two machines.
- Processing times and revenues per unit are given as follows:
  - **Product 1**: 2 hrs on machine 1, 1 hr on machine 2, \$30 revenue per unit.

ensia

Changes in the Right-Hand side

- JOBCO manufactures two products on two machines.
- Processing times and revenues per unit are given as follows:
  - **Product 1**: 2 hrs on machine 1, 1 hr on machine 2, $30 revenue per unit.
  - **Product 2**: 1 hr on machine 1, 3 hrs on machine 2, $20 revenue per unit

Changes in the Right-Hand side

- JOBCO manufactures two products on two machines.
- Processing times and revenues per unit are given as follows:
  - **Product 1**: 2 hrs on machine 1, 1 hr on machine 2, \$30 revenue per unit.
  - **Product 2**: 1 hr on machine 1, 3 hrs on machine 2, \$20 revenue per unit
- Total daily processing time available for each machine is 8 hrs
- x_1 and x_2 represent the daily number of units of products 1 and 2.

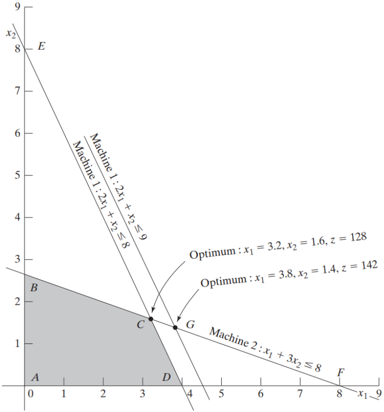
Changes in the Right-Hand side

- JOBCO manufactures two products on two machines.
- Processing times and revenues per unit are given as follows:
  - **Product 1**: 2 hrs on machine 1, 1 hr on machine 2, \$30 revenue per unit.
  - **Product 2**: 1 hr on machine 1, 3 hrs on machine 2, \$20 revenue per unit
- Total daily processing time available for each machine is 8 hrs
- x_1 and x_2 represent the daily number of units of products 1 and 2.

$$
\begin{array}{llcccccc}
\textbf{Maximize} & z = & 30 & x_1 & + & 20 & x_2 & & \\
\textbf{S.t} & & 2 & x_1 & + & & x_2 & \leq & 8 \\
 & & & x_1 & + & 3 & x_2 & \leq & 8 \\
 & & & x_1 & , & & x_2 & \geq & 0
\end{array}
$$

Changes in the Right-Hand side

- Increasing machine 1 capacity from 8 to 9 hrs moves the optimum solution to point *G*.
- $\frac{\text{Rate of revenue change}}{\text{Capacity change}} = \frac{z_G - z_C}{9-8}$.
- $\frac{\$142-\$128}{9-8} = \$14$ \hour
- The point ***G*** should stays between ***B*** and ***F***.
- The dual price for machine 2 capacity is $2/hr.



Dual Prices

- The **dual price** is the rate of change of the objective function per unit change of a resource.
- The abstract name ”dual” or ”shadow” price is standard in LP literature and software packages.
- The dual price of \$14/hr remains valid for changes in machine 1 capacity that move its constraint parallel to itself to any point on the line segment *BF*.
- The dual price is only valid in the **feasibility range** (2.67 hr $\leq$ Machine 1 capacity $\leq$ 16 hr), as calculated at points *B* and *F*.
- Changes outside this range produce a different dual price (worth per unit).

ensia

Changes in the RHS questions

- **Question 1:** If JOBCO can increase the capacity of both machines, which machine should receive priority?

Changes in the RHS questions

- **Question 1:** If JOBCO can increase the capacity of both machines, which machine should receive priority?

- **Response:** Priority should be given to machine 1, as each additional hour of machine 1 increases revenue by $14, as opposed to only $2 for machine 2.

Changes in the RHS questions

- **Question 1:** If JOBCO can increase the capacity of both machines, which machine should receive priority?
- **Response:** Priority should be given to machine 1, as each additional hour of machine 1 increases revenue by $14, as opposed to only $2 for machine 2.
- **Question 2:** A suggestion is made to increase the capacities of machines 1 and 2 at the additional cost of $10/hr for each machine. Is this advisable?

Changes in the RHS questions

- **Question 1:** If JOBCO can increase the capacity of both machines, which machine should receive priority?
- **Response:** Priority should be given to machine 1, as each additional hour of machine 1 increases revenue by $14, as opposed to only $2 for machine 2.
- **Question 2:** A suggestion is made to increase the capacities of machines 1 and 2 at the additional cost of $10/hr for each machine. Is this advisable?
- **Response:** Only machine 1 should be considered for capacity increase, as the additional net revenue per hour is **14-10=$4**, compared to a net of **2-10=$-8** for machine 2.

Changes in the RHS questions

- **Question 3:** If the capacity of machine 1 is increased from 8 to 13 hrs, how will this increase impact the optimum revenue?

Changes in the RHS questions

- **Question 3:** If the capacity of machine 1 is increased from 8 to 13 hrs, how will this increase impact the optimum revenue?

- **Response:** The proposed increase falls within the feasibility range for machine 1 and will result in a **$14(13 - 8) =$70** increase in revenue, from $128 to **$198 (=$128 + $70)**.

Changes in the RHS questions

- **Question 3:** If the capacity of machine 1 is increased from 8 to 13 hrs, how will this increase impact the optimum revenue?

- **Response:** The proposed increase falls within the feasibility range for machine 1 and will result in a **\$14(13 - 8) =\$70** increase in revenue, from \$128 to **\$198 (=\$128 + \$70)**.

- **Question 4:** Suppose that the capacity of machine 1 is increased to 20 hrs, how will this increase affect the optimum revenue?

ensia

Changes in the RHS questions

- **Question 3:** If the capacity of machine 1 is increased from 8 to 13 hrs, how will this increase impact the optimum revenue?

- **Response:** The proposed increase falls within the feasibility range for machine 1 and will result in a **\$14(13 - 8) =\$70** increase in revenue, from \$128 to **\$198 (=\$128 + \$70)**.

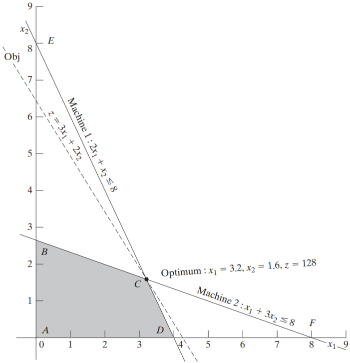
- **Question 4:** Suppose that the capacity of machine 1 is increased to 20 hrs, how will this increase affect the optimum revenue?

- **Response:** The proposed increase falls outside the feasibility range, and further calculations are needed to determine the impact on optimum revenue.

ensia

Objective Coefficient Change

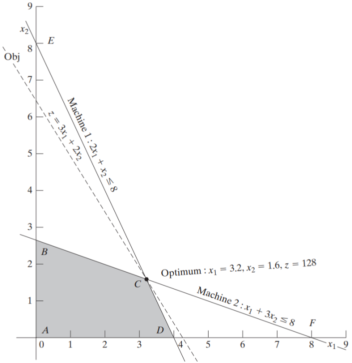
- Maximize $z = c_1x_1 + c_2x_2$.
- How the optimum changes when we change c_1 and c_2.



ensia

Objective Coefficient Change

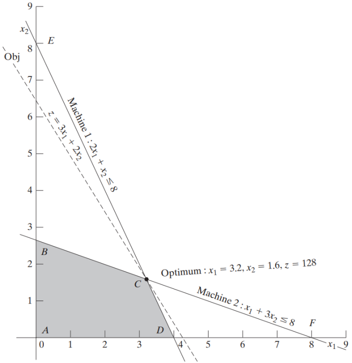
- Maximize $z = c_1 x_1 + c_2 x_2$.
- How the optimum changes when we change c_1 and c_2.
- Changes in objective coefficients change the slope of the isoprofit.



ensia

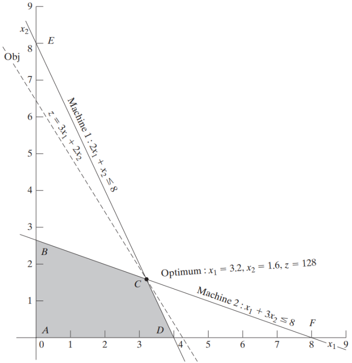
Objective Coefficient Change

- Maximize $z = c_1 x_1 + c_2 x_2$.
- How the optimum changes when we change c_1 and c_2.
- Changes in objective coefficients change the slope of the isoprofit.
- Optimum at *C* remains if objective function is between *BF* and *DE*.



Objective Coefficient Change

- Maximize $z = c_1 x_1 + c_2 x_2$.
- How the optimum changes when we change c_1 and c_2.
- Changes in objective coefficients change the slope of the isoprofit.
- Optimum at C remains if objective function is between BF and DE.
- **Optimality range** for coefficients keeping optimum at C: $\frac{1}{3} \leq \frac{c_1}{c_2} \leq \frac{2}{1}$.



Objective Coefficient Change questions

- **Question 1:** If unit revenues for Products 1 and 2 are changed to \$35 and \$25, respectively, will the current optimum remain the same?
- The solution at C will remain optimal because $\frac{c_1}{c_2} = \frac{35}{25} = 1.4$ remains within the optimality range $(\frac{1}{3}, 2)$.
- **Question 2:** If the unit revenue of Product 2 is fixed at its current value c_2 = \$20, what is the associated optimality range for the unit revenue for Product 1, c_1, that will keep the optimum unchanged?
- The optimality range for c_1 is: $20 \times \frac{1}{3} \leq c_1 \leq 2 \times 20$.

ensia

Conclusion

To sum up, we have covered the following key points:

- The two-phase method provides a viable approach for finding an initial feasible solution for the Simplex method.

- While executing Simplex, one must consider its numerous special cases such as degeneracy, unboundedness, and infeasibility to prevent potential issues.

- Cycling may occur in LP problems with degeneracy, which requires attention to ensure convergence to an optimal solution.

- Graphical sensitivity analysis is a useful tool for investigating the impact of LP parameter changes on the optimal solution, particularly in two dimensions.