Solving Linear Programming Problems

Algebraic Sensitivity Analysis and LP Solvers

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ENSIA/Intelligent Systems Enginnering

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Outline

Key Takeaways from Last Lecture

- The two-phase method provides a viable approach for finding an initial feasible solution for the Simplex method.
- While executing Simplex, one must consider its numerous special cases such as degeneracy, unboundedness, and infeasibility to prevent potential issues.
- Cycling may occur in LP problems with degeneracy, which requires attention to ensure convergence to an optimal solution.
- Graphical sensitivity analysis is a useful tool for investigating the impact of LP parameter changes on the optimal solution, particularly in two dimensions.

TOYCO Production Model

- TOYCO uses three operations to assemble three types of toys: **trains, trucks**, and **cars.**

- Daily available times for the three operations: 430, 460, and 420 mins.

- Revenues per unit:
  - Train: $3
  - Truck: $2
  - Car: $5

- Assembly times unit:
  - Train: (1, 3, 1) mins at each operation.
  - Truck: (2, 0, 4) mins at each operation.
  - Car: (1, 2, 0) mins at each operation.

TOYCO Production Model

Let x1, x2, and x3 be the daily number of units assembled for trains, trucks, and cars, respectively. The LP model is given as:

$$
\begin{array}{llrcrcrcll}
\textbf{Maximize} & z = & 3x_1 & + & 2x_2 & + & 5x_3 & & & \\
\textbf{Subject to} & & x_1 & + & 2x_2 & + & x_3 & \leq & 430 & (\text{ Operation 1 }) \\
 & & 3x_1 & & & + & 2x_3 & \leq & 460 & (\text{ Operation 2 }) \\
 & & x_1 & + & 4x_2 & & & \leq & 420 & (\text{ Operation 3 }) \\
 & & x_1 & , & x_2 & , & x_3 & \geq & 0 &
\end{array}
$$

TOYCO Production Model

The optimum tableau of TOYCO Production Model using simplex is:

Basic	x_1	x_2	x_3	x_4	x_5	x_6	**Solution**
z	4	0	0	1	2	0	1350
x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100
x_3	$-\frac{2}{3}$	0	1	0	$\frac{1}{2}$	0	230
x_6	2	0	0	-2	1	1	20

- The recommended production quantities are:
  - 100 trucks
  - 230 cars
  - 0 trains.
- The associated revenue is $1350.

Sensitivity Analysis in TOYCO Model: RHS changes

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- The original TOYCO model can be modified to reflect these changes.

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\textbf{Maximize} & z = & 3 & x_1 & + & 2 & x_2 & + & 5 & x_3 & & & \\
\textbf{Subject to} & & & x_1 & + & 2 & x_2 & + & & x_3 & \leq & 430 + D_1 & \text{(Operation 1)} \\
& & 3 & x_1 & & & & + & 2 & x_3 & \leq & 460 + D_2 & \text{(Operation 2)} \\
& & & x_1 & + & 4 & x_2 & & & & \leq & 420 + D_3 & \text{(Operation 3)} \\
& & & x_1 & , & & x_2 & , & & x_3 & \geq & 0 &
\end{array}
$$

Sensitivity Analysis in TOYCO Model: : RHS changes

- Rewrite the starting tableau using the new right-hand sides: 430 + D_1, 460 + D_2, and 420 + D_3.

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- Calculate the new optimal solution using the revised simplex method.

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- Rewrite the starting tableau using the new right-hand sides: 430 + D_1, 460 + D_2, and 420 + D_3.
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- Obtain sensitivity analysis information such as dual prices and feasibility ranges from the new optimal solution.

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							Solution			
Basic	x_1	x_2	x_3	x_4	x_5	x_6	*RHS*	D_1	D_2	D_3
z	-3	-2	-5	0	0	0	0	0	0	0
x_4	1	2	1	1	0	0	430	1	0	0
x_5	3	0	2	0	1	0	460	0	1	0
x_6	1	4	0	0	0	1	420	0	0	1

Sensitivity Analysis in TOYCO Model

- The shaded areas are identical.
- The same simplex iterations can be repeated as in the original model.
- The columns in the two highlighted areas will also be identical in the optimal tableau.

							Solution			
Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS	D_1	D_2	D_3
z	4	0	0	1	2	0	1350	1	2	0
x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100	$\frac{1}{2}$	$-\frac{1}{4}$	0
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230	0	$\frac{1}{2}$	0
x_6	2	0	0	-2	1	1	20	-2	1	1

Sensitivity Analysis in TOYCO Model

The new optimum tableau provides the following optimal solution:

$$
\begin{aligned}
z &= 1350 + D_1 + 2D_2 + 0D_3 \\
x_2 &= 100 + \frac{1}{2}D_1 - \frac{1}{4}D_2 \\
x_3 &= 230 + \frac{1}{2}D_2 \\
x_6 &= 20 - 2D_1 + D_2 + D_3
\end{aligned}
$$

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Dual prices in TOYCO Model

- Dual prices can be determined using the optimal solution.
- The objective function can be written as :
  - $z = 1350 + 1 \times D_1 + 2 \times D_2 + 0 \times D3.$

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- The coefficients of slack variables x_4, x_5, and x_6 in the optimal z-row are exactly those of D_1, D_2, and D_3, respectively.

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- The dual prices equal the coefficients of the slack variables in the optimal z-row.

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- The coefficients of slack variables x_4, x_5, and x_6 in the optimal z-row are exactly those of D_1, D_2, and D_3, respectively.
- The dual prices equal the coefficients of the slack variables in the optimal z-row.
- Each slack variable is uniquely identified with a constraint, so there is no ambiguity as to which coefficient applies to which resource.

Feasibility range in TOYCO Model

- To keep the solution feasible, simultaneous changes D_1, D_2, and D_3 must satisfy certain inequalities.
- The new optimum solution can be obtained by substituting the values of D_1, D_2, and D_3.
- Feasibility ranges can be determined by finding the range of simultaneous changes D_1, D_2, and D_3 that keep the solution feasible.

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$$x_2 = 100 + \frac{1}{2}D_1 - \frac{1}{4}D_2 \geq 0$$

$$x_3 = 230 + \frac{1}{2}D_2 \geq 0$$

$$x_6 = 20 - 2D_1 + D_2 + D_3 \geq 0$$

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Feasibility Ranges Example

- New optimum solution found by substituting values of D_1, D_2, and D_3.
- Manufacturing times: 480, 440, and 400 mins for ops 1, 2, and 3.
- $D_1 = 50(480 - 430)$, $D_2 = -20(440 - 460)$, and $D_3 = -20(400 - 420)$.

Feasibility Ranges Example

- New optimum solution found by substituting values of D_1, D_2, and D_3.
- Manufacturing times: 480, 440, and 400 mins for ops 1, 2, and 3.
- $D_1 = 50(480 - 430)$, $D_2 = -20(440 - 460)$, and $D_3 = -20(400 - 420)$.
- Feasibility conditions:

$$x_2 = 100 + \frac{1}{2}D_1 - \frac{1}{4}D_2 = 130 \qquad \Rightarrow \text{feasible}$$

$$x_3 = 230 + \frac{1}{2}D_1 = 220 \qquad \Rightarrow \text{feasible}$$

$$x_6 = 20 - \frac{1}{2}D_1 + D_2 + D_3 = -110 \qquad \Rightarrow \text{infeasible}$$

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x_6 &= 20 - \frac{1}{2}D_1 + D_2 + D_3 = -110 && \Rightarrow \text{infeasible}
\end{aligned}
$$

- The current solution does not remain feasible because $x_6 < 0$.

Feasibility Ranges example

- Alternatively, if the changes in the resources are such that $D_1 = -30$, $D_2 = -12$, and $D_3 = 10$, then

$$x_2 = 100 + \frac{1}{2}D_1 - \frac{1}{4}D_2 = 88 \qquad \Rightarrow \text{feasible}$$

$$x_3 = 230 + \frac{1}{2}D_1 = 224 \qquad \Rightarrow \text{feasible}$$

$$x_6 = 20 - \frac{1}{2}D_1 + D_2 + D_3 = 78 \qquad \Rightarrow \text{feasible}$$

  The new (optimal) feasible solution is $x_2 = 88$, $x_3 = 224$, and $x_6 = 68$ with $z = 31x_1 + 218x_2 + 512x_3 = \$1296$.

- The dual prices D_1 and D_2 can be used to calculate the optimal objective value as $z = 1350 + D_1 + 2D_2 + 0D_3$.

Individual Feasibility Ranges Example

- Given conditions produce feasibility ranges for changing resources one at a time.
- Change in operation 1: $D_2 = D_3 = 0$.
- Simultaneous conditions reduce to expressions for x_2, x_3, and inequality for x_6 in terms of D_1.

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\begin{aligned}
x_2 &= 100 + \frac{1}{2}D_1 \geq 0 \\
x_3 &= 230 - 7D_2 \geq 0 \\
x_6 &= 20 - 2D_1 \geq 0
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- Dual price for operation 1 valid in $-200 \leq D_1 \leq 10$.
- Feasibility ranges for operations 2 and 3: $-20 \leq D_2 \leq 400$, $-20 \leq D_3 \leq \infty$.

Summary of sensitivity for changes in the RHS

			Resource amount (minutes)		
Resource	Dual price($)	Feasibility range	*Minimum*	*Current*	*Maximum*
Operation 1	1	$-200 \leq D_1 \leq 10$	230	430	440
Operation 2	2	$-20 \leq D_2 \leq 400$	440	440	860
Operation 3	0	$-20 \leq D_3 < \infty$	400	420	∞

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			Resource amount (minutes)		
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Dual prices remain valid for feasible simultaneous changes, even if they violate individual ranges.

Algebraic Sensitivity Analysis - Reduced Cost

$$z = 1350 - 4 \times \boldsymbol{x_1} - \boldsymbol{x_4} - 2 \times \boldsymbol{x_5}$$

- Optimal solution does not produce toy trains $(x_1 = 0)$.

Algebraic Sensitivity Analysis - Reduced Cost

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- **Reason:** A unit increase in x_1 (above its current zero value) decreases z by $4.

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- Optimal solution does not produce toy trains $(x_1 = 0)$.
- **Reason:** A unit increase in x_1 (above its current zero value) decreases z by \$4.
- Calculation: $z = 1350 - 4 \times (1) - 1 \times (0) - 2 \times (0) = \$1346$.

Algebraic Sensitivity Analysis - Reduced Cost

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- The **reduced cost** of x_1 is 4.

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- Optimal solution does not produce toy trains $(x_1 = 0)$.
- **Reason:** A unit increase in x_1 (above its current zero value) decreases z by $4.
- Calculation: $z = 1350 - 4 \times (1) - 1 \times (0) - 2 \times (0) = \$1346$.
- The **reduced cost** of x_1 is 4.
- Basic variables have zero reduced costs because increasing their values would violate a constraint.

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Algebraic Sensitivity Analysis - Objective Function

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- $\boldsymbol{d_i}$ is the change in unit revenue for toy i.

$$Max \quad z = (3 + \boldsymbol{d_1}) \times x_1 + (2 + \boldsymbol{d_2}) \times x_2 + (5 + \boldsymbol{d_3}) \times x_3$$

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- d_i is the change in unit revenue for toy i.

$$Max \quad z = (3 + \boldsymbol{d_1}) \times x_1 + (2 + \boldsymbol{d_2}) \times x_2 + (5 + \boldsymbol{d_3}) \times x_3$$

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	$-3 - d_1$	$-2 - d_2$	$-5 - d_3$	0	0	0	0

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Algebraic Sensitivity Analysis - Objective Function

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	$4 - \frac{1}{4}d_2 + \frac{3}{2}d_3 - d_1$	0	0	$1 + \frac{1}{2}d_2$	$2 - \frac{1}{4}d_2 + \frac{1}{2}d_3$	0	$1350 + 100d_2 + 23d_3$
x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230
x_6	$-\frac{1}{4}$	0	0	-2	1	1	20

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z	$4 - \frac{1}{4}d_2 + \frac{3}{2}d_3 - d_1$	0	0	$1 + \frac{1}{2}d_2$	$2 - \frac{1}{4}d_2 + \frac{1}{2}d_3$	0	$1350 + 100d_2 + 23d_3$
x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230
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- Simplex with same sequence of entering/leaving variables as original model.

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z	$4 - \frac{1}{4}d_2 + \frac{3}{2}d_3 - d_1$	0	0	$1 + \frac{1}{2}d_2$	$2 - \frac{1}{4}d_2 + \frac{1}{2}d_3$	0	$1350 + 100d_2 + 23d_3$
x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100
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- Simplex with same sequence of entering/leaving variables as original model.
- New optimal tableau is similar to the original one, except for the reduced costs.

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Algebraic Sensitivity Analysis - Objective Function

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z	$4 - \frac{1}{4}d_2 + \frac{3}{2}d_3 - d_1$	0	0	$1 + \frac{1}{2}d_2$	$2 - \frac{1}{4}d_2 + \frac{1}{2}d_3$	0	$1350 + 100d_2 + 23d_3$
x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100
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- Simplex with same sequence of entering/leaving variables as original model.
- New optimal tableau is similar to the original one, except for the reduced costs.
- Objective-function coefficients changes affect the optimality and not feasability.

Algebraic Sensitivity Analysis - Objective Function

No need to carry out simplex row operation for computing new reduced costs.

		d_1	d_2	d_3	0	0	0	
	Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
1	z	4	0	0	1	2	0	1350
d_2	x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100
d_3	x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230
0	x_6	2	0	0	-2	1	1	20

Algebraic Sensitivity Analysis - Objective Function

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		d_1	d_2	d_3	0	0	0	
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d_2	x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100
d_3	x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230
0	x_6	2	0	0	-2	1	1	20

Reduced cost for $x_1 = \begin{pmatrix} 4 & -\frac{1}{4} & \frac{3}{2} & 2 \end{pmatrix} \begin{pmatrix} 1 & d_2 & d_3 & 0 \end{pmatrix} - d_1$

Reduced cost for $x_1 = 4 - \dfrac{1}{4} d_2 + \dfrac{3}{2} d3 - d_1$

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Algebraic Sensitivity Analysis - Objective Function

- The current BFS remains optimal if the new reduced costs remain nonnegative (maximization case).

Algebraic Sensitivity Analysis - Objective Function

- The current BFS remains optimal if the new reduced costs remain nonnegative (maximization case).
- We thus have the following simultaneous optimality conditions corresponding to nonbasic x_1, x_4, and x_5:

$$4 - \frac{1}{4}d_2 + \frac{3}{2}d_3 - d_1 \geq 0$$
$$1 + d_2 \geq 0$$
$$2 - \frac{1}{4}d_2 + \frac{1}{2}d_3 \geq 0$$

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Simultaneous objective function changes example

- Objective function of TOYCO is changed from $z = 3x_1 + 2x_2 + 5x_3$ to $z = 2x_1 + x_2 + 6x_3$.

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- Objective function of TOYCO is changed from $z = 3x_1 + 2x_2 + 5x_3$ to $z = 2x_1 + x_2 + 6x_3$.
- Substitution in the given conditions yields:
  - $4 - \frac{1}{4}d_2 + \frac{3}{2}d_3 - d_1 = 6.75 \geq 0 \Rightarrow$ **satisfied**
  - $1 + \frac{1}{2}d_2 \geq 0 \Rightarrow$ **satisfied**
  - $2 - \frac{1}{4}d_2 + \frac{1}{2}d_3 = 2.75 \geq 0 \Rightarrow$ **satisfied**

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  - $1 + \frac{1}{2}d_2 \geq 0 \Rightarrow$ **satisfied**
  - $2 - \frac{1}{4}d_2 + \frac{1}{2}d_3 = 2.75 \geq 0 \Rightarrow$ **satisfied**
- The proposed changes will keep the current solution $(x_1 = 0, x_2 = 100, x_3 = 230)$ optimal with a new value of $z = 1350 + 100d_2 + 230d_3 = 1480$.

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  - $1 + \frac{1}{2}d_2 \geq 0 \Rightarrow$ **satisfied**
  - $2 - \frac{1}{4}d_2 + \frac{1}{2}d_3 = 2.75 \geq 0 \Rightarrow$ **satisfied**

- The proposed changes will keep the current solution $(x_1 = 0, x_2 = 100, x_3 = 230)$ optimal with a new value of $z = 1350 + 100d_2 + 230d_3 = 1480$.

- If any condition is not satisfied, a new solution must be determined.

Individual changes in objective function example

- Optimality ranges for a single variable's coefficient can be derived from simultaneous optimality conditions by setting changes in other variables to 0.

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- Similarly, changing x_1's coefficient by $(3 + d_1)$ and x_3's coefficient by $(5 + d_3)$ result in optimality ranges of $d_1 \leq 4$ and $d_3 \geq -\frac{8}{3}$.

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- Similarly, changing x_1's coefficient by $(3 + d_1)$ and x_3's coefficient by $(5 + d_3)$ result in optimality ranges of $d_1 \leq 4$ and $d_3 \geq -\frac{8}{3}$.
- Allowable individual ranges of d_1, d_2, and d_3 may not necessarily satisfy the simultaneous conditions, and vice versa.

LP Solution with Excel Solver

- Input all the coefficients:
  - Objective function coefficients.
  - Constraint coefficients.

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  - Constraint coefficients.
- Write the formulas for the totals based on values of the decision variables.
- Use Excel Solver to find the optimal values of the decision variables that maximize/minimize the objective function, and satisfy the constraints.
- Solver provides the optimal values for the decision variables.
- Check the Solver results for feasibility and sensitivity analysis to see how the optimal solution would be affected by changes in the input parameters.

LP Solution with Excel Solve example

$$
\begin{array}{llrrrrrl}
\textbf{Maximize} & z = & 3x_1 & + 2x_2 & + 5x_3 & & & \\
\textbf{Subject to} & & x_1 & + 2x_2 & + x_3 & \le & 430 & \text{(Operation 1)} \\
 & & 3x_1 & & + 2x_3 & \le & 460 & \text{(Operation 2)} \\
 & & x_1 & + 4x_2 & & \le & 420 & \text{(Operation 3)} \\
 & & x_1 & , x_2 & , x_3 & \ge & 0 &
\end{array}
$$

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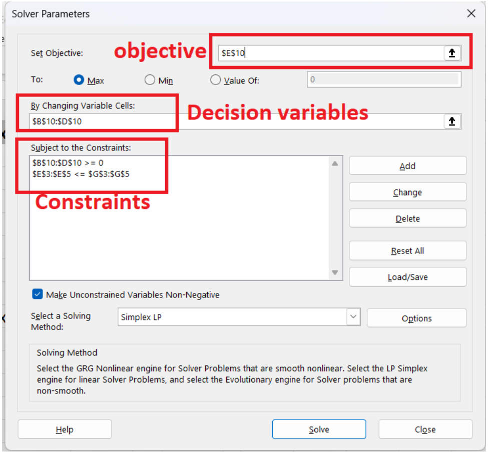
Excel Formulas

- **Total = SUMPRODUCT(Coefficients array, decision variables array)**
  - Total profit = SUMPRODUCT(B2:D2,$B$10:$D$10)
  - Total operation 1 = SUMPRODUCT(B3:D3,$B$10:$D$10)
  - Total operation 2 = SUMPRODUCT(B4:D4,$B$10:$D$10)
  - Total operation 3 = SUMPRODUCT(B5:D5,$B$10:$D$10)

Excel Solver



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Revision