Spherical off-center head model with offset ears

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In analogy to the head-related transfer function, the transfer function of a rigid sphere can be defined by the sound field on the sphere's surface p_s, divided by the sound field p_{ff} at the center of the sphere with the sphere being absent (Note that the sphere's center often coincides with the origin of co-ordinates)

$$H = \frac{p_s}{p_{ff}}. \tag{1}$$

As shown by Duda and Martens (1998) [1], the analytical solution is given by

$$H(\varrho_o, \varrho, \mu, \Theta) = -\frac{\varrho_0}{\mu} \mathrm{e}^{-j\mu\varrho_0} \Psi(\varrho, \mu, \Theta), \tag{2}$$

with

$$\Psi(\varrho, \mu, \Theta) = \sum_{m=0}^{\infty} (2m+1) P_m(\cos(\Theta)) \frac{h_m(\mu\varrho)}{h'_m(\mu)} \tag{3}$$

$$\mu = \frac{\omega}{c} a = ka \tag{4}$$

$$\varrho = \frac{r}{a} \tag{5}$$

$$\varrho_0 = \frac{r_0}{a}, \tag{6}$$

where Θ is the angle of incidence between the source and the point on the sphere (the ear), $\omega = 2\pi f$ the angular frequency, f [Hz] the frequency, c [m/s] the speed of sound, a [m] the sphere's radius, r the distance of the source to the center of the sphere, and r_0 the distance of the source to the origin of co-ordinates. Note that this is the exact notation from [1], with the exception that the two radii r and r_0 are distinguished. In case the sphere's center coincides with the origin of co-ordinates, r equals r_0 (cf. Fig. 1, left).

If H has to be calculated for a spatial sampling grid $s(\varphi, \vartheta, r = \text{constant})$ in spherical coordinates (azimuth $\varphi = 0$: front, $\varphi = 90$: left, $\varphi = 180$: back; elevation $\vartheta = 90$: top, $\vartheta = 0$: front, $\vartheta = -90$: bottom), Θ specifies the great circle distance between each point on the sampling grid, and the left and right 'ear' of the spherical head given by $\varphi_{\mathrm{l,r}}$, and $\vartheta_{\mathrm{l,r}}$

$$\Theta = \arccos\left(\sin(\vartheta)\sin(\vartheta_{\mathrm{l,r}}) + \cos(\vartheta)\cos(\vartheta_{\mathrm{l,r}})\cos(\varphi - \varphi_{\mathrm{l,r}})\right). \tag{7}$$

According to the classical HRTF definition [2], the interaural center (midpoint between the axis through the ears) is considered to be the center of the coordinate system. In the case of offset ears (i.e. $\varphi \neq \pm 90°$, and $\vartheta \neq 0°$) the spherical head thus needs to be moved by $\sin(\varphi_{\mathrm{ear\ l,r}} - \pi/2) * a$ in x-direction (positive x: front, negative x: back), and by $\sin(\vartheta_{\mathrm{ear\ l,r}}) * a$ in z-direction (positive z: up, negative z: down). A simple way of realizing this translation is to apply its opposite (switched signs) to the sampling grid in cartesian coordinates $s(x, y, z)$. As a consequence the angles φ, ϑ, and the radii r now reflect the angle and distance from each point of the sampling grid to the center of the off-center spherical head. This means that r can

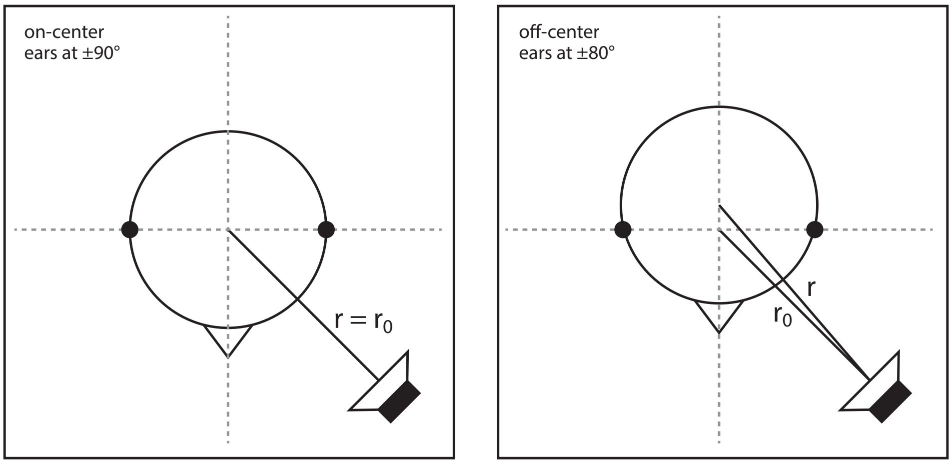

Figure 1: Spherical head model with ears marked by black dots. Left: On-center model with ears at $\pm 90°$ and $r = r_0$. Right: Off-center model with ears at $\pm 80°$ and $r \neq r_0$.

now differ from r_0 (cf. Fig. 1, right). Moreover, the radius r will be different among the points of s in the case of an off-center spherical head model.

Duda and Martens [1] also give an iterative solution for Eq. (2) in Appendix A that speeds up the calcualation, which is used by `AKsphericalHead.m`. If considering the difference between r and r_0 this expression slightly changes to

$$H(\varrho_o, \varrho, \mu, \Theta) = \frac{\varrho_0}{j\mu} \mathrm{e}^{j(\mu\varrho - \mu\varrho_0 - \mu)} \sum_{m=0}^{\infty} (2m+1) P_m(\cos(\Theta)) \frac{Q_m\left(\frac{1}{j\mu\varrho}\right)}{\frac{m+1}{j\mu} Q_m\left(\frac{1}{j\mu}\right) - Q_{m-1}\left(\frac{1}{j\mu}\right)}. \tag{8}$$

References

[1] Richard O. Duda and William L. Martens. Range dependence of the response of a spherical head model. *J. Acoust. Soc. Am.*, 104(5):3048–3058, November 1998.

[2] Henrik Møller. Fundamentals of binaural technology. *Applied Acoustics*, 36:171–218, 1992.