Course: Theory of Computation

HW8： Reducibility

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5.1 Show that the $EQ_{\text{CFG}}$ is undecidable.

Solution: Suppose $EQ_{\text{CFG}}$ is decidable, then there is decider TM $R$ for it, we construct TM $S$ to decide $ALL_{\text{CFG}}$ as following:

S=" on input <$M$>, where $M$ is a TM

1. Run $R$ on input <$M$, $M_1$>, where $M_1$ is a TM that rejects all inputs.
2. If $R$ accepts, accepts; if $R$ rejects, reject."

If R decides $EQ_{\text{CFG}}$, S decides $ALL_{\text{CFG}}$, but $ALL_{\text{CFG}}$ is undecidable by Theorem 5.13, so the supposition is wrong, then $EQ_{\text{CFG}}$ is undecidable.

5.3. Find a match in the following instance of the Post Correspondence Problem.

$$\left\{\left[\frac{ab}{abab}\right],\left[\frac{b}{a}\right],\left[\frac{aba}{b}\right],\left[\frac{aa}{a}\right]\right\}$$

Solution

The match is $\left\{\left[\frac{ab}{abab}\right],\left[\frac{ab}{abab}\right],\left[\frac{b}{a}\right]\left[\frac{b}{a}\right],\left[\frac{aa}{a}\right],\left[\frac{aa}{a}\right]\right\}$

5.4 If A $\leq_m$ B and B is a regular language, does that imply that A is a regular? Why or why not?

Solution: that does not imply A is a regular Language.

For example: for nonregular language $A=\{1^n0^n|n\geq 0\}$, and regular language $B=\{0\}$, we can construct a computation function $f$ as following:

$$f(w)=\begin{cases}0, & w=1^n0^n\\ 1, & w\neq 1^n0^n\end{cases}$$

So there is $w\in A\Leftrightarrow f(w)\in B$, so $A\leq_m B$. Here $B$ is regular Language, however, $A$ is nonregular language.

5.9 Let $T$ = {<$M$>| $M$ is a TM that accepts $w^R$ whenever it accepts $w$}. Show that $T$ is undecidable.

Solution: for any <$M$, $w$>, where $M$ is TM and $w$ is input string, the following TM F computes a reduction $f$(<$M$, $w$ >):

F="on input $x$:

1) if $x$≠01 or 10, reject;
2) if $x$=01 accept;
3) if $x$=10, run $M$ on $w$, if $M$ accepts, accept, if $M$ rejects, reject;

from above, we can get for any<$M$, $w$>,there is <$M$, $w$>$\in A_{TM}\Leftrightarrow f(<M,w>)\in T$, so $A_{TM}\leq_m T$, since $A_{TM}$ is undecidable, according to Corollary 5.23, $T$ is undecidable.

5.16 Let Γ = {0, 1, ␣ } be the tape alphabet for all TMs in this problem. Define the ***busy beaver function*** BB: $\mathcal{N}\rightarrow\mathcal{N}$ as follows. For each value of $k$, consider all $k$-stateTMs that halt when started with a blank tape. Let $BB(k)$ be the maximum number of 1s that remain on the tape among all of these machines. Show that $BB$ is not a computable function.