Course: Theory of Computation

HW7： Decidability

Lina Mi

@01377283

1. Show that the set of all finite strings over the alphabet { a, b, c, d } is countable. Must show a correspondence.

As all strings over the alphabet { a, b, c, d } are finite, namely there are k ($k \in \mathcal{N}$) number of letters which are selected from alphabet { a, b, c, d } in each string, then we can build a correspondence as following

Number of letters in the string	Number of strings
1	$C_4^1 = 4$
2	$C_4^1 * C_4^1 = 4^2$
3	$C_4^1 * C_4^1 * C_4^1 = 4^3$
:	:
n	4^n

It can be seen that there is correspondence between numbers of strings with n letters in the string and set of natural numbers $\mathcal{N}$, so the set of all finite strings over the alphabet { a, b, c, d } has same size of $\mathcal{N}$, according to definition 4.14, this set is countable

2. Show that the set of all strings over the alphabet of { a, b, c, d } is uncountable. Must use diagonalization.
Solution: Since the strings over the alphabet of {a, b, c, d} may have finite length or infinite length, suppose there is correspondence f between $\mathcal{N}$ and the set of string over the alphabet of {a, b, c, d}, let f(1)=abcd, f(2)=dac, f(3)=daa,….,

n	f(n)
1	abcd
2	dac
3	daa
4	bccda
:	:

we can construct a string w with infinite number of letters over the alphabet of {a, b, c, d}, the first letter of w is different from that of $f(1)$, let it be c, so $w \neq f(1)$; the second letter of w is anything different from the second letter of $f(2)$, let it be d, then $w \neq f(2)$; the third letter of w is different from that of $f(3)$, let it be c, so $w \neq f(3)$; continuing this way down the diagonal of the table for f, we obtain all digits of w, as shown in the following table

n	f(n)
1	abcd
2	dac
3	daa
4	bccda
:	:

w=cdca…..

According to the procedure of constructing string w, we know that w is not $f(n)$ for any n, since it differ from $f(n)$ in the nth letter, namely $w \neq f(n)$; so the correspondence between natural number set $\mathcal{N}$ and the set of all string over the alphabet {a, b, c, d} does not exist. So the set of all strings over the alphabet {a, b, c, d} is uncountable.

3. Show that you can create a TM that shows that two REs are equivalent by testing if the REs generate all the strings up to a certain size are equal. What is that size?

4.21 Let $S = \{\langle M \rangle | M \text{ is a DFA that accept } w^R \text{ whenever it accepts } w\}$ show that S is decidable

Solution: let TM G for S and

G="on input $\langle M \rangle$, where *M* is DFA what accept *w*

1 simulate DFA *M* on input w^R

2 if input w^R end M with *accept* state, then *accept*, if input w^R end *M* with *reject* state, then *reject*"

So the TM G can decide *S*, therefore *S* is decidable

4.28 Let $C = \{\langle G, x \rangle | G \text{ is } CFG, x \text{ is a substring of some } y \in L(G)\}$, show that *C* is decidable.

Solution: let TM R is for *C* :

R="on put $\langle G, x \rangle$, where *G* is CFG, and s is a string,

1 mark all the terminals of *G* that are symbols in string *x* ;

2 repeat until no new variables get marked;

3 Mark any variable A where G have a rule $A \to U_1 U_2 \cdots U_k$ and each symbol $U_1, U_2, \cdots U_k$ has already been marked;

4 if start variable is not marked, *reject*; otherwise *accept*

because TM R can decide the problem *C,* so *C* is decidable.