Course: Theory of Computation

HW1

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0.3 let A be the set {x,y,z}, and B be the set {x,y}

a. Is A a subset of B?
   because $z \in A, but\ z \notin B$, A is **not** a subset of B

b. Is B a subset of A?
   Because for all members of B: x and y, there are $x \in B, and\ x \in A; y \in B\ and\ y \in A$, B **is** a subset of A

c. What is $A \cup B$?
   $A \cup B = \{x, y, z\}$

d. What is $A \cap B$?
   $A \cap B = \{x, y\}$

e. What is $A \times B$?
   $A \times B = \{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\}$

f. What is the power set of B?
   Power set of B:$\{\emptyset, \{x\}, \{y\}, \{x, y\}$

0.4 If A has a elements and B has b elements, How many elements are in $A \times B$? Explain your answers.
Suppose $Z = A \times B$, then for the elements of Z, z_i, there is $z_i = (a_i, b_i)$, where $a_i \in A$; $b_i \in B$, so there are a numbers of choices for a_i, and b numbers of choices for b_i, hence there are $a * b$ numbers of choices for z_i, namely there are $a * b$ numbers of elements in $A \times B$

0.5 If C is a set with c elements, how many elements are in the power set of C? Explain your answer
Since there are c number of elements in set C, then the power set of C is $\{\emptyset, \{x_i | i \in [1, c]\}$, $\{x_i, x_j | i, j \in [1, c] and\ i \neq j\}, \cdots, \{x_1, x_2, \cdots x_c\}\}$; so the numbers of power set of C is$N = 1 + \binom{c}{1} + \binom{c}{2} + \cdots \binom{c}{c}$, where $\binom{c}{k} | k \in [1, c]$ is k-combination number of set C with c elements, that means $N = \frac{c!}{0!(c-0)!} + \frac{c!}{1!(c-1)!} + \frac{c!}{2!(c-2)!} + \cdots \frac{c!}{c!1} = 2^c$

0.6 Let X be the set {1, 2,3, 4,5} and Y be the set {6, 7, 8, 9, 10}, The unary function $f: X \rightarrow Y$ and the binary function $g: X \times Y \rightarrow Y$ are described in the following tables

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

a. What is the value of $f(2)$?
since n=2 according to table $f(n)$, $f(2) = 7$

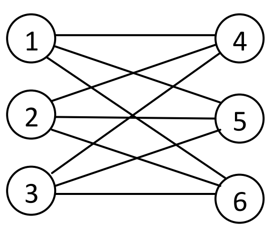
b. What are the range and domain of f?
the range of f is set {6,7}, domain of f is set {1, 2,3, 4,5}

c. What is the value of $g(2,10)$?
According to above table, $g(2,10) = 6$

d. What are range and domain of g?
The range of g is set {6, 7, 8, 9, 10}, the domain of g is set {(1,6), (1,7), (1,8), (1,9), (1,10), (2,6), (2,7), (2,8), (2,9), (2,10), (3,6), (3,7), (3,8), (3, 9), (3, 10), (4,6), (4,7), (4,8), (4,9), (4,10), (5,6), (5,7), (5,8), (5,9), (5,10) }

e. What is the value of $g(4, f(4))$?
Since the $f(4) = 7$, so $g(4, 7) = 8$

0.9 Write a formal description of the following graph



Since there are vertices of 1, 2,3, 4, 5,6 in the graph, and edges of (1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6) in the graph, so the graph can be described as ({1,2,3,4,5,6},{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)})

Problems

0.10 find the error in the following proof that 2=1.

Consider the equation a=b. Multiply both sides by a to obtain a^2=ab. Substract b^2 from both sides to get a^2-b^2=ab-b^2. Now factor each side (a+b)(a-b)=b(a-b), and divide each side by (a-b), to get a+b=b. Finally, let a and b equal 1, which shows that 2=1.

The error in the procedure of above proof is located that diving each side of equation (a+b)(a-b)=b(a-b) by (a-b), there is implicit assumption that $a \neq b$, since if a=b, then a-b=0, it is unreasonable to divide both sides of the equation by 0. In the last step, however, it let a=b=1, it is contradict with the implicit assumption, namely $a \neq b$, thus the previous proof is false.

0.13 Use the Theorem 0.15 to derive a formula for calculating the size of the monthly payment for a mortgage in terms of the principal P, interest rate I, and the number of payments t. Assume that, after t payments have been made, the loan amount is reduced to 0,. Use the formula to calculate the dollar amount of each monthly payment for a 30-year mortgage with 360 monthly payments on an initial loan amount of $100,000 with an 8% annual interest rate.

According to theorem 0.15,

$P_t = PM^t - Y(\frac{M^t-1}{M-1})$ where $M = 1 + I/12$

then

$$P_{t+1} = PM^{t+1} - Y(\frac{M^{t+1}-1}{M-1})$$

Because

$$Y = P_{t+1} - P_t = PM^{t+1} - Y\left(\frac{M^{t+1}-1}{M-1}\right) - \left[PM^t - Y\left(\frac{M^t-1}{M-1}\right)\right]$$

$$Y = PM^t(M-1) - Y\left(\frac{M^{t+1}-1}{M-1} - \frac{M^t-1}{M-1}\right) = PM^t(M-1) - YM^t$$

$$Y(1+M^t) = PM^t(M-1)$$

$$Y = PM^t\left(\frac{M-1}{1+M^t}\right)$$

For 8% annual interest rate, $M = 1 + 8\%/12 = 1.0067$, t=360, P=100000

So monthly payment

$$Y = PM^t\left(\frac{M-1}{1+M^t}\right) = 10^5 * 1.0067^{360} * (\frac{1.0067-1}{1+1.0067^{360}}) \approx \$614.48$$