SP17_Intro to Data Science

Problem 4

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1) You are taking a 10 question multiple choice test. If each question has four choices and you guess on each question, what is the probability of getting one question correct? *[Hint: This is a binomial*
in the form of 10 choose 1 with p=.25.]

Ans:

According to binomial equation: $P(k\ successes\ in\ n\ tirals) = \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$

Here, $p = 0.25$ (four choose one), $n = 10; k = 1$

So the probability of getting one question correct is :

$$P(passing\ test1) = \frac{10!}{1!\,(10-1)!}0.25^1(1-0.25)^{10-1} \approx 0.188 = 18.8\%$$

2)What is the probability of getting seven questions correct? *[Hint: This is a binomial as Q1 with modified choose value.]*

ANS: Here, $p = 0.25$ (four choose one), $n = 10; k = 7$

According to binomial equation, the probability of getting seven questions correct is

$$P(passing\ test2) = \frac{10!}{7!\,(10-7)!}0.25^7(1-0.25)^{10-7} \approx 0.0031 = 0.31\%$$

3) What are your chances of answering seven questions correctly if you can reliably eliminate one possible answer from each question? [Hint: This is a binomial as Q2 with a modified p value.]

ANS: since one possible answer can be eliminated from 4 possible answers, the probability of choosing correct answer for question is three choosing one, namely $p = 1/3$, here, $n = 10; k = 7$

According to binomial equation, the probability is

$$P(passing\ test3) = \frac{10!}{7!\,(10-7)!}\left(\frac{1}{3}\right)^7 (1-\frac{1}{3})^{10-7} \approx 0.0163 = 1.63\%$$

4) Let's say, instead, that the test is an adaptive test; you get to answer more questions based on your previous success.

This test is structured like this:

- First you have to answer three questions and if you are correct on two of them, you get to answer three more questions.
- If two of **those** are correct, then you get three final questions, of which you need to get at least two correct to pass the whole test.

The test details are:

- The first test, T1, has three multiple choice questions with four possible answers each (*p*=0.25 per question).

The second test, T2, has three multiple choice questions with three possible answers each (*p*=0.33 per question).

- The final test, T3, has three questions that are true/false (*p*=0.50 each question).

The test questions are formed as follows:
The questions are in a language you have never seen: a mixture of Navaho, Swahili, Klingon, and Esperanto. So you have to guess on all of the questions and there are no contextual clues to eliminate any answers. This is the first one:
'Arlogh Qoylu'pu'?
Moja: Yel kholgo eeah.
Mbili: Floroj kreskas ĉirkaŭ mia domo. Pe'el!
Tatu: La sandviĉo estos manĝota'mo'tlhIngan maH!
Nne: 'Adeez'a`q eeah.

Problem Hint:

Structure your analysis.

- Figure out the component probabilities: p(passing test 1), p(passing test 2), p(passing test 3).
- Make a table of their proportional contributions of probability to the whole.
- Calculate the total probability: p(Total).
- Continue using Bayes' theorem to calculate the probability of passing test 3 conditional on passing tests 1 and 2.
- Render your interpretation. Use the interpretation in the example as a template, if you are unsure of what to say.

Using the binomial probability rule, the law of total probability and Bayes' theorem:

a) What is the probability of getting two right on each sub-exam? (T1, T2, and T3, separately.)
Ans:
for T1, there is $n = 3, k = 2, and\ p = 0.25$, according to binomial equation, the probability of getting two right is

$$P(2\ corrects\ in\ T1) = \frac{3!}{2!\,(3-2)!}(0.25)^2(1-0.25)^{3-2} = 0.140625 = 14.0625\%$$

$n = 3, k = 3, and\ p = 0.25$The probability of getting three rights is

$$P(3 corrects\ in\ T1) = \frac{3!}{3!\,(3-3)!}0.25^3(1-0.25)^{3-3} = 0.015625 = 1.5625\%$$

So the probability of passing test 1 is:

$$P(passing\ T1) = P(2\ corrects\ in\ T1) + P(3\ corrects\ in\ T1)$$
$$= 14.0625\%+1.5625\%=15.625\%$$

For T2, the probability of getting two rights: $n = 3, k = 2, and\ p = \frac{1}{3}$, according to binomial equation,

$$P(2\ corrects\ in\ T2) = \frac{3!}{2!\,(3-2)!}\left(\frac{1}{3}\right)^2(1-\frac{1}{3})^{3-2} = \frac{2}{9} \approx 22.22\%$$

The probability of getting three rights: $\mathrm{n} = 3, \mathrm{k} = 3, \mathrm{and}\ \mathrm{p} = \frac{1}{3}$, according to binomial equation,

$$P(3\ corrects\ in\ T2) = \frac{3!}{3!\,(3-3)!}\left(\frac{1}{3}\right)^3\left(1-\frac{1}{3}\right)^{3-3} \approx 0.03704 \approx 3.704\%$$

So the probability of passing test2 is:

$P(passing\ T2) = P(2\ corrects\ in\ T2) + P(3\ corrects\ in\ T2)$=22.22%+3.704%=25.924%

For T3, the probability of getting two rights: $n = 3, k = 2, and\ p = 0.5$, according to binomial equation,

$$P(2\ corrects\ in\ T3) = \frac{3!}{2!\,(3-2)!}\left(\frac{1}{2}\right)^2 (1-\frac{1}{2})^{3-2} = 0.375 = 37.5\%$$

The probability of getting three rights: $n = 3, k = 3, and\ p = 0.5$, according to binomial equation:

$$P(3\ corrects\ in\ T3) = \frac{3!}{3!(3-3)!}\left(\frac{1}{2}\right)^3 (1-\frac{1}{2})^{3-3} = 0.125 = 12.5\%$$

The probability of passing test3 is :

$$P(passing\ T3) = P(2\ corrects\ in\ T3) + P(3\ corrects\ in\ T3) = 37.5\% + 12.5\% = 50\%$$

b) What are your overall chances of passing the entire exam?

Table of contributions to total probability:

	probability	%contrib
P(passing T1)	.15625	1/3
P(passing T2)	.25924	1/3
P(passing T3)	.5	1/3

According to law of total probability, the total probability of passing the entire test is:

$$P(Total) = 0.15625 * \frac{1}{3} + 0.25924 * \frac{1}{3} + 0.5 * \frac{1}{3} \approx 0.3052 = 30.52\%$$

c) What are your chances of passing T3 if you first pass T1 and T2?

According to Baye's theorem using total probability as denominator, the probability of passing test3 given first pass T1 &T2

$$P(T3|T1\&T2) = \frac{P(T3) * \%contrib(T3)}{P(Total)} = \frac{0.5 * \frac{1}{3}}{0.3052} \approx 0.5461 = 54.61\%$$

Problem 5: Now, let's say that you know just enough of these obscure languages to translate the first question in T1:

What time is it? (Klingon)

1. (Swahili): From dawn to setting sun. (Navajo)
2. (Swahili): Flowers grow around my house (Esperanto) so all of you may come in. (Klingon)
3. (Swahili): The sandwich will be eaten (Esperanto) because we are Klingons! (Klingon)
4. (Swahili): It's mid-afternoon. (Navajo) **[correct answer]**

Now the probability of passing T1 has changed because you only have to guess correctly on one of the two remaining questions in the first section, a one-in-two chance.
a) What is the new probability for T1?
b) Now what is the overall probability of passing the entire test?
c) And what is the probability of passing section T3, given that you have already passed sections T1 and T2?
d) The kicker: How do you explain the difference between 4c and 5c? Can you relate this to a larger context about conditional probability and making decisions?

Problem Hint:

 Compute the new probability for T1
 Derived the total probability using the new value of T1
 Use Bayes theorem with the updated values to compute new conditional probability of passing T3 given you have passed T1 and T2
 Consider conditional probability and how T1, T2 and T3 are considered a systems

a) What is the new probability for T1?

for T1, there is $n = 2, k = 1, and\ p = 0.25$, according to binomial equation, the probability of getting two right is

$$P(2\ corrects\ in\ T1) = \frac{2!}{1!\,(2-1)!}(0.25)^1(1-0.25)^{2-1} = 0.375 = 37.5\%$$

$n = 2, k = 2, and\ p = 0.25$The probability of getting three rights is

$$P(3corrects\ in\ T1) = \frac{2!}{2!\,(2-2)!}0.25^2(1-0.25)^{2-2} = 0.0625 = 6.25\%$$

So the probability of passing test 1 is:

$$P1(passing\ T1) = P(2\ corrects\ in\ T1) + P(3\ corrects\ in\ T1)$$
$$= 37.5\%+6.25\%=43.75\%$$

b) Now what is the overall probability of passing the entire test?

Table of contributions to total probability:

	probability	%contrib
P1(passing T1)	.4375	1/3
P(passing T2)	.25924	1/3
P(passing T3)	.5	1/3

According to law of total probability, the new total probability of passing the entire test is:

$$P1(Total) = 0.4375 * \frac{1}{3} + 0.25924 * \frac{1}{3} + 0.5 * \frac{1}{3} \approx 0.3989 = 39.89\%$$

c) And what is the probability of passing section T3, given that you have already passed sections T1 and T2?
According to Baye's theorem using total probability as denominator, the new probability of passing test3 given first pass T1 &T2

$$P1(T3|T1\&T2) = \frac{P(T3) * \%contrib(T3)}{P1(Total)} = \frac{0.5 * \frac{1}{3}}{0.3989} \approx 0.4178 = 41.78\%$$

d) The kicker: How do you explain the difference between 4c and 5c? Can you relate this to a larger context about conditional probability and making decisions?

Ans: Comparing the results of 4c, which is 54.61%, and 5c, which is 41.78%, it can be seen that the improvement of probability of passing test 1 leads to the decrease of conditional probability of passing test3 given past test1 and test2.