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TEST 1 SUMMER 2017

Please prove the following statements:

1. 18^n-3^n is divisible by 5 for any n >0

   1)for n=1, $18^1 - 3^1$=18-3=15 it is divisible by 5.

   2)Assuming that the statement holds for $n=k$, that means:

   18^k-3^k is divisible by 5 is true;

   3)for $n=k+1$,

   $18^{k+1}-3^{k+1}=18^1*18^k - 3^1*3^k = (15+3)*18^k - 3*3^k = 15*18^k + 3*(18^k - 3^k)$

   Since 15 is divisible by 5, then the first part $15*18^k$ is divisible by 5;

   Based on assumption in step (2), namely 18^k-3^k is divisible by 5, the second part $3*(18^k - 3^k)$ is divisible by 5;

   So $18^{k+1}-3^{k+1}$ is divisible by 5, it can be concluded that the claim 18^n-3^n is divisible by 5 for any n >0 is true

2. 1+5+9+…+(4n-3) = n(2n-1) for any n >0

   (1) For n=1, 4n-3=1, and n(2n-1)=1, so 4n-3=n(2n-1) is true for n=1;

   (2) Assuming for n=k, the claim is true, namely 1+5+9+…+(4k-3) = k(2k-1)

   (3) For n=k+1;

   According the assumption in step(2), 1+5+9+…+(4k-3)= k(2k-1),

   The left part of the statement

   1+5+9+…+(4k-3)+(4(k+1)-3)

   = k(2k-1) +(4(k+1)-3)

   $=2k^2-k+4k+4-3 =2k^2+3k+1$

   =2k(k+1)+(k+1)

   =(k+1)(2k+1)

   =(k+1)(2(k+1)-1)= the right part of statement

   So it can be concluded that the claim is true

3. 12^n-2^n is divisible by 5 for any n >0

   (1) For n=1: 12^1-2^1=10, it is divisible by 5;

   (2) Assuming the claim is true for n=k:

Namely 12^k-2^k is divisible by 5;

(3) For n=k+1:

12^{k+1}-2^{k+1}

$=12*12^k -2*2^k$

$=(10+2)*12^k-2*2^k$

$=10*12^k+2*12^k -2*2^k$

$=10*12^k+2*(12^k - 2^k)$

Since 10 is divisible by 5, then $10*12^k$ is divisible by 5, based on assumption in step (2), $(12^k - 2^k)$ is divisible by 5, therefore 12k+1-2k+1 is also divisible by 5.

According to previous analysis, the statement is true.

4. 4+8+12+…+4n=2n(n+1) for any n>0

(1) For n=1:

left side of claim:4*1=4;

right side of claim: 2*1*(1+1)=4;

left side = right side, so the claim is true;

(2)Assuming the claim is true for n=k:

Namely statement 4+8+12+…+4k=2k(k+1) is true;

(3)For n=k+1:

Based on previous assumption

The left side of statement:

4+8+12+…+4k+4(k+1)

=2k(k+1)+4(k+1)

=2(k+1)(k+2)

=2(k+1)((k+1)+1)

=right side of statement

According to previous analysis, the claim is true.

5. 2^{2n} -1 is divisible by 3 for any n >0

(1)For n=1: 2^{2*1}-1=3, it is divisible by 3;

(2)Assuming the claim is true for n=k:

Namely 2^{2k}-1 is divisible by 3;

(3)For n=k+1:

$2^{2(k+1)}-1$

$=2^2*2^{2k} -1$

$=4*2^{2k} – 4+3$

$=4*(2^{2k}-1)+3$

Based on the assumption in step (2), $4*(2^{2k}-1)$ is divisible by 3, and it is obviously that 3 is divisible by 3, since both $4*(2^{2k}-1)$ and 3 are divisible by 3, then $4*(2^{2k}-1)+3$ is divisible by 3, therefore $2^{2n}-1$ is divisible when n=k+1.

Based on previous analysis, it can be concluded that $2^{2n}-1$ is divisible by 3, the claim is true

6. $2+6+10+....+2(2n-1)=2n^2$ for any n>0

(1)For n=1:

left side of claim:$2(2*1-1)=2$;

right side of claim: $2*1^2=2$;

left side = right side, so the claim is true;

(2)Assuming the claim is true for n=k:

Namely the statement $2+6+10+...+2(2k-1)=2k^2$ is true;

(3)For n=k+1:

Based on previous assumption

The left side of statement:

$2+6+10+\ldots+2(2k-1)+2(2(k+1)-1)$

$=2k^2+2(2(k+1)-1)$

$=2k^2+4k+4-2$

$=2(k^2+2k+1)$

$=2(k+1)^2$

=right side of statement

According to previous analysis, the claim is true.

7. Please simplify ((pΛ f) v q) Λ t and prove correctness of your simplification. Use truth tables or reasoning by contradiction

Solution:

According to distribution law, there is

((pΛ f) v q) Λ t= ((pΛ f) Λ t) ∨ (q Λ t)=(p Λ f Λ t) ∨(q Λ t)

Using truth table to prove that the previous two statements are logically equivalent as following:

((pΛ f) v q) Λ t and (p Λ f Λ t) ∨(q Λ t)

p	q	f	t	pΛ f	(pΛ f) v q	((pΛ f) v q) Λ t
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	0	0	0

0	0	1	1	0	0	0
0	1	0	0	0	1	0
0	1	0	1	0	1	1
0	1	1	0	0	1	0
0	1	1	1	0	1	1
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	1	0
1	0	1	1	1	1	1
1	1	0	0	0	1	0
1	1	0	1	0	1	1
1	1	1	0	1	1	0
1	1	1	1	1	1	1

(p Λ f Λ t) v(q Λ t)

p	q	f	t	pΛ f	pΛ f Λ t	q Λ t	(p Λ f Λ t) v(q Λ t)
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	0	1	0	0	1	1
0	1	1	0	0	0	0	0
0	1	1	1	0	0	1	1
1	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	1	0	1
1	1	0	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	0	1	0	0	0
1	1	1	1	1	1	1	1

The shaded columns in previous two truth table have same truth value, therefore the two statements are logically equivalent.

8. Please simplify (¬pΛp) v (¬q v ¬q) and prove correctness of your simplification Use truth tables or reasoning by contradiction

Solution: according to the truth table of "and" and "not":
¬pΛp= 0,
then (¬pΛp) v (¬q v ¬q) = 0 v (¬q v ¬q),
since the truth value of 0 v (¬q v ¬q) is depend on the truth value of (¬q v ¬q), there is
so (¬pΛp) v (¬q v ¬q) = 0 v (¬q v ¬q)=¬q v ¬q;
in the statement (¬q v ¬q), ¬q is same as ¬q, according to truth table of "or" operation, there is
¬q v ¬q= ¬q,
Therefore, the original statement can be simplified as ¬q;
Namely ,(¬pΛp) v (¬q v ¬q)= ¬q

The truth table used to prove the correctness of previous result is shown as following:

p	q	¬p	¬pΛp	¬q	¬q v ¬q	(¬pΛp) v (¬q v ¬q)
0	0	1	0	1	1	1
0	1	1	0	0	0	0
1	0	0	0	1	1	1
1	1	0	0	0	0	0

The shaded columns have same values, so the simplification result is correct.

9. Please write down the following statement as a Boolean formula: "if p implies q, then negation of q implies negation of p". Does your Boolean formula express a logical law? Please prove correctness of your answer. Use truth tables or reasoning by contradiction
Solution: "p implies q" can be expressed as p=> q,
"Negation of q implies negation of p" can be presented as ¬q => ¬p
Then statement "if p implies q, then negation of q implies negation of p" can be expressed as (p=> q) => (¬q => ¬p)
This Boolean formula expressed a logical law, it can be proved using truth table.

p	q	p=> q	¬q	¬p	¬q => ¬p	(p=> q) => (¬q => ¬p)
0	0	1	1	1	1	1
0	1	1	0	1	1	1
1	0	0	1	0	0	1
1	1	1	0	0	1	1

From the truth table above, it can be seen that the columns with yellow shade always have same truth values in different cases and the column with blue shade is always true, so the statement (p=> q) => (¬q => ¬p) is always true. Therefore the conclusion that statement (p=> q) => (¬q => ¬p) expressed a logical law is correct.

10. Please write down the following statement as a Boolean formula: "**IF** p implies q, and q implies r, and not s implies not r and s implies m, **THEN** not m implies not p. Use truth tables or reasoning by contradiction

Solution: "p implies q" can be expressed as p=>q;
"q implies r" as q=> r;
"not s implies not r" as ¬s=>¬r;
"s implies m" as s=>m;
"not m implies not p" as ¬m=>¬p;

Then the statement can be expressed as following Boolean formula
((p=>q)∧(q=> r)∧(¬s=>¬r)∧(s=>m)) => (¬m=>¬p)

Proof by contradiction:

(1) Assume ((p=>q)∧(q=> r)∧(¬s=>¬r)∧(s=>m))=>(¬m=>¬p) is false, namely (((p=>q)∧(q=>r)∧(¬s=>¬r)∧(s=>m))=>(¬m=>¬p))=0
(2) ((p=>q)∧(q=>r)∧(¬s=>¬r)∧(s=>m))=1 and (¬m=>¬p)=0 (from (1) and the definition of implication)
(3) (p=>q)=1 and (q=>r)=1 and (¬s=>¬r)=1 and (s=>m)=1 and m=0 and p=1 (from (2) and definition of conjunction and implication and negation)
(4) ①From (p=>q)=1and p=1, according to the definition of implication, then q=1;
②From (q=>r)=1 and q=1, based on the definition of implication, then r=1
③From (¬s=>¬r)=1 and r=1, according to the definition of implication and negation, there are s=1
④From (s=>m)=1 and s=1, based on the definition of implication, There are m=1;
(5) there is contradiction between (3) (m=0) and (4) (m=1), then the assumption (1) is false, and ((p=>q)∧(q=> r)∧(¬s=>¬r)∧(s=>m))=>(¬m=>¬p) is always true

11. Please provide your own example how you can simplify a Boolean formula using De-Morgan law $\neg(A \land B)$ is equivalent to $(\neg A \lor \neg B)$. You may also other laws, like double negation or whatever you might need.
Example: $\neg(\neg (A \lor C) \land \neg B) = \neg(\neg(A \lor C)) \lor \neg(\neg B) = (A \lor C) \lor B = A \lor C \lor B$

12. Please provide your own example how you can simplify a Boolean formula using De-Morgan law $\neg(A \lor B)$ is equivalent to $(\neg A \land \neg B)$. You may also other laws, like double negation or whatever you might need.
Example: $\neg(A \lor \neg B) \land \neg B$

$$= (\neg A \land \neg(\neg B)) \land \neg B$$
$$= (\neg A \land B) \land \neg B$$
$$= \neg A \land B \land \neg B \quad (\text{since } B \land \neg B = 0)$$
$$= \neg A \land 0$$
$$= 0$$