6) Assume that you have a TM such that the tape goes to infinity in both directions. At some point the tape is completely blank except for a cell with has the symbol \$. The head is at some blank cell somewhere. You want to find the \$ and transition to state p. How would you do it?

a) With a NTM?

b) With a DTM? (4points)

Solution: since the TM go on tape infinitely in both directions, the symbol \$ on the tape when head is at some blank cell somewhere, we can not decide symbol is on which side of current location of head, if we search deterministicly on one side of head, it is possible to search infinitely on this direction if symbol \$ is not on this side of head. In order to find symbol \$ on the tape and transit to state p without trapping into infinite search, we should search on both side of head undeterministicly from current location of head

5) Define $MIN(L) = \{x \in L | xw \notin L\ for\ \forall w\}$

Show that CFL are not closed under MIN

Hint: you can use $\{0^i 1^j 2^k | i \le k, or j \le k\}$ (7points)

Solution: we can give a counter example to prove that CFL is not closed under MIN(L), language L=$\{0^i1^i$ | i greater than or equal to 0$\}$is CFL, language M=$\{0^i1^i2^i$ | i is greater than or equal to 0$\}$ is belong to MIN(L), since M=$0^i1^i2^i$=xw, where x= 0^i1^i belong to L and w=2^i , and xw=$0^i1^i2^i$ do not belong to L, according example 2.36 in the text book, M is not CFL. Since L is CFL and M belong to MIN(L) and M is not CFL, so CFL is not closed under MIN

4) We showed in class that for CFGs there is a Chomsky normal form, where all rules are of the form
A →BC
or A→a

Where A,B,C are variables and 'a' is a terminal. The only epsilon production possible was from the start symbol.

Now, can we do the same thing with all production in the following form?
A →BCD
or A→a

Where A,B,C,D are variables and 'a' is a terminal. The only epsilon production possible was from the start symbol.

How is it possible, or why isn't it possible. Please prove either way. (4points)

Solution: Chomsky normal form can not be in expressed as productions in the following form:

A →BCD

or A→a

Since Chomsky form is one of simplest form of CFG, in the rules of A →BC or A→a , each substitution step can only have one or two branches **which are possible smallest branches on each substitution step; the rules of** A →BCD or A→a, however, have one or three branches on each substation step, which is definitely not the simplest form of CFG.

3) Let $L_1, L_2, ..., L_k$ be a collection of languages over the alphabet Σ. such that

1. **For all $i \neq j$, $L_i \cap L_j = \emptyset$**
2. **$L_1 \cup L_2 \cup ... L_k = \Sigma^*$**
3. **Each L_i is recognizable**

Prove that each language is decidable.

Hint: you can use the other languages to prove a specific language is decidable. (10 points)

Solution:

because each language L_i (i=1,2….k) is recognizable, there are several cases:

(1) if L_i is DFA recognizable, namely L_i is regular, then there is a DFA B_i to recognize it, and according to theorem4.1 Language A_{DFA} is decidable, let TM M_i be the Turing machine which decide A_{DFA} , constructs a TM T_i as following:

T_i= "on input w, where $w \in L_i$ and L_i is a language over alphabet Σ, L_i is recognizable by DFA B_i"

1. Run M_i on input<B_i, L_i> ;
2. if Turing Machine M_i accepts, accept; if it rejects, reject.

Since TM T_i can decide L_i, according Definition 3.6, language L_i is decidable.

(2) If L_j is NFA recognizable, namely L_j is regular, then there is a NFA F_j recognizes it. According Theorem 4.2, A_{NFA} is decidable, Let TM N_j decide A_{NFA}, we design TM T_j that can decide L_i as following:

T_j= "on input w, where $w \in L_j$ and L_j is a language over alphabet Σ and is recognizable by NFA F_j"

1. Run N_j on input<F_j, L_j> ;
2. if Turing Machine T_j accepts, accept; if it rejects, reject.

Since T_j can decide L_j, according Definition 3.6, language L_j is decidable.

(3) if L_k is CFG recognizable, then it is CFL, according to Theorem 4.9, L_k is decidable.

Combine all previous cases, it can be concluded that each language Li is decidable.

2) Are deciders closed under intersections? (3point)

Prove that they are, or show how they are not.

Solution: deciders are closed under intersection. Let A_1 are decidable language and TM M_1 decides it; and A_2 is decidable language and TM M_2 decides it; let $A_3=A_1 \cap A_2$, now design a TM M as following:

TM M= "on input *w*, where $w \in A_3$,

1. Simulate M_1 on *w*, if M_1 accepts *w*, go to step 2, if M_1 rejects *w*, REJECT;
2. Simulate M_2 on *w*, if M_2 accepts *w*, ACCEPT, if M_2 rejects *w*, REJECT;

Since TM M can decide $\forall w \in A_3$, then A_3 is decidable, therefore deciders are closed under intersection.

1) Give a context-free grammar for $\{a^i b^j | i \leq j \leq 2i\}$ (2point)

S=aSb|aSbb|ε