$A+A^{-1})A$

A^2+E

$A+dA^{-1}+E$

Q2

(2)

$$\begin{vmatrix} 9 & 5 & 4 & 3 \\ 4 & 18 & 2 & 8 \\ 6 & -4 & 3 & -2 \\ 9 & 13 & 4 & 7 \end{vmatrix} \xrightarrow{4行-1行} \begin{vmatrix} 9 & 5 & 4 & 3 \\ 4 & 18 & 2 & 8 \\ 6 & -4 & 3 & -2 \\ 0 & 8 & 0 & 4 \end{vmatrix}$$

$$\xrightarrow[4行\times\frac{1}{4}]{2行\times\frac{1}{2}} 8\begin{vmatrix} 9 & 5 & 4 & 3 \\ 2 & 9 & 1 & 4 \\ 6 & -4 & 3 & -2 \\ 0 & 2 & 0 & 1 \end{vmatrix}$$

$$\xrightarrow{3行-2行\times 3} 8\begin{vmatrix} 9 & 5 & 4 & 3 \\ 2 & 9 & 1 & 4 \\ 0 & -31 & 0 & -14 \\ 0 & 2 & 0 & 1 \end{vmatrix} = 8\left(9\begin{vmatrix} 9 & 1 & 4 \\ -31 & 0 & -14 \\ 2 & 0 & 1 \end{vmatrix} - 2\right.$$

$$= 8(27-24)$$

$$= \underline{24}_{//}$$

$$2\begin{vmatrix} 9 & 5 & 4 & 3 \\ 2 & 9 & 1 & 4 \\ 6 & -4 & 3 & -2 \\ 9 & 13 & 4 & 7 \end{vmatrix} = 2\begin{vmatrix} 9 & 5 & 4 & 3 \\ 2 & 9 & 1 & 4 \\ 6 & -4 & 3 & -2 \\ 0 & 8 & 0 & 4 \end{vmatrix}$$

$$= 8\begin{vmatrix} 9 & 5 & 4 & 3 \\ 2 & 9 & 1 & 4 \\ 6 & -4 & 3 & -2 \\ 0 & 2 & 0 & 1 \end{vmatrix}$$

$$= 8\begin{vmatrix} 9 & -1 & 4 & 3 \\ 2 & 1 & 1 & 4 \\ 6 & 0 & 3 & -2 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$= 8\begin{vmatrix} 9 & -1 & 4 \\ 2 & 1 & 1 \\ 6 & 0 & 3 \end{vmatrix} = 24\begin{vmatrix} 9 & -1 \\ 2 & 1 \\ 2 & 0 \end{vmatrix}$$

$$= 24\begin{vmatrix} 1 & -1 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 24\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 2$$

$A^2 = dA + A^{-1}$ $\quad a^2+bc = ad+d$

$\therefore a^2 = \underbrace{ad-bc}_{=1} + d$

$\therefore \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix} = \begin{pmatrix} ad+d & bd-b \\ cd-c & d^2+a \end{pmatrix}$

各成分比較し、$ad-bc=1$に注意して

$\begin{cases} a^2 = d+1 \\ ab = -b \\ ac = -c \\ a = bc \end{cases}$

← ここで、$b=0$のとき $a=b=0$となり$ad-bc\neq 1$なので $b\neq 0$といえる。

$ab=-b$より$a=-1$、$a^2=$より$d=0$

$A^3 = A^2A = (dA+A^{-1})A$
$= dA^2+E$
$= d^2A+dA^{-1}+E$
$= \begin{pmatrix} ad^2+d^2+1 & d^2b-db \\ d^2c-dc & d^3+ad+1 \end{pmatrix}$

$a=-1, d=0$を用いて

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ //

Q5 A^{-1}か

$A^2=A$

$1+x=$

よって

$(1+x)-$

$A^2=((($

$A^2=A$よ

Q5 A^{-1}が存在すると仮定

$A^2=A$ より $A=E$

$1+x=1$ かつ $2+x=0$

よって矛盾.

$(1+x)-(1+y)(2+x)=0$

$A^2=((1+x)+1)A$

$A^2=A$ より $(x+2)A=A$

$(x+1)A=O$

$A \neq O$ より

$x+1=0$

$x=-1$

$y=-1$

Jul.12.2024

Exercises In Engineering§16

Reference No.(NOT Student ID No.) 24 Name 真庭正宗

$96 \to 2$

$\cancel{3+4}$ + + + +

Q1 Find the inverse of $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ by using the cofactors.

$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

$A_{11} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3$ $A_{21} = \begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} = 0$ $A_{31} = \begin{vmatrix} 0 & 0 \\ 2 & 1 \end{vmatrix} = 0$

$|A| = 1 \times 3 + 0 + 0 = 3$

$A_{12} = \begin{vmatrix} 0 & 1 \\ 2 & 2 \end{vmatrix} = (-1)(-2) = 2$ $A_{22} = \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} = 2$ $A_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1$

$\therefore A^{-1} = \frac{1}{3}\begin{pmatrix} 3 & 0 & 0 \\ 2 & 2 & -1 \\ -4 & -1 & 2 \end{pmatrix}$

$A_{13} = \begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = -4$ $A_{23} = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = -1$ $A_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2$

Q2 Find the values of the following determinats :

(1) $\begin{vmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{vmatrix} = -\begin{vmatrix} -1 & 2 & -1 & 0 \\ 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{vmatrix} \overset{\text{1}\times 2}{=} -\begin{vmatrix} -1 & 2 & -1 & 0 \\ 0 & 3 & -2 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{vmatrix}$

$= \begin{vmatrix} 3 & -2 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix}$

$= (12) - (4 + 3) = 5$

$9 - \frac{10}{9} = \frac{71}{9}$

-27

(2) $\begin{vmatrix} 9 & 5 & 4 & 3 \\ 4 & 18 & 2 & 8 \\ 6 & -4 & 3 & -2 \\ 9 & 13 & 4 & 7 \end{vmatrix} = 2\begin{vmatrix} 9 & 5 & 4 & 3 \\ 2 & 9 & 1 & 4 \\ 6 & -4 & 3 & -2 \\ 0 & 8 & 0 & 4 \end{vmatrix} = 2\begin{vmatrix} 9 & 5 & 4 & 3 \\ 0 & \frac{71}{9} & \frac{1}{9} & \frac{30}{9} \\ 0 & -31 & 0 & -14 \\ 0 & 8 & 0 & 4 \end{vmatrix}$

80

32

-1

120

$$\begin{array}{r} 124 \\ 112 \\ \hline 12 \end{array}$$

$= 2 \cdot \begin{vmatrix} 71 & 1 & 30 \\ -31 & 0 & -14 \\ 8 & 0 & 4 \end{vmatrix}$

$= 2\{(-112) - (-124)\} = 2 \times 12 = 24$

Q3 Matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is satisfied with the equation $\cancel{A^2 = dA = A^{-1}}$. Find A^3 where $ad - bc = 1$.

$A^2 = dA + A^{-1}$

$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$A^2 = dA + A^{-1}$

両辺に左から A をかける

$A^3 = AdA + E$

$= \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} ad & bd \\ cd & d^2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} a^2d + bcd & abd + bd^2 \\ acd + cd^2 & bcd + d^3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$A^3 = \begin{pmatrix} a^2b + bcd + 1 & abd + bd^2 \\ acd + cd^2 & bcd + d^3 + 1 \end{pmatrix}$

Q4 Answer the following questions for the matrix $A = \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ \cancel{0} & \cancel{0} & \cancel{c} \end{pmatrix}$.

(1) Find the condition that A is regular.

$$|A| = adc - bc^2 = c(ad - bc)$$

$\therefore$ $c \neq 0$ ~~または~~ $ad - bc \neq 0$

$c(ad-bc) \neq 0$

(2) Find the inverse A^{-1} when A is regular.

$A_{11} = \begin{vmatrix} d & 0 \\ 0 & c \end{vmatrix} = dc$ $A_{21} = \begin{vmatrix} b & 0 \\ 0 & c \end{vmatrix} = bc$ $A_{31} = \begin{vmatrix} b & 0 \\ d & 0 \end{vmatrix} = 0$

$A_{12} = \begin{vmatrix} c & 0 \\ 0 & c \end{vmatrix} = c^2$ $A_{22} = \begin{vmatrix} a & 0 \\ 0 & c \end{vmatrix} = ac$ $A_{32} = \begin{vmatrix} a & 0 \\ c & 0 \end{vmatrix} = 0$

$A_{13} = \begin{vmatrix} c & d \\ 0 & 0 \end{vmatrix} = 0$ $A_{23} = \begin{vmatrix} a & b \\ 0 & 0 \end{vmatrix} = 0$ $A_{33} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$$A^{-1} = \frac{1}{c(ad-bc)} \begin{pmatrix} dc & -bc & 0 \\ -c^2 & ac & 0 \\ 0 & 0 & ad-bc \end{pmatrix} = \frac{1}{ad-bc} \begin{pmatrix} d & -b & 0 \\ -c & a & 0 \\ 0 & 0 & \frac{ad-bc}{c} \end{pmatrix}$$

Q5 Matrix $A = \begin{pmatrix} 1+x & 1+y \\ 2+x & 1 \end{pmatrix}$ is satisfied with the equation $A^2 = A$. Find x and y.

$$A^2 = \begin{pmatrix} 1+x & 1+y \\ 2+x & 1 \end{pmatrix}\begin{pmatrix} 1+x & 1+y \\ 2+x & 1 \end{pmatrix} = \begin{pmatrix} (1+x)^2 + (1+y)(2+x) & (1+x)(1+y) + 1+y \\ (2+x)(1+x) + 2+x & (2+x)(1+y) + 1 \end{pmatrix}$$

$$= \begin{pmatrix} (1+x)\left\{(1+x) + (1+y)\frac{2+x}{1+x}\right\} & (1+y)\{(1+x)+1\} \\ (2+x)\{(1+x)+1\} & 1\{(2+x)(1+y)+1\} \end{pmatrix}$$

$$\therefore \begin{cases} 1+x+(1+y)\dfrac{2+x}{1+x} = 1 \\ x+2 = 1 \\ x+2 = 1 \\ (2+x)(1+y)+1 = 1 \end{cases} \Rightarrow \begin{aligned} x &= -1 \\ (1)(1+y)+1 &= 1 \\ y+2 &= 1 \\ y &= -1 \end{aligned}$$

A. $x = -1$, $y = -1$