Exercises In Engineering§9

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Q1 Find the values of the following definete integrals:

(1) $\displaystyle\int_0^3 \sqrt{3-t}\,dt$

$\dfrac{dx}{dt} = -1 \quad dt = -dx$

$x = 3-t$ とする.

$\displaystyle\int_3^0 -\sqrt{x}\,dx = \left[-\dfrac{2}{3}x^{\frac{3}{2}}\right]_3^0 = (0) - \left(-\dfrac{2}{3} \times 3\sqrt{3}\right)$

$= \underline{2\sqrt{3}} \#$

(2) $\displaystyle\int_{-1}^1 x^2\sqrt{1+x^3}\,dx$

$t = 1+x^3, \quad \dfrac{dt}{dx} = 3x^2, \quad dx = \dfrac{dt}{3x^2}$

$\displaystyle\int_0^2 \sqrt{t} \times \dfrac{1}{3x^2}\,dt \times x^2 = \dfrac{1}{3}\int_0^2 \sqrt{t}\,dt = \dfrac{1}{3}\left[\dfrac{2}{3}t^{\frac{3}{2}}\right]_0^2 = \dfrac{1}{3}\left(\dfrac{2}{3}2\sqrt{2} - 0\right)$

$= \underline{\dfrac{4\sqrt{2}}{9}} \#$

(3) $\displaystyle\int_{-2}^2 |x^2-1|\,dx$

$= 2\displaystyle\int_0^2 |x^2-1|\,dx$

$= 2\left\{\displaystyle\int_0^1 1-x^2\,dx + \int_1^2 x^2-1\,dx\right\}$

$= 2\left\{\left[x-\dfrac{1}{3}x^3\right]_0^1 + \left[\dfrac{1}{3}x^3-x\right]_1^2\right\}$

$= 2\left\{\left(1-\dfrac{1}{3}\right) + \left(\dfrac{8}{3}-2\right) - \left(\dfrac{1}{3}-1\right)\right\}$

$= 2\left\{\dfrac{2}{3} + \dfrac{2}{3} + \dfrac{2}{3}\right\}$

$= 2 \times \dfrac{6}{3}$

$= \underline{4} \#$

Q2 Find the constants a, b and c that

$$\int_{-1}^1 f(x)\,dx = af(-1) + bf(0) + cf(1)$$

for any quadratic function $f(x)$.

$f(x) = \alpha x^2 + \beta x + \gamma$ とする.

$\displaystyle\int_{-1}^1 f(x)\,dx = \int_{-1}^1 \alpha x^2 + \beta x + \gamma\,dx$

$= \dfrac{2}{3}\alpha + 2\gamma$

$\Rightarrow \dfrac{2}{3}\alpha + 2\gamma = a(\alpha-\beta+\gamma) + b\gamma + c(\alpha+\beta+\gamma)$

$\begin{cases} \dfrac{2}{3} = a+c \\ 0 = -a+c \\ 2 = a+b+c \end{cases}$

より これより. $\quad a = c = \dfrac{1}{3}$

$b = \dfrac{4}{3}$

$af(-1) = a(\alpha-\beta+\gamma)$

$b(f(0)) = b\gamma$

$cf(1) = c(\alpha+\beta+\gamma)$