10.17

水面の高さをhとする

$h\leqq 1$のとき

$$V(h)=\pi\int_0^h y^2\,dy$$

$$V(h)=\frac{1}{3}\pi h^3$$

$h>1$のとき

$$V(h)=\pi\int_0^1 y^2\,dy+\pi\int_1^h\left(\frac{1}{y}\right)^2 dy$$

$$=\frac{4}{3}\pi-\frac{\pi}{\ }$$

(1) $0\leq t\leq\frac{\pi}{3}$ のとき

t秒後の体積はt

$$t=\frac{1}{3}\pi h^3 \qquad H(t)=\sqrt[3]{\frac{3t}{\pi}}$$

$\frac{\pi}{3}<t<\frac{4}{3}\pi$のとき

$$t=\frac{4}{3}\pi-\frac{\pi}{h}$$

$$H(t)=\frac{3\pi}{4\pi-3t}$$

(2) $0\leq t\leq\frac{1}{3}\pi$のとき

$$H'(t)=\left(\sqrt[3]{\frac{3}{\pi}}\sqrt[3]{t}\right)'$$

$$H'(t)=\frac{1}{\sqrt[3]{9\pi t^2}}$$

$\frac{\pi}{3}<t<\frac{4}{3}\pi$

$$H'(t)=\frac{-3\pi-3}{(4\pi-3t)^2} \qquad H'(t)=\frac{9\pi}{(4\pi-3t)^2}$$

Q3.

absolut

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absolute

determinat

↓

determinant

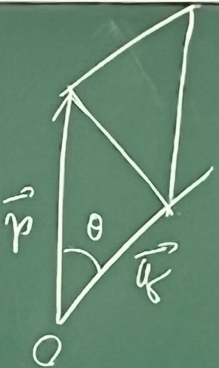
Determine

↓

determine

Q4

$F \to f$



$$S = \frac{2}{2} \cdot |\vec{p}| \cdot |\vec{q}| \cdot \sin\theta$$

$$S^2 = |\vec{p}|^2 \cdot |\vec{q}|^2 \cdot (1 - \cos^2\theta)$$

$$= |\vec{p}|^2 \cdot |\vec{q}|^2 - \underbrace{(|\vec{p}||\vec{q}|\cos\theta)^2}_{\text{内积}}$$

$$= (p_1^2 + p_2^2)(q_1^2 + q_2^2) - (p_1 q_1 + p_2 q_2)^2$$

$$= (p_1 q_2 - p_2 q_1)^2$$

$$\therefore S = \pm (p_1 q_2 - p_2 q_1) = \pm \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} \quad (> 0$$

$$S = \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix}$$

(2) $Ap = \begin{pmatrix} ap_1 + bp_2 \\ cp_1 + dp_2 \end{pmatrix}$, $Aq = \begin{pmatrix} aq_1 + bq_2 \\ cq_1 + dq_2 \end{pmatrix}$

$$S' = \left| \begin{vmatrix} Ap & Aq \end{vmatrix} \right|$$

$$= \left| (ap_1 + bp_2)(cq_1 + dq_2) - (aq_1 + bq_2)(cp_1 + dp_2) \right|$$

$$= \left| adp_1q_2 - bcp_1q_2 + bcp_2q_1 - adp_2q_1 \right|$$

$$= \left| ad - bc \right| (p_1q_2 - p_2q_1)$$

0)

元と等しくなるためには、$|ad - bc| = 1$,

よって $\det A = \pm 1$ //

$$|AB| = |A||B|$$

$$\left| \begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} \right| = \left| \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} \right|$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} = \pm \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \pm 1$$

3.

solut
↓
solute

terminat
↓
terminant

Determine
↓
determine

Q4

$F \to f$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ m \end{pmatrix} = \begin{pmatrix} 1 \\ m \end{pmatrix}$$

$a + bm = 1$

$c + dm = m$

$b = \dfrac{1-a}{m}$

$c = m(a-1)$

$d = 2 - a$

$ad - bc = 1$

lと並行でPを通る直線 $y = m(x-1)$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x' \\ m(x'-1) \end{pmatrix}$$

$c = m(a-1)$

$-bp_2$ / p_2), $Aq =$

$np_2)(cq_1 +$

$-bcp_1q$

$(p_1q_2$

3たび

det

$Q(x_0, y_0)$ としての像 $Q'(x_0', y_0')$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x_0' \\ y_0' \end{pmatrix}$$

$$x_0' = ax_0 + \frac{1-a}{m}y_0$$

$$y_0' = m(a-1)x_0 + (2-a)y_0$$

Qを通り l と平行な直線

$y - y_0 = m(x - x_0)$ に x_0' を代入すると

$$y = m(a-1)x_0 + (2-a)y_0$$

Jul.5.2024

Exercises In Engineering§14, §15

Reference No.(NOT Student ID No.) 24 Name 真鹿 正宗

Q1 Let $B_1 = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$, $B_2 = \begin{pmatrix} \cos\theta & \sin\theta \end{pmatrix}$, $C_1 = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$, $C_2 = \begin{pmatrix} -\sin\theta & \cos\theta \end{pmatrix}$. Moreover a and b is real numbers but not zero, A is square matrix of order 2 and $A = aB_1B_2 + bC_1C_2$.

(1) Find A^n.

$$A = \begin{pmatrix} a\cos^2\theta + b\sin^2\theta & (a-b)\cos\theta\sin\theta \\ (a-b)\cos\theta\sin\theta & a\sin^2\theta + b\cos^2\theta \end{pmatrix}$$

$$A^2 = \begin{pmatrix} a^2\cos^2\theta + b^2\sin^2\theta & (a^2-b^2)\cos\theta\sin\theta \\ (a^2-b^2)\cos\theta\sin\theta & a^2\sin^2\theta + b^2\cos^2\theta \end{pmatrix}$$

帰納法で示せる

$$A^n = a^n B_1B_2 + b^n C_1C_2 \quad //$$

(2) Find the inverse A^{-1}.

$$|A| = ab$$

$$\therefore A^{-1} = \frac{1}{ab}\begin{pmatrix} a\sin^2\theta + b\cos^2\theta & (b-a)\cos\theta\sin\theta \\ (b-a)\cos\theta\sin\theta & a\cos^2\theta + b\sin^2\theta \end{pmatrix} \quad //$$

Q2 Find the following matrix for the 2×2matrix $A = \begin{pmatrix} \alpha & 1-\alpha \\ 0 & 1 \end{pmatrix}$ $(0 < \alpha < 1)$:

(1) A^{-1}

$0-\alpha$

$$\begin{pmatrix} \alpha & 1-\alpha \\ 0 & 1 \end{pmatrix}^{-1} = \frac{1}{\alpha}\begin{pmatrix} 1 & \alpha-1 \\ 0 & \alpha \end{pmatrix} = \begin{pmatrix} -\frac{1}{\alpha} & \frac{1}{\alpha}-1 \\ 0 & -1 \end{pmatrix}$$

$$\frac{1}{\alpha}\begin{pmatrix} 1 & \alpha-1 \\ 0 & \alpha \end{pmatrix} =$$

(2) $A^2 - (\alpha+1)A + \alpha E$

$$= \begin{pmatrix} \alpha & 1-\alpha \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \alpha & 1-\alpha \\ 0 & 1 \end{pmatrix} - (\alpha+1)\begin{pmatrix} \alpha & 1-\alpha \\ 0 & 1 \end{pmatrix} + \alpha E$$

$$= \begin{pmatrix} \alpha^2 & 1-\alpha^2 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \alpha^2+\alpha & 1-\alpha^2 \\ 0 & \alpha+1 \end{pmatrix} + \alpha E = \begin{pmatrix} -\alpha & 0 \\ 0 & -\alpha \end{pmatrix} + \alpha E = O$$

(3) $\lim_{n\to\infty} A^n$ by deriving A^n

$\alpha^2 - \alpha^3 + 1 - \alpha^2$

$$A^n = \begin{pmatrix} \alpha & 1-\alpha \\ 0 & 1 \end{pmatrix}^n$$

$$\lim_{n\to\infty} A^n = \begin{pmatrix} \alpha^n & 1-\alpha^n \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$