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Exercises In Engineering§10

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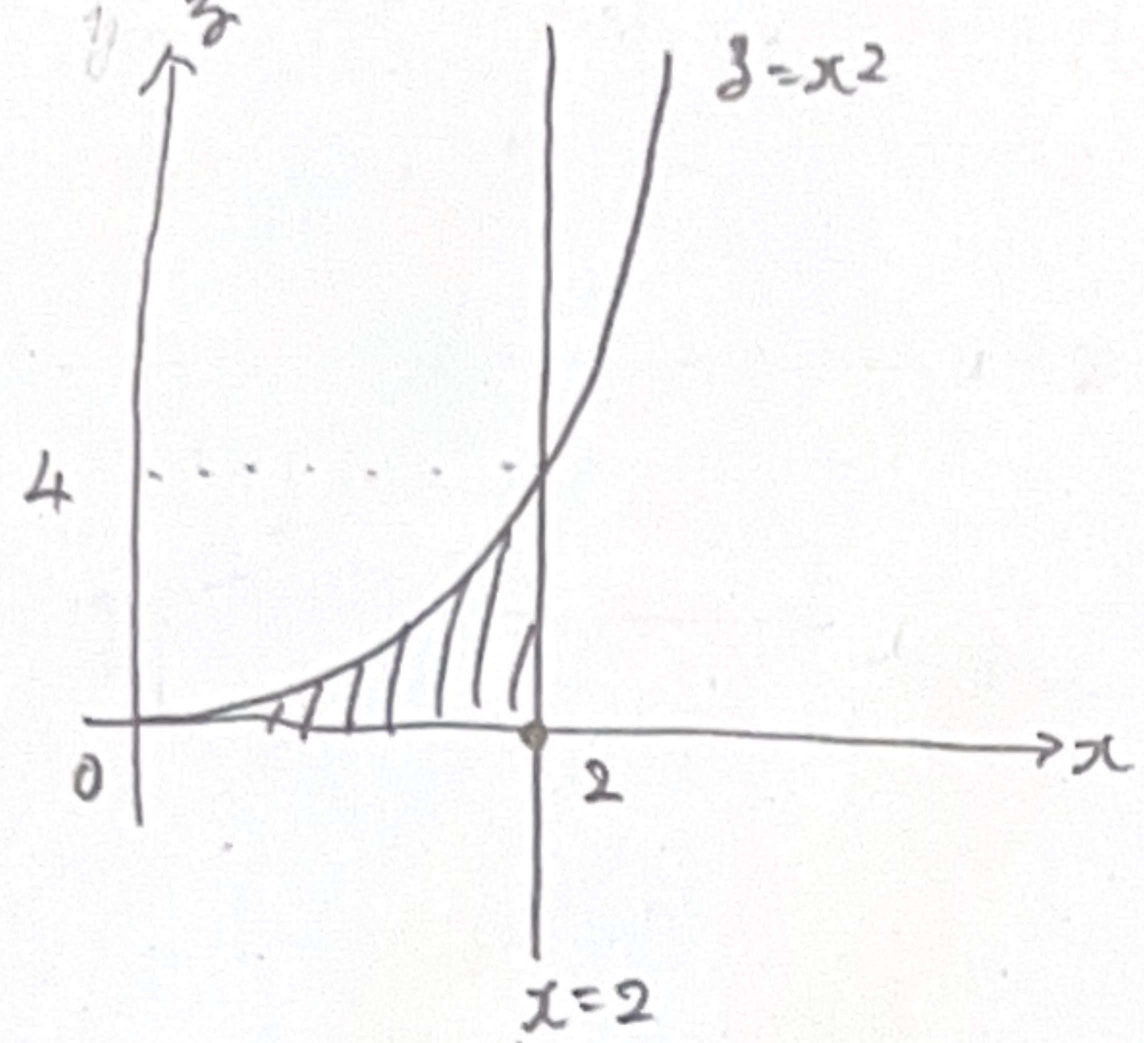
Q1 Find the area of the figure enclosed by the following curves and lines.:

(1) $y = x^2$, $x = 2$, $y = 0$



$$\int_0^2 x^2\,dx = \left[\frac{1}{3}x^3\right]_0^2$$
$$= \frac{8}{3}$$

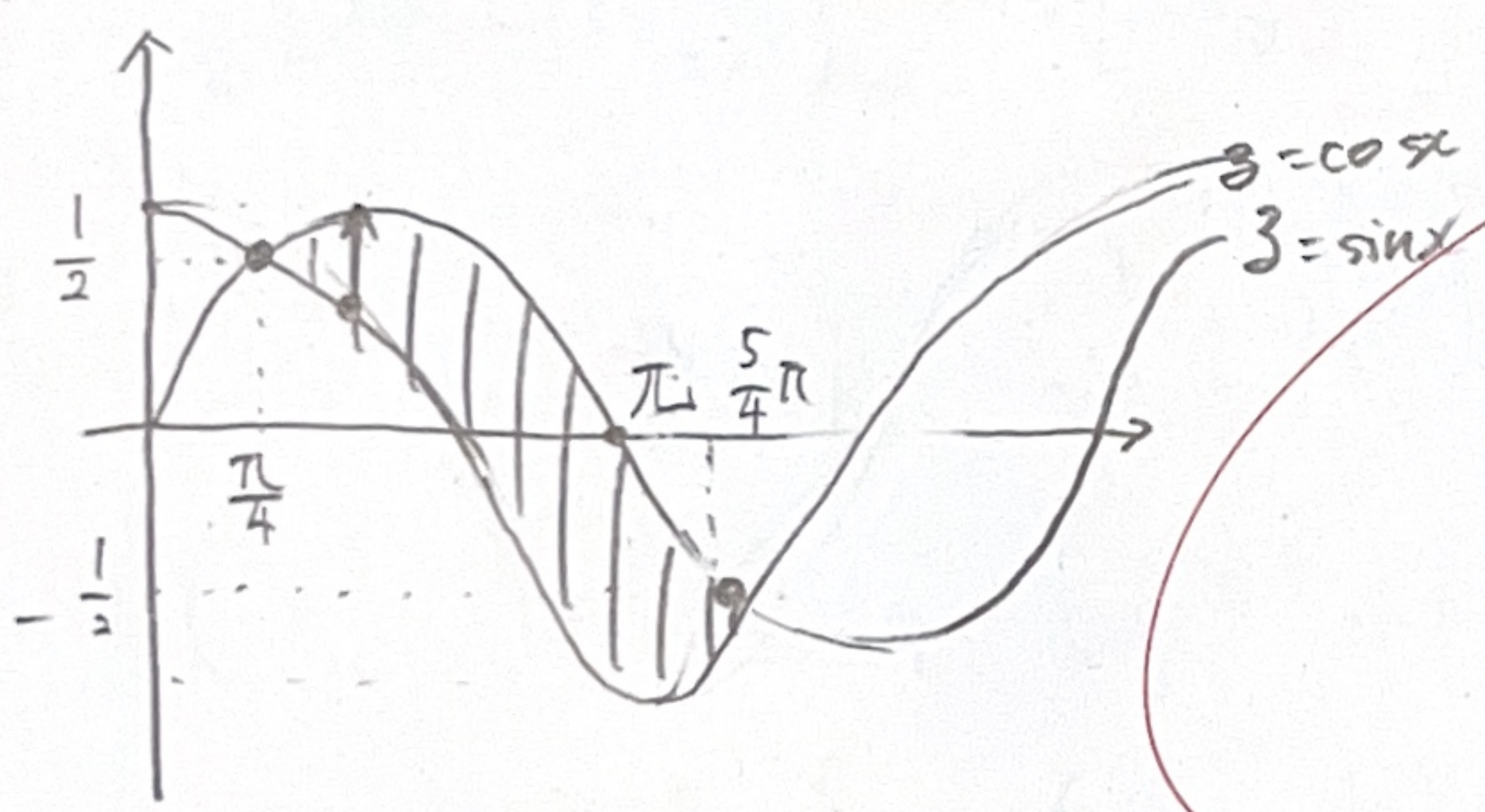
$(\tan x)' = \frac{1}{\cos^2 x}$

$\tan x = \frac{1}{\cos^2 x}$

$(\cos x)^2$

$$\int \cos^2 x\,dx = \int \frac{\cos 2x + 1}{2}\,dx$$
$$= \frac{1}{2}\left[\frac{1}{2}\sin 2x + x\right]$$
$$\frac{1}{4}\sin 2x + \frac{1}{2}x$$

(2) $y = \sin x$, $y = \cos x$, $\left(\frac{\pi}{4} \leq x \leq \frac{5}{4}\pi\right)$

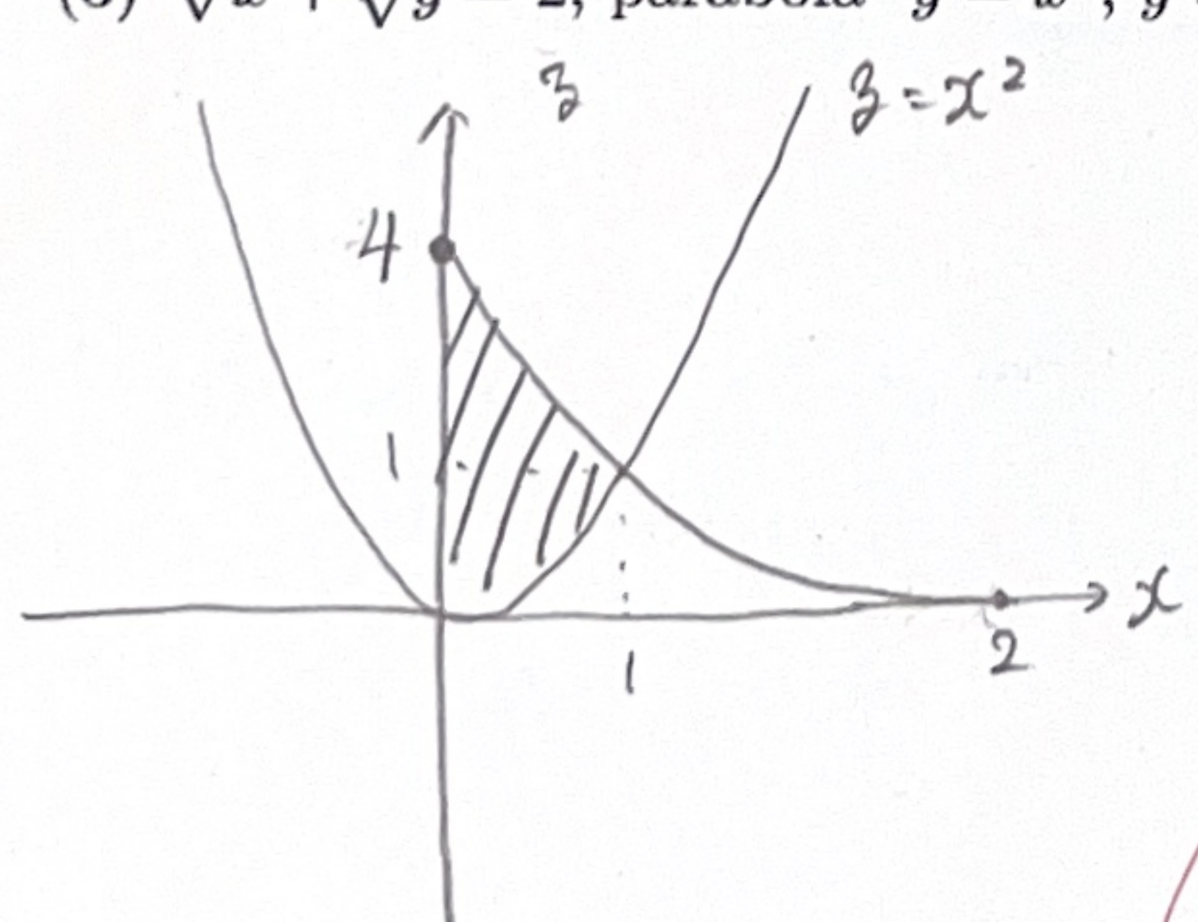


$$\int_{\frac{\pi}{4}}^{\frac{5}{4}\pi} \sin x\,dx + \int_{\frac{\pi}{4}}^{\frac{5}{4}\pi} \cos x\,dx$$
$$= \left[-\cos x\right]_{\frac{\pi}{4}}^{\frac{5}{4}\pi} + \left[\sin x\right]_{\frac{\pi}{4}}^{\frac{5}{4}\pi}$$
$$= \left(\frac{2}{\sqrt{2}}\right) + \left(\frac{2}{\sqrt{2}}\right)$$
$$= 2\sqrt{2}$$

$4x^{\frac{1}{2}}$

$\sqrt{y} = 2 - \sqrt{x}$ $y = 4 - 4\sqrt{x} + x$ $(x^{\frac{3}{2}})' = \frac{3}{2}x^{\frac{1}{2}}$

(3) $\sqrt{x} + \sqrt{y} = 2$, parabola $y = x^2$, y-axis



$$\int_0^1 (4 - 4\sqrt{x} + x)\,dx - \int_0^1 x^2\,dx$$
$$= \left[4x - \frac{8}{3}x^{\frac{3}{2}} + \frac{1}{2}x^2\right]_0^1 - \left[\frac{1}{3}x^3\right]_0^1$$
$$= \left(4 - \frac{8}{3} + \frac{1}{2}\right) - \left(\frac{1}{3}\right)$$
$$= \left(\frac{24 - 16 + 3 - 2}{6}\right)$$
$$= \frac{9}{6}$$
$$= \frac{3}{2}$$

$\frac{8}{3}$

$\frac{11}{\cancel{8+3}}$

10.15

(1) $y = xe^{x^2}$

$y' = e^{x^2} + x \cdot 2x e^{x^2} = e^{x^2} + 2x^2 e^{x^2} = (2x^2+1)e^{x^2}$

$y - e = (2+1)e \quad (x-1)$

$y = 3e(x-1) + e$

$= 3ex - 2e$

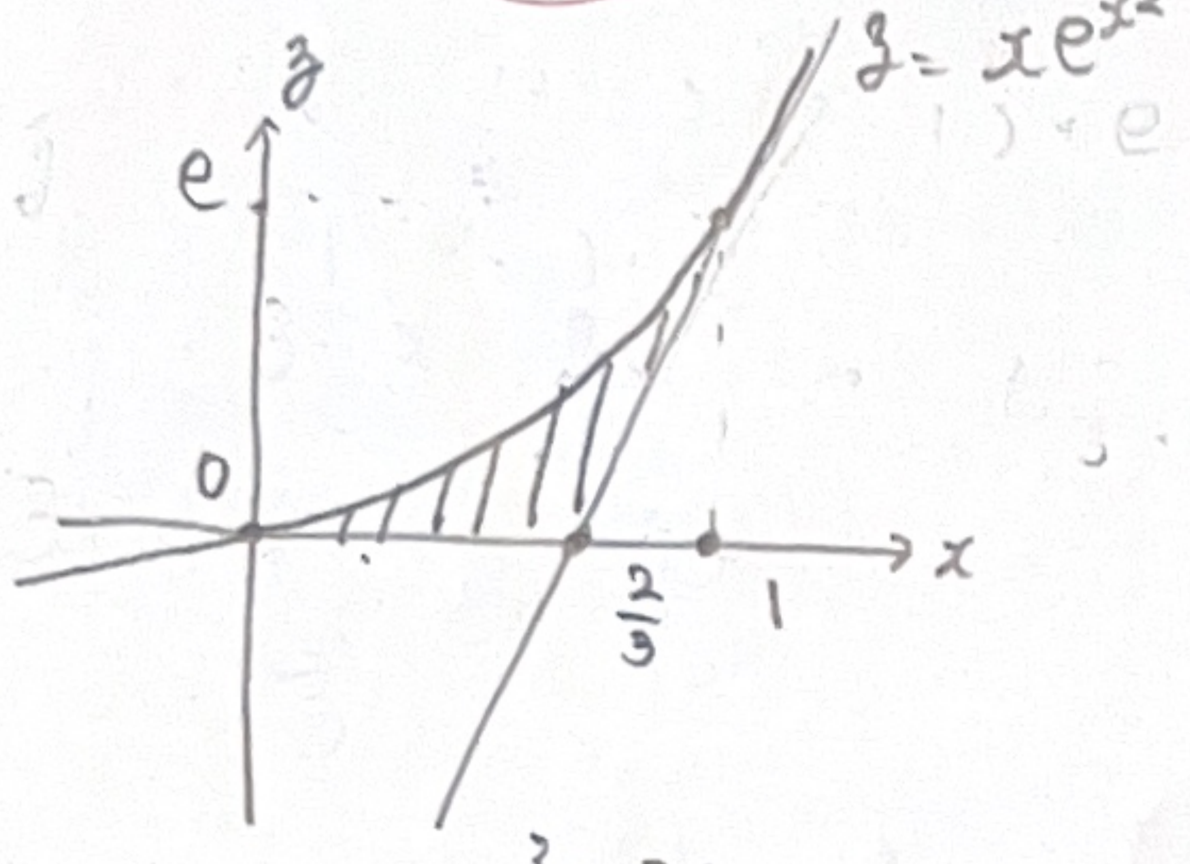
$x - 1 + 2x^3 - 2x^2$

$-12e - 2e$

$(e^{x^2})' = 2xe^{x^2}$

$-4e^{16}$

(2)



$y = 3ex - 2e$

$$\int_0^1 xe^{x^2}\,dx - \int_{\frac{2}{3}}^1 3ex - 2e\,dx$$

$\frac{2}{3}e - \frac{4}{3}$

$-e$

$\frac{3-4}{2}$

$$= \left[\frac{1}{2}e^{x^2}\right]_0^1 - \left[\frac{3}{2}ex^2 - 2ex\right]_{\frac{2}{3}}^1$$

$$= \left(\frac{1}{2}e - \frac{1}{2}\right) - \left\{\left(\frac{3}{2}e - 2e\right) - \left(\frac{3}{2}e \times \frac{4}{9} - \frac{4}{3}e\right)\right\}$$

$$= \left(\frac{1}{2}e - \frac{1}{2}\right) - \left\{-\frac{1}{2}e - \left(-\frac{2}{3}e\right)\right\}$$

$$= \left(\frac{1}{2}e - \frac{1}{2}\right) - \left\{\frac{1}{6}e\right\}$$

$$= \frac{1}{3}e - \frac{1}{2}$$

$-\frac{1}{2} + \frac{2}{3} = \frac{-2+4}{6} = \frac{2}{6}$

$xe^{x^2} = 3ex - 2e$

$\frac{2}{3}e - \frac{4}{3}e$

$\frac{3}{6}$

$-\frac{2}{3}e$

$-\frac{1}{2}e + \frac{2}{3}e$

$= \frac{-3+4}{6}e = \frac{1}{6}e$

$\cancel{\frac{e}{2} - \frac{3}{2}e} + 2e + \cancel{\frac{2}{3}e - \frac{4}{3}e}$

$= -e + 2e - \frac{2}{3}e$

$= e - \frac{2}{3}e$

$= \frac{1}{3}e$

$y-f(a)=f'(a)(x-a)$

$f(x)=xe^{x^2}$

$f'(x)=e^{x^2}+x(e^{x^2})'$

$y=e^{x^2},\ \log y=x^2\log e=x^2$

$\frac{y'}{y}=2x,\ y'=2xe^{x^2}$

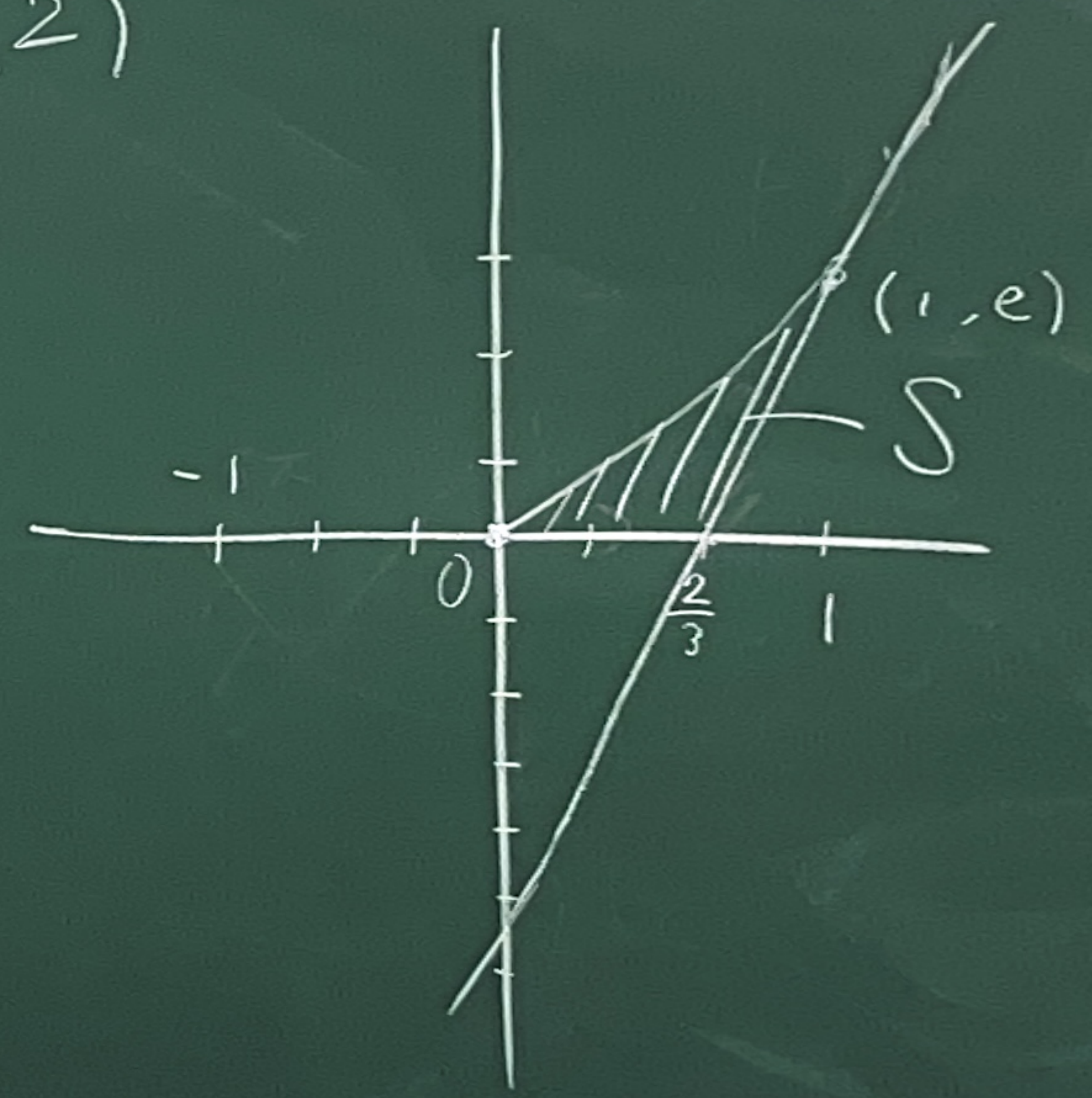
○ $f'(x)=e^{x^2}+2x^2e^{x^2}$

$f(a)=e+2e=3e$

$y-e=3e(x-1)$

$y=3ex-2e$ //

(2)



・$y=3ex-2e$ に代入して

$(0,-2e),(\frac{2}{3},0)$ が求まる

・$y=xe^{x^2}$ に代入して

$(0,0),(1,e)$ が求まる

$S=\int_0^1 xe^{x^2}dx-\int_{\frac{2}{3}}^1(3ex-2e)dx$

$=\left[\frac{1}{2}e^{x^2}\right]_0^1-\left[\frac{3}{2}ex^2-2ex\right]_{\frac{2}{3}}^1$

$=(\frac{1}{2}e-\frac{1}{2})-$

$\{(\frac{3}{2}e-2e)-(\frac{2}{3}e-\frac{4}{3}e)\}$

$=\frac{1}{3}e-\frac{1}{2}$ //