Exercises In Engineering§14, §15

Reference No.(NOT Student ID No.) Name

Q1 Let $B_1 = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$, $B_2 = \begin{pmatrix} \cos\theta & \sin\theta \end{pmatrix}$, $C_1 = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$, $C_2 = \begin{pmatrix} -\sin\theta & \cos\theta \end{pmatrix}$. Moreover a and b is real numbers but not zero, A is square matrix of order 2 and $A = aB_1B_2 + bC_1C_2$.

(1) Find A^n.

(2) Find the inverse A^{-1}.

Q2 Find the following matrix for the 2×2matrix $A = \begin{pmatrix} \alpha & 1-\alpha \\ 0 & 1 \end{pmatrix}$ $(0 < \alpha < 1)$:

(1) A^{-1}

(2) $A^2 - (\alpha + 1)A + \alpha E$

(3) $\lim_{n \to \infty} A^n$ by deriving A^n

Q3 Show that the area S of parallelogram with vectors $\boldsymbol{p} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$ as adjacent sides is equal to the absolut value of the determinat $\begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix}$. Then Determine the conditions that S is equal to the area of parallelogram with vectors $A\boldsymbol{p}$ and $A\boldsymbol{q}$ as adjacent sides, where $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

Q4 Let f be a linear transformation on plane and denoted by a matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, and l be a straight line $y = mx$ $(m \neq 0)$. f is satisfied with the following 2 conditions:

(i) f does not transfer each point on l.

(ii) f transfers the point $P(1, 0)$ to a point a straight line that includes P and parallel to l.

(1) Find $ad - bc$.

(2) Show that F transfers any point Q on the plane to a point on a straight line that includes Q and parallel to l.