

## STA567 HW4

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Model for 110m hurdles	Model Fitting Details	RMSE from LOOCV
Backward Stepwise Regression from AIC	Selected variables list: x100m, long, shotput, high	0.4322
Ridge Regression	tuning parameter value = 0.1	0.4667538
LASSO Regression	tuning parameter value = 0.2684	0.4605
Elastic Net Regression	tuning parameter value is fraction= 0.35, lambda=0.1	0.4517
Principal Component Regression	number of selected components = 4	0.4220
Partial Least Squares Regression	number of selected components = 1	0.4527
(567) Backward Stepwise Regression from RMSE	Selected variables list: x100m, long, high	0.4233

Model for 1500m run	Model Fitting Details	MSE from 5 <sup>th</sup> fold CV
Backward Stepwise Regression	Selected variables list: x100m + long + shotput + x400m	12.5098
Ridge Regression	tuning parameter value = 0.03162	12.3727
LASSO Regression	tuning parameter value = 0.3105	12.2904
Elastic Net Regression	tuning parameter value = fraction=0.3 / lambda= 0	12.3041
Principal Component Regression	number of selected components = 5	12.3967
Partial Least Squares Regression	number of selected components = 2	9.7365
(567) Backward Stepwise Regression from RMSE	Selected variables list: x100m, long, shotput, x400m	12.003

```
setwd("C:\\Users\\linal\\Desktop\\Miami2019\\STA567\\Homework\\Homework4")
load(file="Decathlons.Rdata")
head(london)

library(tidyverse)
library(caret)
```

### Remove missing values

```
london <- london %>%
  select(x110m, x1500m, x100m, long, shotput, high, x400m) %>%
```

```
filter(!is.na(x110m)) %>%
filter(!is.na(x1500m))
```

## (1) Backward Stepwise Regression from AIC

### Model for 110m hurdles

*# Backward Stepwise*

```
mod1 <- lm(x110m ~ .-x1500m,data=london)
```

```
stepBackward <- step(mod1)
```

```
## Start: AIC=-38.93
```

```
## x110m ~ (x1500m + x100m + long + shotput + high + x400m) - x1500m
```

```
##
```

	Df	Sum of Sq	RSS	AIC
- x400m	1	0.02100	3.6874	-40.782
<none>			3.6664	-38.931
- shotput	1	0.39180	4.0582	-38.291
- high	1	0.41787	4.0843	-38.125
- long	1	0.48077	4.1472	-37.727
- x100m	1	1.26339	4.9298	-33.233

```
##
```

```
## Step: AIC=-40.78
```

```
## x110m ~ x100m + long + shotput + high
```

```
##
```

	Df	Sum of Sq	RSS	AIC
<none>			3.6874	-40.782
- shotput	1	0.38049	4.0679	-40.229
- long	1	0.46014	4.1476	-39.725
- high	1	0.50006	4.1875	-39.476
- x100m	1	2.65562	6.3430	-28.679

```
stepBackward
```

```
##
```

```
## Call:
```

```
## lm(formula = x110m ~ x100m + long + shotput + high, data = london)
```

```
##
```

```
## Coefficients:
```

	x100m	long	shotput	high
(Intercept)	1.4761	1.5817	0.6215	-0.1582

*# Cross Validation*

```
set.seed(12345)
```

```
mlr1 <- train(x110m ~ x100m + long + shotput + high,
              data=london,
              method="lm",
              trControl=trainControl(method="cv",number = 5),
              preProcess = c("center", "scale"))
```

```
# RMSE for 5th-fold cross validation
```

```
min(mlr1$results$RMSE)
```

```
## [1] 0.4321686
```

## Model for 1500m run

```
# Backward Stepwise Regression from AIC
```

```
mod2 <- lm(x1500m ~ .-x110m,data=london)
```

```
stepBackward <- step(mod2)
```

```
## Start: AIC=128.63
```

```
## x1500m ~ (x110m + x100m + long + shotput + high + x400m) - x110m
```

```
##
```

```
##           Df Sum of Sq    RSS    AIC
```

```
## - high      1      18.79 2326.5 126.84
```

```
## <none>                2307.7 128.63
```

```
## - x100m     1     294.16 2601.9 129.75
```

```
## - long      1     332.69 2640.4 130.14
```

```
## - x400m     1     482.55 2790.3 131.57
```

```
## - shotput   1      609.47 2917.2 132.73
```

```
##
```

```
## Step: AIC=126.84
```

```
## x1500m ~ x100m + long + shotput + x400m
```

```
##
```

```
##           Df Sum of Sq    RSS    AIC
```

```
## <none>                2326.5 126.84
```

```
## - x100m     1     498.11 2824.6 129.89
```

```
## - x400m     1     569.98 2896.5 130.54
```

```
## - long      1     571.41 2897.9 130.56
```

```
## - shotput   1     626.86 2953.4 131.05
```

```
stepBackward
```

```
##
```

```
## Call:
```

```
## lm(formula = x1500m ~ x100m + long + shotput + x400m, data = london)
```

```
##
```

```
## Coefficients:
```

```
## (Intercept)      x100m          long      shotput      x400m
```

```
##      247.765      -25.048      -19.308        6.426        7.174
```

```
# Cross Validation
```

```
set.seed(12345)
```

```
mlr2 <- train(x1500m ~ x100m + long + shotput + x400m,  
              data=london,  
              method="lm",  
              trControl=trainControl(method="cv",number = 5),  
              preprocess = c("center", "scale"))
```

```
# RMSE for 5th-fold cross validation
min(mlr2$results$RMSE)

## [1] 12.00308
```

## (2) Lasso regression

### Model for 110m hurdles

```
# Set seed for reproducibility
set.seed(12345)
# Train the model
lasso_mod1<-train(x110m ~ .-x1500m ,
                  data=london,
                  method="lasso",
                  # Set up repeated k-fold cross-validation
                  trControl=trainControl(method="cv",number=5),
                  preProcess = c("center", "scale"),
                  tuneLength=20)

lasso_mod1$bestTune

##      fraction
## 5 0.2684211

mean(lasso_mod1$resample$RMSE)

## [1] 0.460513
```

### Model for 1500m run

```
#### x1500m
set.seed(12345)
lasso_mod2<-train(x1500m ~ .-x110m ,
                  data=london,
                  method="lasso",
                  trControl=trainControl(method="cv",number=5),
                  preProcess = c("center", "scale"),
                  tuneLength=20)

lasso_mod2$bestTune

##      fraction
## 6 0.3105263

mean(lasso_mod2$resample$RMSE)

## [1] 12.29043
```

### (3) Ridge regression

#### Model for 110m hurdles

```
set.seed(12345)
ridge_mod1 <- train(x110m ~ .-x1500m ,
                    data=london,
                    method="ridge",
                    trControl=trainControl(method="cv", number = 5),
                    preProcess = c("center", "scale"),
                    tuneLength=20)

ridge_mod1$bestTune

##      lambda
## 20      0.1

min(ridge_mod1$results$RMSE)

## [1] 0.4667538
```

#### Model for 1500m run

```
set.seed(12345)
ridge_mod2 <- train(x1500m ~ .-x110m ,
                    data=london,
                    method="ridge",
                    trControl=trainControl(method="cv", number = 5),
                    preProcess = c("center", "scale"),
                    tuneLength=20)

# depending on the model, criteria to choose bestTune is different? not
# RMSE??

ridge_mod2$bestTune

##      lambda
## 17 0.03162278

min(ridge_mod2$results$RMSE)

## [1] 12.37265
```

### (4) Elastic net

#### Model for 110m hurdles

```
set.seed(12345)
enet_mod1 <- train(x110m ~ .-x1500m ,
                  data=london,
                  method="enet",
```

```

trControl=trainControl(method="cv",number = 5),
preProcess = c("center", "scale"),
tuneLength=20)

enet_mod1$bestTune

##      fraction lambda
## 387      0.35      0.1

min(enet_mod1$results$RMSE)

## [1] 0.4517055

```

## Model for 1500m run

```

set.seed(12345)
enet_mod2 <- train(x1500m ~ .-x110m ,
                  data=london,
                  method="enet",
                  trControl=trainControl(method="cv",number = 5),
                  preProcess = c("center", "scale"),
                  tuneLength=20)

enet_mod2$bestTune

##      fraction lambda
## 6          0.3      0

min(enet_mod2$results$RMSE)

## [1] 12.30414

```

## (5) Principal Component Regression

### Model for 110m hurdles

```

set.seed(12345)
pcr_mod1 <- train(x110m ~ .-x1500m,
                 data=london,
                 method="pcr",
                 preProcess=c("center","scale"),
                 trControl = trainControl(method="cv",number = 5),
                 tuneGrid = data.frame(ncomp=1:6))

pcr_mod1

## Principal Component Analysis
##
## 26 samples
## 6 predictor

```

```
##
## Pre-processing: centered (5), scaled (5)
## Resampling: Cross-Validated (5 fold)
## Summary of sample sizes: 20, 22, 20, 22, 20
## Resampling results across tuning parameters:
##
##   ncomp  RMSE      Rsquared  MAE
##   1      0.4534019  0.3626028  0.3725647
##   2      0.4493509  0.4479443  0.3762921
##   3      0.4534782  0.4427727  0.3819616
##   4      0.4220778  0.4592301  0.3521685
##   5      0.4974491  0.2602942  0.4362889
##   6      0.4974491  0.2602942  0.4362889
##
## RMSE was used to select the optimal model using the smallest value.
## The final value used for the model was ncomp = 4.

pcr_mod1$bestTune

##   ncomp
## 4      4

min(pcr_mod1$results$RMSE)

## [1] 0.4220778
```

## Model for 1500m run

```
set.seed(12345)
pcr_mod2 <- train(x1500m ~ .-x110m,
                  data=london,
                  method="pcr",
                  preProcess=c("center", "scale"),
                  trControl = trainControl(method="cv", number = 5),
                  tuneGrid = data.frame(ncomp=1:6))

pcr_mod2$bestTune

##   ncomp
## 5      5

min(pcr_mod2$results$RMSE)

## [1] 12.3967
```

## (6) Partial Least Squares Regression

### Model for 110m hurdles

```
set.seed(12345)
pls_mod1 <- train(x110m ~ .-x1500m,
```



```

        data=london,
        method="pls",
        preProcess=c("center","scale"),
        trControl = trainControl(method="cv"),
        tuneGrid = data.frame(ncomp=1:6))

plsr_mod1$bestTune

##      ncomp
## 1         1

min(plsr_mod1$results$RMSE)

## [1] 0.4527919

```

## Model for 1500m run

```

set.seed(12345)
plsr_mod1 <- train(x1500m ~ .-x110m,
                   data=london,
                   method="pls",
                   preProcess=c("center","scale"),
                   trControl = trainControl(method="cv"),
                   tuneGrid = data.frame(ncomp=1:6))

plsr_mod1$bestTune

##      ncomp
## 2         2

min(plsr_mod1$results$RMSE)

## [1] 9.736572

```

## (7) Backward Stepwise Regression from RMSE

### full model

```

set.seed(12345)
mod <- train(x110m ~ x100m+ long+ shotput+ high+ x400m,
             data=london,
             method="lm",
             trControl=trainControl(method="cv",number = 5),
             preProcess = c("center", "scale"))
min(mod$results$RMSE)

## [1] 0.4974491

```

## STEP1

### Drop one variable from the full model

```
set.seed(12345)
mod1 <- train(x110m ~ x100m+ long+ shotput+ high,
              data=london,
              method="lm",
              trControl=trainControl(method="cv",number = 5),
              preProcess = c("center", "scale"))
RMSE1<-min(mod1$results$RMSE)

set.seed(12345)
mod2 <- train(x110m ~ x100m+ long+ shotput+ x400m,
              data=london,
              method="lm",
              trControl=trainControl(method="cv",number = 5),
              preProcess = c("center", "scale"))
RMSE2<-min(mod2$results$RMSE)

set.seed(12345)
mod3 <- train(x110m ~ x100m+ long+ high+ x400m,
              data=london,
              method="lm",
              trControl=trainControl(method="cv",number = 5),
              preProcess = c("center", "scale"))
RMSE3<-min(mod3$results$RMSE)

set.seed(12345)
mod4 <- train(x110m ~ x100m+ shotput+ high+ x400m,
              data=london,
              method="lm",
              trControl=trainControl(method="cv",number = 5),
              preProcess = c("center", "scale"))

RMSE4<-min(mod4$results$RMSE)

set.seed(12345)
mod5 <- train(x110m ~ long+ shotput+ high+ x400m,
              data=london,
              method="lm",
              trControl=trainControl(method="cv",number = 5),
              preProcess = c("center", "scale"))
RMSE5<-min(mod5$results$RMSE)

RMSE_list<-c(RMSE1,RMSE2,RMSE3,RMSE4,RMSE5)
RMSE_list

## [1] 0.4321686 0.4732444 0.4893315 0.5027613 0.4781007
```

```
min(RMSE_list)
```

```
## [1] 0.4321686
```

*mod 1 has least RMSE 0.4322. Now our improved model is  $lm(x_{110m} \sim x_{100m} + long + shotput + high)$*

## STEP2

**Drop one variable from the improved model from STEP1.**

```
set.seed(12345)
```

```
mod2_1 <- train(x110m ~ x100m + long + shotput,  
               data=london,  
               method="lm",  
               trControl=trainControl(method="cv", number = 5),  
               preProcess = c("center", "scale"))  
RMSE2_1 <- min(mod2_1$results$RMSE)
```

```
set.seed(12345)
```

```
mod2_2 <- train(x110m ~ x100m + long + high,  
               data=london,  
               method="lm",  
               trControl=trainControl(method="cv", number = 5),  
               preProcess = c("center", "scale"))  
RMSE2_2 <- min(mod2_2$results$RMSE)
```

```
set.seed(12345)
```

```
mod2_3 <- train(x110m ~ x100m + shotput + high,  
               data=london,  
               method="lm",  
               trControl=trainControl(method="cv", number = 5),  
               preProcess = c("center", "scale"))  
RMSE2_3 <- min(mod2_3$results$RMSE)
```

```
set.seed(12345)
```

```
mod2_4 <- train(x110m ~ long + shotput + high,  
               data=london,  
               method="lm",  
               trControl=trainControl(method="cv", number = 5),  
               preProcess = c("center", "scale"))  
RMSE2_4 <- min(mod2_4$results$RMSE)
```

```
RMSE2_list <- c(RMSE2_1, RMSE2_2, RMSE2_3, RMSE2_4)
```

```
RMSE2_list
```

```
## [1] 0.4354856 0.4233306 0.4396006 0.5540580
```

```
min(RMSE2_list)
```

```
## [1] 0.4233306
```

The second model  $\text{lm}(x_{110m} \sim x_{100m} + \text{long} + \text{high})$  in step2 has the least RMSE as 0.4233. So, our improved model is  $\text{lm}(x_{110m} \sim x_{100m} + \text{long} + \text{high})$ .

### STEP3

Drop one variable from the improved model from STEP2.

```
set.seed(12345)
mod3_1 <- train(x110m ~ x100m+ long,
               data=london,
               method="lm",
               trControl=trainControl(method="cv", number = 5),
               preProcess = c("center", "scale"))
RMSE3_1<-min(mod3_1$results$RMSE)

set.seed(12345)
mod3_2 <- train(x110m ~ x100m+ high,
               data=london,
               method="lm",
               trControl=trainControl(method="cv", number = 5),
               preProcess = c("center", "scale"))
RMSE3_2<-min(mod3_2$results$RMSE)

set.seed(12345)
mod3_3 <- train(x110m ~long+ high,
               data=london,
               method="lm",
               trControl=trainControl(method="cv", number = 5),
               preProcess = c("center", "scale"))
RMSE3_3<-min(mod3_3$results$RMSE)

RMSE3_list<-c(RMSE3_1,RMSE3_2,RMSE3_3)
RMSE3_list

## [1] 0.4268190 0.4305779 0.5550041

min(RMSE3_list)

## [1] 0.426819
```

*All of the model in step3 has larger RMSE than the RMSE of the final model in step2, ( $\text{lm}(x_{110m} \sim x_{100m} + \text{long} + \text{high})$ ). Therefore, our final model is  $\text{lm}(x_{110m} \sim x_{100m} + \text{long} + \text{high})$ , and RMSE is 0.4233.*

### 1500m

full model

```
set.seed(12345)
mod <- train(x1500m ~ x100m+ long+ shotput+ high+ x400m,
             data=london,
             method="lm",
```

```

        trControl=trainControl(method="cv",number = 5),
        preProcess = c("center", "scale"))
min(mod$results$RMSE)
## [1] 12.3967

```

## STEP1

### Drop one variable from the full model

```

set.seed(12345)
mod1 <- train(x1500m ~ x100m+ long+ shotput+ high,
              data=london,
              method="lm",
              trControl=trainControl(method="cv",number = 5),
              preProcess = c("center", "scale"))
RMSE1<-min(mod1$results$RMSE)

```

```

set.seed(12345)
mod2 <- train(x1500m ~ x100m+ long+ shotput+ x400m,
              data=london,
              method="lm",
              trControl=trainControl(method="cv",number = 5),
              preProcess = c("center", "scale"))
RMSE2<-min(mod2$results$RMSE)

```

```

set.seed(12345)
mod3 <- train(x1500m ~ x100m+ long+ high+ x400m,
              data=london,
              method="lm",
              trControl=trainControl(method="cv",number = 5),
              preProcess = c("center", "scale"))
RMSE3<-min(mod3$results$RMSE)

```

```

set.seed(12345)
mod4 <- train(x1500m ~ x100m+ shotput+ high+ x400m,
              data=london,
              method="lm",
              trControl=trainControl(method="cv",number = 5),
              preProcess = c("center", "scale"))

RMSE4<-min(mod4$results$RMSE)

```

```

set.seed(12345)
mod5 <- train(x1500m ~ long+ shotput+ high+ x400m,
              data=london,
              method="lm",
              trControl=trainControl(method="cv",number = 5),
              preProcess = c("center", "scale"))
RMSE5<-min(mod5$results$RMSE)

```

```

RMSE_list<-c(RMSE1,RMSE2,RMSE3,RMSE4,RMSE5)
RMSE_list

## [1] 12.90815 12.00308 12.65103 13.06753 12.91901

min(RMSE_list)

## [1] 12.00308

```

*mod 2 has least RMSE 12.003. Now our improved model is  $lm(x_{1500m} \sim x_{100m} + long + shotput + x_{400m})$*

## STEP2

**Drop one variable from the improved model from STEP1.**

```

set.seed(12345)
mod2_1 <- train(x1500m ~ x100m + long + shotput,
                data=london,
                method="lm",
                trControl=trainControl(method="cv",number = 5),
                preProcess = c("center", "scale"))
RMSE2_1<-min(mod2_1$results$RMSE)

set.seed(12345)
mod2_2 <- train(x1500m ~ x100m + long + x400m,
                data=london,
                method="lm",
                trControl=trainControl(method="cv",number = 5),
                preProcess = c("center", "scale"))
RMSE2_2<-min(mod2_2$results$RMSE)

set.seed(12345)
mod2_3 <- train(x1500m ~ x100m + shotput + x400m,
                data=london,
                method="lm",
                trControl=trainControl(method="cv",number = 5),
                preProcess = c("center", "scale"))
RMSE2_3<-min(mod2_3$results$RMSE)

set.seed(12345)
mod2_4 <- train(x1500m ~ long + shotput + x400m,
                data=london,
                method="lm",
                trControl=trainControl(method="cv",number = 5),
                preProcess = c("center", "scale"))
RMSE2_4<-min(mod2_4$results$RMSE)

```

```
RMSE2_list<-c(RMSE2_1,RMSE2_2,RMSE2_3,RMSE2_4)  
RMSE2_list
```

```
## [1] 12.16900 12.42921 12.63966 12.85012
```

```
min(RMSE2_list)
```

```
## [1] 12.169
```

*All of the model in step2 has larger RMSE than the RMSE of the first model(lm(x1500m ~ x100m+ long+ shotput+ x400m)). Therefore, our final model is lm(x1500m ~ x100m+ long+ shotput+ x400m), and RMSE is 12.003*