Assignment 7: Modelling (9 marks)

```r
library(deSolve)
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1. In this exercise, we will be working with the Lotka-Volterra competition model introduced in class. Use the following parameter set in this question. (3 marks)

- `r` $= 0.5$
- `K` $= 1000$
- `alpha12` $= 3$
- `alpha21` $= 1.5$

   a. Solve the Lotka-Volterra competition model and plot the trajetory of the population for 100 time steps. Briefly explain the population dynamics of these species. (1 mark)

   b. Catastrophe hits population 2 at t=10, such that their numbers were drastically decreased to a quarter of what it was (hint: at t=10, N2 reset to 1/4 its non-catastrophe value)! Use a simulation to show the trajetory of these species over the 100 time steps. Explain what you saw. (2 marks)

2. In this exercise, we will be working with the malaria dynamics model we worked with in class, and we will be thinking of ways in which we can "implement" various mosquito control methods in this hypothetical population. For each of the following mosquito control strategies, describe how you would implement them in terms of math. For example, you may wish to modify some parameter, add a parameter, or out right change the strucutre of the model. Explain your choices, and include any new parameters or new equations where applicable (e.g., include an equation to show where a new parameter would appear). (3 marks)

   a. Use of insecticide to kill off adult mosquitoes. (1 mark)
   b. Use of bednet to reduce contact between mosquito and host. (1 mark)
   c. Provide hosts with vaccines. (1 mark)

3. The Allee Effect (3 marks)

   Generally, as population size increases, a population will experience a decreased growth rate due to greater competition for resources. This is a negative density-dependent growth rate, and one example of this is the logistic model.

   The Allee effect introduces positive density dependence, where increases in population size result in increased growth rates over a certain range of population sizes. One way to incorporate the Allee effect into the logistic growth equation is as follows:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)\left(\frac{N-A}{K}\right)$$

   Here r represents the growth rate of the population, K is the carrying capacity, and A is the critical population size above which the total growth rate is positive.

   a. Take $r = 1$, $A = 10$, and $K = 50$. Plot $\frac{dN}{dt}$ vs. N for $0 \leq N \leq 55$. For which values of N is the growth rate ($\frac{dN}{dt}$) positive or negative? (0.5 marks)

   b. Plot the **per capita** growth rate ($\frac{1}{N}\frac{dN}{dt}$) vs. N for this model of the Allee effect and for the logistic growth model: $\frac{dN}{dt} = rN(1 - \frac{N}{K})$. (1 marks)

   c. What do you notice about the density (N) dependence of the per capita growth rate in each case? Hint: in the logistic model, the growth rate **per capita** (per organism) decreases in a straight line as N increases. (0.5 marks)

d. What happens to the Allee effect as A decreases? Plot curves for $A = 0$ and a few values of $A > 0$. (0.5 marks)

e. Describe two biological situations in which you might expect to see an Allee effect (either weak or strong). (0.5 marks)