ASTR 1404 Stars, Galaxies, and Cosmology

Problem Set 2

June 6, 2016

The law of skinny triangles may be written

$$a = \theta D$$

with a, D in the same distance units and θ in radians. The definition of the parsec allows us to write it in the form

$$a(\mathrm{AU}) = \theta(\mathrm{arcsec}) \cdot D(\mathrm{parsec})$$

The definition of a parsec, a length of 1 AU subtends an angle of arcsec if viewed from a distance of 1 parsec, is clearly expressed by this relation.

Problem 1. More Skinny Triangle

1 (a).

The distance to the binary is

$$D(\mathrm{parsec}) = \frac{a(\mathrm{AU})}{\theta(\mathrm{arcsec})} = \frac{1}{0.1} = 10\,\mathrm{parsec}$$

Here $a = 1\,\mathrm{AU}$ is the Earth-Sun distance and θ is the parallax, $\theta = 0.1\,\mathrm{arcsec}$. The skinny triangle is:

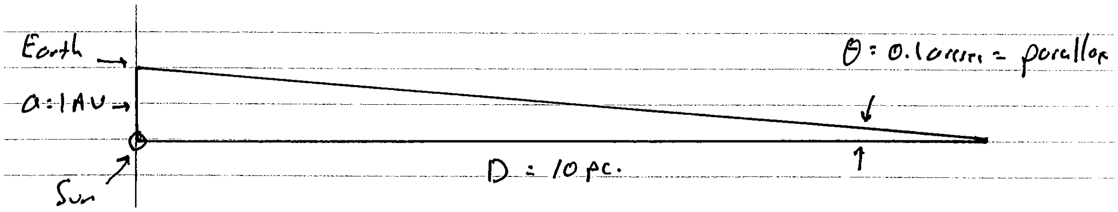

Figure 1: Skinny triangle of Earth-Sun and the binary.

If this seems easy, it is. This ease is why the parsec is defined the way it is.

1 (b).

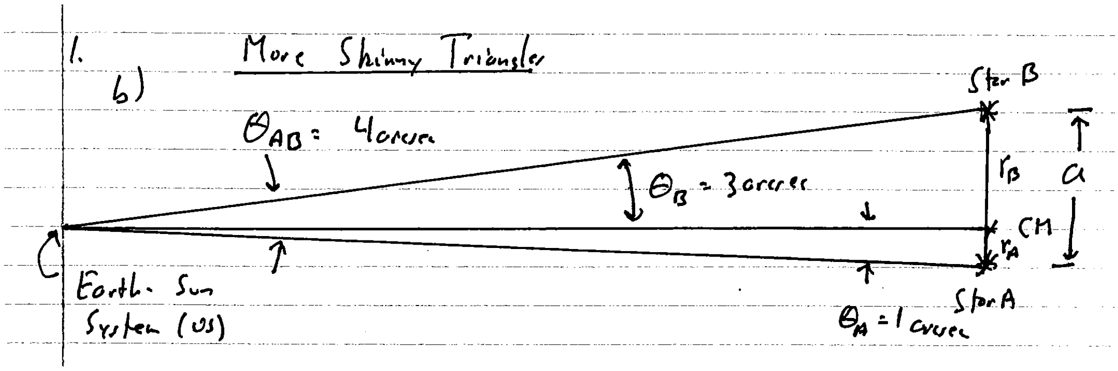

Figure 2: Skinny triangle of Earth-Sun and the binary.

The physical separation in AU is given by:

$$a(\text{AU}) = \theta(\text{arcsec}) \cdot D(\text{parsec})$$

with

$D = 10\,\text{parsec} = \text{Distance to Binary}$

$\theta = \theta_{AB} = 4\,\text{arcsec} = \text{Separation between Star A and Start B}$

$a(\text{AU}) = 4 * 10 = 40\,\text{AU} = \text{Physical separation between Star A and Star B}$

1 (c).

Kepler's Third Law may be written:

$$p^2(\text{yr}) = \frac{a^2(\text{AU})}{M_T(M_{M_\odot})}$$

where $P(\text{yr})$ is the orbital period in years, $a(\text{AU})$ is the separator in AU, and $M_T = M_A + M_B$ is the total mass in $M_\odot$. This gives:

$$M_T = \frac{a^3(\text{AU})}{p^2(\text{yr})} = \frac{40^3}{100^2} M_\odot = 6.4 M_\odot$$

1 (d).

The teeter-totter law (definition of the center of mass) is

$$M_A r_A = M_B r_B$$

with

$$r_A + r_B = a$$

$$M_A + M_B = M_T$$

A bit of algebra produces the result:

$$M_A = \frac{r_B}{a} M_T$$

$$M_B = \frac{r_A}{a} M_T$$

The law of skinny triangles gives ($D =$ Distance to Binary)

$$a = \theta_{AB} \cdot D$$

$$r_A = \theta_A \cdot D$$

$$r_B = \theta_B \cdot D$$

Hence,

$$M_A = \frac{r_B}{a} M_T = \frac{\theta_B}{\theta_{AB}} M_T$$

$$M_B = \frac{r_A}{a} M_T = \frac{\theta_A}{\theta_{AB}} M_T$$

The masses M_A and M_B are:

$$M_A = \frac{3}{4} M_T = \frac{3}{4} 6.4 M_\odot = 4.8 M_\odot$$

$$M_B = \frac{1}{4} M_T = \frac{1}{4} 6.4 M_\odot = 1.6 M_\odot$$

Note that $\theta_A = 1\,\text{arcsec}$, $\theta_B = 3\,\text{arcsec}$, $\theta_{AB} = 4\,\text{arcsec}$.

Problem 2. Alpha Centauri

2 (a).

$$D(\text{parsec}) = \frac{1}{\text{parallax in arcsec}} = \frac{1}{0.752} = 1.33\,\text{parsec}$$

2 (b).

Recall that $a(\text{AU}) = \theta(\text{arcsec}) * D(\text{parsec})$. We know that $\theta = \theta_{AB} = 17.6\,\text{arcsec}$. Then,

$$a(\text{AU}) = 17.6 * 1.33 = 23.4\,\text{AU}$$

2 (c).

Recall that $p^2(\text{yr}) = \frac{a^3(\text{AU})}{M_T(M_\odot)}$. Then,

$$M_T(M_\odot) = \frac{a^3(\text{AU})}{p^2(\text{yr})} = \frac{23.4^3}{80.1^2} = 2.00M_\odot$$

2 (d).

$$A = M(\alpha\text{CenA}) = \frac{\theta_B}{\theta_{AB}} M_T = \frac{9.7}{17.6} M_T = 1.10M_\odot$$

$$A = M(\alpha\text{CenB}) = \frac{\theta_A}{\theta_{AB}} M_T = \frac{7.9}{17.6} M_T = 0.90M_\odot$$

Problem 3. Procyon

3 (a).

Given that the parallax of Procyon is 0.29 parsec, the distance D is

$$D = \frac{1}{0.29} = 3.45\,\text{parsec}$$

3 (b).

The separation a is

$$a = \theta_{AB} D = 4.5 * 3.45 = 15.5\,\text{AU}$$

3 (c).

The total mass M_T is

$$M_T = \frac{a^3(\text{AU})}{p^2(\text{yr})} M_\odot = \frac{15.5^3}{40.6^2} = 2.27M_\odot]$$

3 (d).

$$M_A = \frac{\theta_B}{\theta_{AB}} M_T = \frac{3.3}{4.5}(2.27)M_{\odot} = 1.66M_{\odot}$$

$$M_A = \frac{\theta_A}{\theta_{AB}} M_T = \frac{1.2}{4.5}(2.27)M_{\odot} = 0.61M_{\odot}$$