ASTR 1404 Stars, Galaxies, and Cosmology

Problem Set 1 Solutions

June 5, 2016

Problem 1.

One full circle is 2π rad and 360 degrees. Thus the number of degrees in a rad is:

$$1 \text{ Circle} = 2\pi \,\text{rad} = 360\,\text{degrees}$$

Then,

$$1\,\text{rad} = \frac{360}{2\pi}\,\text{degrees}$$

1 (a).

$$1\,\text{rad} = 57.3\,\text{degrees}$$

1 (b).

$$\begin{aligned}
1\,\text{degree} &= 60\,\text{arcmin} \\
1\,\text{rad} &= \frac{360}{2\pi}\,\text{degrees} = \frac{360}{2\pi} \times 60\,\text{arcmin} \\
1\,\text{rad} &= 3.44 \times 10^3\,\text{arcmin}
\end{aligned}$$

1 (c).

$$
\begin{aligned}
1\,\text{arcmin} &= 60\,\text{arcsec} \\
1\,\text{rad} &= 3.44 \times 10^{3}\,\text{arcmin} = (3.44 \times 10^{3}) \times (60)\,\text{arcsec} \\
1\,\text{rad} &= 2.06 \times 10^{5}\,\text{arcsec}
\end{aligned}
$$

1 (d).

$$
\begin{aligned}
1\,\text{degree} &= \frac{2\pi}{360}\,\text{rad} \\
1\,\text{degree} &= 0.0175\,\text{rad} = 1.75{-}2\,\text{rad}
\end{aligned}
$$

1 (e).

$$
\begin{aligned}
1\,\text{arcmin} &= \frac{1}{60}\,\text{deg} \\
1\,\text{arcmin} &= \frac{2\pi}{360} \times \frac{1}{60}\,\text{rad} \\
1\,\text{arcmin} &= 2.91 \times 10^{-4}\,\text{rad}
\end{aligned}
$$

1 (f).

$$
\begin{aligned}
1\,\text{arcmin} &= 2.91 \times 10^{-4}\,\text{rad} \\
1\,\text{arcsec} &= \frac{1}{60}\,\text{arcmin} = 2.91 \times 10^{-4} \times \left(\frac{1}{60}\right)\,\text{rad} \\
1\,\text{arcsec} &= 4.85 \times 10^{06}\,\text{rad}
\end{aligned}
$$

Problem 2.

The angular separation of the Earth-Moon system as seen from the Sun is

$$
\begin{aligned}
\theta &= \frac{\text{Earth-Moon distance}}{1\,\mathrm{AU}} \\
&= \frac{3.84 \times 10^{10}\,\mathrm{cm}}{1.5 \times 10^{13}\,\mathrm{cm}} \\
&= 2.56 \times 10^{-3}\,\mathrm{rad} \\
&= 0.147\,\text{degrees} = 8.8\,\text{arcmin}
\end{aligned}
$$

Remember that $1\,\mathrm{AU} = 1.5 \times 10^{13}\,\mathrm{cm}$ and $D_M = 384,000\,\mathrm{km} = 3.84 \times 10^{10}\,\mathrm{cm}$

Problem 3.

3 (a).

$$
\begin{aligned}
\theta_J &= \frac{D_J}{4\,\mathrm{AU}} \\
&= \frac{1.43 \times 10^{10}\,\mathrm{cm}}{6 \times 10^{13}\,\mathrm{cm}} = \text{ Angular diameter of Jupiter} \\
&= 2.38 \times 10^{-4}\,\mathrm{rad} = 1.73 \times 10^{-2}\,\text{degrees} \\
&= 0.82\,\text{arcmin} = 49\,\text{arcsec}
\end{aligned}
$$

3 (b).

A human with 20/20 vision can resolve about one arcmin, so Jupiter is a bit too small for us to see as a disk on the sky. It is pretty easy to resolve with a small telescope. See the attached drawings by Galileo and others from 1610.

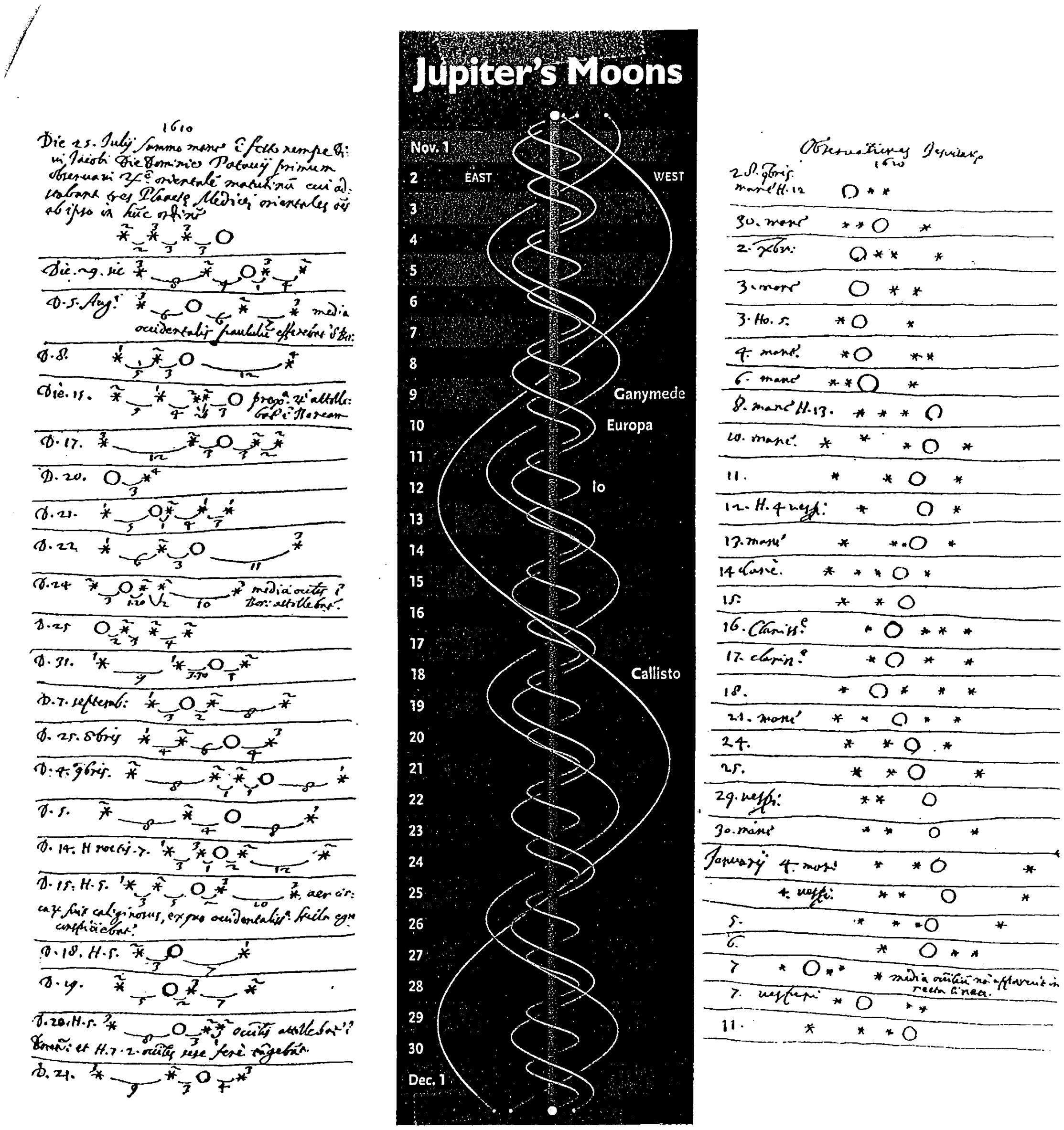

Figure 1: Galileo's drawings of Jupiter and its satellites

Problem 4.

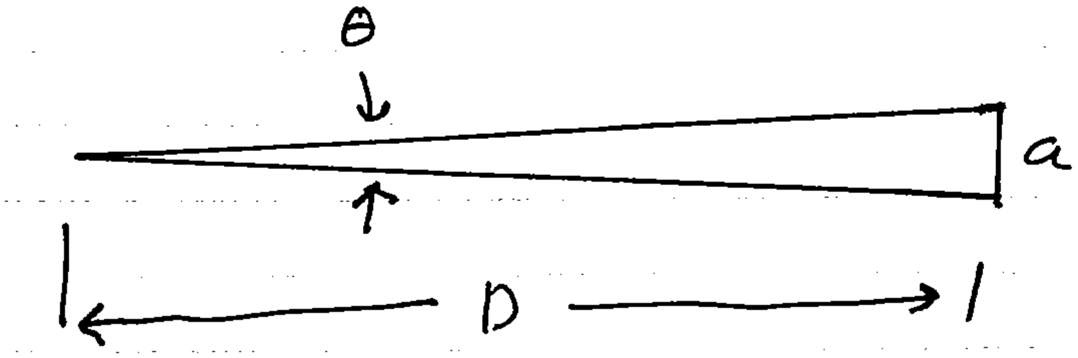

Figure 2: Law of Skinny Triangles where $a = \theta \cdot D$.

Law of Skinny Triangles: $a = \theta \cdot D$. Here,

- $a =$ Short Side $=$ Diameter (physical of Saturn's rings)
- $D =$ Long Side $=$ Distance to Saturn
- $\theta =$ Small angle (in rad)

Convert θ to radians:

$$\theta = 39\,\text{arcsec} = \left(4.85 \times 10^{-6}\frac{\text{rad}}{\text{arcsec}}\right) = 1.89 \times 10^{-4}\,\text{rad}$$

$$D = 1.3 \times 10^{14}\,\text{cm} = \text{ Distance to Saturn}$$

Diameter of rings $= \theta \times D = \left(1.89 \times 10^{-4}\right) \times \left(1.3 \times 10^{14}\right)\text{cm} = a$. Then, $a = 2.46 \times 10^{10}\,\text{cm} = 246{,}000\,\text{km}$.

Problem 5.

The Law of Skinny Triangles is $a = \theta D$. Here the short side is the baseline of the two observers on Earth.

$$
\begin{aligned}
a &= R_E = 6.37 \times 10^8\,\text{cm} = \text{ Radius of Earth} \\
\theta &= 31.3\,\text{arcsec} = 1.52 \times 10^{-4}\,\text{rad} \\
D &= 0.28\,\text{AU} = \text{ Earth-Venus distance at Transit}
\end{aligned}
$$

This gets

$$
\begin{aligned}
D &= \frac{R_E}{\theta} = \frac{6.37 \times 10^8\,\text{cm}}{1.52 \times 10^{-4}} = 4.2 \times 10^{12}\,\text{cm} \\
&= 0.28\,\text{AU}
\end{aligned}
$$

This implies that $1\,\text{AU} = \frac{4.2 \times 10^{12}\,\text{cm}}{0.28} = 1.50 \times 10^{13}\,\text{cm}$

Problem 6.

We have $a = \theta D$ where a is the separation of $19,600\,\text{km}$ and D is $40\,\text{AU} = 6 \times 10^{14}\,\text{cm}$. Then $\theta = \frac{a}{D}$

$$
\theta = \frac{1.96 \times 10^9\,\text{cm}}{6 \times 10^{14}\,\text{cm}} = 3.27 \times 10^{-6}\,\text{rad} = 0.67\,\text{arcsec}
$$

The Pluto-Charon separation is barely resolvable from the ground. The resolution of a ground based telescope is a bit less than one arcsecond due to the distortion of images produced by the atmosphere. Note: The image in Figure 3, first recorded in 1978, resulted in Pluto's eventual fall from "Planet" status.

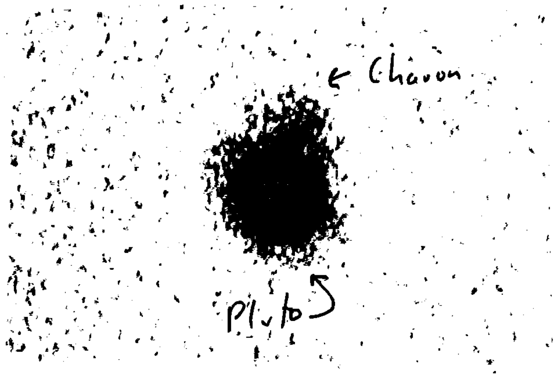

Figure 3: Highly enlarged image of Pluto on photograph made at the U.S. Naval Observatory, Flagstaff Station. The “bump” on the upper right is Pluto’s satellite, Charon. (Courtesy, U.S. Naval Observatory)

Problem 7.

The angular separation of the satellite and the center of Jupiter’s dish is given by

$$\theta = \frac{a}{D}$$

where $a =$ Moon-Jupiter Separation, $D =$ Earth-Jupiter Distance $= 6 \times 10^{13}\,\mathrm{cm}$

Moon	Moon Jupiter Distance	θ
Io	$4.22 \times 10^{10}\,\mathrm{cm}$	$7.03 \times 10^{-4}\,\mathrm{rad} = 2.42\,\mathrm{arcmin}$
Europa	$6.71 \times 10^{10}\,\mathrm{cm}$	$1.21 \times 10^{-3}\,\mathrm{rad} = 3.84\,\mathrm{arcmin}$
Ganymede	$1.07 \times 10^{11}\,\mathrm{cm}$	$1.78 \times 10^{-3}\,\mathrm{rad} = 6.13\,\mathrm{arcmin}$
Callisto	$1.88 \times 10^{11}\,\mathrm{cm}$	$3.13 \times 10^{-3}\,\mathrm{rad} = 10.8\,\mathrm{arcmin}$

Table 1: Angular separations (θ) of Jupiter’s moons and the center of Jupiter’s dish.

Problem 8.

The law of skinny triangles may be written

$$a = \theta D$$

with a, D in the same distance units and θ in radians. The definition of the parsec allows us to write it in the form

$$a(\mathrm{AU}) = \theta(\mathrm{arcsec}) \cdot D(\mathrm{parsec})$$

The definition of a parsec, a length of 1 AU subtends an angle of arcsec if viewed from a distance of 1 parsec, is clearly expressed by this relation.

$$\begin{aligned} a(\mathrm{AU}) &= \text{ short side expressed in AU} \\ \theta &= \text{ Small angle expressed in arcseconds} \\ D(\mathrm{parsec}) &= \text{ Long side in parsecs} \end{aligned}$$

Conversion factors are:

$$\begin{aligned} 1\,\mathrm{parsec} &= 2.06 \times 10^5\,\mathrm{AU} \\ &= 3.09 \times 10^{18}\,\mathrm{cm} = 3.09 \times 10^{16}\,\mathrm{m} \\ &= 3.26\,\text{light-years} \end{aligned}$$

For stellar parallax observed from Earth, $a(\mathrm{AU}) = 1$, so $D(\mathrm{parsec}) = \frac{1}{\theta(\mathrm{arcsec})}$.

8 (a).

$$\theta = 0.37\,\mathrm{arcsec} \text{ for Sirius}$$

Then,

$$D(\mathrm{parsec}) = \frac{1}{0.37} = 2.7\,\mathrm{parsec}$$

8 (b).

$$D(\mathrm{cm}) = 3.09 \times 10^{18}\,\mathrm{cm} \cdot D(\mathrm{parsec}) = 8.35 \times 10^{18}\,\mathrm{cm} = 8.35 \times 10^{16}\,\mathrm{m}$$

8 (c).

$$D(\text{light-years}) = 3.26 D(\mathrm{parsec}) = 8.8\,\text{light-years}$$

Problem 9.

9 (a).

Distance to αCen in parsec is $D(\text{parsec}) = \frac{1}{\text{parallax in arcsec}} = \frac{1}{0.75}\,\text{parsec} = 1.33\,\text{parsec}$.

9 (b).

$$\theta = \frac{a(\text{AU})}{D(\text{parsec})} = \frac{5.2}{1.33} = 3.9\,\text{arcsec}$$