# Analysis of Trajectory Tracking Control Algorithms for Wheeled Mobile Robots

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***Abstract*—Trajectory tracking is a key component of autonomous robot navigation. Trajectory tracking control algorithms control the robots effectively to track the desired trajectory that is predetermined by the trajectory planning module. Since the control performance changes significantly depending on the scenarios, choosing an appropriate controller is a non-trivial task. A simulation study of trajectory tracking controllers for non-holonomic mobile robots is presented in this work. All the control algorithms are simulated and compared on a two wheeled differential drive mobile robot. The control algorithms are implemented on MATLAB/SIMULINK 2020a and the performance of control algorithms is evaluated by tracking circular trajectory as the reference trajectory.**

***Index Terms*—wheeled mobile robots, PID, TID, pure pursuit, FOPID, LQR controller, kinematics, dynamics**

## I. Introduction

Over the past two decades wheeled mobile robots are gaining more and more attraction by researchers. The ability of mobile robots to work in hazardous, highly accurate, and complex environments resulted in the wide spread usage in various applications such as industries, hospitals, military, mines, etc. Mobile robots are classified as wheeled mobile robots and legged robots. Wheeled robots are more energy efficient, easy to control, less complex, and are easy to build. Wheeled mobile robots maybe two, three, or four wheeled. The two wheeled robot (segway) requires Gyro-Sensor to remain stable. The three wheeled robot consists of two differential drive wheels plus a castor wheel for balancing. Two continuously moving rear wheels and ordinary front wheels are used in a four wheeled mobile robot [1]. The changes in robot state for a given sequence of inputs can be predicted using motion models. The autonomous mobile robot navigation systems rely on both kinematic and dynamic models. Wheeled mobile robot navigation systems rely critically on kinematic and dynamic models especially in the absence of a global positioning system [2]. In [3], kinematic modeling is done by assigning frames in a transformation graph following the Sheth-Uicker convention and the Jacobians relating wheel to body velocities are obtained using symbolic differentiation. A velocity based approach is proposed in [4]. Even though wheeled mobile robots slip significantly in low traction conditions most of the kinematic models do not consider slip. The kinematic model considering slip is proposed in [5]. Dynamic modeling of the wheeled mobile robot considering the constraints of no slip and constant length is derived in [6] using the Lagrange approach. The robot dynamics model including the presence of friction in kinematic pairs and the electro mechanical model of servomotor drive unit is presented in [7]. The model is formulated in a linear form with respect to parameters and can be used for simulation based investigations of control systems. The mobile robots are supposed to navigate autonomously in restricted environments for which highly robust control algorithms are required. Planning a robot trajectory satisfying obstacle avoidance and developing an appropriate method to control trajectory tracking are important research problems in robotics. An effective trajectory tracking controller should be able to track the desired trajectory with minimum tracking error, lesser control input, and minimum total control effort.

Trajectory tracking controllers are classified into two types. Type-1 considers only the robot kinematics whereas the type-2 focuses on both robot kinematics and dynamics. In [8], a trajectory tracking controller based on the robot kinematics for a differential drive robot is proposed and the stability is demonstrated using Lyapunov's direct method. Motion errors are compensated in [9] by using cross coupling control method which further improves the stability. The feedback linearisation approach is presented in [10] for the trajectory control of a differential drive wheeled robot and has better performance while compared to conventional linear control system design. A simpler and modular control structure is described in [11] which uses a backstepping- like feedback linearisation. All these control methods consider only the robot kinematics. Due to the kinematic imperfection odometry error is caused. Control algorithms that consider the robot dynamics are more efficient than the kinematic based controllers when the velocity requirement is high. A control approach based on the robot dynamics is discussed in [12]. Here the actuator dynamics is also considered for developing the controller which increased the complexity of this method. In [13], a highly robust and flexible fuzzy logic controller is implemented for the trajectory tracking of a wheeled mobile robot. Based on analogy with a human driving, a car the FLC rules are made and experimentation is done to optimize the controller. In order to have more accuracy, this method requires more fuzzy grades which increases the run time and complexity of the system. Adaptive trajectory tracking control algorithms considering the parametric uncertainties is proposed in [14]. Here. the parameters are updated online which ensures better performance and ultimately bounded control errors. The authors of [15] proposed a method that combines the A* algorithm with an adaptive window approach for robot trajectory tracking in dynamic environments. The tracking is achieved using the adaptive window based approach. In [16], a linear model-predictive control (LMPC) method is proposed for the trajectory tracking of robots and uses a linearized robot model. A semi smooth Newton method is used to discretized linear-quadratic optimal control problems which are fast and smooth. However, this method fails to achieve precise tracking under uncertainties and disturbances. The instability issue in certain motion-coordination task of multi robots is addressed in [17]. The standard broadcast control framework is proposed in [17] for solving the instability issue and also improved the convergence time. The differential drive wheeled mobile robot is a non linear system. Conventional control algorithms fail to model the nonlinearities and parameter uncertainties. An effective trajectory tracking controller should have minimum tracking error and low control input.

In this paper, a simulation study of different trajectory tracking controllers is presented. PID, LQR controller, pure pursuit tracker, and a fractional order PID controller are simulated. The efficacy of these controllers is evaluated by tracking a predefined circular trajectory. A comparative assessment of these controllers is done in terms of tracking accuracy and control input.

The paper is organized as follows. In section II kinematics of the differential drive robot is described. Trajectory tracking control algorithms are presented in section IV. The simulation results are discussed in section V and the work is concluded in section VI.

## II. Kinematic Model of Mobile Robot

Kinematics is the branch of classical mechanics that describes the motion of a point (object's center) without considering the forces that have caused the motion [18]. In order to design appropriate mobile robots for particular tasks and to build control software, it is essential to know about the robot kinematics. The kinematic model of the robot is used to determine the robot's motion as a function of time. Two wheeled differential drive mobile robot with a castor wheel considered in this work is shown in Fig. 1. The state of the mobile robot is defined as a three-element vector, $\begin{bmatrix} x & y & \theta \end{bmatrix}^T$. The kinematic equations of motion can be modeled as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} vcos\theta \\ vsin\theta \\ \omega \end{bmatrix} \tag{1}$$

where $v$ and $\omega$ are the control variables. By integration of the kinematic model, the robot pose at time t can be obtained which is known as odometry or dead reckoning. Forward kinematics refers to the determination of the robot pose for given control variables. Inverse kinematics refers to the determination of control variables to drive the robot to the desired robot pose. The robot has many possible solutions to arrive at the desired pose. Assuming that the robot has a perfect kinematic model and there is no disturbances, the linear velocity $v(t)$ can be determined by

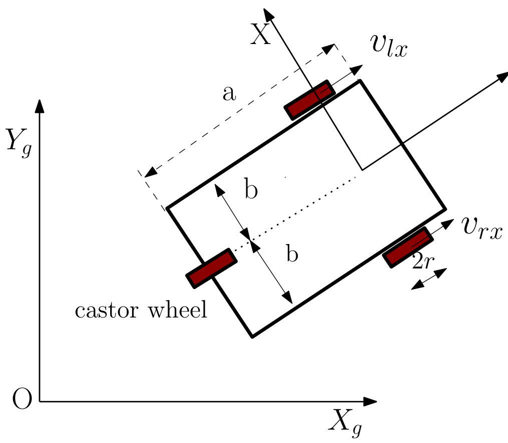



Fig. 1: Differential drive wheeled mobile robot

$$v(t) = \sqrt{\dot{x}^2(t) + \dot{y}^2(t)} \tag{2}$$

and the angular velocity is given by

$$\omega(t) = \frac{\dot{x}(t)\ddot{y}(t) - \dot{y}(t)\ddot{x}(t)}{\dot{x}^2(t) + \dot{y}^2(t)} \tag{3}$$

## III. Trajectory Tracking

Let $(x, y, \theta)$ and $(x_d, y_d, \theta_d)$ be the current and desired pose of the mobile robot. In the trajectory tracking problem, we have to generate the control variables $v$ and $\omega$ so that the desired trajectory is tracked. The error vector which is the difference between the desired and actual pose of the robot is given by

$$e(t) = \begin{bmatrix} e_x(t) \\ e_y(t) \\ e_\theta(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & \sin\theta(t) & 0 \\ -\sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_d(t) - x(t) \\ y_d(t) - y(t) \\ \theta_d(t) - \theta(t) \end{bmatrix} \tag{4}$$

The architecture of trajectory tracking control algorithm is shown in Fig. 2.

The input to the system is the reference pose $(x_d, y_d, \theta_d)$ and the output will be the actual pose of the robot $(x, y, \theta)$. The purpose of the trajectory tracking controller is to converge the error vector to zero.

## IV. Trajectory Tracking Control Algorithms

A comparative assessment of various trajectory tracking control algorithms is performed in this paper. The performance evaluation of each controller is done by tracking reference trajectories. The control algorithms are explained subsequently.

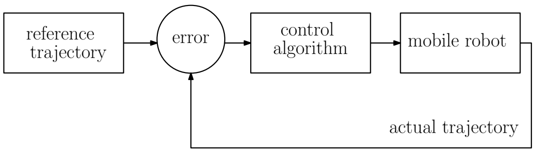



Fig. 2: Trajectory tracking problem

### A. PID and fractional order PID Controller

PID controller is one of the common and widely used trajectory tracking controllers in mobile robots. Two PID controllers, for generating the linear and angular velocities are used in [20]. The inputs to the PID controllers are the error in position and heading angle. The PID controller output is given by

$$u(t) = k_p e(t) + k_i \int e(t) + k_d \frac{de(t)}{d(t)} \quad (5)$$

where $k_p$, $k_i$, and $k_d$ are the PID parameters. Zeiger Nicholas method which is an experimentation procedure is commonly used for tuning the PID parameters. Although this method is simple, the obtained closed loop response is not appropriate for most of the process. Tuning of PID parameters is a highly challenging task, when the stability and performance of the plant are to be considered. Stability, precision of tracking, and anti interference are the advantages of genetic algorithms based PID and Fuzzy PID. Wheeled mobile robots undergo continuous parametric changes and external disturbances. So it is desirable to use online tuning methods such as neural networks which promise better results as the system dynamics or operating points change.

To improve the performance of a conventional PID controller, a fractional order is used in the integration and differentiation parts of PID controller. A fractional order PID (FOPID) is used in [21] to track the reference trajectory. Particle Swarm Optimization is used to tune the gains of FOPID. The FOPID is represented as $PI^{\lambda}D^{\mu}$, where $\lambda$ is the integration and $\mu$ is the differentiation gain. By making the gain fractional orders, the controller becomes less sensitive to parameter changes and has better control over dynamic systems. The differential equation of the FOPID is given by

$$u(t) = K_p e(t) + K_i D^{-\lambda} e(t) + K_d D^{\mu} e(t) \quad (6)$$

where e(t) is the error, u(t) is the control input. The Laplace transform is

$$G(s) = K_p + K_i S^{-\lambda} + K_d S^{\mu} \quad (7)$$

The altitude control of an unmanned aerial vehicle is done using a tilt integral derivative (TID) controller in [22]. The TID controller is similar to that of a PID controller, but the proportional gain is substituted with a tilted mode consisting of a transfer function $\frac{1}{s^{\frac{1}{s}}}$. The stability and speed of the controller response are high. The optimal tuning of the TID gains resulted in better altitude control of a UAV. While compared to the conventional PID controller, the TID controller controls the altitude of UAV with less tracking error. The dynamic and steady state characteristics of the optimized TID is better.

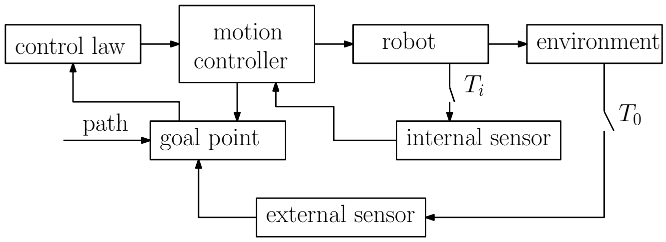



Fig. 3: Pure pursuit tracker

### B. Pure Pursuit Controller for Tracking Mobile Robots

The planned navigation of ground vehicles is important in various applications. Pure pursuit algorithm is a simple and efficient control algorithm. In early literature, the pure pursuit concept is used in the problem of missile pursuing a target [23] [24].

During every control interval, the curvature set point of the robot is changed. It is done by fitting a circumference arc to the goal point at a certain look ahead distance. The pure pursuit controller is implemented as a cascade controller in [25] is shown in Fig. 3. Speed and steering set-points are translated into references for the actuators, by the inner loop and it is called the motion controller. According to a maximum acceleration the inner loop, that is the motion controller filters abrupt speed set point changes. Dead-reckoning is updated with a period $T_i$ by the motion controller. The outer loop has a longer control period of the outer loop is $T_o$ and it implements the pure pursuit control law. For both explicit and implicit paths this control structure can be used. Tracking accuracy is reduced when the look ahead distance is longer but the steering control is made smoother. On the other hand, tracking errors are reduced with shorter look ahead distance, but steering commands increase and the robot's motion can become unstable. The response of a pure pursuit tracker is similar to first order system response. Even though the pure pursuit tracker is simple and easy to implement it cannot track the path at high speeds.

### C. Linear Quadratic Regulator (LQR) controller

LQR provides an optimal solution for a control problem by minimizing a quadratic cost function in an infinite horizon [26]. The control input $u(t)$ is generated using the state space representation of the system. Thus an optimal closed-loop feedback system is formed using the control input. The state space representation of the wheeled mobile robot is stated as

$$\dot{x} = Ax + Bu \quad (8)$$

where $x \in \Re^{3\times1}$, $u \in \Re^{2\times1}$, $A \in \Re^{3\times3}$ and $B \in \Re^{3\times2}$ The LQR controller gain K is calculated such that the performance index function defined as

$$J = \frac{1}{2}\int_0^{\infty} (x^T Q x + u^T R u) dt \quad (9)$$

is minimized. The feedback control law is computed as

$$u(t) = Ke(t) \tag{10}$$

The LQR controller is effective for tracking the desired trajectory if the gains are chosen in an optimal manner. However, the performance is satisfactory only for the linearized plant and it will not work if the plant deviates from the operating points. An LQR controller is incapable of providing an optimal performance when the model parameters are changed.

## V. Results and Discussions

Different trajectory control algorithms are studied and simulated using SIMULINK MATLAB 2020a. The kinematics and dynamics of the wheeled mobile robot are implemented using SIMULINK blocks. A circular trajectory given by $(x_d(t), y_d(t))$=$(\text{cost}, \text{sint})$ is used as the reference trajectory. The PID parameters are tuned using the Zeiger Nicholas method and are set as $K_p$= 10.66, $k_i$= 4.21, and $k_d$=-0.212. From Fig. 4(a), it can be demonstrated that the PID controller is able to track the circular trajectory even though there is a small tracking error at the initial stage of tracking. But the maximum control input required is high. The LQR controller parameters $Q$ and $R$ are set as identity matrices. The trajectory tracking performance of the LQR controller is depicted in Fig. 5. The tracking error is large for the LQR controller but the maximum control input is less while compared to the PID controller. The pure pursuit controller computes the controller commands for the differential drive robot so that the desired circular trajectory is tracked. The maximum angular velocity of the controller is limited as 1 rad/s and the look ahead distance is set as 0.1 m. It can be illustrated from Fig. 6 that the pure pursuit tracker is capable of tracking the desired circular trajectory with less control input but the tracking error is more. The fractional order PID controller is used to track the desired circular trajectory. The controller parameters are set as: (i) $k_p$= 3.31 (ii) $k_i$= 4.2 (iii) $k_d$=1.1 (iv) $\lambda = 0.8$ (v) $\mu = 1.12$. The fractional order PID controller tracks the desired trajectory with the least tracking error as compared to the other controllers and is shown in Fig. 7(a). The control input is smooth with the fractional order PID controller. A comparison of controllers is shown in Table I. Various performance measures such as integral squared error, maximum absolute error, and maximum control input are taken for comparison. The integral squared error (ISE) is computed by

$$ISE = \int_0^T (e_x(t) + e_y(t))^2$$

Maximum absolute error (MAE) is given by

$$MAE = \max(\max(e_x(t)), \max(e_y(t)))$$

The maximum linear velocity generated by each controller is calculated from the control history depicted in Figs. 4(b), 5(b), 6(b), and 7(b). The maximum linear velocity is generated by the PID controller whereas the LQR controller produces the least control input. The control input should be within the saturation limits for the better performance of motors. The tracking accuracy is high for the fractional order PID as compared to other controllers.

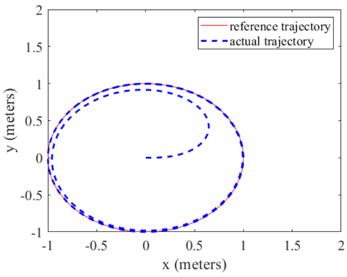
TABLE I: Comparison of controllers

| Controllers | ISE | MAE | Maximum control input (m/s) |
|---|---|---|---|
| PID | 3.342 | 1.02 | 0.9 |
| Pure pursuit | 5.021 | 2.7 | 0.8 |
| LQR | 6.763 | 3.12 | 0.55 |
| FOPID | 2.321 | 0.86 | 0.7 |

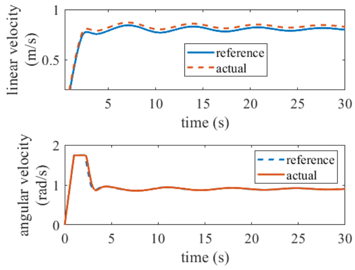
PID controller is one of the simplest and easily available controller using in industrial applications. But in cases where better robustness and transient performance is required, PID controller is not advisable. This problem can be solved by using a fractional order PID. Stability and robustness is guaranteed while using a FOPID. The tilt integral derivative with a filter have better performance while compared to the PID and FOPID. Stability and speed of response is increased. The effect of high frequency noise is eliminated by the first order filter. All these control algorithms need to be tuned. Experimental tuning methods such as Zeiger Nicholas method is not advisable for systems with dominant delay. Fine adjustment is needed when parameters are tuned by ZN method because the closed loop system's response can present an overshoot of 25%. Particle swarm optimization, Fuzzy rules etc are used to find the optimal values of controller parameters. Pure pursuit algorithm is an effective tracking algorithm for wheeled mobile robots. The geometric relationship is used in a pure pursuit algorithm to obtain the steering angle. While compared to other kinematic controllers geometric relations are more stable and easy to implant. The steering angle depends only on the look ahead distance for a given pose of the robot. Even a great tracking error exists a reasonable response is made in a pure pursuit algorithm. That is the reason why while turning a curved path the corners are cut.

## VI. Conclusion

Wheeled mobile robots are used in many applications for their distinct advantages over other unmanned ground vehicles. Trajectory tracking control is a very important issue in the field of robotics. In this paper, a comparative study of different trajectory tracking control algorithms PID, LQR, pure pursuit tracker, and fractional order PID is done by tracking a predefined circular trajectory. The kinematics and dynamics of the wheeled mobile robot that describes the robot motion are implemented using MATLAB. Performance and efficiency of each controller is evaluated. The tuning of PID and FOPID controllers are time consuming even though these controllers tracked the circular trajectory with less error. The trajectory tracking controllers are compared in terms of the tracking accuracy and control input. The fractional PID controller has the least tracking error whereas an optimal control input is produced by the LQR controller.

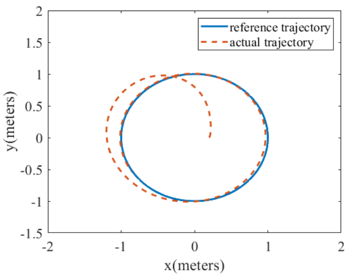



(a) Trajectory tracking

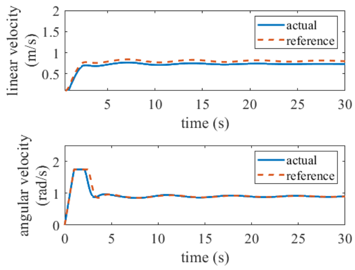



(b) Control history

Fig. 4: Performance of PID controller

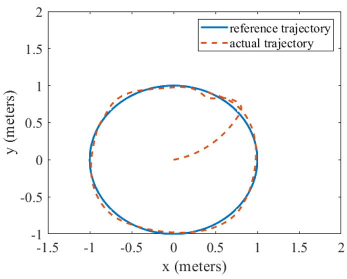



(a) Trajectory tracking

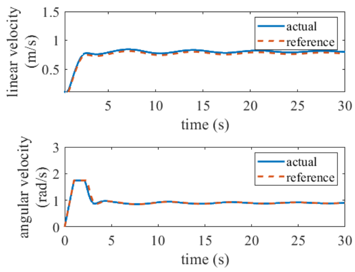



(b) Control history

Fig. 5: Performance of LQR controller

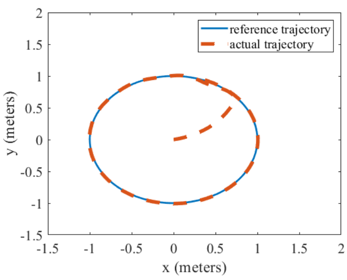



(a) Trajectory tracking

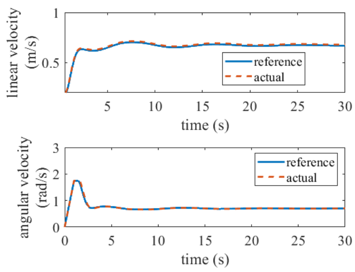



(b) Control history

Fig. 6: Performance of pure pursuit tracker




(a) Trajectory tracking




(b) Control history

Fig. 7: Performance of fractional order PID

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