Design of Fractional-order PID Controller for Path Tracking of Wheeled Mobile Robot

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2021 China Automation Congress (CAC) | 978-1-6654-2647-3/21/$31.00 ©2021 IEEE | DOI: 10.1109/CAC53003.2021.9727828

***Abstract*—In this paper, a path tracking control scheme based on the fractional PID controller is proposed to solve the path tracking problem of differential drive wheeled mobile robot. The velocity-based kinematic dynamic model are established for the differential drive wheeled mobile robot first, and then the preliminaries of fractional calculus theory and particle swarm algorithm (PSO) are introduced. Moreover, a mobile robot path tracking control system based on the fractional PID controller is designed, the controller parameters are optimized through an improved PSO algorithm. The simulation results can verify the superiority and effectiveness of the proposed control scheme.**

***Index Terms*—particle swarm optimization, fractional PID controller, differential drive wheeled mobile robot**

I. INTRODUCTION

With the rapid development of robotics, the wheeled mobile robot (WMR), which is a class of strongly coupled and highly nonlinear time-varying dynamic system, has attracted the worldwide attentions and intensive investigations [1]. The path tracking control of WMR is of great importance to the fast and accurate task execution in the increasing wide range of application scenarios.

In recent years, a new type of controller, i.e. fractional PID (FPID) controller, has been proposed and widely investigated. With the two additional adjustable parameters, the controller design can achieve a larger flexibility, and thus obtain better control performance. Therefore, the FPID has been widely used in the design of mobile robot control system recently. For example, a path tracking controller has been designed based on the kinematic model using integer-order PD controller and fractional-order PD controller in [2], and the simulation results demonstrate that the fractional controller can acquire more satisfying performance in comparing with the integer controller. In [3], a fractional-order controller has been employed to control a nonholonomic autonomous ground vehicle to achieve the tracking of desired path, where the particle swarm optimization (PSO) algorithm is used for the parameters optimization of the controller. In [4], a method to regulate the parameters using the firefly algorithm has been proposed to obtain the optimal path tracking controller. Although the above-mentioned works have obtained good results, there are still much room to further improve some shortcomings of them, e.g. the lack of unified model of mobile robot concerning kinematics and dynamics, and the premature and local trapping phenomena of the intelligent optimization algorithms in the controller design.

This work was supported in part by the Qunxing Project of Shandong University of Science and Technology under Grant QX2018M33 and the National Natural Science Foundation of China under Grant 61703242.
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To address the aforementioned problems, a path tracking control scheme has been proposed for the differential drive mobile robot based on the fractional PID controller optimized by an improved PSO algorithm. The contributions of this paper lie in threefold: *(1) A unified model combined with kinematics and dynamics is adopted to design the controller of the differential drive mobile robot; (2) A fractional PID controller is designed through an improved PSO variant; (3) Several simulations are carried out to confirm the superiority and effectiveness of the proposed approach.*

The paper is structured as follows. Section 2 denotes to the modelling of the differential drive mobile robot. In Section 3, the design of path tracking controller is addressed in detail. Section IV presents the simulation results of the path tracking controller in different scenes. Finally, the paper is concluded in the last section.

II. MODELLING OF DIFFERENTIAL DRIVE MOBILE ROBOT

A. Model of Mobile Robot Kinematics

In this section, we assume that the mobile robot moves in an ideal plane, its structure and force analysis are sketched in Fig. 1, where ω andν stand for the angular and linear velocities of the mobile robot respectively; G is the mobile robot's mass center; h denotes the point of simplified mobile robot in the X-Y plane, whose coordinates are indicated as (x, y); ϕ represents the directional angle of the mobile robot; a indicates the distance between h and B, where B is the middle of axis of the drive wheel; b is the distance from G to B; and d denotes the two rear wheels' distance; and C is a caster that plays the role of supporting and maintaining balance, so it is not considered in the model.

The model of mobile robot kinematics can be expressed as follows [5]:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & -a\sin\phi \\ \sin\phi & a\cos\phi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \nu \\ \omega \end{bmatrix} \tag{1}$$

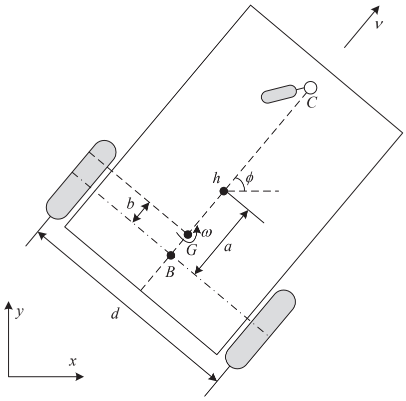

Fig. 1: Structure and force analysis of differential drive mobile robot

B. Model of Mobile Robot Dynamics

For the above-mentioned mobile robot, a robot dynamics model with the speed command as the control input is adopted in this paper. According to Fig. 1, the force analysis is done for the x- and y-coordinates of the mobile robot, and the moment analysis is done for the point G. Considering the motor model, the model of mobile robot dynamics can be obtained as follows:

$$\begin{bmatrix} \dot{\nu} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{\theta_3}{\theta_1}\omega^2 - \frac{\theta_4}{\theta_1}\nu \\ -\frac{\theta_5}{\theta_2}\nu\omega - \frac{\theta_6}{\theta_2}\omega \end{bmatrix} + \begin{bmatrix} \frac{1}{\theta_1} & 0 \\ 0 & \frac{1}{\theta_2} \end{bmatrix} \begin{bmatrix} \nu_{ref} \\ \omega_{ref} \end{bmatrix} + \begin{bmatrix} \bar{\delta}_\nu \\ \bar{\delta}_\omega \end{bmatrix} \tag{2}$$

where ω_{ref} and ν_{ref} are the desired angular and linear velocities; $[\bar{\delta}_\omega, \bar{\delta}_\nu]^T$ is the uncertainty vector that can be ignored if the wheel slippage and the forces of the tool and caster are not considered. The dynamics parameters θ_1–θ_6 can be expressed as follows [6]:

$$\begin{aligned} \theta_1 &= \frac{\frac{R_a}{k_a}\left(mR_t r + 2I_e\right) + 2rk_{DT}}{2rk_{PT}}, \\ \theta_2 &= \frac{\frac{R_a}{k_a}\left(I_e d^2 + 2R_t r\left(I_z + mb^2\right)\right) + 2rdk_{DR}}{2rdk_{PR}}, \\ \theta_3 &= \frac{R_a}{k_a}\frac{mbR_t}{2k_{PT}}, \\ \theta_4 &= \frac{\frac{R_a}{k_a}\left(\frac{k_a k_b}{R_a} + B_e\right)}{rk_{PT}} + 1, \\ \theta_5 &= \frac{R_a}{k_a}\frac{mbR_t}{dk_{PR}}, \\ \theta_6 &= \frac{R_a}{k_a}\left(\frac{k_a k_b}{R_a} + B_e\right)\frac{d}{2rk_{PR}} + 1. \end{aligned} \tag{3}$$

The dynamics parameters of the mobile robot are the functions of some physical parameters, such as the distances b and d, the radius r of the wheels, the moment of inertia I_z at point G, the moment of inertia I_e of each group rotor-reduction gear-wheel, the electromotive constant k_b of the motors, the constant of torque k_a of the motors, the mass m, the electrical resistance R_a of the motors, and the coefficient of friction B_e. We assume that the robot servos use PD controllers with proportional gains $k_{PT} > 0$, $k_{PR} > 0$ and derivative gains $k_{DT} \geq 0$, $k_{DR} \geq 0$ to control the velocities of each motor. In this paper, we also assume that the motors of both driven wheels have the same characteristics, where the inductances of the wheels are neglectable. The specific values of each dynamic parameter can be obtained by a suitable identification method.

III. Design of Path Tracking Controller

A. Fractional Calculus

In this section, the FPID controller is designed for the path tracking control of mobile robot, where the Grunwald-Letnikov definition of fractional calculus is adopted and expressed as follows:

$${}_{t_0}\mathscr{D}_t^\lambda f(t) = \lim_{h\to 0} h^{-\lambda} \sum_{k=0}^{\left[\frac{t-t_0}{h}\right]} (-1)^k \binom{\lambda}{j} f(t-kh), \tag{4}$$

where h indicates the step size of the numerical calculation; and $[\cdot]$ denotes the rounding operation; and the differential operator ${}_{t_0}\mathscr{D}_t^\lambda$ can be defined as follows:

$${}_{t_0}\mathscr{D}_t^\lambda = \begin{cases} d^\lambda/dt^\lambda & \lambda > 0, \\ 1 & r = 0, \\ \int_{t_0}^t (d\tau)^{-\lambda} & \lambda < 0, \end{cases} \tag{5}$$

where $\lambda \in \mathbb{R}$ denotes the fractional order; t_0 and t stands for the lower and upper limits of the operator respectively.

In the following simulation, the fractional differential operator is usually approximated by a modified Oustaloup filter with a frequency band of (ω_b, ω_h) expressed as follows [7]:

$$s^\lambda \approx \left(\frac{dw_h}{b}\right)^\lambda \left(\frac{bw_h s + ds^2}{d\lambda + bw_h s + d(1-\lambda)s^2}\right) \prod_{k=1}^{n} \frac{s + w_k'}{s + w_k}, \tag{6}$$

where $w_k = w_b w_u^{\frac{2k-1+\lambda}{N}}$, $w_k{}' = w_b w_u^{\frac{2k-1-\lambda}{N}}$, $w_u = \sqrt{\frac{w_h}{w_b}}$, n denotes the order of the filter, λ means the order of fractional, w_h and w_b indicate the lower and upper limits of the selected fitting frequency. Normally, the weighting parameters are taken as $b = 10$ and $d = 9$.

B. Improved PSO Algorithm

The idea of PSO algorithm comes from the simulation of the birds swarm's activity . PSO algorithm uses a great amount of random particles moving in the search space to find the optimal solution, and outputs the optimal result after reaching a termination condition or a specified number of iterations. The updating equations of PSO can be expressed as

$$\begin{aligned} \vec{V}_{k+1}^i = \omega\vec{V}_k^i + &+ c_1 r_1(\vec{P}_k^i - \vec{X}_k^i) \\ &+ c_2 r_2(\vec{G}_k - \vec{X}_k^i), \end{aligned} \tag{7}$$

$$\vec{X}_{k+1}^i = \vec{X}_k^i + \vec{V}_{k+1}^i, \tag{8}$$

where $\vec{V}_k^i$ and $\vec{X}_k^i$ stand for the vectors of velocity and position of the i-th particle at the k-th iteration, respectively; ω indicate the inertia weights of the velocity $\vec{V}_k^i$ with $0 < \omega < 1$; c_1 and c_2 are the cognitive and social parameters that are usually called acceleration factors; r_1 and r_2 denote the random numbers subject to the uniform distribution between 0 and 1; $\vec{P}_k^i$ and $\vec{G}_k$ are respectively, the personal best position (denoted as *pbest*) of the i-th particle and the global best position (denoted as *gbest*) of the whole particle swarm till the k-th iteration.

Till now, several variants of PSO have been developed by scholars at home and abroad, they mainly focus on the suitable regulations of the inertia weight that can balance the global and local search abilities of the algorithm. In this section, the inertia weight is updated by using a nonlinear decreasing approach as shown in (9):

$$w = \frac{w_{\max} - w_{\min}}{2} \cos\left(\pi \cdot \left(\frac{t}{T_{\max}}\right)^2\right) + \frac{w_{\max} + w_{\min}}{2}, \quad (9)$$

where T_{max} indicates the maximal iteration number; w_{min} means the minimal value of the inertia weight; w_{max} indicates the maximal value of the inertia weight; and t denotes the current iteration.

C. Path Tracking Control System

During the operation of the mobile robot, its path tracking controller usually first gets the target position coordinates from the path planning scheme. Then, it compares its current position with the desired position to obtain the position error, and calculates the desired angular and linear velocities for the mobile robot to track the desired path. The expected speed will be transmitted to the robot dynamic model as an input signal, and the actual linear speed and angular speed commands will be output to the robot kinematic model. Finally, the actual pose state of the mobile robot is generated from the kinematic model. The structure for the path tracking control system of the mobile robot can be described as Fig. 2.

According to the improved PSO variant, the block diagram for the parameter tuning of fractional-order PI controller is depicted as Fig. 3, where the ITAE index, which can produce smaller oscillations and overshoot, is utilized for the performance optimization of the system. The ITAE criteria can be described as follows:

$$J_{ITAE} = \int_0^{\infty} t\,|e(t)|dt, \quad (10)$$

where $e(t) = \sqrt{(x - x_d)^2 + (y - y_d)^2}$ is the current position error of the mobile robot.

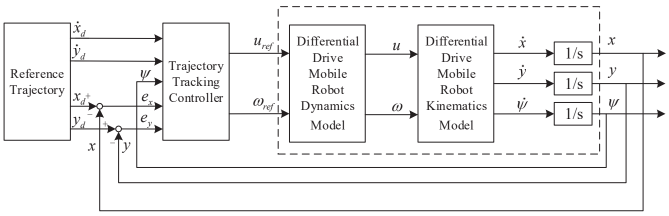

Fig. 2: Structure of the mobile robot path tracking control system.

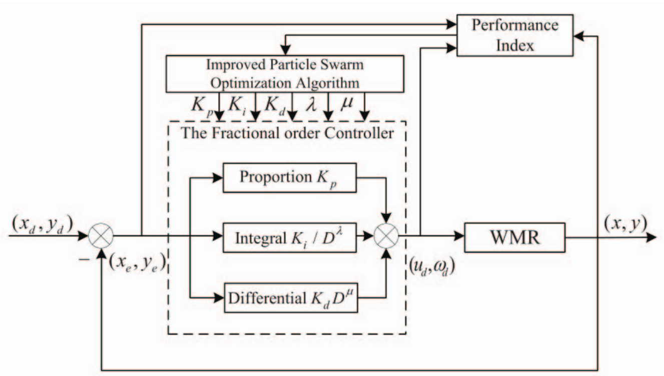

Fig. 3: Parameter tuning of fractional-order PID controller

IV. Simulation Results

In this paper, we apply the proposed path tracking control scheme to the two-wheel differential drive mobile robot model presented in Section 2.

The mobile robot dynamics parameters are as follows: $\theta_1 = 0.3625s$, $\theta_2 = 0.2609s$, $\theta_3 = -0.0005sm/rad^2$, $\theta_4 = 1.0122$, $\theta_5 = 0.0031s/m$, $\theta_6 = 1.0986$. The path tracking controller uses fractional PID controller and integer PID controller respectively, and the parameters of the controllers are optimized and tuned through the improved PSO algorithm. The parameters of PSO algorithm are taken as follows: $c_1 = c_2 = 1.5$, $w_{\min} = 0.4$, $w_{\max} = 0.9$. For sake of convenient comparison, the mobile robot starts at the position $(0.2, 0.0)$ with an orientation of 0 degrees in the following simulations.

A. Circular Trajectory

The mathematical expression for the circular trajectory is as follows:

$$\begin{cases} x_d = R\cos(2\pi f t) \\ y_d = R\sin(2\pi f t) \end{cases} \quad (11)$$

where $f = 0.04$, $R = 1$. Figs. 4 and 5 show the tracking results of the two controllers respectively. The comparison of tracking error is shown in Fig. 6.

B. 8-shape Trajectory

The mathematical expression for the 8-shape trajectory is shown as (12):

$$\begin{cases} x_d = A_x\sin(2ft) \\ y_d = A_y\sin(ft) \end{cases} \quad (12)$$

where $f = 0.04$, $Ax = Ay = 1$. The tracking results of the two controllers can be shown in Figs. 7 and 8. The comparison of tracking error is shown in Fig. 9.

The results of the simulation experiments show that the fractional PID controller can achieve faster tracking speed and smaller tracking error than the integer PID controller when the mobile robot tracks the same desired paths.

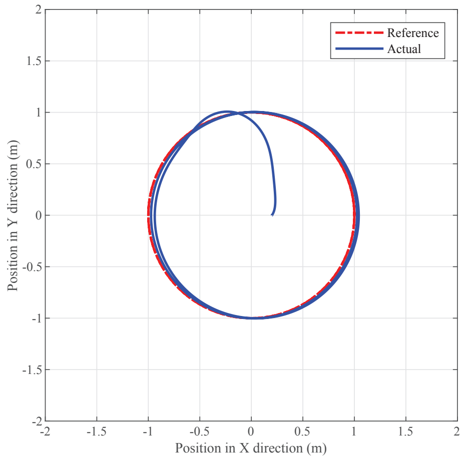

Fig. 4: The result of integer-order PID controller tracking circular trajectory

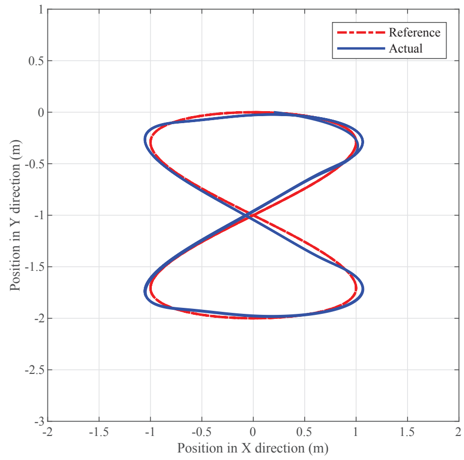

Fig. 7: The result of integer-order PID controller tracking 8-shape trajectory

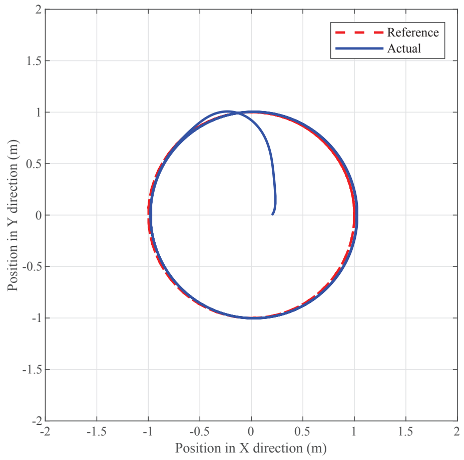

Fig. 5: The result of fractional-order PID controller tracking circular trajectory

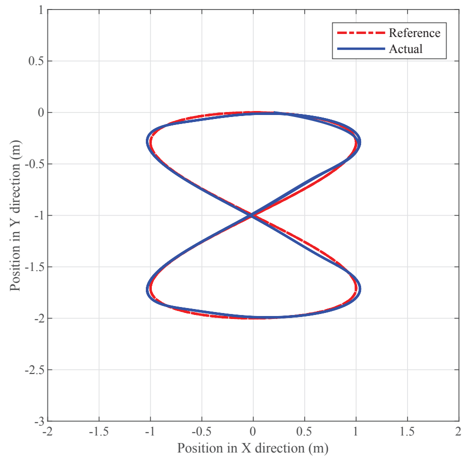

Fig. 8: The result of fractional-order PID controller tracking 8-shape trajectory

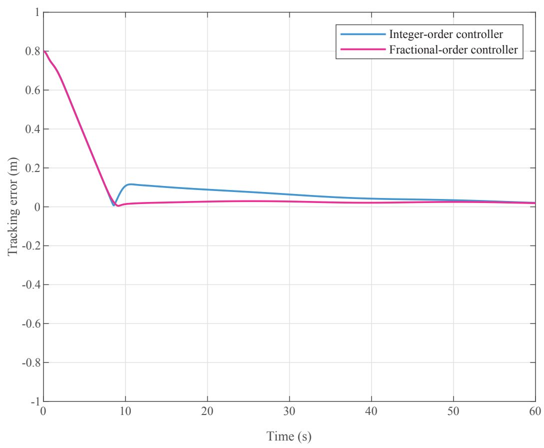

Fig. 6: Error comparison of integer and fractional controllers in tracking circular trajectory

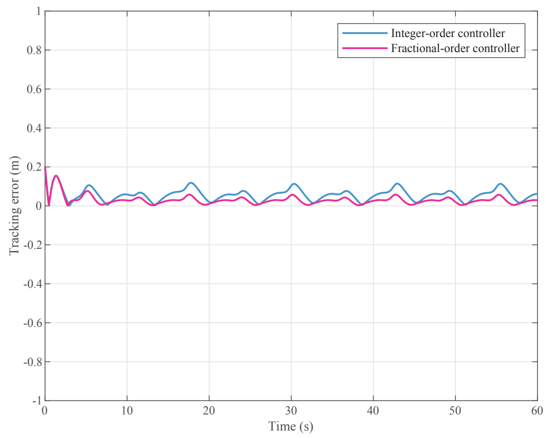

Fig. 9: Error comparison of integer and fractional controller in tracking 8-shape trajectory

V. Conclusions

In this paper, the mathematical model with desired speed as the control signal is established for the path tracking problem of differential drive wheeled mobile robot. Then, a path tracking control scheme based on the fractional PID controller is designed, and the controller is improved and optimized via the improved PSO algorithm. The superiority of this control scheme can be verified through the comparison of the tracking results of integer and fractional PID controllers on MATLAB/Simulink.

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