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Finite time tracking control of mobile robot based on non-singular fast terminal sliding mode

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ABSTRACT

To solve the tracking problem of mobile robot, a hybrid control algorithm based on back-stepping control and non-singular fast terminal sliding mode control is proposed in this paper. The algorithm not only ensures the pose tracking of the robot but also realizes the finite time tracking of the optimal speed by designing the non-singular terminal sliding mode surface. The controller solves the singularity problem in the general terminal sliding mode control. And the advantage of the controller over the general sliding mode controller is that the control input is continuously and approaching to zero. On this basis, we used an RBF neural network to estimate the uncertainties of the system, the stability of the system is proved by Lyapunov stability theory and Barbalat theorem. Finally, the simulation results show that the controller is correct and effective.

1. Introduction

In recent years, the control problem of non-holonomic systems has been a hot topic of research (Kolmanovsky & McClamroch, 1995). As a typical non-holonomic system, mobile robots are widely used for industrial, military, scientific and commercial purposes (Temel & Ashrafiuon, 2015). The wheeled mobile robot is a typical type of non-holonomic system. The control problem of mobile robots with non-holonomic constraints has attracted wide attention in the academic community. Due to the uncalibrated parameters, the existence of external disturbances, and the uncertainties of measurement and unmodelled dynamics, it is difficult to obtain an accurate kinematic model of a robot system in practice, which brings great difficulties to research. The general solution is to use the adaptive control, variable structure control, dynamic feedback control and other control methods combined with back-stepping technique and Lyapunov stability theory to study the trajectory tracking problem of non-holonomic mobile robots with unknown parameters. In this paper (Jiang & Nijmeijer, 1997; Wu, Chen, & Wang, 2001), the Lyapunov direct method and the integral back-stepping method are used to study the trajectory tracking problem of mobile robots, the parameter trajectories which satisfy certain conditions can be globally tracked. The paper (Wang, Li, & Zhu, 2010) combines two methods, inversion and dual adaptive neural

sliding mode robust dynamic control method, it cancels the uncertainty problem in the control through the sliding mode control and uses the sliding mode parameters in the neural sliding mode control to eliminate jitter phenomenon. The adaptive exponential sliding mode control is proposed to reduce the jitter problem and external interference in the tracking of a mobile robot by Mehrjerdi, Zhang, and Saad (2012). The simulation results show that the controller has certain advantages. However, the controller is given by the transformation matrix. In order to enhance the robustness of sliding mode control and reduce external disturbance, the integral sliding mode design method is adopted in the paper (Asif, Khan, & Cai, 2014). Since the dynamic model of the mobile robot is derived from Euler–Lagrange equation, the study of Euler–Lagrange system has a certain significance. The paper (Wang, Wang, Cai, & Ji, 2017) investigates leaderless and leader-following consensus control problems for a group of Euler–Lagrange systems with unknown identical control directions under an undirected connected and time-invariant graph in the presence of parametric uncertainties. For both leaderless and leader-following consensus cases, distributed adaptive controllers are presented using the back-stepping technique and a Nussbaum-type function. The paper (Wang, Wang, & Shen, 2018) addresses the leader-following consensus problem of networked Lagrangian systems with unknown control

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directions and uncertain dynamics. For undirected graphs and directed graphs, two types of distributed control protocols are proposed without assuming that the leader's position information is linearly parameterized. It is proven that all signals in the closed-loop system are bounded, and a leader-following consensus can be achieved with the proposed corresponding protocols.

For the finite time tracking problem of mobile robots, the terminal sliding mode is designed to study it (Man, Paplinski, & Wu, 1994). The paper (Huang & Zhai, 2017) divides the mobile robot dynamic system into two subsystems related to line speed and angular speed. An adaptive fast terminal sliding mode controller is designed to converge the attitude angle error to an arbitrarily small neighbourhood within a finite time, and then it designs a line speed controller to ensure the convergence of the position tracking errors. It uses the potential linear structure of the chain system, combined with the terminal sliding mode method to design the discontinuous control law, and realizes the finite time stabilization of the wheeled mobile robot system (Zhu, Dong, & Hu, 2006).

In this paper, the hybrid algorithm based on backstepping control and non-singular fast terminal sliding mode control is used to study the tracking problem of mobile robots, and the uncertainties of the system are estimated by RBF neural network. Due to the various uncertainties in the actual situation, the system often cannot track the optimal speed. The hybrid algorithm in the paper can make the actual speed of the system track to the expected speed to achieve the performance of the system. The non-singular terminal sliding surface is introduced in this paper, which eliminates the singular problems in the terminal sliding mode controller and improves the discontinuity of the general sliding mode controller. The fractional form in its controller ensures continuity and convergence of the control law. Moreover, the controller can eliminate the jitter in sliding mode control. Since the uncertainties in this paper cannot be solved by adaptive control, we use neural network to estimate them. The first two sections introduce the kinematics model and dynamics model of mobile robot. The third section is mainly to design the controller, and use the RBF neural network to estimate uncertainties. Finally, a simulation experiment was performed on the controller. The simulation results also prove the effectiveness of the method.

2. Kinematics model of mobile robot

In this section, we will give the kinematics model of the mobile robot, and the tracking error model is solved for the tracking problem of a mobile robot. On this basis, we give the linear speed and angular speed outputted by the speed tracking controller. A planar mobile robot with two
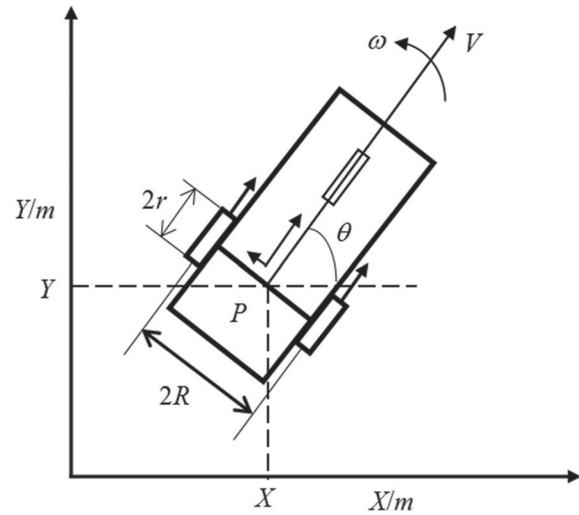
Figure 1. The schematic diagram of mobile robot.

coaxial drive wheels and one support wheel is considered, as shown in Figure 1.

In the figure, r represents the radius of the robot drive wheel, $2R$ represents the distance between the two drive wheels, and the state of the robot is represented by the position of the midpoint P of the axes of the two drive wheels in the coordinate system and the heading angle. In addition, (x, y) represents the position of the centre of the mass of the mobile robot in the coordinate system, θ represents the angle between the motion direction of the mobile robot and the X-axis, v and ω represent the linear speed and angular speed of the mobile robot. The non-holonomic constraints introduced by the pure rolling of the wheel and the ground limit the direction of movement of the mobile robot, i.e. the mobile robot can only move in a direction perpendicular to the drive axle.

The pose of the mobile robot in the coordinate system can be expressed as follows.

$$q(t) = \begin{bmatrix} x(t) & y(t) & \theta(t) \end{bmatrix}^{\mathrm{T}} \tag{1}$$

$$\dot{q}(t) = \begin{bmatrix} \dot{x}(t) & \dot{y}(t) & \dot{\theta}(t) \end{bmatrix}^{\mathrm{T}} \tag{2}$$

Since there is no side slip between the wheel of the mobile robot and the ground, there is a non-holonomic constraint that exists during the movement as shown below.

$$\dot{y}\cos\theta - \dot{x}\sin\theta = 0 \tag{3}$$

The kinematic model can be obtained from Figure 1 of the mobile robot and the above constraint (3), as follows.

$$\dot{q} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} = S(q)V \tag{4}$$

For the tracking problem, a reference pose $q_r(t) = [\, x_r(t)\ y_r(t)\ \theta_r(t)\,]^T$ is given to indicate the pose of the reference trajectory at each moment in the coordinate system, and the corresponding speed $V_r = (v_r\quad \omega_r)^T$ is to represent the reference linear speed and angular speed in the reference pose. By designing the control inputs, when $t \to \infty, q \to q_r$, the tracking is achieved. Therefore, the error of actual pose and reference pose of the robot is defined as $q_e = q_r - q = [\, (x_r-x)\ (y_r-y)\ (\theta_r-\theta)\,]^T$ then

$$q_e = \begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{pmatrix} \quad (5)$$

By simultaneously deriving both ends, the differential equations of pose error can be obtained as follows.

$$\dot{q}_e = \begin{pmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{pmatrix} = \begin{pmatrix} y_e\omega - v + v_r\cos\theta_e \\ -x_e\omega + v_r\sin\theta_e \\ \omega_r - \omega \end{pmatrix} \quad (6)$$

For the kinematics tracking problem of the mobile robot, a linear speed controller and an angular speed controller are generally designed to make the system state errors approach zero. Here, a kinematics controller is designed by the back-stepping method and Lyapunov stability theory. The specific form is given in the paper (Chen & Li, 2017), as follows.

$$V_b = \begin{pmatrix} v_b \\ \omega_b \end{pmatrix} = \begin{pmatrix} v_r\cos\theta_e + k_1 x_e \\ \omega_r + k_2 v_r y_e + k_3 v_r\sin\theta_e \end{pmatrix} \quad (7)$$

where v_b and ω_b are the linear speed and angular speed outputted by the speed tracking controller; k_1, k_2 and k_3 are positive constants.

Compared with the kinematics model of mobile robots, the dynamic model is more complicated, and there are more variables in the equation. The controller is more troublesome to solve, and the dynamic equation is the fundamental of mobile robot control. In practical applications, the torque input is also more extensive. Therefore, the focus of this paper is on the dynamics of mobile robots. We will introduce the dynamics model of the mobile robot below.

3. Dynamic model of mobile robot

In this section, the dynamics model of the mobile robot will be given, and the speed state equation is solved through a series of transformations.

The dynamic equation mainly describes the relationship between the external force and the pose, velocity and acceleration of the mobile robot. What we need to control are the left and right wheel torques. The dynamic model of a non-holonomic mobile robot can generally be described by the generalized dynamic system with non-holonomic constraints. As shown below (Fan & Lv, 2017).

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_a = B(q)\tau + A^T(q)\lambda \quad (8)$$

$$A(q)\dot{q} = 0 \quad (9)$$

where $M(q) \in R^{3\times3}$ is the inertial matrix of the system and it is symmetric positive definite; $C(q,\dot{q}) \in R^{3\times3}$ is the Coriolis-central matrix; $F(\dot{q}) \in R^{3\times1}$ is the sliding friction matrix; $G(q) \in R^{3\times1}$ is the gravity vector; $\tau_a \in R^{3\times1}$ are the bounded unknown perturbations of unstructured unmodelled dynamic; $B(q) \in R^{3\times2}$ is the input transformation matrix; $\tau \in R^{2\times1}$ is the control input torque; $A(q) \in R^{3\times1}$ is the non-holonomic constraint matrix; λ is the constraint term.

For the mobile robot with two driving wheels in the paper, the variables in Equation (8) are shown in the following equation.

$$M(q) = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{pmatrix}, \quad A^T(q) = \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix},$$

$$B(q) = \frac{1}{r}\begin{pmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ R & -R \end{pmatrix}, \quad \tau = \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix}$$

$$C(q,\dot{q}) = G(q) = 0$$

where m is the mass of the trolley, I is the moment of inertia of the mobile robot about its centre, τ_1 and τ_2 are the torques produced by the left and right wheels respectively. Since $F(\dot{q})$ and τ_a are unknown disturbances of the system, it is possible to set $\tau_d(\dot{q}, t) = \tau_a + F(\dot{q})$ as the unknown disturbance of the model. We can simplify it and ignore the uncertain interference, i.e. $\tau_d = 0$. Then the dynamic model can be reduced as follows.

$$M(q)\ddot{q} = B(q)\tau + A^T(q)\lambda \quad (10)$$

$$A(q)\dot{q} = 0 \quad (11)$$

Let us derive both sides of (3) and substitute it into (10), then the left and right sides of (10) are multiplied by $S^T(q)$ to the left. We can get the following formula.

$$\dot{V}(t) = E \cdot \tau(t) \quad (12)$$

where $V = (v, \omega)^T, \tau = (\tau_1, \tau_2)^T$

$$E = \bar{M}^{-1}(q)B(q) = \frac{1}{m \cdot I}\begin{pmatrix} I\cos\theta & I\sin\theta & 0 \\ 0 & 0 & m \end{pmatrix}$$

$$\cdot \frac{1}{r}\begin{pmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ R & -R \end{pmatrix}$$

$$= \frac{1}{m \cdot r \cdot I}\begin{pmatrix} I & I \\ Rm & -Rm \end{pmatrix}$$

In the following section, the dynamic model (8) and (4) of the mobile robot will be used to design the controller by the back-stepping method and the non-singular fast terminal sliding mode control method to achieve the purpose of tracking.

4. Design of non-singular fast terminal sliding mode controller

In this section, we will design the sliding mode controller to make the speed tracking errors approach zero, and use an RBF neural network to estimate uncertainties. On this basis, we use Lyapunov stability theory to illustrate the effectiveness of the controller.

The trajectory tracking of the mobile robot is generally to find the speed control amount through the kinematics model to achieve the purpose of tracking. However, due to the various uncertainties in the actual situation, the system often cannot track the optimal speed. In this paper, the bounded dynamics controller $\tau(t)$ is mainly designed, so that the actual speed of the system is tracked to the expected speed to achieve the performance of the system.

For this reason, we introduce the speed tracking errors as follows.

$$e = V - V_c = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} v - v_c \\ \omega - \omega_c \end{pmatrix} \tag{13}$$

where V is the actual speed, V_c is the expected speed, we can make V_c as the following formula.

$$V_c = \begin{pmatrix} v_c \\ \omega_c \end{pmatrix} = V_b = \begin{pmatrix} v_r\cos\theta_e + k_1 x_e \\ \omega_r + k_2 v_r y_e + k_3 v_r \sin\theta_e \end{pmatrix}$$

Consider the various uncertainties in the system during the actual process. In order to ensure that the actual speed of the system tracks the expected speed, the influence of uncertainty τ_d needs to be considered.

Considering the uncertainty of parameters and external disturbances, Equation (12) can be expressed as follows.

$$\dot{V}(t) = E \cdot \tau(t) + E' \cdot \tau_d(\dot{q}, t) \tag{14}$$

where $V = (v, \omega)^T$, $\tau = (\tau_1, \tau_2)^T$, $\tau_d = (\tau_{d1}, \tau_{d2}, \tau_{d3})^T$

$$E = \bar{M}^{-1}(q)B(q) = \frac{1}{m \cdot r \cdot I}\begin{pmatrix} I & I \\ Rm & -Rm \end{pmatrix}$$

$$E' = \frac{1}{m \cdot I}\begin{pmatrix} I\cos\theta & I\sin\theta & 0 \\ 0 & 0 & m \end{pmatrix}$$

Let $\delta(q, \dot{q}, t) = E'(q) \cdot \tau_d(\dot{q}, t)$, then Equation (14) can be expressed as follows.

$$\dot{V}(t) = E \cdot \tau(t) + \delta(q, \dot{q}, t) \tag{15}$$

where $\delta(q, \dot{q}, t) = \begin{pmatrix} \delta_1(q, \dot{q}, t) & \delta_2(q, \dot{q}, t) \end{pmatrix}^T$.

Assumption 4.1:

(1) There are non-negative actual numbers t_a and t_b, τ_a are the disturbances applied to system at any time interval $[t_a, t_b]$.
(2) There is a continuous bounded positive function $\bar{\bar{\delta}}(q, \dot{q})$ that makes the following equation true.

$$||\delta(q, \dot{q}, t)|| \leq \bar{\delta}(q, \dot{q})$$

where $|| \bullet ||$ is the norm in Euclidean space.

Define non-singular fast terminal sliding surfaces $s = \begin{pmatrix} s_1 & s_2 \end{pmatrix}^T$, as shown in the following equation (Hua, Sun, Chi, Guo, & Liu, 2017).

$$\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} + \frac{1}{\alpha}\begin{pmatrix} e_1^{g/h} \\ e_2^{g/h} \end{pmatrix} + \frac{1}{\beta}\begin{pmatrix} \dot{e}_1^{p/q} \\ \dot{e}_2^{p/q} \end{pmatrix} \tag{16}$$

where g, h, p, q are positive odd numbers, and $1 < p/q < 2$, $g/h < p/q$, α and β are positive constants.

In order to ensure that the speed tracking errors converge to the sliding surfaces, we can define the following formula.

$$\dot{e} = \begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \end{pmatrix} = \dot{V} - \dot{V}_c = \begin{pmatrix} \left(-\frac{\beta}{\alpha}e_1^{g/h} - \beta e_1\right)^{q/p} \\ \left(-\frac{\beta}{\alpha}e_2^{g/h} - \beta e_2\right)^{q/p} \end{pmatrix} \tag{17}$$

If the reference linear speed and reference angular speed are assumed to be constants, the differential equation of V_c is shown in the following equation.

$$\dot{V}_c = \begin{pmatrix} \dot{v}_r\cos\theta_e - v_r\dot{\theta}_e\sin\theta_e + k_1\dot{x}_e \\ \dot{\omega}_r + k_2\dot{v}_r y_e + k_2 v_r\dot{y}_e + k_3\dot{v}_r\sin\theta_e + k_3 v_r\dot{\theta}_e\cos\theta_e \end{pmatrix}$$

$$= \begin{pmatrix} -v_r\dot{\theta}_e\sin\theta_e + k_1\dot{x}_e \\ k_2 v_r\dot{y}_e + k_3 v_r\dot{\theta}_e\cos\theta_e \end{pmatrix}$$

Let us substitute (17) into (16), then we can get that $s = \begin{pmatrix} s_1 & s_2 \end{pmatrix}^T = 0$. Therefore, the speed tracking errors can converge to zero within a finite time $T = \max(t_1, t_2)$. The time T is as follows.

$$T = \max\left\{ 2\lambda_1^{-q/p}\frac{p}{p-q}V_i(0)^{(p-q)/2p} \right. \\ \left. F\left(A, B, C, -\frac{\lambda_2}{\lambda_1}V_i(0)^{(g-h)/2h}\right) \right\}, i = 1, 2 \tag{18}$$

where $\lambda_1 = 2^{(p+q)/2q}\beta$, $\lambda_2 = 2^{p/2q+g/2h}(\beta/\alpha)$, $V(t) = (1/2)e^2(t)$, $A = q/p$, $B = ((p-q)h)/(p(g-h))$,

$C = (pg - qh)/(p(g - h))$, $F(\bullet)$ is a Gaussian hypergeometric function, and the specific expression of $F(\bullet)$ is as follows (Li, Li, Wang, & Gao, 2009).

$$F(A, B, C, z) = \sum_{k=0}^{+\infty} \frac{(A)_k (B)_k}{(C)_k k!} z^k$$

So, we can get the following controller through (15), (16) and (17).

$$\tau = E^{-1} \left[\begin{pmatrix} \left(-\frac{\beta}{\alpha} e_1^{g/h} - \beta e_1\right)^{q/p} \\ \left(-\frac{\beta}{\alpha} e_2^{g/h} - \beta e_2\right)^{q/p} \end{pmatrix} \right.$$
$$\left. + \begin{pmatrix} -v_r \dot{\theta}_e \sin\theta_e + k_1 \dot{x}_e \\ k_2 v_r \dot{y}_e + k_3 v_r \dot{\theta}_e \cos\theta_e \end{pmatrix} - \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \right] \quad (19)$$

where $\delta_1 = \delta_1(q, \dot{q}, t)$, $\delta_2 = \delta_2(q, \dot{q}, t)$.

In order to satisfy the tracking of optimal speed to achieve the performance of the system, we introduce RBFNN to estimate unknown function $\delta(q, \dot{q}, t)$. Let's define the speed tracking errors as the inputs of the radial basis neural network, i.e. $x_i = e_i$, $i = 1, 2$. The outputs of the radial basis neural network is as follows.

$$\delta_i = W_i^T \cdot H_i(x) = w_{i1} h_{i1} + w_{i2} h_{i2} + \cdots + w_{in} h_{in} \quad (20)$$

where $W_i = (w_{i1} \ w_{i2} \ \cdots \ w_{in})^T$ is the weight vector of the neural network and $H_i = (h_{i1} \ h_{i2} \ \cdots \ h_{in})^T$ is the radial basis vector function, h_{ij} is a Gaussian function.

$$h_{ij} = \exp\left(-\frac{\|x_i - c_{ij}\|}{2 b_{ij}^2}\right), i = 1, 2. j = 1, 2 \cdots n \quad (21)$$

where c_{ij} and b_{ij} are the centre position and base width parameter of the Gaussian function, and both c_{ij} and b_{ij} are constants greater than zero.

In order to estimate uncertainties by neural network, we hope that the weights of the neural network is the optimal weights. However, due to the deviations in actual operation, the accurate network weights are often not obtained. Adaptive control can be used to estimate the optimal weights. So we assume that the neural network outputs with the optimal weights is as follows.

$$\delta^*(x) = \begin{pmatrix} \delta_1^* & \delta_2^* \end{pmatrix}^T = \begin{pmatrix} W_1^{*T} H_1 \\ W_2^{*T} H_2 \end{pmatrix} = W^{*T} H(x) \quad (22)$$

where $W^* = diag\{W_1^*, W_2^*\}$, W_i^* is the optimal weight vector; $H(x) = (\ H_1 \ \ H_2 \ )^T$.

The network outputs after estimating the uncertainties by the neural network is as follows.

$$\hat{\delta}(x) = \begin{pmatrix} \hat{\delta}_1 & \hat{\delta}_2 \end{pmatrix}^T = \begin{pmatrix} \hat{W}_1^T H_1 \\ \hat{W}_2^T H_2 \end{pmatrix} = \hat{W}^T H(x) \quad (23)$$

where $\hat{W}$ is the estimate of W^*, $\tilde{W}$ is the estimate error, and $\tilde{W} = \hat{W} - W^*$.

In order to ensure that the neural network can arbitrarily approximate nonlinear functions, we make the following assumption.

Assumption 4.2: *For any small positive number ψ, there is an optimal network weight vector W_i^*, which satisfies the following formula.*

$$|\varepsilon_i(x)| = |W_i^{*T} H_i(x) - \delta_i| < \psi \quad (24)$$

where $\varepsilon_i(x), i = 1, 2$ are the network approximation errors.

Through the above analysis, we can get the following theorem.

Theorem 4.1: *For the dynamics model (8) of mobile robot, we choose the back-stepping controller (7) and the non-singular terminal sliding mode controller (25), which enables the robot to track the reference pose while meeting the optimal speed for limited time tracking.*

$$\tau = E^{-1} \left[\begin{pmatrix} \left(-\frac{\beta}{\alpha} e_1^{g/h} - \beta e_1\right)^{q/p} \\ \left(-\frac{\beta}{\alpha} e_2^{g/h} - \beta e_2\right)^{q/p} \end{pmatrix} \right.$$
$$\left. + \begin{pmatrix} -v_r \dot{\theta}_e \sin\theta_e + k_1 \dot{x}_e \\ k_2 v_r \dot{y}_e + k_3 v_r \dot{\theta}_e \cos\theta_e \end{pmatrix} - \hat{W}^T H \right]$$

$$\dot{\hat{W}} = F H e^T \quad (25)$$

where $F = diag\{F_1, F_2\}$ is the adaptive parameter matrix, and $F_i > 0$.

Proof: For the hybrid algorithm in this paper, we need to define two Lyapunov functions to prove the above theorem, as follows.

$$\xi = \xi_1 + \xi_2$$

where ξ_1 is the Lyapunov function of the back-stepping control of the kinematics system, and ξ_2 is the Lyapunov function of the non-singular terminal sliding mode control of the dynamic system.

(1) The proof of back-stepping control in kinematics system

$$\xi_1 = \frac{1}{2}(x_e^2 + y_e^2) + \frac{1}{k_2}(1 - \cos\theta_e)$$

$$\dot{\xi}_1 = x_e\dot{x}_e + y_e\dot{y}_e + \frac{1}{k_2}\sin\theta_e\dot{\theta}_e$$

$$= x_e(y_e\omega - v + v_r\cos\theta_e) + y_e(-x_e\omega + v_r\sin\theta_e)$$

$$+ \frac{1}{k_2}\sin\theta_e(\omega_r - \omega)$$

$$= -k_1 x_e^2 - \frac{k_3}{k_2}v_r\sin^2\theta_e$$

∎

If we assume that the reference linear speed $v_r \geq 0$, then we can get that $\dot{\xi}_1 \leq 0$. So ξ_1 is bounded, q_e and $\dot{q}_e$ are both bounded. Therefore, $\dot{\xi}_1$ is bounded too. We can get that $\xi_1 \to 0$ by Barbalat theorem. So we prove that $x_e, y_e, \theta_e \to 0$.

(2) The proof of non-singular terminal sliding mode control in dynamic system

$$\xi_2 = \frac{1}{2}(e_1^2 + e_2^2) + \frac{1}{2}tr(\tilde{W}^T F^{-1}\tilde{W})$$

$$\dot{\xi}_2 = e_1\dot{e}_1 + e_2\dot{e}_2 + tr(\tilde{W}^T F^{-1}\dot{\hat{W}})$$

$$= (e_1 \quad e_2)(E\tau + \delta - \dot{V}_c) + e^T\tilde{W}^T H$$

$$= (e_1 \quad e_2)(E\tau + \delta - \dot{V}_c + \tilde{W}^T H)$$

$$= (e_1 \quad e_2)\begin{pmatrix}\left(-\frac{\beta}{\alpha}e_1^{g/h} - \beta e_1\right)^{q/p} \\ \left(-\frac{\beta}{\alpha}e_2^{g/h} - \beta e_2\right)^{q/p}\end{pmatrix}$$

$$+ (e_1 \quad e_2)\left(\begin{pmatrix}\delta_1 \\ \delta_2\end{pmatrix} - \begin{pmatrix}W_1^{*T} H_1 \\ W_2^{*T} H_2\end{pmatrix}\right)$$

$$\leq (e_1 \quad e_2)\begin{pmatrix}\left(-\frac{\beta}{\alpha}e_1^{g/h} - \beta e_1\right)^{q/p} \\ \left(-\frac{\beta}{\alpha}e_2^{g/h} - \beta e_2\right)^{q/p}\end{pmatrix}$$

$$+ |e_1\psi| + |e_2\psi|$$

$$= \frac{\left(-\frac{\beta}{\alpha}e_1^{g/h} - \beta e_1\right)^{q/p} \cdot e_1^{q/p}}{e_1^{(q-p)/p}}$$

$$+ \frac{\left(-\frac{\beta}{\alpha}e_2^{g/h} - \beta e_2\right)^{q/p} \cdot e_2^{q/p}}{e_2^{(q-p)/p}} + |e_1\psi| + |e_2\psi|$$

$$= \frac{\left(-\frac{\beta}{\alpha}e_1^{(g+h)/h} - \beta e_1^2\right)^{q/p}}{e_1^{(q-p)/p}}$$

$$+ \frac{\left(-\frac{\beta}{\alpha}e_2^{(g+h)/h} - \beta e_2^2\right)^{q/p}}{e_2^{(q-p)/p}} + |e_1\psi| + |e_2\psi|$$

$$= -\frac{\left(\frac{\beta}{\alpha}e_1^{(g+h)/h} + \beta e_1^2\right)^{q/p}}{e_1^{(q-p)/p}}$$

$$- \frac{\left(\frac{\beta}{\alpha}e_2^{(g+h)/h} + \beta e_2^2\right)^{q/p}}{e_2^{(q-p)/p}} + |e_1\psi| + |e_2\psi|$$

By the definition of the non-singular terminal sliding mode surface, we can know that p, q, g, h all are positive odd numbers. So $g + h$ and $q - p$ are even numbers, when $e_i \neq 0 (i = 1, 2)$, it is known from the nature of the power exponent that $e_i^{(g+h)/h} > 0 (i = 1, 2)$, $e_i^{(q-p)/p} > 0 (i = 1, 2)$. Since $\alpha > 0$, $\beta > 0$, therefore

$$\frac{\left(\frac{\beta}{\alpha}e_1^{(g+h)/h} + \beta e_1^2\right)^{q/p}}{e_1^{(q-p)/p}} > 0, \quad \frac{\left(\frac{\beta}{\alpha}e_2^{(g+h)/h} + \beta e_2^2\right)^{q/p}}{e_2^{(q-p)/p}} > 0$$

Since

$$\left|\frac{\left(\frac{\beta}{\alpha}e_i^{(g+h)/h} + \beta e_i^2\right)^{q/p}}{e_i^{(q-p)/p}} \cdot \frac{1}{e_i}\right| > 0, i = 1, 2$$

And by Assumption 4.2, we can know that ψ is any small position number. So we can know that

$$\left|\frac{\left(\frac{\beta}{\alpha}e_i^{(g+h)/h} + \beta e_i^2\right)^{q/p}}{e_i^{(q-p)/p}} \cdot \frac{1}{e_i}\right| > \psi, i = 1, 2$$

So

$$\left|\frac{\left(\frac{\beta}{\alpha}e_i^{(g+h)/h} + \beta e_i^2\right)^{q/p}}{e_i^{(q-p)/p}}\right| > |e_i\psi|, i = 1, 2$$

So we can get that $\dot{\xi}_2 \leq 0$. When $e_1 = 0$ or $e_2 = 0$, it is easy for us to get that $\dot{\xi}_2 \leq 0$. According to the Lyapunov stability theory, $e = (e_1 \quad e_2)^T \to 0$. So the system can track the optimal speed.

5. Simulation

If we assume that the mobile robot is moving on the horizontal plane, its dynamics model is as follows.

$$\begin{cases} \ddot{x} = \frac{1}{rm}(\tau_1 + \tau_2)\cos\theta - \frac{\lambda}{m}\sin\theta - \frac{\tau_{d1}}{m} \\ \ddot{y} = \frac{1}{rm}(\tau_1 + \tau_2)\sin\theta + \frac{\lambda}{m}\cos\theta - \frac{\tau_{d2}}{m} \\ \ddot{\theta} = \frac{R}{rl}(\tau_1 - \tau_2) - \frac{\tau_{d3}}{l} \end{cases} \quad (26)$$

where $\lambda = -m(\dot{x}\cos\theta + \dot{y}\sin\theta)\dot{\theta}$.

In order to verify the effectiveness of the controller, the wheeled mobile robot model such as (4) and (8) was constructed under Matlab, then we begin to simulate. The parameters of the wheeled mobile robot are as follows. $R = 2, r = 1, m = 1, l = 1$. The initial error of the pose and the speed of the mobile robot are as follows $(x_e, y_e, \theta_e, e_1, e_2) = (0.1, -0.2, -0.1, 0.1, 0.2)$. The reference speed is as follows. $v_r = 2, \omega_r = 5$. The parameters in the controller are as follows, $k_1 = 1, k_2 = 2, k_3 = 2,$ $\alpha = \beta = 1, p = 7, q = 5, g = 11, h = 3$. The initial weight
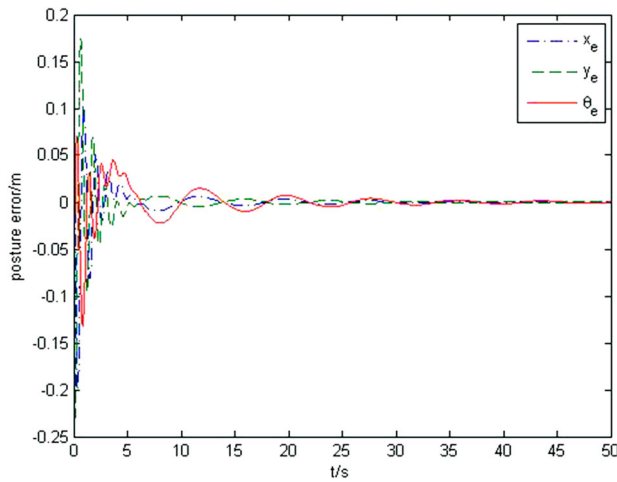

Figure 4. The control input v, ω.
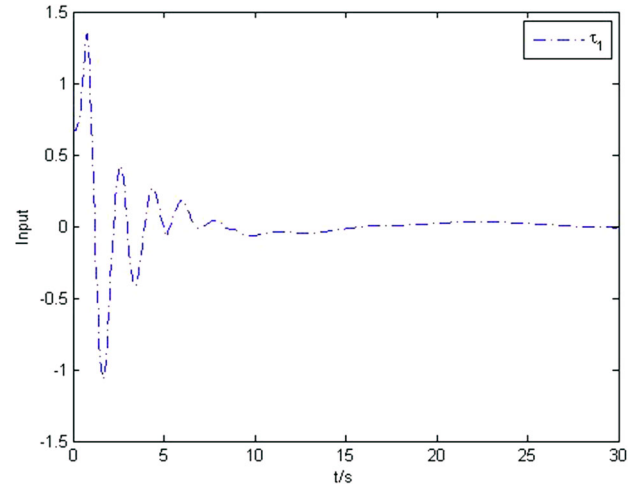
Figure 5. The left wheel control torque τ_1.
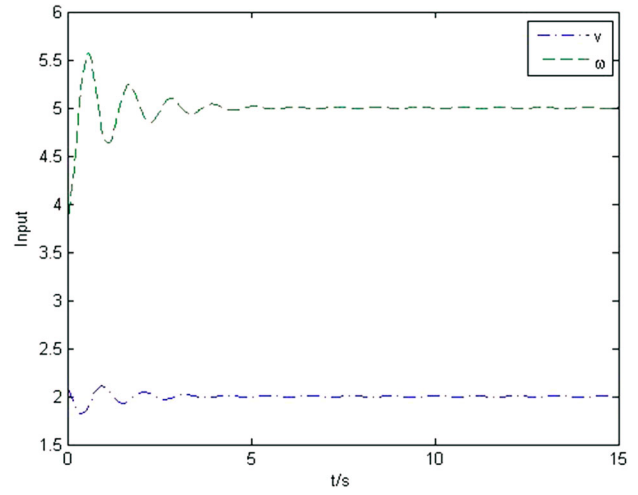

Figure 2. The pose tracking errors for x_e, y_e, θ_e.

**Figure 3.** The speed tracking errors for $e_i (i = 1, 2)$.

of neural network is 0, the width of each radial basis function is 0.2, the centre point is randomly selected in $(-2, 2)$, the adaptive parameters are all 1, the number of neural network nodes is 5. The expected reference trajectory is as follows $x_r(t) = t, y_r(t) = \sin(t)$.

Let us substitute the controllers (7) and (25) into the model to get the following simulation diagram. Figure 2 shows the tracking errors of the pose, Figure 3 shows the tracking errors of optimal speed, Figures 4–6 show the speed controllers and the torque controllers, Figure 7 shows the expected trajectory and the actual trajectory, Figures 8 and 9 show the torque controllers solved by the P-I sliding mode surface. We can know that the actual speed of the system can track to the expected speed from Figure 3, and by comparison Figures 5, 6, 8 and 9, we can find that the sliding mode controller given in this paper is continuous and convergence and the jitter problem in the sliding mode control is eliminated.
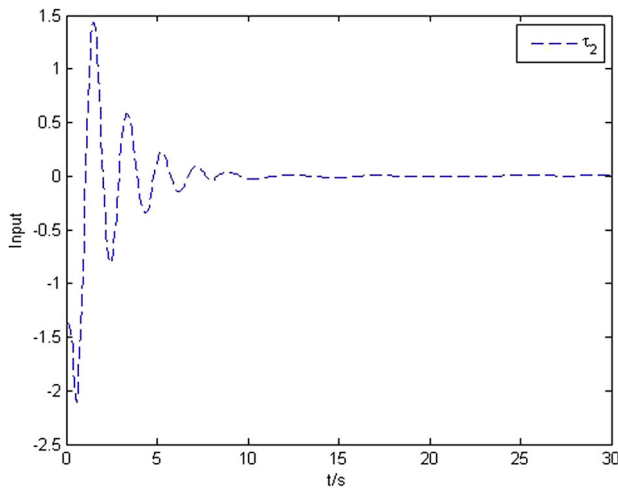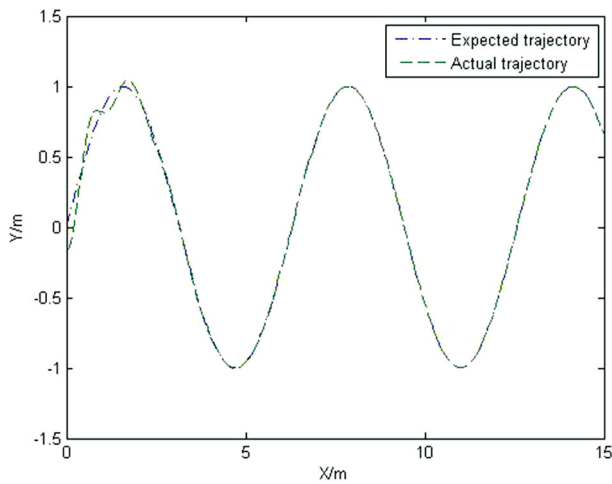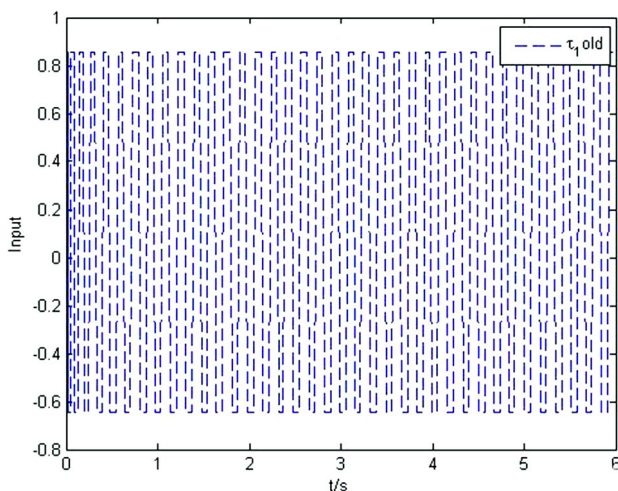
Figure 6. The right wheel control torque τ_2.
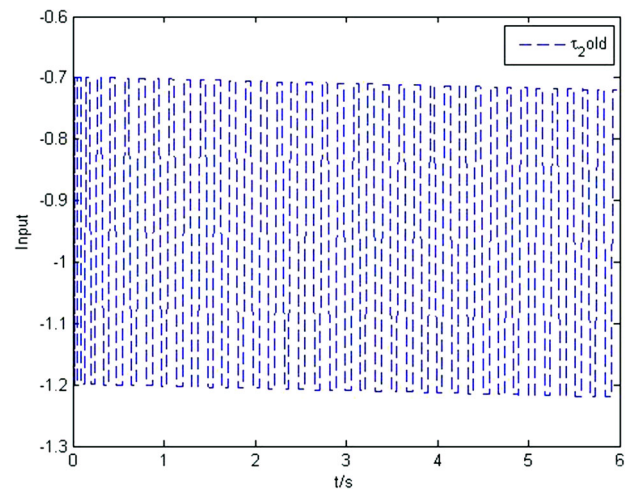
Figure 9. The right wheel control torque τ_{2old}.

Figure 7. The tracking trajectory.

Figure 8. The left wheel control torque τ_{1old}.

6. Conclusion

In this paper, a hybrid control algorithm combining back-stepping control and non-singular terminal sliding mode control is proposed for the dynamic model of the wheeled mobile robot. The back-stepping controller ensures the tracking of the mobile robot's pose, and the non-singular terminal sliding mode controller ensures the tracking of the optimal speed. The simulation results show that the hybrid control algorithm is effective. The main advantages of sliding mode control in this paper over other sliding mode control are as follows.

(1) The sliding mode controller is continuous and convergent.
(2) The controller ensures that the initial point is on the sliding surface and it can eliminate the jitter.

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