Design of Feedback Controller for Path Tracking of Mobile Robot with Differential Drive

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2023 25th International Conference on Digital Signal Processing and its Applications (DSPA) | 979-8-3503-3723-5/23/$31.00 ©2023 IEEE | DOI: 10.1109/DSPA57594.2023.10113453

***Abstract*—The paper describes a control system for cargo two- and four-wheeled robots capable of moving indoors in an autonomous mode without the use of satellite navigation signals. The control system includes the creation of a binary occupancy grid with an array of probable waypoints, the calculation of the probabilistic optimal roadmap of the mobile robot from the start and end waypoints, and the path tracking controller. The control algorithm uses a single control parameter – the difference in torque between the right and left driving wheels of the differential drive. In this case, feedback is used with microwave sensors measuring the linear speeds of the wheels.**

Keywords—mobile robot, microwave sensor, Doppler Effect, differential drive, occupancy grid, probabilistic roadmap

I. Introduction

A mobile robot (MR) is usually called an unmanned vehicle controlled remotely or autonomously, while performing any useful actions, from moving goods to monitoring the environment, advertising or marketing campaigns. For any unmanned vehicles, as a rule, external navigation is required or, when operating offline, some technical vision systems (for example, Lidar, video monitor) with elements of artificial intelligence. However, the MR coverage area is limited to premises and other areas where it is impossible to use satellite global navigation signals due to their weakening or unstable reception [1]. On the other hand, since the MR is designed to perform routine operations on the ground with a digitized map, the use of expensive control systems in most cases is not justified.

Existing MR's more often use two- or four-wheeled (according to the Ackerman scheme) platforms with differential separate drive controlled by electric motors for each of the two rear driving wheels [2-3], omnidirectional platforms are less often used [4]. Despite the fact that the operating conditions of the MR assume a flat surface without a height difference, control systems are not always able to provide initial signals that ensure accurate movement from the start to end waypoint of the local coordinate system, following the selected route.

The paper sets the task of creating a control system for a two-wheeled or four-wheeled mobile robot with a differential electric drive [5]. At the same time, the system should provide a truly autonomous control mode with an integrated approach – from creating and entering a binary employment grid, to launching after entering the start and end points of the movement. For this, a dynamic model of a differential drive is considered with the rationale for the need for direct speed measurement. Then a digital binary grid of the coverage area is created, and the optimal path is calculated using the found waypoints. An algorithm for following the trajectory is proposed, which should maintain a given course of rectilinear movement between waypoints and set the required angle of rotation when changing course between them when controlled by one parameter. To solve this problem, it is proposed to use feedback from microwave sensors [6], which measure the real linear speeds of the wheels. Their data must be built into the system to correct the signals that control the movement.

II. Dynamic Robot Model with Differential Drive

We consider the dynamic model of the robot using the example of a two-wheeled differential circuit, the kinematic diagram of which is shown in Figure 1. The considered robot is equipped with two wheels that have one degree of freedom with their own differential drive and a third point of support for contact.

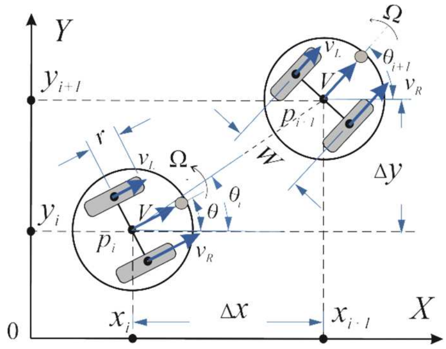

Fig.1. Kinematic model of a two-wheeled mobile robot.

From the relationship between the linear speeds and the speeds of rotation of the right and left wheels of the robot V_R and V_L and ω_r and ω_l, we can write:

$$V_R = r\omega_r\,,\quad V_L = r\omega_l \tag{1}$$

Here r is the radius of the wheels. If W is the distance between the wheels, then the linear V and angular Ω speed of the robot at the point P, located in the middle between the wheels, is expressed through the equations:

$$V = (V_R + V_L)/2\,,\quad \Omega = (V_R - V_L)/W \tag{2}$$

We write the kinematic model of the robot describing the movement from the point $P = [x_P, y_P, \theta_P]^{\mathrm{T}}$ in matrix form [7]:

$$\begin{bmatrix} \dot{x}_P \\ \dot{y}_P \\ \dot{\theta}_P \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ \Omega \end{bmatrix} \tag{3}$$

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Here orientation θ_i = arctg[$(y_{i+1}–y_i)/(x_{i+1} – x_i)$] is the direction of the MP to move to the next waypoint (Fig. 1).

We define the translational and rotational dynamics of the robot by the following equations:

$$\begin{aligned} M\dot{V} &= F - B_V V \\ J\dot{\Omega} &= T - B_\Omega \Omega \end{aligned} \quad (4)$$

where M is the mass of the robot, J is the moment of inertia, F are the forces applied to the system, T is the friction force, B_v is the friction coefficient of the translational motion, B_ω is the coefficient of the rotational friction [8]. With the distribution of forces F on the right F_R and the left wheel F_L with the coefficient l, we get:

$$F = F_R + F_L, T = l(F_R - F_L) \quad (5)$$

After transformations (4) and (5), taking into account the sums of the moments of inertia m_R and m_L at the corresponding control voltages on the motors, we obtain a system of differential equations, which are used to determine the angular Ω and linear V velocity of the mobile robot:

$$\begin{aligned} dV &= k_V V + a(m_R + m_L) \\ d\Omega &= k_\Omega \Omega + b(m_R - m_L) \end{aligned}$$

The coefficients of this system of equations k_V, a, k_Ω, and b are unknown. To find them, you need to create several systems by conducting a series of experiments with different values of torques, obtaining linear and angular velocities. Obviously, in the case of using robots for transporting goods, when the mass can change arbitrarily, the calculation of angular and linear velocities becomes impossible. That is, there is no exact correspondence between the control voltages that provide torques for the required linear speeds of the wheels. At the same time, the use of external navigation signals for their measurement is impossible due to the working conditions in the premises of production workshops and warehouses. Therefore, in this case, a direct measurement of the linear speeds of the wheels is necessary.

III. Direct Measurement of Linear Wheel Speeds

The use of inertial sensors for these purposes is unsuitable due to a significant cumulative error and the lack of the possibility of constant correction. It is possible to use rotation sensors or odometer sensors based on encoders installed on each of the wheels, with subsequent calculation of linear speeds according to (1). However, these sensors do not take into account the effects of slippage and changes in wheel traction, which introduce distortions into the speed measurement.

The most suitable methods for direct measurement of velocities are Doppler sensors for sound or electromagnetic waves. Odometers and video cameras with frame-by-frame image processing (visual odometers) can also be used. Of these sensors, microwaves are the most suitable, since the readings of sound sensors depend on humidity and other changes in the composition of the atmosphere, visual odometers require deep learning and the presence of markers.

The microwave sensor uses continuous radiation of microwave radio waves. The antenna radiates forward and at an angle α to the direction of movement of the vehicle or away from it in the vertical plane of microwave waves with a frequency f_0. The microwave waves reflected from the road surface are received by the same antenna. Then they are mixed with a part of the emitted waves, producing a Doppler signal with a frequency f. This frequency will be proportional to the linear velocity v of the vehicle:

$$f = 2v\cos(\alpha)/\lambda_0 \quad (6)$$

Here $\lambda_0 = c/f_0$ is the wavelength of electromagnetic waves, c is the speed of light. From here, from (6), the linear velocity of the MR is equal to:

$$v = \lambda_0 f / 2\cos(\alpha) \quad (7)$$

In this case, after substituting the values of the speeds of the right and left wheels from (7) into (2) and (3), we obtain expressions for the linear and angular speeds:

$$V = \lambda_0 (f_L + f_R)/4\cos(\alpha) \quad \Omega = \lambda_0 (f_L - f_R)/2W\cos(\alpha) \quad (8)$$

Taking into account (8), we obtain the expression of the kinematic model in terms of Doppler frequencies:

$$\begin{bmatrix} \dot{x}_P \\ \dot{y}_P \\ \dot{\theta}_p \end{bmatrix} = \frac{\lambda_0}{2\cos\alpha} \begin{bmatrix} \cos\theta/2 & \cos\theta/2 \\ \sin\theta/2 & \sin\theta/2 \\ 1/W & -1/W \end{bmatrix} \begin{bmatrix} f_R \\ f_L \end{bmatrix} \quad (9)$$

IV. Kinetic Model of a Four-Wheeled Warehouse Robot

For warehouse transportation, a four-wheeled robot with Ackerman steering is more suitable. Steering is also done with rear wheel differential drive, but with all wheels turning in such a way that they travel around the circles from the same center O (see Fig. 2).

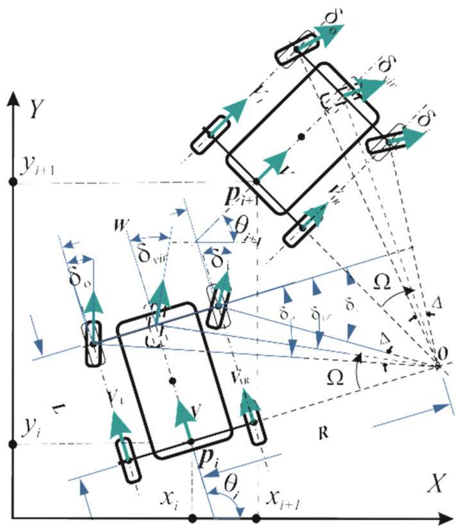

Fig.2. Kinematic model of a four-wheeled mobile robot.

The calculated waypoint **P** = $[x_P, y_P, \theta_P]^T$ is also located in the middle of the rear axis of the mobile robot. The following relation can be deduced from Fig.2:

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$$W = Lctg(\delta_o) - Lctg(\delta_i)$$

Here δ_o and δ_i are the steering angles of the outer and inner wheels, W is the distance between the wheels, L is the length or base of the mobile robot. From this follows the Ackerman condition [7]:

$$ctg(\delta_o) - ctg(\delta_i) = W / L$$

As can be seen from Figure 2, the instantaneous virtual steering angle δ_{vir} corresponding to a bicycle (two-wheeled) scheme can be determined from the formula:

$$tg(\delta_{vir}) = L / R \tag{10}$$

Here R is the radius of movement of the central point of the rear axis of the mobile robot. Similarly, it is possible to derive the values of the outer δ_o and inner δ_i steering angle of the front wheels of the vehicle (see Fig. 2):

$$\delta_o = arctg\left[\frac{Ltg(\delta_{vir})}{L - 0,5Wtg(\delta_{vir})}\right] \quad \delta_i = arctg\left[\frac{Ltg(\delta_{vir})}{L + 0,5Wtg(\delta_{vir})}\right] \tag{11}$$

The linear velocities of the left and right rear wheels V_L and V_R (see Fig. 2) can be expressed as functions of the angular velocity MP Ω:

$$V_L = \Omega(R + W/2), \; V_R = \Omega(R - W/2) \tag{12}$$

Substituting the value of Ω and V from (2) into (12), we obtain the value of R:

$$R = VW / (V_L - V_R) = VW / \Delta V \tag{13}$$

Here $\Delta V = V_L - V_R$ or via Doppler frequencies:

$$R = (f_L + f_R)W / 2(f_L - f_R) \tag{14}$$

Thus, if we substitute the expression for R from (13) and (14) into (10), we calculate the value of the steering angle δ_{vir} through the Doppler frequencies:

$$\delta_{vir} = arctg\left(\frac{\Delta V}{V}\frac{L}{W}\right) = arctg\left(2\frac{f_L - f_R}{f_L + f_R}\frac{L}{W}\right) \tag{15}$$

According to [6] kinematic equations for the Ackerman model [7] taking into account the calculation of the speeds of the left and right wheels (8), (9) and the angle δ_{vir} (15) through the Doppler frequencies of the corresponding sensors (7), (10):

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\delta}_{vir} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ \tan(\delta_{vir})/L & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ \dot{\delta}_{vir} \end{bmatrix} = \begin{bmatrix} \frac{\lambda_0}{4\cos\alpha}\cos(\theta) & 0 \\ \frac{\lambda_0}{4\cos\alpha}\sin(\theta) & 0 \\ 2\frac{(f_L - f_R)}{(f_R + f_L)W} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_R + f_L \\ \dot{\delta}_{vir} \end{bmatrix} \tag{16}$$

Thus, the entire trajectory of movement of mobile robots with a differential drive for two and four-wheeled versions is determined through the Doppler frequencies of the sensors on the right and left rear wheels (8) and (16). These frequencies are used to determine the angular and linear velocities of the robot along the entire trajectory of the route. In the second option, the steering angles (outer δ_o and inner δ_i) of the front wheels are additionally calculated using formulas (11) and (16) for precise control when turning the front wheels.

V. Control Algorithm for Path Tracking of a Mobile Robot with a Differential Drive

The process starts with entering data into the control system and creating a safe path for the robot to move around the room. For this, a generated binary grid is used (a free zone for the movement of the robot) and the calculation of the optimal trajectory by waypoints without the risk of collisions. The following steps must be taken.

- Creation of a binary occupancy grid of the room (**MAP**) by scanning, direct measurement or technical documentation from different formats.
- Input of the robot specification W, L, sensor parameters – λ_0, α.
- Inflating the size of restricted areas (walls, columns, etc.), according to the size of the robot to avoid the risk of collisions, getting a new MAP grid.
- Input the beginning and end of the route in the new **MAP** grid in coordinates $P_0 = [x_0, y_0, \theta_0]$, $P_N = [x_N, y_N, \theta_N]$.
- Determination of a probabilistic roadmap for choosing the optimal path from a network of connected nodes randomly distributed in the allowed zone. Selection of the number of nodes N_{nod} in accordance with the complexity of the map and the desire to find the most efficient path. Setting b_i criterion for waypoints.

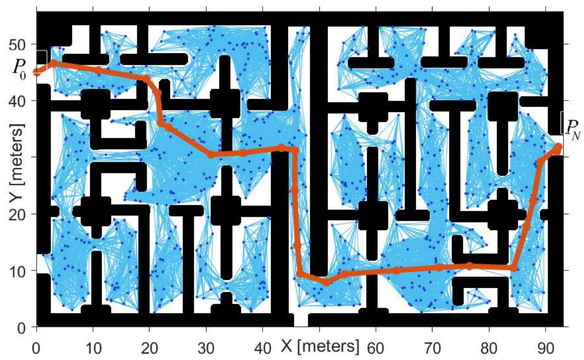

Fig.3. Binary occupancy grid with computed probabilistic road path.

To simplify the control system, we choose the speed of the robot movement constant due to the constant control voltage U of the total moment of inertia of the right and left wheels. Then, to implement a right or left turn, we will apply $U/2 + \Delta U(t)$, and $U/2 - \Delta U(t)$ to the corresponding control inputs, or vice versa. Then the total moment will always be constant and there will be a constant linear speed of the robot when changing the angle of rotation. Figure 4 shows a block diagram of the control system for a mobile wheeled robot.

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The essence of the motion control algorithm is to sequentially move the MR from one waypoint to the next target point, beginning from the start to the goal waypoint:

$$p_i = [x_i, y_i, \theta_i]^T \Rightarrow p_{i+1} = [x_{i+1}, y_{i+1}, \theta_{i+1}]^T$$

The algorithm operation is illustrated by the diagram in Figure 4.

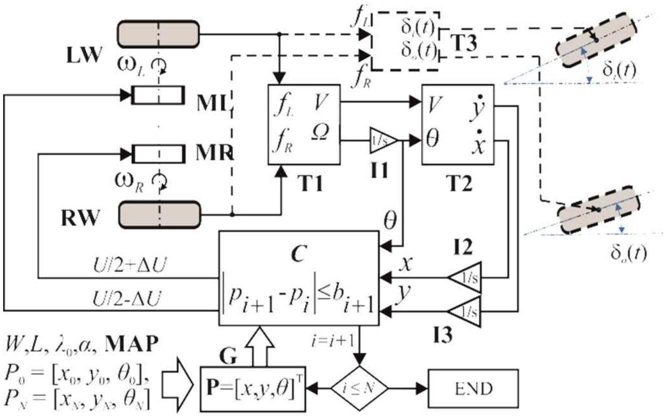

Fig.4. Block diagram of the control system of a wheeled mobile robot.

At the start command, the first waypoint of the array P from the route generator **G** goes to the controller **C**. The controller supplies control voltages to the electric motors of the left and right wheels **ML** and **MR**. As a result, moments are created with rotation speeds ω_L and ω_R from the starting point to the first one. Thanks to the microwave speed sensors of the right and left wheels, the Doppler frequencies of the sensors enter the $T1$ unit, where they are converted into linear V and angular Ω speed. Then V directly and Ω, after the transformation in the integrator $I1$ into θ, go to the converter $T2$. This converter, together with integrators $I2$ and $I3$, calculates the current coordinates x and y, which enter the controller C. θ also comes there from the output $I1$. The controller calculates the current coordinates in accordance with the kinematic model (9). After comparison with the specified coordinates of the target p_{i+1}, **C** generates a signal ΔU that controls the difference in the rotation speeds of the wheels or their linear speeds to minimize trajectory deviations between waypoints. After reaching the condition:

$$|p_{i+1} - p_i| \le b_{i+1},$$

MR moves to the next target waypoint. The movement continues until the last point with number N is reached, and the control process is completed (Fig.5).

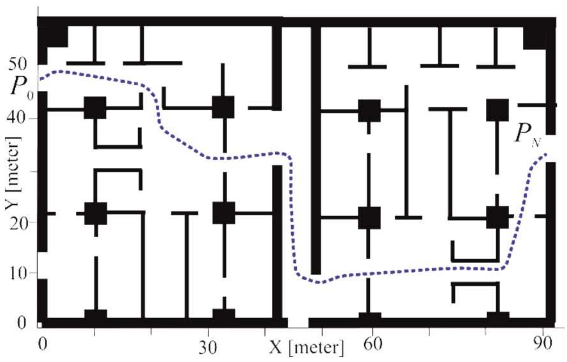

Fig. 5. The trajectory of the mobile robot.

In the case of using a four-wheeled robot circuit, the outer δ_o and internal δ_i steering angles are additionally calculated according to the formulas (11) in the $T3$ block, with the corresponding signals being sent to the front wheel turning devices.

It should be noted that the controller at a constant linear speed maintains movement along the route by adjusting one parameter ΔU. This parameter is controlled only by the difference between the Doppler frequencies of the sensors of the right and left wheels, and this allows you to get rid of the influence of the instability of other parameters.

VI. Conclusion

In this work, a kinetic model of a robot with a differential drive with direct measurement of its linear and angular velocity is implemented. In this case, the Doppler frequencies of the sensors of the linear speed of the robot's wheels are used. The difference between these frequencies at a constant speed was included in the controller feedback. This made it possible to achieve confident autonomous following without external navigation along a given route indoors. The route consists of an array of waypoints selected from the free zone probability distribution of the binary grid converted from the room map. The use of feedback on the linear speeds of the wheels made it possible to optimize the algorithm of movement along a given trajectory using a single control parameter.

The development of the results of the study will achieve target the established goal of creating an inexpensive system of autonomous cargo transportation for many routine operations in the premises of warehouses and manufacturing enterprises.

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