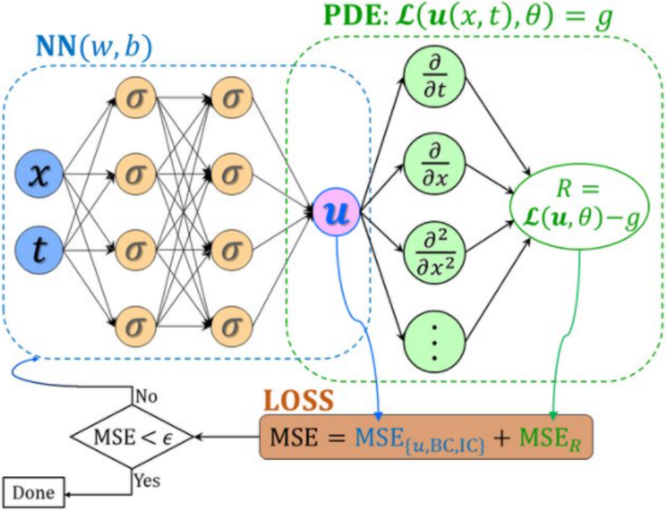
Preliminary Experimental Results

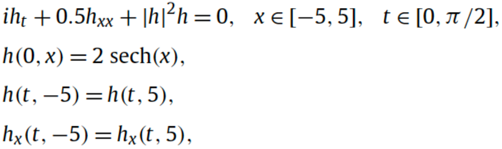
June 28, 2021

Physics-informed neural networks (PINN)


PDE: $\mathcal{L}(\boldsymbol{u}(x,t),\theta) = g$
NN(w, b)
x
t
σ
$\boldsymbol{u}$
$\frac{\partial}{\partial t}$
$\frac{\partial}{\partial x}$
$\frac{\partial^2}{\partial x^2}$
$\vdots$
$R = \mathcal{L}(\boldsymbol{u},\theta) - g$
LOSS
MSE = $\mathrm{MSE}_{\{u,\mathrm{BC},\mathrm{IC}\}} + \mathrm{MSE}_R$
MSE $< \epsilon$
No
Yes
Done

PINN vs. Quasi-Binarized PINN (QBPINN)

The nonlinear Schrödinger equation along with periodic boundary conditions



$$
\begin{aligned}
&ih_t + 0.5h_{xx} + |h|^2 h = 0, \quad x \in [-5,5], \quad t \in [0, \pi/2],\\
&h(0,x) = 2\,\text{sech}(x),\\
&h(t,-5) = h(t,5),\\
&h_x(t,-5) = h_x(t,5),
\end{aligned}
$$

where $f := ih_t + 0.5h_{xx} + |h|^2 h$

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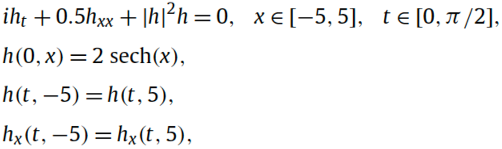

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type	#I&B	#collocation	relative L_2 norm
PINN	50	20000	1.97^{-3}
QBPINN	50	20000	0.918

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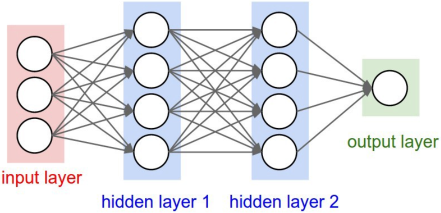
- With different number of initial, boundary (50, 100, 150) and collocation points (25000, 30000), we have similar results.
- Pre-training with full-precision PINN, then train with QBPINN, we cannot improve the accuracy. This is confirmed by several papers.

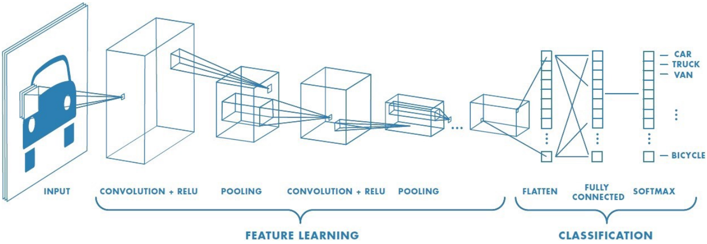
Naive FFNN (Larq)



type	example	#training	#validation	L_2	MSE
QBPINN	Schrödinger	20150	51456	0.918	–
BFFNN	Schrödinger	41456	10000	4.37	0.0839
BFFNN	BB1	220000	30000	0.395	3.385
BFFNN	BB2	220000	30000	0.156	74.5

- architecture (2, 100, 100, 100, 50, 1)
- BB1: with bouncing transitions (height 3, step 0.005)
- BB2: without bouncing transitions (height 60, step 0.005)

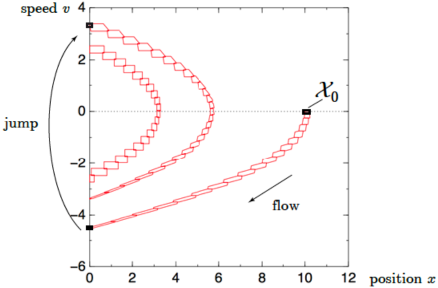
Naive CNN



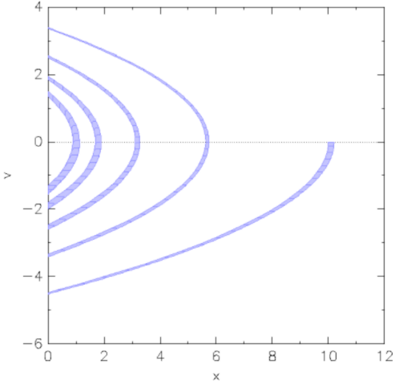
type	example	#training	#validation	L_2	archi
B-CNN1	BB	9000	1000	0.232	A1
B-CNN2	BB	9000	1000	9.43^{-2}	A2
B-CNN3	BB	9000	1000	9.39^{-2}	A3

- A1: archi ((10,10,2), (64,2), (64,2), (64,2), 64, 100)
- A2: ((10,10,2), (64,2), (64,2), (128,2), (128,2), (256,2), (256,2), 512, 512, 100)
- A3: ((10,10,2), (128,2), (128,2), (256,2), (256,2), (512,2), (512,2), 1024, 1024, 100)

Comparison

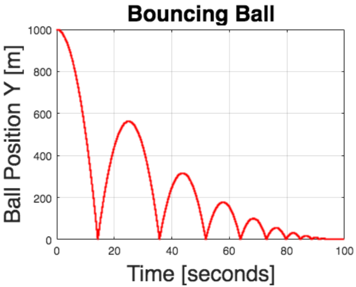


Comparison



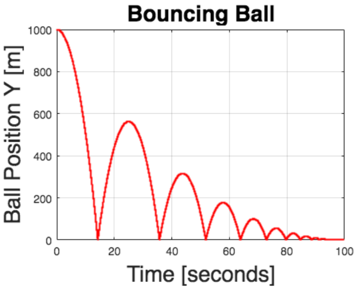
- initial set: $x \in [10, 10.2]$, $v = 0$
- property: the set of bad states is the set of all states where $v \geq 10.7$

Another comparison



- initial set: $x \in [10000, 15000]$, $v = 0$
- property on interval time between transitions (or between peakheight).

Another comparison



- initial set: $x \in [10000, 15000]$, $v = 0$
- property on interval time between transitions (or between peakheight).

type	example	#training	#validation	L_2	archi
BFFNN	BB	20000	5000	0.176	A1

A1: 2, 100, 100, 100, 100, 100, 1