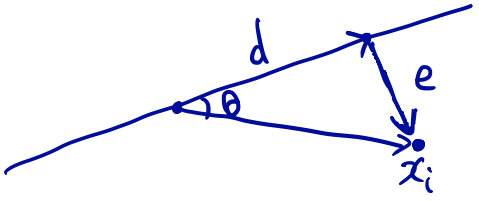

$a \quad (^{*}\|a\|_2 = 1)$

$$\begin{cases} a^T x_i = \|a\|_2 \cdot \|x_i\|_2 \cdot \cos\theta \\ \cos\theta = \dfrac{\|d\|_2}{\|x_i\|_2} \end{cases}$$

$$\Rightarrow a^T x_i = 1 \cdot \|x_i\|_2 \cdot \frac{\|d\|_2}{\|x_i\|_2}$$

$$\Rightarrow \|d\|_2 = a^T x_i$$

$$\Rightarrow d = a(a^T x_i)$$

$$\Rightarrow e = x_i - d = x_i - a(a^T x_i)$$

P2)

$$\min J(a) = \|X - a(a^T X)\|_F^2$$

$$= \mathrm{Tr}(\underbrace{X^T X}_{\text{constant}} - a^T X X^T a)$$

$$\Rightarrow \max -J(a)$$

$$\Rightarrow \max_a \; a^T X X^T a \quad \text{s.t.} \quad a^T a = 1 \quad (\|a\|_2 = 1)$$

Lagrangian function:

$$\mathcal{L}(a, \lambda) = a^T X X^T a - \lambda(a^T a - 1), \quad \lambda > 0$$

$$\frac{\partial \mathcal{L}}{\partial a} = 2XX^T a - 2\lambda a = 0$$

$$\Rightarrow XX^T a = \lambda a$$

$$\Rightarrow \max_a \; a^T X X^T a = a^T(\lambda a) \Rightarrow \boxed{\max \lambda}$$

$\Rightarrow a$ is eigvec(XX^T) with **largest** eigvalue λ

Centerize the data:

$x_1, x_2, \cdots\cdots, x_n$

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the mean

$$\begin{cases} x_1' = x_1 - \bar{x} \\ x_2' = x_2 - \bar{x} \\ \vdots \\ x_n' = x_n - \bar{x} \end{cases}$$

new mean: $\bar{x'} = \frac{1}{n}\sum_{i=1}^{n} x_i'$

$$= (x_1 - \bar{x}) + \cdots + (x_n - \bar{x})$$

$$= (x_1 + \cdots + x_n) - n\bar{x}$$

$$= 0$$

$$\mathrm{cov}[z_2, z_1]$$

$$= (a_1^T X) \cdot (a_2^T X)^T$$

$$= a_1^T X X^T a_2$$

$$= a_1^T S a_2 \qquad (S a_1 = \lambda a_1)$$

$$= \lambda a_1^T a_2 = 0$$

$$\Rightarrow a_1^T a_2 = 0$$

$$\Rightarrow a_1, a_2 : \text{orthogonal}.$$

$$\mathcal{L} = a_2^T S a_2 - \lambda(a_2^T a_2 - 1) - \phi a_2^T a_1$$

$$\frac{\partial \mathcal{L}}{\partial a_2} = 2Sa_2 - 2\lambda a_2 - \phi a_1 = 0$$

$$\frac{\partial \mathcal{L}}{\partial a_1} = \phi a_2 = 0 \Rightarrow \phi = 0 \quad (\overset{*}{a_2} \neq 0)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = a_2^T a_2 - 1 = 0 \Rightarrow a_2^T a_2 = 1$$

$$\Rightarrow 2Sa_2 - 2\lambda a_2 - 0 \cdot a_1 = 0$$

$$\Rightarrow \boxed{Sa_2 = \lambda a_2}$$

$$\Rightarrow a_2^T (Sa_2) = a_2^T \lambda a_2$$

$$\Rightarrow a_2^T S a_2 = \lambda a_2^T a_2 = \lambda \cdot 1$$

$$\Rightarrow \boxed{\lambda = a_2^T S a_2}$$