

Intelligent Systems Principles and Programming

Xiaoqing Zheng
zhengxq@fudan.edu.cn



Introduction

- Classical *Bivalent* or *two-valued* logic.
- L. A. Zadeh's *fuzzy* set.
- Pawlak's *Rough* set.

Information table

U	a	b	c	d	e
(1)	1	2	2	1	2
(2)	3	2	2	2	1
(3)	3	2	3	3	3
(4)	2	1	3	1	2
(5)	2	3	2	1	3
(6)	1	2	2	2	3
(7)	2	1	2	1	2
(8)	3	3	1	1	1

Rough set

- Let $I = (U, A)$ be an *information system* (attribute-value system), where U is a non-empty set of finite objects (the universe) and A is a non-empty, finite set of attributes such that $a: U \rightarrow V_a$ for every $a \in A$. V_a is the set of values that attribute a may take.
- The information table assigns a value in V_a to each attribute a of each object in universe U .

Universe $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

Associated equivalence relation R :

$$E_1 = \{x_1, x_4\}$$

$$E_2 = \{x_3, x_8\}$$

$$E_3 = \{x_2, x_5, x_7\}$$

$$E_4 = \{x_6\}$$

Partition $U/R = \{E_1, E_2, E_3, E_4\}$, and $X = \{x_1, x_4, x_2\}$

Lower approximation $R_*(X) = \{x_1, x_4\}$

Upper approximation $R^*(X) = \{x_1, x_4, x_2, x_5, x_7\}$

Universe $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

Associated equivalence relation R :

$$E_1 = \{x_1, x_4\}$$

$$E_2 = \{x_3, x_8\}$$

$$E_3 = \{x_2, x_5, x_7\}$$

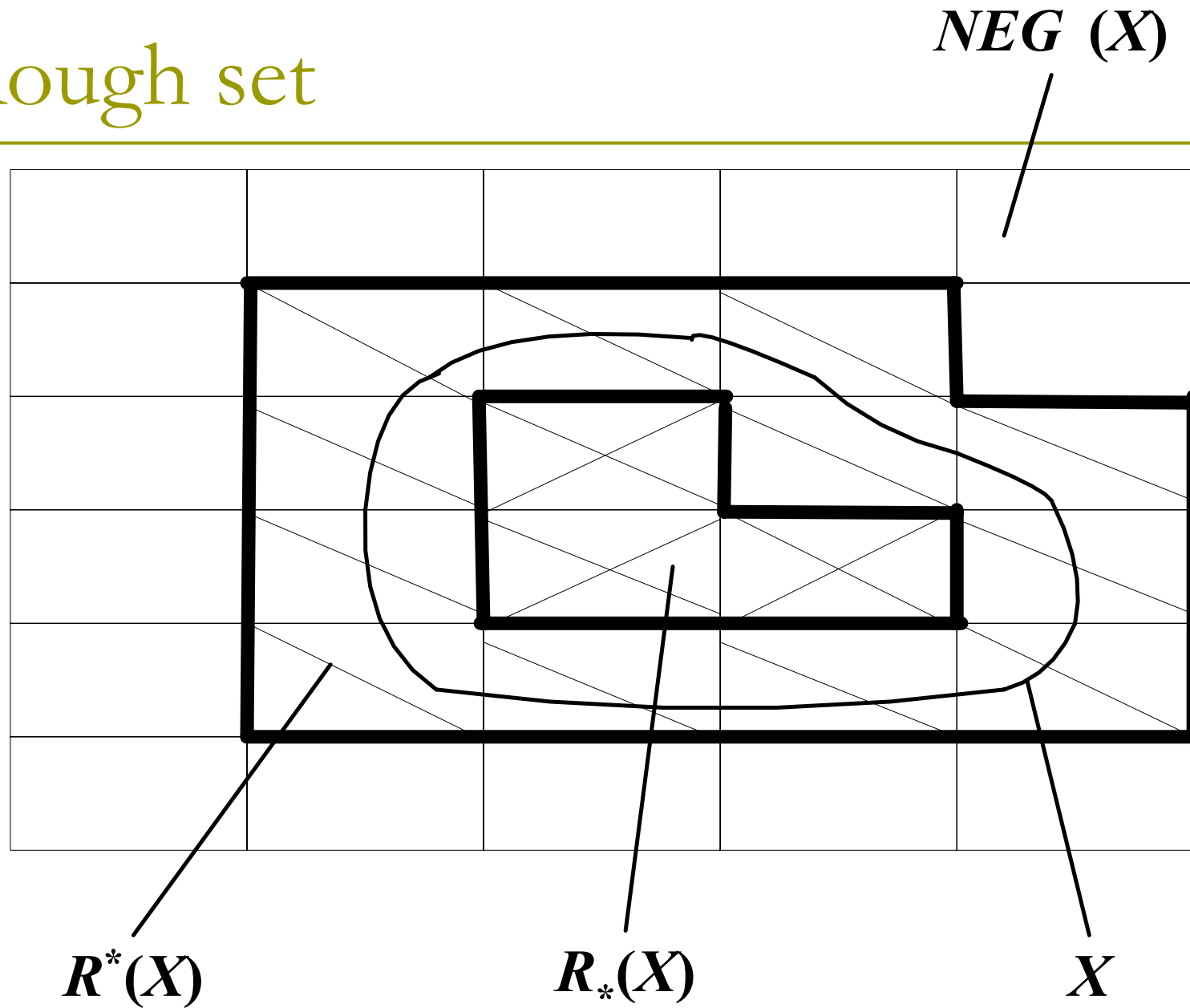
$$E_4 = \{x_6\}$$

Partition $U/R = \{E_1, E_2, E_3, E_4\}$, and $X = \{x_1, x_4, x_2\}$

Lower approximation $R_*(X) = \{x_1, x_4\}$

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Membership function

$$\mu_X^R(x) = \text{card}(X \cap \mathbf{R}(x)) / \text{card}(\mathbf{R}(x))$$

$$\mu_X^R(x_1) = 2 / 2 = 1.00$$

$$\mu_X^R(x_2) = 1 / 3 = 0.33$$

$$\mu_X^R(x_3) = 0 / 2 = 0.00$$

Universe $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

A set of **Associated equivalence relation** $R = \{P, Q, W\}$

$$U/P = \{\{x_1, x_3, x_4, x_5, x_6, x_7\}, \{x_2, x_8\}\}$$

$$U/Q = \{\{x_1, x_3, x_4, x_5\}, \{x_2, x_6, x_7, x_8\}\}$$

$$U/W = \{\{x_1, x_5, x_6\}, \{x_2, x_7, x_8\}, \{x_3, x_4\}\}$$

Then $U/IND(R) = \{\{x_1, x_5\}, \{x_3, x_4\}, \{x_2, x_8\}, \{x_6\}, \{x_7\}\}$

Associated equivalence relation S (~~Decision Attribute~~)

$$U/S = \{\{x_1, x_5, x_6\}, \{x_3, x_4\}, \{x_2, x_7\}, \{x_8\}\}$$

Positive region:

$$POS_R(S) = \{x_1, x_3, x_4, x_5, x_6, x_7\}$$

Universe $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

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Then $U/IND(R) = \{\{x_1, x_5\}, \{x_3, x_4\}, \{x_2, x_8\}, \{x_6\}, \{x_7\}\}$

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Positive region:

$$POS_R(S) = \{x_1, x_3, x_4, x_5, x_6, x_7\}$$

Information table

U	a	b	c	d	e
(1)	1	0	2	1	1
(2)	2	1	0	1	0
(3)	2	1	2	0	2
(4)	1	2	2	1	1
(5)	1	2	0	0	2

Rules:

$$a_1 b_0 c_2 \rightarrow d_1 e_1$$

$$a_2 b_1 c_0 \rightarrow d_1 e_0$$

$$a_2 b_1 c_2 \rightarrow d_0 e_2$$

$$a_1 b_2 c_2 \rightarrow d_1 e_1$$

$$a_1 b_2 c_0 \rightarrow d_0 e_2$$

Problem

- An interesting question is whether there are attributes in the information system which are more *important* to the knowledge represented in the equivalence class structure than other attributes. .
- We wonder whether there is a subset of attributes which can, by itself, fully characterize the knowledge in the database; such an attribute set is called a *reduct*.

Remove attribute a

U	b	c	d	e
(1)	0	2	1	1
(2)	1	0	1	0
(3)	1	2	0	2
(4)	2	2	1	1
(5)	2	0	0	2

**Legitimate
reduct**

$$a_2b_1c_2 \rightarrow d_0e_2$$

$$a_1b_2c_2 \rightarrow d_1e_1$$

$$a_1b_2c_0 \rightarrow d_0e_2$$

Remove attribute b

U	a	c	d	e
(1)	1	2	1	1
(2)	2	0	1	0
(3)	2	2	0	2
(4)	1	2	1	1
(5)	1	0	0	2

**Legitimate
reduct**

$$a_2b_1c_2 \rightarrow d_0e_2$$

$$a_1b_2c_2 \rightarrow d_1e_1$$

$$a_1b_2c_0 \rightarrow d_0e_2$$

Remove attribute c

U	a	b	d	e
(1)	1	0	1	1
(2)	2	1	1	0
(3)	2	1	0	2
(4)	1	2	1	1
(5)	1	2	0	2

Rules:

$$a_1b_0c_2 \rightarrow d_1e_1$$

$$a_2b_1c_0 \rightarrow d_1e_0$$

$$a_2b_1c_2 \rightarrow d_0e_2$$

$$a_1b_2c_2 \rightarrow d_1e_1$$

$$a_1b_2c_0 \rightarrow d_0e_2$$

Remove attribute c

U	a	b	d	e
(1)	1	0	1	1
(2)	2	1	1	0
(3)	2	1	0	2
(4)	1	2	1	1
(5)	1	2	0	2

**Causing
collapse**

$$a_2b_1c_2 \rightarrow d_0e_2$$

$$a_1b_2c_2 \rightarrow d_1e_1$$

$$a_1b_2c_0 \rightarrow d_0e_2$$

Remove attribute c

U	a	b	d	e
(1)	1	0	1	1
(2)	2	1	1	0
(3)	2	1	0	2
(4)	1	2	1	1
(5)	1	2	0	2

**Causing
collapse**

$a_2b_1c_2 \rightarrow d_0e_2$

$a_1b_2c_2 \rightarrow d_1e_1$

$a_1b_2c_0 \rightarrow d_0e_2$

Information table after removing a

U	b	c	d	e
(1)	0	2	1	1
(2)	1	0	1	0
(3)	1	2	0	2
(4)	2	2	1	1
(5)	2	0	0	2

Rules:

$$b_0c_2 \rightarrow d_1e_1$$

$$b_1c_0 \rightarrow d_1e_0$$

$$b_1c_2 \rightarrow d_0e_2$$

$$b_2c_2 \rightarrow d_1e_1$$

$$b_2c_0 \rightarrow d_0e_2$$

Information table after removing b

U	a	c	d	e
(1)	1	2	1	1
(2)	2	0	1	0
(3)	2	2	0	2
(4)	1	2	1	1
(5)	1	0	0	2

Rules:

$$a_1c_2 \rightarrow d_1e_1$$

$$a_2c_0 \rightarrow d_1e_0$$

$$a_2c_2 \rightarrow d_0e_2$$

$$a_1c_2 \rightarrow d_1e_1$$

$$a_1c_0 \rightarrow d_0e_2$$

Information table after removing b

U	a	c	d	e
(1)	1	2	1	1
(2)	2	0	1	0
(3)	2	2	0	2
(4)	1	2	1	1
(5)	1	0	0	2

Rules:

$$a_1c_2 \rightarrow d_1e_1$$

$$a_2c_0 \rightarrow d_1e_0$$

$$a_2c_2 \rightarrow d_0e_2$$

$$a_1c_2 \rightarrow d_1e_1$$

$$a_1c_0 \rightarrow d_0e_2$$

Information table after removing b

U	a	c	d	e
(1)	1	2	1	1
(2)	2	0	1	0
(3)	2	2	0	2
(5)	1	0	0	2

Rules:

$$a_1c_2 \rightarrow d_1e_1$$

$$a_2c_0 \rightarrow d_1e_0$$

$$a_2c_2 \rightarrow d_0e_2$$

$$a_1c_0 \rightarrow d_0e_2$$

Records

序号	住宅区名	地段	房型	面积 (m ²)	结构	价格 (元/ m ²)
1	站前路小区	站前路	2/2, 3/2	90~140	一般	10000
2	高新区	高新区	2/2, 3/2, 4/2	83~170	框架	7000
3	远东花园	洪城路	3/2, 4/2	120以上	复式	7800
4	曙光小区	洪都南大道	3/2	103以上	一般	9000
5	怡鑫花园	洪都中大道	2/1, 2/2, 3/2, 4/2	87~230	一般	12000
6	文化大楼	沿江路	2/1, 3/1, 3/2, 2/2	99~180	框架	18000
7	玉达住宅	二七北路	2/2	97~107	一般	9800
8	洪都新村	洛阳东路	2/1, 3/1, 3/2, 1/1	53~123	一般	7000

Discretization

- 地段分类：站前路和沿江路被数字化为1；洪都大道和二七路被数字化为2；洪城路、高新区和洛阳东路被数字化为3。
- 房型分类：房型单一被数字化为1；房型中等被数字化为2；房型多样化被数字化为3。
- 面积分类：80m²以下面积被数字化为1；80~120m²为中等面积，被数字化为2；120m²以上为大面积，被数字化为3。
- 结构分类：一般结构被数字化为1；框架结构被数字化为2；复式结构被数字化为3。
- 价格分类：9000元/m²以下被数字化为1；9000~12000元/m²被数字化为2；12000元/m²被数字化为3。

Information table

U	a	b	c	d	e
(1)	1	2	2	1	2
(2)	3	2	2	2	1
(3)	3	2	3	3	3
(4)	2	1	3	1	2
(5)	2	3	2	1	3
(6)	1	2	2	2	3
(7)	2	1	2	1	2
(8)	3	3	1	1	1

Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

Remove attribute *a*

<i>U</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
(1)	2	2	1	2
(2)	2	2	2	1
(3)	2	3	3	3
(4)	1	3	1	2
(5)	3	2	1	3
(6)	2	2	2	3
(7)	1	2	1	2
(8)	3	1	1	1

Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

Remove attribute *a*

<i>U</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
(1)	2	2	1	2
(2)	2	2	2	1
(3)	2	3	3	3
(4)	1	3	1	2
(5)	3	2	1	3
(6)	2	2	2	3
(7)	1	2	1	2
(8)	3	1	1	1

Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

Remove attribute b

U	a	c	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	3	3	3
(4)	2	3	1	2
(5)	2	2	1	3
(6)	1	2	2	3
(7)	2	2	1	2
(8)	3	1	1	1

Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

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$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

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(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	3	3	3
(4)	2	3	1	2
(5)	2	2	1	3
(6)	1	2	2	3
(7)	2	2	1	2
(8)	3	1	1	1

Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

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$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

Remove attribute c

U	a	b	d	e
(1)	1	2	1	
(2)	3	2	2	
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(7)	2	1	1	2
(8)	3	3	1	1

**Legitimate
reduct**

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

Remove attribute d

U	a	b	c	e
(1)	1	2	2	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	3	2
(5)	2	3	2	3
(6)	1	2	2	3
(7)	2	1	2	2
(8)	3	3	1	1

Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

Remove attribute d

U	a	b	c	e
(1)	1	2	2	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	3	2
(5)	2	3	2	3
(6)	1	2	2	3
(7)	2	1	2	2
(8)	3	3	1	1

Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

Information table after removing C

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(7)	2	1	1	2
(8)	3	3	1	1

Information table after removing C

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(7)	2	1	1	2
(8)	3	3	1	1

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

Analyze rule (1):

$$[1]_a = \{1, 6\},$$

$$[1]_b = \{1, 2, 3, 6\},$$

$$[1]_d = \{1, 4, 5, 8\},$$

$$[1]_e = \{1, 4\}$$

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

Analyze rule (1):

$$[1]_a = \{1, 6\},$$

$$[1]_b = \{1, 2, 3, 6\},$$

$$[1]_d = \{1, 4, 5, 8\},$$

$$[1]_e = \{1, 4\}$$

$$[1]_{\{a, b\}} = [1]_a \cap [1]_b = \{1, 6\} \not\subseteq [1]_e$$

d_1 can not be removed.

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

Analyze rule (1):

$$[1]_a = \{1, 6\},$$

$$[1]_b = \{1, 2, 3, 6\},$$

$$[1]_d = \{1, 4, 5, 8\},$$

$$[1]_e = \{1, 4\}$$

$$[1]_{\{a, d\}} = [1]_a \cap [1]_d = \{1\} \subseteq [1]_e$$

b_2 can be removed.

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

Analyze rule (1):

$$[1]_a = \{1, 6\},$$

$$[1]_b = \{1, 2, 3, 6\},$$

$$[1]_d = \{1, 4, 5, 8\},$$

$$[1]_e = \{1, 4\}$$

$$[1]_{\{b, d\}} = [1]_b \cap [1]_d = \{1\} \subseteq [1]_e$$

a_1 can be removed.

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

Analyze rule (1):

$$[1]_a = \{1, 6\},$$

$$[1]_b = \{1, 2, 3, 6\},$$

$$[1]_d = \{1, 4, 5, 8\},$$

$$[1]_e = \{1, 4\}$$

d_1 is core value, then we get

$$a_1 d_1 \rightarrow e_2$$

$$b_2 d_1 \rightarrow e_2$$

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

Analyze rule (2):

$$[2]_a = \{2, 3, 8\},$$

$$[2]_b = \{1, 2, 3, 6\},$$

$$[2]_d = \{2, 6\},$$

$$[2]_e = \{2, 8\}$$

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

Analyze rule (2):

$$[2]_a = \{2, 3, 8\},$$

$$[2]_b = \{1, 2, 3, 6\},$$

$$[2]_d = \{2, 6\},$$

$$[2]_e = \{2, 8\}$$

$$[2]_{\{a, b\}} = [2]_a \cap [2]_b = \{2, 3\} \not\subseteq [1]_e$$

d_2 can not be removed.

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

Analyze rule (2):

$$[2]_a = \{2, 3, 8\},$$

$$[2]_b = \{1, 2, 3, 6\},$$

$$[2]_d = \{2, 6\},$$

$$[2]_e = \{2, 8\}$$

$$[2]_{\{a, d\}} = [2]_a \cap [2]_d = \{2\} \subseteq [1]_e$$

b_2 can be removed.

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

Analyze rule (2):

$$[2]_a = \{2, 3, 8\},$$

$$[2]_b = \{1, 2, 3, 6\},$$

$$[2]_d = \{2, 6\},$$

$$[2]_e = \{2, 8\}$$

$$[2]_{\{b, d\}} = [2]_a \cap [2]_b = \{2, 6\} \not\subseteq [1]_e$$

a_3 can not be removed.

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

Analyze rule (2):

$$[2]_a = \{2, 3, 8\},$$

$$[2]_b = \{1, 2, 3, 6\},$$

$$[2]_d = \{2, 6\},$$

$$[2]_e = \{2, 8\}$$

a_3 and d_2 is core, then we get

$$a_3 d_2 \rightarrow e_1$$

Results of reduction

U	a	b	d	e
(1)	1	—	1	2
(1')	—	2	1	2
(2)	3	—	2	1
(3)	—	—	3	3
(4)	—	1	—	2
(5)	2	3	—	3
(6)	1	—	2	3
(8)	3	—	1	1
(8')	3	3	—	1

Rules:

$$a_1 d_1 \rightarrow e_2 \text{ or } b_2 d_1 \rightarrow e_2$$

$$a_3 d_2 \rightarrow e_1$$

$$d_3 \rightarrow e_3$$

$$b_1 \rightarrow e_2$$

$$a_2 b_3 \rightarrow e_3$$

$$a_1 d_2 \rightarrow e_3$$

$$a_3 d_1 \rightarrow e_1 \text{ or } a_3 b_3 \rightarrow e_1$$

Results of reduction

Rules:

$$a_1d_1 \rightarrow e_2 \text{ or } b_2d_1 \rightarrow e_2$$

$$a_3d_2 \rightarrow e_1$$

$$d_3 \rightarrow e_3$$

$$b_1 \rightarrow e_2$$

$$a_2b_3 \rightarrow e_3$$

$$a_1d_2 \rightarrow e_3$$

$$a_3d_1 \rightarrow e_1 \text{ or } a_3b_3 \rightarrow e_1$$

Results of reduction

Rules:

$$a_1d_1 \rightarrow e_2 \text{ or } b_2d_1 \rightarrow e_2$$

$$a_3d_2 \rightarrow e_1$$

$$d_3 \rightarrow e_3$$

$$b_1 \rightarrow e_2$$

$$a_2b_3 \rightarrow e_3$$

$$a_1d_2 \rightarrow e_3$$

$$a_3d_1 \rightarrow e_1 \text{ or } a_3b_3 \rightarrow e_1$$

Final Rules:

$$a_3d_2 \vee a_3d_1 \vee a_3b_3 \rightarrow e_1$$

$$a_1d_1 \vee b_2d_1 \vee b_1 \rightarrow e_2$$

$$d_3 \vee a_2b_3 \vee a_1d_2 \rightarrow e_3$$

Results of reduction

Rules:

$$a_1d_1 \rightarrow e_2 \text{ or } b_2d_1 \rightarrow e_2$$

$$a_3d_2 \rightarrow e_1$$

$$d_3 \rightarrow e_3$$

$$b_1 \rightarrow e_2$$

$$a_2b_3 \rightarrow e_3$$

$$a_1d_2 \rightarrow e_3$$

$$a_3d_1 \rightarrow e_1 \text{ or } a_3b_3 \rightarrow e_1$$

Final Rules:

$$a_3d_2 \vee a_3d_1 \vee a_3b_3 \rightarrow e_1$$

$$a_1d_1 \vee b_2d_1 \vee b_1 \rightarrow e_2$$

$$d_3 \vee a_2b_3 \vee a_1d_2 \rightarrow e_3$$

自然语言解释：

(1) 3类地段框架结构或3类地段一般结构或3类地段房型多样的地产价格便宜；

(2) 1类地段一般结构或房型中等一般结构或房型单一的地产价格适中；

(3) 复式结构或2类地段房型多样或1类地段框架结构的地产价格昂贵

Any question?



Xiaoqing Zheng
Fudan University